Slope Stability Assessment of Layered Soil

by

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Submitted in fulfilment of the requirements for the degree of
Doctor of Philosophy (Engineering)

Deakin University
February, 2016
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The papers presented in this thesis are as follows:

1. Paper 1 (Chapter 3):

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ABSTRACT

Slope stability is a crucial problem in geotechnical engineering. Numerous methodologies have been developed for slope stability analyses such as the limit equilibrium method, the numerical analysis method and the limit analysis method. This research uses the finite element limit analysis methods to investigate three-dimensional (3D) effects on the stability of slopes with layered soil, which are typically encountered in embankment slopes. Specifically, this thesis investigates three types of fill slopes. Results of this research are presented in the form of comprehensive stability charts, which can be used for quick first estimate of slope safety especially during preliminary design. For comparison purposes, results based on a 2D analysis are also presented. In fact, the results obtained show that the factors of safety obtained from a 3D analysis are larger than those obtained from a 2D analysis. In fact, detailed observations reveal that the depth and size of the failure mechanisms are also affected by the 3D boundary of the slopes. Additionally, various factors affecting the stability of the fill slopes are also investigated and discussed in this research. For instance, for the case of two-layered undrained clay slopes, one of the main governing factors of the stability of the slopes is the ratio of the undrained shear strength between the two layers ($c_{u1}/c_{u2}$ ratios). It can be observed that depending on the $c_{u1}/c_{u2}$ ratios, the failure mechanisms obtained can be of a toe failure or a base failure mode. Furthermore, this thesis also shows that for a certain range of $c_{u1}/c_{u2}$ ratios, the stability of the slopes can be the same. For fill slopes that are constructed by placing frictional fill materials on undrained clay, it has been found that the effect of 3D boundary on the factor of safety is much more significant when the difference between the friction and slope angles is larger. Furthermore, it can also be observed that the failure mechanisms obtained using a 2D analysis and that of a 3D analysis are also different in terms of size. On the other hand, for the case where the foundation clay shear strength increases with depth, results from this study show that the stability of the slopes is also governed by a few factors particularly the rate of strength increment, $\rho$. In fact, it can be seen that due to the increasing shear strength with depth, the failure mechanisms and the stability numbers are almost unchanged when the depth factor $d/H \geq 2$. 
In fact, apart from the main findings above, this thesis shows that many other different factors also affect the stability of the fill slopes and the degree of influence of these factors are different for the different types of fill slopes investigated. Furthermore, 3D effects are also shown to be affected by the factors. Hence, the significance of appropriate consideration of 3D effects in the analyses of fill slopes is thoroughly demonstrated in this thesis. The failure mechanisms of the slopes are also discussed in this thesis. Similarly, as briefly discussed above the failure mechanisms are also shown to be affected by various factors. For instance, for slopes that are highly restricted by 3D physical boundaries, the failure mechanisms are smaller than those less restricted.

In addition to the above deterministic-based stability analyses, a probabilistic-based investigation is also performed in this thesis. The results of the probabilistic analysis quantify the relationship between the probability of failure and the factor of safety of the fill slope investigated. Various coefficients of variation (COVs) of soil properties are investigated in this probabilistic-based study. In fact, the results from this study show that the probability of failure of the fill slope investigated is affected by a number of factors. Particularly, it can be observed that depending on the $c_{u1}/c_{u2}$ ratios and COVs, different relationships of the probability of failure versus the mean factor of safety can be obtained. Additionally, the results also provide guidance on the use of the stability number proposed for the deterministic study.

Further to the probabilistic-based investigation, this thesis also develops a robust and convenient artificial neural network (ANN) scheme to predict the stability of fill slopes. The primary aim of this ANN scheme is to solve the issue of having many different inputs to the probabilistic-analysis above. For instance, in this study, the ANN scheme has been developed for two-layered undrained clay slopes where the different influence factors of slope stability include the slope angle, the $c_{u1}/c_{u2}$ ratios, the COVs of the shear strength parameters and the unit weight of the soil and the depth factor. Hence, using the developed ANN scheme, a less-cumbersome method of obtaining the factor of safety and the probability of failure of fill slopes is established. As demonstrated in the parametric example, the factors of safety can be obtained relatively easy whenever the input parameters are changed. Furthermore, a few categories for convenient risk classification of the
slopes have also been proposed. In fact, the results presented show great promises for the adoption of ANNs and the proposed extreme learning machine in slope stability analyses.
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**Symbols**

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<td>Coefficient of variation</td>
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<td>(c_u)</td>
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<td>(c_{u0})</td>
<td>The undrained shear strength of the soil at the top of Region 2 (Chapter 5)</td>
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<td>(c_{u1})</td>
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<tr>
<td>(d)</td>
<td>Vertical length from the top of the slope to the bottom of the foundation</td>
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<td>(d/H)</td>
<td>Depth factor</td>
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<tr>
<td>(F)</td>
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\( H \) Height of the slope

\( L \) Into the page (transverse) length of slope

\( L/H \) Ratio of \( L \) over \( H \)

\( N_{2c} \) Stability number for Chapter 3

\( N_{ci} \) Stability number for Chapter 5

\( N_{sc} \) Stability number for Chapter 4

\( P_f \) The probability of failure

\( R \) Coefficient of correlation

\( \beta \) Slope angle

\( E' \) The young modulus of soil

\( \bar{x} \) The mean value of the soil properties in Chapter 6 and 7.

\( \gamma \) The unit weight of the soil

\( \rho \) The rate of increment of the soil undrained shear strength with depth

\( \sigma \) The standard deviation of the soil properties in Chapter 6 and 7

\( \varphi \) Friction angle of fill material

\( \delta_{max} \) Maximum nodal displacement in an FEM analysis
Abbreviations

AI          Artificial Intelligence
ANN         Artificial Neural Network
BP          Back Propagation
ELM         Extreme Learning Machine
FEM         Finite Element Method
FORM        First Order Reliability Method
FOSM        First Order Second Method
LEM         Limit Equilibrium Method
MFOSTM      Mean Value First Order Second Method
MLE         Maximum Likelihood Estimation
MLP         Multi-Layer Perception
NN          Neural Network
PNN         Probabilistic Neural Network
RFEM        Random Finite Element Method
SRM         Strength Reduction Method
Chapter 1 INTRODUCTION

1.1 INTRODUCTION

In general, geotechnical engineering problems include trench stability (Fox 2004; Li et al. 2014), tunnelling (Lee et al. 2006; Yamamoto et al. 2011) and slope stability (Bishop 1955; Griffiths and Lane 1999). Particularly, slope stability is a crucial and ongoing problem for geotechnical engineers and thus many investigations have been performed (Morgenstern and Price 1965; Azzouz and Baligh 1978; Leshchinsky and Baker 1986; Gens et al. 1988; Michalowski 1995; Jiang and Magnan 1997; Chang 2002; Li et al. 2010; Gao et al. 2012b; Sun and Zhao 2013; Zhang et al. 2013).

Traditionally, majority of slope stability analyses have been performed using a plane strain analysis (two-dimensional (2D)). However, in recent years, slope stability analyses have been rigorously investigated using three-dimensional (3D) analyses. From these 3D analyses, it has been found that results based on 2D analyses are underestimated and hence may lead to uneconomical design. Therefore based on 3D analyses, the stability of fill slopes is investigated in this thesis. Additionally, the results from the analyses are presented using stability charts, which are convenient tools during preliminary designs.

Many methods have been developed to investigate slope stability. In this thesis, the recently developed finite element limit analysis methods, which are robust, and can be applied to various geotechnical engineering applications have been employed.

It is well-known that a deterministic approach does not explicitly account for the uncertainties in soil properties. Therefore, a probabilistic study considering various coefficients of variation in soil properties is also performed in this research. This research then adopts an artificial neural network (ANN) with Extreme Learning Machine (ELM) to develop a convenient and efficient tool in assessing fill slope stability. ANNs are well known for their capability to predict the result for a given set of inputs.
1.2 RESEARCH MOTIVATIONS

The above introduction has briefly described the scopes and directions of this thesis. The following discussion details the objectives and plans of this research. This research is based on fill slopes, which are commonly found in the constructions of embankments or highways (Indraratna et al. 1992; Al-Homoud et al. 1997; Richards and Reddy 2005). Embankment slopes, where fill materials are placed on a soil foundation with similar or different properties are very common in geotechnical engineering. It was found that to date, fill slopes have not been investigated extensively especially their failure mechanisms. Additionally, while chart solutions have been produced for natural/cut slopes, such charts are limited for fill slopes. In fact, stability charts as highlighted by Michalowski (2002) and Li et al. (2009, 2010) are valuable tools for preliminary design of slopes. Examples of these stability charts can be seen in Fig. 1-1. Therefore, considering the lack of investigations and chart solutions of fill slopes, this research has identified a gap that needs to be filled. It is hoped that through this study, a better understanding of the behaviour of fill slopes can be achieved and rigorous stability solutions for design engineers can be developed.

Three specific cases of fill slopes are investigated: 1) two-layered undrained clay slopes, 2) frictional fill materials placed on undrained clay slopes; and 3) frictional fill materials placed on undrained clay with strength increases with depth slopes. In addition, to compare with the results obtained using a 3D analysis, results from a 2D analysis are also presented in this thesis. As mentioned earlier, this research utilizes the finite element limit analysis methods developed by Lyamin and Sloan (2002a, 2002b) and Krabbenhoft et al. (2005). These methods have been applied in various slope stability problems (Yu et al. 1998; Kim et al. 2002; Li et al. 2009, 2010).

It is important to highlight that the results from the above studies are presented in the form of stability numbers. Due to the different definition of $F$ in the stability number, the safety factors obtained from the stability number are generally different from the $F$ obtained from the conventional Limit Equilibrium Method (LEM) or the Strength Reduction Method (SRM). This has also been noted by Li et al. (2012). Therefore to calibrate the new safety factor, a probabilistic slope
stability analysis is performed. In this instance the calibration is done for the case of two-layered undrained clay slopes. The influence from the two layers of undrained clay on the probability of failure of the slope is worth investigating due to the large coefficient of variation of the undrained shear strength reported for undrained clay. Similarly, it has been reported in the literature that undrained shear strength can highly affect the probability of failure of slopes. This part of study takes into account the uncertainties in the soil properties and charts relating the probability of failure and the factor of safety are proposed. Factors affecting this relation are also investigated. Hence, these plots such as those seen in Fig. 1-2 will provide guidance for the appropriate selection of the factor of safety to achieve the desired probability of failure. These plots will also allow a more rational design to be made (Li et al. 2012). It should be noted that these plots are only valid for a similar slope configuration (Alonso 1976).

Then to further simplify and extend the above study, an artificial neural network (ANN) is adopted. Similarly, this investigation is based on two-layered undrained clay slopes. The core motivation of adopting an ANN is that an ANN can be trained using a set of training data and once trained, due to its robustness and extrapolation capabilities, it can be used to predict an output from a set of inputs. Hence, in this thesis, an ANN is trained to predict the probability of failure for the two-layered undrained clay slopes while also capable of providing the stability numbers for the slopes. In fact, the objective is to use the trained ANN to extend the probabilistic study from above where users of the trained ANN are not required to manually inter/extrapolate the results, therefore improving accuracies and efficiency, considering the different degree of uncertainties in the soil properties and also factors affecting the probability of failure. Also considering the high number of inputs (factors influencing the probability of failure of the slopes) required, presentations based on charts would be messy and inconvenient.

1.3 RESEARCH OBJECTIVES

Based on the above discussion, the objective of this research is to provide a better understanding of fill slopes and the influence of 3D boundary on this type of slope. Additionally, this research aims to provide quick and easy solutions for the
preliminary assessment of fill slopes. In summary, the objectives of this research are as follows:

- Develop convenient and comprehensive stability charts for three types of fill slopes namely: (1) Two-layered undrained clay slopes, (2) Fill materials placed on undrained clay slopes and (3) Fill materials placed on undrained clay with increasing strength slope.
- Present the influence of 3D boundary on the above types of fill slopes, compare the results with traditional 2D analyses and seek to reduce over-design of the slopes due to the use of the traditional conservative 2D analyses.
- Adoption of the finite element limit analysis methods to provide more accurate solutions compared to the conventional LEM and the limit analysis methods.
- Provide simple and convenient probabilistic-analysis-based solutions for the design of two-layered undrained clay slopes while also give a better understanding of the influence of uncertainties in the undrained shear strength of the clays.
- Provide guidance on the selection of the appropriate factor of safety obtained with the stability numbers proposed in this study based on the desired probability failure for two-layered undrained clay slopes.
- Propose the use of an ANN to solve two-layered undrained clay slopes stability problems both deterministically and probabilistically.

The bottom line of this research is to simplify fill slope stability assessment in the form of stability charts while also study and present the impact of the 3D boundary on slope stability. In fact, using the stability charts users can quickly look up for the dimensionless stability numbers and calculate the factor of safety for a given slope without the need for modelling the slope. Also, based on the 3D analysis results, it is hoped that more accurate and economical judgements and designs can be made such as on the selection of fill materials and the design of slope height. In addition, the aim of the probabilistic analysis in this study is to establish the relationship between the probability of failure and the mean factor of safety for two-layered undrained clay and superficially to study the impact of the
variation of the soil shear strength on the relationship. Lastly, this research intends to incorporate artificial intelligence to further simplify fill slope stability analysis. By training the ANN tool to give both factor of safety and probability of failure, the preliminary design of the slope can be easily determined and thus more time and effort can be directed to detailed design.

1.4 BRIEF DESCRIPTION OF KEY COMPONENTS

1.4.1 Finite element limit analysis

As mentioned earlier, this thesis utilizes the finite element limit analysis methods (Lyamin and Sloan 2002a, 2002b; Krabbenhoft et al. 2005) to conduct all of the analyses. Thanks to the methods’ robustness, they can be utilized to solve various geotechnical engineering problems (Sloan 2013). In fact, these methods have also been applied to solve a number of slope stability problems (Kim et al. 2002; Li et al. 2009, 2010). As shown by these studies, these numerical limit theorems are capable of handling both 2D and 3D analyses.

Unlike the conventional limit analysis methods, the finite element limit analysis methods are based on the combination of the limit theorems with finite elements. These methods have an advantage over the conventional limit analysis and the more popular LEM because they do not require any assumptions on the mode of failure. Also, no statical assumptions are required. The conventional limit analysis method is known for having difficulty in computing the lower bound stress field. Thus many studies utilizing the limit analysis methods have been based on the upper bound method. However, the finite element limit analysis methods do not have a similar issue.

1.4.2 Three-dimensional slope stability analysis

Slope stability problems have traditionally been assessed using a two-dimensional analysis. However, in reality not all slopes are infinitely wide especially in embankment slopes where the width of the slopes can be limited. As a matter of fact, 3D effects do influence the stability of slopes (Gens et al. 1988; Arellano and Stark 2000). For instance, 3D failure mechanisms with different curved ends (Fig. 1-3) were investigated by Gens et al. (1988). This finding was further strengthened by Leschinsky and Huang (1992b) and Stark and Eid (1998) who proved that the factor of safety obtained from a 2D analysis is underestimated. In
other words the available shear strength can be overestimated during back analysis if 3D effects are not considered.

Hence, it can be concluded that slopes that are restricted by physical boundaries should be analysed using a 3D analysis. This is because using plane strain conditions to assess slope stability problems where 3D conditions prevail will incur excessive costs due to the underestimation of the factor of safety (Baligh and Azzouz 1975; Azzouz and Baligh 1978; Ugai 1985) and potentially requiring material with higher strength or reinforcement, which leads to an overdesigned slope. In fact, through appropriate consideration of 3D effects, the slope can be designed to be steeper or higher, hence reducing the volume of excavation.

1.4.3 Chart solutions in slope stability assessments

Stability charts are convenient tools for geotechnical engineers in assessing slope stability. A number of stability charts have been produced for various slope stability problems using various methods described above (Gens et al. 1988; Michalowski 2002; Loukidis et al. 2003; Viratjandr and Michalowski 2006; Li et al. 2009, 2010; Michalowski 2010). Some of these charts even consider the effect of seismic force or pore pressure Fig. 1-4 (Michalowski 2002; Loukidis et al. 2003; Viratjandr and Michalowski 2006; Gao et al. 2012b; Gaopeng et al. 2014; Tang et al. 2015) while some are for natural or cut slopes (Li et al. 2009, 2010).

The symbol $r_u$ and $k_h$ in Fig. 1-4 stand for the magnitude of pore water pressure and earthquake coefficient respectively. Stability charts can be quick and efficient tools in slope stability analyses where the factors of safety can be read off the charts without the need for iteration. Hence, this ultimately saves the computational effort and time during the preliminary designs of slopes. That said, this doesn’t mean a thorough and detailed investigation for the final design would no longer be required. However, the charts would at least allow the engineers to allocate the time and resources – saved during the preliminary design – towards more important tasks such as displacement analyses and site investigations.

1.4.4 Uncertainties in soil properties and their implications

In-situ soil properties are difficult to measure accurately and there will be a certain degree of uncertainties in the measured values. In fact, Ching and Phoon (2014) highlighted that different magnitudes of uncertainties can be obtained
depending on the number of soil investigations performed. Hence, a probabilistic analysis would be more appropriate in such instance compared to the use of a deterministic approach because a probabilistic analysis takes into account these uncertainties, and can return the probability of failure or the reliability index of the slope investigated. As a result, this allows for more rational and effective designs to be made. In fact, it is unwise to use the same factor of safety for all slope designs because this will result in different levels of risk depending on the magnitude of variability of the soil properties. In other words, different precautions/actions can be taken for different risks. For instance, more precautionary measures can be made or more soil investigations can be performed for slopes with high risk of failure. However, none of the information on risk would be known if a deterministic analysis were used, hence leaving the slopes exposed to high risk of a slope failure. As known, slope failure is undesirable because it may result in massive damage and loss of lives.

1.4.5 Artificial Neural Network (ANN)

In recent years, ANNs have been used to solve various geotechnical engineering problems (Goh 1994; Lee and Lee 1996; Mayoraz et al. 1996; Pradhan and Lee 2010b; Abdalla et al. 2015). An ANN, once trained, has the advantage of being able to predict the output from a set of inputs. In fact, Shahin et al. (2001) highlighted that the results predicted by ANNs for certain geotechnical problems may even be better than those calculated from theoretical formulations.

Additionally, ANNs have also been used to investigate landslide susceptibility as well as assessing slope stability (Sakellariou and Ferentinou 2005; Chauhan et al. 2010; Poudyal et al. 2010; Abdalla et al. 2015; Gelisli et al. 2015). Examples of the ANN models and the results obtained can be seen in Fig. 1-5. It should be noted that the study by Sakellariou and Ferentinou (2005) was based on real slope data. Hence, based on the fundamentals of the ANNs, an ANN can be used to provide the same function as the chart solutions. In fact, it may prove to be more convenient because manual reading of the charts – which may be cumbersome – is no longer necessary.
1.4.6 Computer codes
The finite element limit analysis computer code used in this analysis was developed by Lyamin and Sloan (2002a, 2002b) and Krabbenhoft et al. (2005) of The University of Newcastle. Based on the source code, appropriate modifications have to be made to prepare the code for this research such as the selection of suitable domains and other technical specifications. Additionally, a framework of constructing the finite element mesh is also developed in this research to optimize the analysis. On the other hand, to conduct the probabilistic analysis, a MATLAB code – using which the finite element limit analysis methods can be incorporated within – has to be written. In fact, to obtain more accurate results, the number of realizations of the Monte Carlo simulations used in the probabilistic analysis is also programmed into the MATLAB code. Finally, regarding the development of the ANN tool, a MATLAB code has also been written based on the mathematical derivations and theory found in Chapter 7.

1.5 THESIS OUTLINE
This thesis is made up of 8 distinct chapters. The first chapter is the introduction to this thesis, which comprises the objectives and key components of this thesis. The second chapter presents the previous studies in the literature. It is important to highlight that the writings on previous studies found in Chapter 3–7 have been omitted from this chapter to avoid repetition.

Chapter 3–7, which is the main body of this thesis presents the results and papers of the studies of this research. The chapters are as follows:

- Chapter 3: Slope stability assessment of two-layered undrained clay
  In this chapter, 3D boundary effects on two-layered undrained clay slopes are investigated. Various factors influencing the stability of this type of slope – such as $c_{u1}/c_{u2}$ ratios, 3D boundary ($L/H$ ratios) and depth factors ($d/H$ ratios) – are investigated and discussed. From the discussion, the influence of one parameter on the effect of another parameter on the slope stability is also observed. Additionally, the influence of the above parameters on the failure mechanisms is also discussed.

- Chapter 4: Slope stability assessment of frictional fill materials placed on undrained clay
This chapter presents the results from the 2D and 3D analyses of frictional fill materials placed on uniform undrained clay slopes. In this chapter, the results from LEM analyses are also presented for comparison. Again various variables are investigated in this chapter to study the effect of the variables on this type of slope. The parameters investigated include fill material’s friction angles $\phi'$, $L/H$ ratios and $d/H$ ratios. The failure mechanisms obtained from 2D and 3D analyses are also discussed.

- **Chapter 5: Slope stability assessment of frictional fill materials placed on undrained clay with increasing strength**
This chapter presents the results for slopes with frictional materials placed on inhomogeneous clay foundation. Parameters similar to Chapter 4 are investigated taking into consideration the degree of strength increment with depth. The influence of the strength increment on slope stability is clearly presented. Similarly, the influence on the failure mechanisms is also presented and discussed. Both 2D and 3D analyses are conducted.

- **Chapter 6: Probabilistic analysis of two-layered undrained clay**
In this chapter, the effects of uncertainties in undrained clay on the stability of two-layered undrained clay slopes are investigated. From the investigation of various parameters which include coefficients of variation (COVs), mean $c_u$ ratios and mean $c_y$ values, different relationships between the probability of failure and the mean factor of safety are obtained and presented. Results of this investigation are also used to discuss the implication of the conventionally used design factor of safety of 1.5.

- **Chapter 7: Fill slope stability assessment based on artificial neural network (ANN)**
A trained artificial neural network capable of predicting the stability of two-layered undrained clay slopes both deterministically and probabilistically is developed in this chapter. The convenience of using the trained ANN tool is demonstrated for a range of application examples with different parameters’ values.

After these 5 chapters, the thesis finishes off with a chapter of conclusion and the Appendix. Relevant materials to this thesis are presented in the appendix.
1.6 REFERENCES


1.7 FIGURES

(a) An example of slope stability chart for cut slopes (Li et al. 2009)

(b) An example of chart solution for uniform slopes (Michalowski 2002)

Fig. 1-1. Examples of chart solutions
Fig. 1-2. Plot of the probability of failure vs the mean factor of safety by (Alonso 1976)

Fig. 1-3. A three-dimensional cylindrical failure surface with curved ends (Gens et al. 1988)
(a) Stability charts that include the effect of pore water pressure and seismic force (Michalowski 2002)

(b) The critical seismic coefficient for various depth ratios using two distinct methods of analysis (Loukidis et al. 2003)

**Fig. 1-4.** Examples of stability charts considering external factors
(a) The architecture of a Multi-Layer Perceptron ANN model used by Abdalla et al. (2015)

(b) The results of ANN-predicted safety factors compared to the safety factors obtained using the LEM (Sakellariou and Ferentinou 2005)

Fig. 1-5. Examples of the adoption of ANNs in slope stability assessments
Chapter 2 PREVIOUS STUDIES

2.1 SLOPE STABILITY

Although many studies have been performed using the limit equilibrium method (LEM), the shortcomings and limitations of the method are prevalent. For instance the method requires assumptions – on statics and slip surface – and if the assumptions are satisfied, then the method is deemed adequate. That said, the limit equilibrium method is easy to use and the factor of safety can usually be obtained by calculating the amount of shear strength available to resist the mobilized shear stress.

Similarly, the use of the conventional limit analysis method also requires an assumption of a trial slip surface. None the less, the limit analysis method has an advantage over the LEM because no static assumptions are required by the conventional limit analysis method. Additionally, the limit analysis method is also capable of producing two distinct solutions namely the upper and lower bound. However, many slope stability analyses using the limit theorems have been based on upper bound only (Donald and Chen 1997; Michalowski 2002; Viratjandr and Michalowski 2006) because of the difficulty and issues of simulating the stress field, which is a core requirement of the lower bound limit analysis. This argument was also supported by Michalowski (1989) and Sutcliffe et al. (2004) who stated that the stress field used in the lower bound approach is cumbersome.

In light of the limitations of the limit analysis method, new formulations called the finite element limit analysis methods were developed (Lyamin and Sloan 2002a, 2002b; Krabbenhoft et al. 2005). These new formulations do not need a pre-defined failure surface and are capable of simulating the stress field used in the lower bound. The method of incorporating a finite element mesh and the limit analysis method together can be traced back to the studies by Anderheggen and Knöpfel (1972) and Bottero et al. (1980).

Apart from that, another method called the finite element method (FEM) has also been commonly used in slope stability analyses. The FEM is capable of capturing the stress state and deformation in soil. Unlike the LEM, the FEM does not require prior assumptions on statics or a pre-defined failure surface because the
failure occurs naturally and can be judged by observing the shear strain in the soil model. The strength reduction method (SRM) is commonly used with the FEM to determine the factor of safety of a slope. The factor of safety obtained using the SRM is defined as how much the shear strength of soil must be divided to bring the slope to failure. However, the main issues with the FEM are the computation time required and its convergence issues.

Due to the difficulties and costs associated with full scale testing, physical modelling is also very popular in slope stability analyses because the method is capable of simulating the actual slope in a controlled environment albeit in a scaled down form. In fact, physical modelling is capable of producing the physical visualisation of the failure mechanism of a slope. However, the cost of performing the physical model test is expensive and it is known to very be time-consuming as well.

Due to the extensive discussion in the literature sections in this thesis and to avoid repetition, the following review will focus on discussing about things that have not been discussed in the papers.

### 2.1.1 Limit equilibrium method

While the LEM remains popular among geotechnical engineers, there are inevitable shortcomings with the method namely the need of assumptions such as the inter-column shear force (Bishop 1955; Janbu 1973); the translation of inter-column shear force into functions (Morgenstern and Price 1965; Spencer 1967); the validity of the force/moment equilibrium or the need for a pre-defined slip surface. An example of the forces in the LEM can be seen in Fig. 2-1. Having said that, Bishop’s simplified method of slices (Bishop 1955) as shown in Fig. 2-2, which utilizes a moment equilibrium remains one of the most popular LEMs to date due to its simplicity. However the method is only suitable for slope failures with circular slip surfaces.

In fact, Fredlund and Krahn (1977) showed that the inter-column force function has little influence when used with moment equilibrium regardless of the shape of the slip surface. However, Krahn (2003) rebutted this theory and stated that the negligence of inter-column shear force could have an effect on the factor of safety for planar slip surface analyses. Krahn (2003) stated that the increase in the inter-
column shear force function relation may lead to a higher factor of safety. In light of the assumption issues, Leshchinsky (1990) and Leshchinsky and Huang (1992b) developed a variational limit equilibrium method that do not need an assumption on statics. However, the assumption of a pre-defined slip surface is still required within the variational method.

In light of the limitations, more studies have been performed to optimize the searching process and determination of the critical slip surface (Arai and Tagyo 1985; Van Uu 1985; Greco 1996; Malkawi et al. 2001; Zhu 2001; Sarma and Tan 2006; Cheng 2007; Cheng et al. 2008; Kahatadeniya et al. 2009; Hajiazizi and Tavana 2013). Apart from the use of mathematics in optimization, Greco (1996) and Malkawi et al. (2001) showed that the Monte Carlo method can also be utilized to search for the critical slip surface. However, due to the complexity of soil and the different geometries of different slopes, methods that can automatically produce the failure mechanisms such as the finite element method would be more preferable in slope stability analyses.

Most of LEM-based 3D analyses have been extended from the existing available two dimensional (2D) methods. For example the study by Gens et al. (1988) was extended from the Swedish Circle method (Fellenius 1936); (Hungr 1987; Hungr et al. 1989) are an extension of (Bishop 1955; Janbu 1973) respectively and Leshchinsky and Huang (1992b) is an extension from Leshchinsky and Huang (1992). Thus the inherent limitations from 2D limit equilibrium methods were brought forward into the 3D methods. An example of a 3D analysis based on the LEM can be seen in Fig. 2-3 where a sliding block system was adopted. As highlighted by Chang (2005), the method required the sliding mass to be assumed as a block system. A similar system (Fig. 2-4) was also used by Seed et al. (1990) who studied a failed landfill slope in California using a 3D analysis. Huang and Tsai (2000) and Huang et al. (2002) discovered that the use of a symmetrical force in asymmetrical conditions may lead to incorrect safety evaluations. It was found that an analysis that accounts for asymmetrical forces produced lower factors of safety as compared to the safety factors obtained by Hungr (1987) and Lam and Fredlund (1993). Additionally, Huang and Tsai (2000) and Huang et al. (2002) incorporated the sliding direction of the sliding mass as part of the calculation. This was seen as beneficial in the context of the LEM as the need for prior
assumptions of the sliding direction could then be removed. However, the technique was found to have an issue because the implementation of different directions of the sliding of different soil columns may lead to convergence issues (Cheng and Yip 2007). Hence this again proves the shortcoming of the limit equilibrium method.

2.1.2 Finite element method

The strength reduction method (SRM) is commonly used in the FEM to obtain the factor of safety. This technique was originally developed by Matsui and San (1992). Several slopes were investigated by Matsui and San (1992) and the results obtained showed that the safety factors generally agreed well with the LEM (Bishop’s method and Fellanius’ method) except for the excavation slope where the SRM’s factor of safety was slightly higher than that using the Bishop’s method. That said, according to Cheng et al. (2007), the SRM of the finite element analysis is sensitive and may not be suitable for slopes with a soft band of frictional soil (shown as blue band in Fig. 2-5). Additionally, the limit load or the factor of safety has to be determined subjectively. For example, according to Fig. 2-6, the factor of safety denoted by the symbol FOS in the figure is determined when there is a difficulty in convergence and a rapid increase in displacement (Griffiths and Lane 1999; Griffiths and Marquez 2007). The displacement in this instance is represented by a dimensionless displacement \( E'\delta_{\text{max}}/\gamma H^2 \) where \( E' \) is the young modulus of the soil, \( \delta_{\text{max}} \) is the maximum nodal displacement, \( \gamma \) is the soil unit weight and \( H \) is the height of the slope. However, according to Zheng et al. (2005), these non-convergence criteria to identify a failure are ambiguous because non-convergence in soil can be caused by many factors and not necessarily due to a slope failure. For instance, Zheng et al. (2005) demonstrated an example where the use of an inappropriate Poisson ratio led to an underestimation of the factor of safety when the non-convergence option was used as the failure criterion. In the example, it was shown that the user’s selected iteration limit for convergence affected the computation of the results. This argument was further supported by Wei et al. (2009) who highlighted that the SRM can be very sensitive to the convergence criterion, boundary conditions and the mesh design of the slope investigated.
Regarding 3D analysis, several homogeneous slopes were investigated by Griffiths and Marquez (2007) using the FEM and the study showed that the results obtained using the FEM were slightly more conservative compared to those obtained using the LEM. Additionally, it was also highlighted that the FEM is capable of investigating complex geometries and different boundary conditions. Similar view was also expressed by several other investigators who studied the influence of different shapes of 3D slopes (Jeremić 2000; Zhang et al. 2013).

2.1.3 Limit analysis

Similar to the LEM, the limit analysis method requires the failure mechanisms – such as rigid translational/rotational failure mechanisms or block/wedge failure mechanisms – to be pre-determined. Although the conventional limit analysis method has been used to solve 3D slope problems (Farzaneh and Askari 2003; Michalowski 2010), the studies also required the failure mechanism to be assumed (i.e. dividing the failure into several blocks or assuming a 3D rotational mechanism). Just recently, Gao et al. (2012) extended the study by Michalowski and Drescher (2009) – who used a toe failure with a rotational failure mechanism in their study – to include face and base failures. As highlighted by Gao et al. (2012), slopes’ failure mechanisms (i.e. base or toe failures) such as those shown in Fig. 2-7 are influenced by the slopes’ geometry and the soil properties. Hence, this shows the limitation of the limit analysis method where the method is unable to “seek out” the critical failure mechanism. Similar problem regarding failure mechanisms was also found in the study by Chen et al. (2001) who divided the failure mass of a slope into prisms – such as those shown in Fig. 2-8. In fact, it was highlighted by the author that the proposed method can be quite time consuming in terms of the preparation of the data files and the method was occasionally affected by convergence problems.

In addition, the use of an associated flow rule in the limit analysis method was shown to influence the results when compared to the non-associated flow rule used by the FEM. Chen (1975) showed that the collapse load based on a non-associated flow rule would be smaller than that of an associated flow rule. Such finding was also shown by Manzari and Nour (2000) who demonstrated that the effect of soil dilatancy on stability number \( N/\gamma H \) increased as the friction angle increased. Furthermore, as previously mentioned, the majority of slope stability
analyses have been performed utilizing only the upper bound limit analysis method because of the difficulty in computing the stress field required by the lower bound limit analysis method.

2.1.4 Physical modelling

Centrifuge models have also been used to study slope stability (Chen et al. 2007; Chen and Liu 2007; Ling et al. 2009; Hu et al. 2010; Kitazume and Takeyama 2013). Through proper scaling, a centrifuge model can be used to match the configuration of an actual slope and subsequently be used for slope stability assessment. For example, Chen and Liu (2007) investigated the characteristics of the failure of a slope made up of sand particles by observing the movement of the assembly of aluminium rods in a titling box. Additionally, the authors also investigated the effectiveness of soil nail stabilization and found that the optimal installation angle of soil nails was along the minor principal stress direction. On the other hand, Hu et al. (2010) investigated the behaviour of geotextile-reinforced-cohesive slopes and concluded that the deformation of the slopes was affected by the inclination of the slopes and the length of the reinforcement.

In the study by Ling et al. (2009), it was observed that the failure surface obtained from centrifuge modelling agreed well with Bishop’s circular failure mechanism. It was also observed that the failure of slopes under rainfall may be interpreted as a reduction in the apparent cohesion of the soil. Shown in Fig. 2-9 is an example of a failed slope in a centrifuge modelling analysis. Similarly, using centrifuge modelling, Kitazume and Takeyama (2013) investigated different slope heights and the effect of ground improvement on the critical height of an embankment. The authors showed that the results from the centrifuge tests agreed well with the results obtained using the LEM (Fellenius 1936). However Kitazume and Takeyama (2013) only considered a single slope inclination.

Despite the fact that physical modelling has been used in practice, it requires more effort to build the physical model compared to performing a computer simulation and thus is time consuming as well as tedious. Apart from being able to replicate the real situation of a slope, the accuracy of completely matching every properties of the real and actual slope such as consolidation, spatial variation and others proved to be a difficult procedure.
2.2 EMBANKMENTS

Fill slopes are commonly encountered in the construction of embankments (Indraratna et al. 1992; Al-Homoud et al. 1997; Richards and Reddy 2005). A simple illustration of an embankment can be seen in Fig. 2-10. Many conventional methods such as the LEM, the FEM and centrifuge modelling have been used to investigate embankment stability. For instance, many studies have been performed to investigate the behaviour and performance of embankment on soft soils (Almeida et al. 1985; Indraratna et al. 1992; Kaniraj and Abdullah 1992; Chai and Carter 2009; Chai et al. 2013). In fact, Almeida et al. (1985) who used centrifuge tests in their study showed that the method is capable of measuring displacement and pore water pressure. On the other hand, Indraratna et al. (1992) who used the finite element method found that the FEM had the upper hand over the LEM where the shear band obtained from the FEM compared well with the actual observed slip surface. Additionally, the authors also noted that the failure surfaces predicted from the LEM were not as accurate. The FEM also has another advantage, where it is able to model progressive failure of slopes, as shown in the literature (Lechman and Griffiths 2000; Troncone 2005; Chai and Carter 2009). Apart from that, comparisons between the results obtained using the LEM and those from the limit analysis method have also been made in the literature (Jiang and Magnan 1997; Kim et al. 2002) and it was found that the results from the limit analysis method were more conservative compared to those from the LEM.

2.2.1 Limit equilibrium method

Using a semi-analytical procedure based on a moment equilibrium, Low (1989) investigated the stability of embankments constructed on soft clay. However, it was noted in the study that a circular slip surface was used and a 2D analysis was adopted. Additionally, it was discovered by Low (1989) that Bishop’s simplified method may not be suitable for problems where the embankment material is highly cohesive or there are slices with bases at steep angles. That said, many investigators have since extended the study by Low (1989) to investigate reinforced embankments (Low et al. 1990; Kaniraj and Abdullah 1992; Fattah et al. 2012). However, the extended method proposed by Kaniraj and Abdullah (1992) requires a trial-and-error approach and hence can be cumbersome. Also based on the limit equilibrium method, Kaniraj (1994) proposed simple equations...
to compute the stability of both unreinforced and reinforced embankments. However, one of the drawbacks of the proposed method is that the assumption of a circular slip surface is required. Additionally, it was also shown that the solutions obtained were required to satisfy a few conditions imposed and if not satisfied – as shown in one of the examples – the factor of safety obtained may be inaccurate. Similarly, the proposed method is also not applicable for problems where an excavation is close to the embankment.

To study the differences between the two methods, Cala and Flisiak (2001) conducted investigations on layered soil slopes using the FEM and the LEM. The results obtained using the two methods were in fact different for some cases. For example, for the embankment case of two layered soil, Cala and Flisiak (2001) noted that the factors of safety obtained using the LEM were found to be overestimated when compared to those from the FEM for large slope angles. In addition, a case study of a slope in Poland was also studied and significantly different failure surfaces and factors of safety were obtained using the two methods for a large scale complex geology slope. For instance, the factor of safety obtained by the FEM was 1.18 while the factor of safety obtained using the LEM was 2.1. The difference in the failure surfaces obtained using the two methods can be seen in Fig. 2-11. Hence, the study showed that the LEM may significantly overestimate the factor of safety. The discrepancy may be because the FEM does not need a prior assumption of the failure surface. Additionally, Indraratna et al. (1992) investigated a failed embankment slope in Malaysia and compared the results obtained using the LEM and the FEM and found that the failure surfaces predicted using the LEM are shallower than that obtained from the FEM as well as the actual failure surface. The failure surfaces can be seen in Fig. 2-12. Therefore, in terms of reinforcement, the results obtained from the LEM would have provided a misleading guidance since the LEM would indicate an inaccurate position for the reinforcement.

2.2.2 Limit analysis method

A comparison was also made between the results obtained from the limit analysis method and the LEM for the embankment problem found in Low (1989). Particularly, the factor of safety obtained using the limit analysis was slightly lower than that obtained using the LEM (ordinary method of slices). As suggested
by Jiang and Magnan (1997), this might be because the limit analysis method employed the plasticity theory and it was shown that there was a large zone of plasticity outside the slip circle obtained by Low (1989). However, as the ordinary method of slices did not provide any information on the state of plasticity of the soil, it was not immediately clear that the discrepancy between the factors of safety could be correlated in such a way. Another interesting finding was discovered regarding the factor of safety when the friction angle of the fill material was zero. Particularly, it can be observed that the factor of safety obtained from the limit analysis method remained unchanged even though the friction angle became zero. Therefore, it may be concluded that the limit analysis method is less ‘sensitive’ to the change in friction angle compared to the ordinary method of slices. However, as noted by Jiang and Magnan (1997), this could be due to the material being highly cohesive.

Similar problem to the above was also investigated using the finite element limit analysis methods (Kim et al. 2002). It can be observed from Fig. 2-13 that the failure mechanisms obtained from the conventional LEM (Spencer’s method) and the upper bound finite element limit analysis are quite similar. However, from the results obtained, it can be seen that the lower bound numerical analysis method produced the lowest factor of safety among the results, which included results from the moment-equilibrium based semi-analytical method used in the original study, Spencer’s method and the upper bound methods (from this study and the study by Jiang and Magnan (1997)). However, the results from the upper bound methods (finite element limit analysis and conventional limit analysis) were in good agreement with each other. This ultimately shows that the LEM-based method proposed by Low (1989) may produce results that are not conservative and overestimated.

2.2.3 Finite element method

Comparing the results obtained between the FEM and the conventional LEM, Bakır and Akış (2005) who undertook a case study of an embankment failure in Turkey (Fig. 2-14) found that the results were similar while the failure surfaces obtained using the two methods were also in reasonably good agreement. In fact, it is known that the FEM is capable of computing the displacement and settlement of slopes. Hence, using the FEM, Chai et al. (2013) did a case study of an
embankment constructed on soft clay (as shown in Fig. 2-15). The authors, who compared the results obtained using the FEM with the field measurements data found that in order to better predict the behaviour of such a structure (embankment on soft clay), proper testing and interpretation of soil test results are important.

As mentioned earlier, the FEM also has an advantage over the other methods because it is capable of simulating the progressive failure of slopes. For instance, Chai and Carter (2009) used the FEM to study the progressive failure of an embankment constructed on soft clay in Japan. Their study revealed large deformations were produced when the strain-softening behaviour of the clay was considered in the analysis. Additionally, the authors also highlighted that due to strain-softening, the residual strength of the soil may highly affect a slope design and thus proper care should be taken. From the two case studies above, it can be seen that the FEM is suitable for simulating more complex slopes. However, it is known that the FEM is time-consuming and therefore, would not be suitable for preliminary design.

Regarding the effects of 3D boundaries on embankments, a reference can be made to the study by Cala et al. (2006). It was found in the study that the factor of safety obtained using a 3D analysis was higher than that from a 2D analysis. Additionally, it was also found that the factor of safety from the 3D analysis decreased towards plane strain condition’s factor of safety as the width of slope increased. However, it was not immediately clear if the investigation was based on smooth or rough lateral boundaries. Speaking of boundaries, different boundary conditions and configurations will yield different failure mechanisms (Chugh 2003; Zhang et al. 2013). For example, cylindrical-shaped failure mechanisms are more likely to be obtained using smooth boundaries compared to rough boundaries (Zhang et al. 2013). Additionally, Zhang et al. (2013) who studied different geometry and boundary conditions of slopes showed that the results obtained using smooth-smooth boundary conditions were not affected when the width of the slope changed. However, when rough-rough boundary conditions were used, the factors of safety were shown to increase with a decrease in the slope width (smaller 3D boundary). In fact, the study by Chugh (2003) showed that the factor of safety obtained using smooth-smooth boundary
conditions were in fact very similar to that obtained from a 2D analysis. That said, Chugh (2003) also showed that it is possible to construct the 3D boundaries of a slope using two end blocks that represent abutments. However, as shown by the results, the factors of safety obtained using this configuration were less conservative. Therefore, based on the above discussion on boundary conditions, different 3D boundary conditions can be adopted depending on the slope applications.

2.2.4 Centrifuge modelling
While there are numerous embankment studies based on centrifuge modelling, the majority of the studies were regarding the behaviour of the embankments such as deformation, hence, will not be discussed in detail here. Using centrifuge modelling, Almeida et al. (1985) investigated the behaviour of embankments constructed on soft clay foundations. In their study, both strengthened and unstrengthened foundations were considered. From the results presented, it can be seen that several type of data can be obtained using centrifuge modelling such as pore pressure, pore pressure dissipation and displacement. In fact, it was also shown that the recorded pore pressure data can also be utilized in stability analyses, hence providing more realistic analyses of the slopes. Additionally, the results from the above study were also compared with those obtained from numerical modelling (Almeida et al. 1986). The results presented by Almeida et al. (1986) showed that the displacements obtained from the centrifuge modelling matched those from the numerical modelling, particularly near the ground surface under the embankment. However, it was highlighted by Almeida et al. (1986) that the results predicted using the numerical modelling were less accurate at a deeper depth. Additionally, comparing between theoretical results and centrifuge tests results, it was shown that the amount of settlement based on theoretical calculations was underestimated (Jin et al. 2014). Therefore, this shows that physical modelling may better capture the behaviour of slopes than theoretical calculations.

2.3 PROBABILISTIC ANALYSIS
It is well known that the conventional deterministic slope stability analysis does not explicitly consider the uncertainties in soil properties. As a result, the use of
the factor of safety in the deterministic approach does not disclose the risk of failure of the slope investigated. In fact, it is recognized that slopes with the same factor of safety may have different risk levels or probability of failure depending on the degree of uncertainties in the soil properties (Liang et al. 1999; Duncan 2000; Cho 2007). In light of that, many probabilistic-based analysis methods have been developed. Some of the reliability analysis methods used are the First Order Reliability Method (FORM), the First Order Second Moment (FOSM) method and the Monte Carlo simulation method (Low et al. 1998; Malkawi et al. 2000; Low 2003; Hong and Roh 2008; Mbarka et al. 2010; Suchomel and Mašić 2010; Griffiths et al. 2011). In fact, these reliability analysis methods are commonly used together with the conventional slope stability analysis methods such as the LEM and the FEM. That being said, it was found that the first order methods may lead to inaccurate probability of failure (Ching et al. 2009; Zevgolis and Bourdeau 2010; Griffiths et al. 2011). In addition, Silva et al. (2008) showed that the probability of failure can also be determined based on historical slope data. In fact, using the probability of failure charts developed by the authors (Fig. 2-16), the corresponding probability of failure can be obtained for projects of different degree of importance and consequences of failure. However, this technique relies heavily on the users’ judgement.

As the field of slope stability analysis advances, study of the influence of slope heterogeneity or the spatial variability of the soil properties in a slope has also gained popularity (Griffiths and Fenton 2000; Hicks and Samy 2004; Cho 2007; Low et al. 2007; Griffiths et al. 2009; Hicks and Spencer 2010; Huang et al. 2013; Hicks et al. 2014). In fact, Griffiths et al. (2011) – who used the random finite element method – highlighted that a lower probability of failure could be obtained using methods that do not properly account for spatial variability. However, for smaller coefficients of variation (COVs), the probability of failure obtained will be more conservative if spatial variability is ignored. On a separate study, Suchomel and Mašić (2010) showed that the results obtained using the FOSM method can be improved using simple and appropriate modifications. In fact, it was also shown by Suchomel and Mašić (2010) that results obtained from the extended FOSM were in fact closer to those obtained from the RFEM compared to the basic FOSM. Additionally, Cho (2009a) showed that the LEM can also be used with the
random field theory to search for the critical failure surface while considering the
different failure mechanisms caused by the spatial variability of the soil properties
in the slope. In fact, it was also highlighted by Cho (2009a, 2009b) that the
probability of failure of the critical failure surface may not be the highest. Similar
view was also expressed by Hassan and Wolff (1999) who stated that the failure
surface with the minimum factor of safety and the failure surface with the lowest
reliability index may be different. Hence, Hassan and Wolff (1999) proposed an
algorithm that search for the overall lowest reliability index rather than the
reliability index of the critical failure surface. The following discussion will be
based on the different methods used in probabilistic studies found in the literature.

2.3.1 Limit equilibrium method

Based on the FOSM method, Christian et al. (1994) performed a probabilistic
study of an embankment case study and produced a chart of the nominal
probability of failure versus the factor of safety for the problem. However, as
highlighted by Alonso (1976), the relationship between the probability of failure
and the factor of safety is unique to problems with similar configurations.
Therefore, the chart produced by Christian et al. (1994) is only applicable to
similar problems. However, the key point here is that the probability of failure
was shown to vary with the factor of safety and as highlighted by Christian et al.
(1994), the reliability index would provide a more meaningful evaluation of slope
stability compared to the use of only the factor of safety. Similar view was also
expressed by Duncan (2000) who used the Taylor series method in his
probabilistic study of a failed slope in San Francisco. As noted by Duncan (2000),
while the factor of safety of the slope was 1.17, the probability of failure was
calculated to be 18% which wasn’t known at the time of design. Therefore,
Duncan (2000) suggested that both the deterministic and the probabilistic analyses
should be performed during design. Another example of a case study can be found
in study by Rathod and Rao (2012). In their study, it was also found that slopes
that have factors of safety higher than 1 may still be susceptible to failure except
for slopes with factors of safety that are reasonably larger than 1 (i.e. 1.3 and
above). However, it was not indicated in the study what magnitude of $COV$ was
used.
Based on FORM and an extended method of slices, Hong and Roh (2008) investigated the influence of soil uncertainties on various layered soil slopes. Particularly, a sensitivity analysis was performed and it was shown that the influence of the uncertainties in unit weight was relatively small compared to the influence of the uncertainties in cohesion and friction angle. Similar view regarding the influence of uncertainties in cohesion was also expressed by Wang et al. (2010).

It is important to highlight that apart from the FEM, the random field theory can also be applied in a LEM-based investigation. For example, Cho (2009a) extended the traditional method of slices to investigate the uncertainties and spatial variation of soil strength parameters. An important contribution of the study was that various failure mechanisms can be considered and the critical failure surface corresponding to the input random field of soil properties can be located through a search algorithm. Particularly, it was found that the probability of failure may be unconservative (underestimated) using a fixed critical surface for slopes with an undrained condition. In fact, traditionally, the probability of failure were only computed based on the input random field of soil properties along the predetermined critical failure surface (Cho 2009a; Griffiths et al. 2009). Hence, the results obtained from the traditional LEM-based random field theory analysis may not be as accurate.

2.3.2 Finite element method
The study by Mbarka et al. (2010) showed that the reliability index of a slope can be influenced by various factors. In their study, various slope stability analysis methods and probabilistic models were utilized. Particularly, it was shown that based on the FEM strength reduction method, the reliability index obtained using FORM was higher than that using the Mean-Value-First-Order-Second-Moment method (MFOSM). Most importantly, it was shown that different results were also obtained using different slope stability analysis methods. For example, based on the MFOSM method, the reliability index was overestimated using the Bishop method by 10% when compared to the FEM strength reduction method. It is understood that this study did not consider the spatial variation in soil properties.
Using the random field theory with the FEM, Griffiths and Fenton (2000) studied the influence of soil strength spatial variability of an undrained clay and found that results obtained from the Single Random Variable approach may be unconservative when compared to those from the random field approach. It was also highlighted by Griffiths and Fenton (2000) that larger COVs always led to undesirable slope stability. In a more recent paper, Griffiths et al. (2009) compared the performance of the FEM combined with FORM and the more advanced random finite element method (RFEM) and reiterated that using a probabilistic analysis in which the spatial variation of the properties is not accounted for may lead to unconservative estimates of the probability of failure. In addition, Griffiths et al. (2009) highlighted that this might be because the RFEM allows the failure mechanism to be located along the weakest path of the heterogeneous soil mass. However, it was also emphasized that the influence of spatial variation of the soil properties may be the opposite below a certain COV of the input shear strength parameters. In other words, ignoring spatial variability may produce a conservative outcome (Griffiths et al. 2009).

Based on a case study of a slope failure in Norway, Suchomel and Maši´n (2010) utilized the finite element method with three different probabilistic methods to investigate the slope and found some interesting results. For instance, the modified FOSM method which involved the averaging of soil properties along the potential failure surface yielded results that were close to those obtained using the finite element with random field theory. However, it was also highlighted that the extended FOSM method was only applicable for material properties with a small standard deviation and large correlation lengths. That being said, it should be noted that due to its complex implementation, the RFEM has largely been applied to simple cohesive slope geometry.

### 2.3.3 Probabilistic studies based on the finite element limit analysis methods

Apart from the above studies, in recent years, the finite element limit analysis methods have also been utilized in probabilistic-based slope stability analysis (Li et al. 2012; Huang et al. 2013). Based on Monte Carlo simulations, the uncertainties in soil properties can be considered using the finite element limit analysis methods. In fact, the study by Huang et al. (2013) showed that the volume of sliding mass can also be computed using the finite element limit
analysis methods and based on the volume of sliding mass, the consequences of a landslide can be evaluated.

It was mentioned earlier that the factor of safety is not a good indication or representative of the risk of failure of a slope. Similar view was also expressed by Duncan (2000) and Cherubini et al. (2001). Additionally, it was also highlighted that various factors can affect the degree of the uncertainties in soil properties and it was stated that for soil with a large coefficient of variation (COV), it is reasonable for a larger factor of safety to be applied to offset this large uncertainty (Ching and Phoon 2014; Phoon and Ching 2014). Hence, motivated by these issues, Li et al. (2012) investigated and presented the relationships between the factor of safety and the probability of failure for rock slopes for various COVs of rock properties (based on Hoek-Brown failure criterion). It can be seen from the results that the probability of failure is larger for larger COVs. Additionally, a decrease in the probability of failure can also be observed as the mean factor of safety increases. In fact, as highlighted by Alonso (1976), the relationship between the factor of safety and the probability will be useful for preliminary design or evaluation of risk. However, Alonso (1976) also stated that the relationships were only applicable for slopes with similar geometry and configurations. Therefore, the relationship established by Li et al. (2012) for rock slopes are not applicable for soil slopes or fill slopes. Therefore, in light of this and the lack of such relationship for fill slopes, this study aims to investigate the effect of the uncertainties in soil properties on fill slopes and to establish relationships of the factor of safety versus the probability of failure for fill slopes.

2.3.4 Practical application of reliability analysis
The above studies documented the studies in the present literature that investigated slope stability using a probabilistic approach based on the different available slope stability analysis methods. However, it is also worth discussing one of the reasons that a probabilistic analysis is required: uncertainties. To date, many methods have been developed to determine the undrained shear strength of clay. However, as Ching and Phoon (2014) noted, there may be errors if the result from only one test is used. This is due to the use of only univariate information. Hence, the authors proposed a series of transformation equations that convert practical undrained shear strength test results into applicable mean values and
COVs. It was also highlighted by Phoon and Ching (2014) that by combining the estimates from different tests, the COV of the parameter can be potentially reduced. In fact, Ching and Phoon (2014) noted that the COV of undrained shear strength could be more than 0.3 if no undrained shear strength test is performed but the COV could be reduced to less than 0.1 if multiple tests were performed. Additionally, it is understood that the COVs of soil properties may be affected by many factors such as systematic error (statistical, transformation, measurement) and data scatter (spatial variation). However, this can all be considered in the calculation of the total variance (Christian et al. 1994; Xu and Low 2006).

In the context of hazard and risk assessment, Chowdhury and Flentje (2003) discussed the importance of reliability analysis and suggested a range of indicative values of acceptable reliability index or probability of failure corresponding to the types and locations of the slopes. Just recently, Javankhoshdel and Bathurst (2014) produced a set of stability charts using which the probability of failure and the factor of safety of cohesive-frictional soil slopes can be obtained. It should also be highlighted that for layered soil slopes, there are no readily available quick solutions. Hence, it is the aim of this study to propose quick and convenient solutions for fill slopes for the specific case of two-layered undrained clay.

2.4 NEURAL COMPUTING

The advancement of neural networks has enabled the use of neural network computing in geotechnical engineering (Shahin et al. 2001; Lu and Rosenbaum 2003; Ermini et al. 2005; Samui and Kumar 2006). In fact, neural network computing has been applied in a number of geotechnical engineering applications such as displacement prediction, modelling soil behaviour, slope stability and pile capacity prediction, to name a few. In fact, neural networks have gained considerable attention in slope stability investigations particularly in recent years.

2.4.1 Landslide assessment

In recent years, neural networks have been increasingly utilized in the field of landslide susceptibility assessment (Yesilnacar and Topal 2005; Yilmaz 2009; Poudyal et al. 2010; Pradhan and Lee 2010b). As highlighted by Ermini et al. (2005), an artificial neural network (ANN) has an advantage over a statistical
approach because a statistical approach is time-consuming and complex while also requires assumptions to set up a rigorous statistical-based model.

The study by Pradhan and Lee (2010a) showed that neural network models can be used to study the effect of different landslide triggering factors and this can be particularly useful in mitigation efforts. To assist in the planning of the location of a new section of pipeline after the old one was damaged in a landslide, Yesilnacar and Topal (2005) utilized an ANN to develop a landslide susceptibility map. In the study, the ANN model was also used to rank the inputs (landslide triggering parameters) according to the degree of their contribution (importance). The ranking showed that elevation and slope inclination are among the parameters with the highest influence.

Additionally, Poudyal et al. (2010) who utilized the frequency ratio and an ANN to produce landslide susceptibility maps for the Nepal Himalaya stated that these maps can actually be helpful to engineers. For example, as reported in the study, some slopes, as shown by the maps were more prone to landslide occurrences. Hence using these information, the engineers can select the most ideal site for development or mitigate the risk of slope failures for particularly high risk slopes.

### 2.4.2 Slope stability prediction

Apart from landslide predictions, ANNs have also been used to predict slope stability (Sakellariou and Ferentinou 2005; Samui and Kumar 2006). For instance, Sakellariou and Ferentinou (2005) showed that the factors of safety predicted by the trained ANN were reasonably accurate. Particularly it was shown that in terms of the number of training data used, a smaller training set actually produced better results than a larger training set. Using an ANN and the data from a previous study of homogeneous slopes, Samui and Kumar (2006) predicted the stability numbers for two-layered soil slopes. It is important to highlight that the study also included the consideration of pore water pressure. The training data were obtained from the study by Michalowski (2002) who used the upper bound limit analysis method to study the stability of homogeneous slopes. However, it should be noted that the use of only the upper bound limit analysis method is not conservative. Just recently, Abdalla et al. (2015) demonstrated the applicability of a neural network in predicting the factors of safety of simple slopes. For instance, the
results obtained showed good agreement between the ANN predicted safety factors and the calculated factors of safety. It was also highlighted that relative errors between the majority of the calculated outputs and the ANN predicted outputs were less than 6%. Success of using an ANN to predict the stability of slopes with circular failure surface was also reported by Lu and Rosenbaum (2003). In fact, Lu and Rosenbaum (2003) also reported that results obtained using the ANN were more accurate than those using the Maximum Likelihood Estimation (MLE) technique.

Therefore, from the above studies, it can be seen that neural networks are suitable for slope stability analysis. Particularly, ANNs are convenient and time-efficient as they do not require the users to run a full slope stability analysis or computation as long as the inputs are similar to those of the trained ANN. Essentially, this means that a trained ANN is only applicable to slopes with similar conditions. Hence, at this moment, there is no available ANN model for fill slopes. Therefore, one of the aims of this thesis is to develop a neural network that is capable of conveniently predicting the stability of two-layered undrained clay slopes both deterministically and probabilistically.

2.5 FINITE ELEMENT LIMIT ANALYSIS METHODS

2.5.1 Numerical formulations

Owing to the upper and lower bound plastic theorems developed by Drucker et al. (1952), many geotechnical engineering problems can be solved using the limit analysis method. The theory of limit analysis is based on an assumed perfectly plastic model with an associated flow rule. The associated flow rule, or often known as the normality rule, implies that the plastic strain rates are normal to the yield surface. Therefore, velocities and strain rates, instead of displacements and strains are used within the limit analysis framework. Within the limit analysis theory, there are (1) the lower bound limit analysis method which is based on statically admissible stress fields and (2) the upper bound limit analysis method which is based on kinematically admissible velocity fields. That being said, due to the difficulty in manually constructing those fields, the accuracy of simple hand solutions is often questioned. Along the way, the finite element limit analysis formulations have been developed. Inheriting the advantages of the finite element
method, the finite element limit analysis formulations are capable of modelling various types of geotechnical engineering problems and even consider different boundary conditions.

### 2.5.2 Lower bound finite element limit analysis

Although the finite element lower bound analysis was first developed in 1970 (Lysmer 1970), there were various limitations with the formulation such as computational inefficiency and the solutions produced were often thought to be non-rigorous. Further development of the finite element lower bound theorem can be found in (Anderheggen and Knöpfel 1972; Pastor 1978; Bottero et al. 1980). However, because the proposed formulations were based on linear programming, the computational performance was limited and hence the methods could only solve relatively small problems. Considering the drawback, Sloan (1988) proposed a more efficient linear-programming-based formulation. A number of lower bound methods based on non-linear programming have been developed to date (Basudhar et al. 1979; Lyamin and Sloan 2002a). However, the former study was proven to be unsuitable for large-scale geotechnical problems. Significant improvements in the lower bound theorem were made in the latter study. Based on the formulation developed by Lyamin and Sloan (2002a), computational time has been greatly improved. As shown by Lyamin and Sloan (2002a), a significant reduction in CPU time can be achieved over the linear programming approach.

### 2.5.3 Upper bound finite element limit analysis

The development of finite element formulations of the upper bound theorem was first made by Pastor and Turgeman (1976) and Bottero et al. (1980). However, similar to the lower bound finite element theorem, the computational efficiency of the formulations was undesirable. Hence, further development in the finite element upper bound theorem was made (Sloan 1989; Yu et al. 1994; Sloan and Kleeman 1995). Particularly, the need to specify the location and direction of shearing for each discontinuity in an upper bound analysis was removed in the latter study. Additionally, many upper bound theorems based on non-linear programming have also been developed (Jiang 1995; Liu et al. 1995; Capsoni and Corradi 1997; Lyamin and Sloan 2002b; Krabbenhoft et al. 2005). Particularly, the formulations proposed in the latter two studies have been rigorously used in slope stability investigations (Li et al. 2008, 2009; Qian et al. 2014). Furthermore,
the modified formulation developed by Krabbenhøft et al. (2005) can be used with non-linear failure envelopes such as the Hoek-Brown failure criterion.

2.5.4 Applications

Since the studies in this thesis utilize the finite element limit analysis methods, this section is dedicated to discussing some of the recent studies on slope stability based on the finite element limit analysis methods. In fact, other than slope stability, the methods have been used in various other geotechnical engineering applications (Shiau et al. 2003; Sutcliffe et al. 2004; Merifield et al. 2005, 2006; Merifield and Sloan 2006).

To date, various types of slopes have been investigated using the finite element limit analysis methods (Yu et al. 1998; Kim et al. 1999; Kim et al. 2002; Li et al. 2008, 2009, 2010). Particularly, it should be noted that studies that consider 3D effects have only been recently published. Additionally, it should also be highlighted that studies that consider 3D effects only investigated natural or cut slopes. Therefore, there are no studies of fill slopes that consider 3D effects.

Some interesting results were found in the study by Yu et al. (1998). For instance, while the results obtained using the LEM are similar to those obtained using the upper bound method for steep slopes, the results from the LEM were significantly underestimated for gentle slopes. In fact, the results obtained by the LEM were lower than those from the upper and lower bound methods. Hence, proper engineering judgement would be required if the LEM is used to analyse slopes similar to those in Yu et al. (1998). On the other hand, for cohesive-frictional slopes, the results obtained using the LEM and the finite element limit analysis were in good agreement where most of the results obtained using the LEM were in between those from the upper and lower bound methods. Similar findings were also found for undrained clay slopes (Li et al. 2009). Li et al. (2009) showed that the results obtained by Gens et al. (1988) who used the LEM were in between those obtained using the upper and lower bound theorems. Prior to the development of the finite element limit analysis methods, studies based on the more conservative lower bound theorem had been limited.

Comparing the results obtained from the upper limit analysis method (Michalowski 2002) with those using the finite element limit analysis methods for
cohesive-frictional soil slopes, Li et al. (2010) showed good agreement between the results from those methods. In fact, the results obtained by Michalowski (2002) were shown to be in between those obtained using the upper and lower bound finite element limit analysis methods. While the studies discussed above presented the obtained results in the form of stability charts, it is unfortunate that no 3D-analysis-based stability charts have been proposed for fill slopes. Therefore, considering the robustness of the finite element limit analysis methods and the lack of study and quick solutions for fill slopes, comprehensive investigations of different types of fill slopes considering 3D effects are performed in this thesis using the finite element limit analysis methods.
2.6 REFERENCES


2.7 FIGURES

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Chapter 3 THREEDIMENSIONAL SLOPE STABILITY ASSESSMENT OF TWO-LAYERED UNDRAINED CLAY

K. Lim, A. J. Li and A. V. Lyamin

3.1 INTRODUCTION

Geotechnical engineering problems such as trench stability (Fox 2004; Li et al. 2014), tunnelling (Lee et al. 2006; Yamamoto et al. 2011) and slope stability (Bishop 1955; Griffiths and Lane 1999) have been investigated for years. Slope stability is generally influenced by the slope’s physical properties, the strength parameters of the soil and the slope geometry while a slope’s profile is affected by its construction approach. In general, a slope can be classified as a cut slope, natural slope or fill slope based on the method of construction. Duncan et al. (2008) showed that two layered of undrained clay slopes are commonly encountered in the construction of embankments or levees.

Slope stability has traditionally been analysed considering plane strain condition. However, various investigators have pointed out that the stability of a slope can be influenced by the 3D boundary of the slope. In the study by Cavounidis (1987), it was shown that the factors of safety ($F$) from three-dimensional (3D) analyses are higher than those from two-dimensional (2D) analyses. Additionally, Gens et al. (1988) indicated that a 2D back analysis will overestimate the mobilized shear strength and thus lead to an unsafe prediction. As such, many 3D slope stability analysis methods have been developed in recent years. However, many of the methods were extended from the existing 2D limit equilibrium method (LEM). In addition, majority of the 3D assessments utilizing limit analysis methods have been based on the upper bound method alone (Michalowski 1989; Farzaneh and Askari 2003; Michalowski 2010; Gao et al. 2012a). This is due to the inherent difficulty in manually constructing the statically admissible stress fields for lower bound limit analysis.

Although many methods have been developed for slope stability assessment, in comparison stability charts produced to date are not as many (Gens et al. 1988; Michalowski 1997, 2002; Baker et al. 2006; Kumar and Samui 2006; Li et al.
Slope Stability Assessment of Layered Soil

2009, 2010; Michalowski 2010; Michalowski and Martel 2011). Furthermore, only some of those charts have considered 3D boundary effects. The conventional limit equilibrium analysis is the most popular approach to assess slope stability and thus a number of the slope stability charts have been proposed based on the LEM (Taylor 1937; Gens et al. 1988; Baker et al. 2006; Sun and Zhao 2013). Gens et al. (1988) and Taylor (1937) both investigated slopes with purely cohesive soils. In particular, Gens et al. (1988) proposed the first set of 3D slope stability charts. In recent years, stability charts have also been developed using the limit analysis method. Michalowski (2002), Kumar and Samui (2006) and Viratjandr and Michalowski (2006) performed their study based on a 2D analysis while Michalowski (2010) considered 3D effects in his study. However, their studies only used the less conservative upper bound limit analysis. As a result, rigorous solutions have not been widely applied to slope designs.

Unfortunately, stability charts for fill slopes are limited. Therefore, this study aims to investigate the 3D stability of fill slopes using both the upper and lower bound finite element limit analysis methods developed by Lyamin and Sloan (2002a, 2002b) and Krabbenhoft et al. (2005). The results obtained are presented in the form of comprehensive and convenient chart solutions that can be used in practice by engineers. These charts are particularly useful for quick first assessments of slopes. It should be noted that 3D boundary effects on slope stability are investigated thoroughly in this chapter. Additionally, the 3D failure mechanisms of the slopes are also discussed. In this chapter, the fill slopes investigated are made up of two layers of undrained clay.

3.2 PREVIOUS STUDIES

As previously mentioned, several LEM based stability charts have been developed to date (Taylor 1937; Gens et al. 1988; Baker et al. 2006; Sun and Zhao 2013). For instance, Gens et al. (1988) considered 3D effects in their study of purely cohesive clay slope stability and presented their results in the form of stability charts. The study indicated that the difference between the results from 2D and 3D analyses can range from 3-30% with an average of 13.9%. Although LEM is also widely applied to embankment problems (Spencer 1967; Low 1989; Kaniraj 1994; Long et al. 1996), there are no chart solutions for this type of slopes. Additionally,
although many LEM based 3D analysis methods have been developed to date, most of them were extended from the existing 2D methods (Leshchinsky and Huang 1992b; Huang et al. 2002; Xie et al. 2006). Thus, the inherent limitations of LEM still exist within the formulations.

Numerical modelling is another approach commonly used in slope stability analyses (Matsui and San 1992; Griffiths and Lane 1999; Manzari and Nour 2000; Zheng et al. 2005; Griffiths and Marquez 2007). For example, Lane and Griffiths (2000) who are probably the only group, produced a set of stability charts for slopes subjected to water drawdown conditions. On the other hand, Griffiths and Marquez (2007) investigated 3D boundary effects by considering inclined sloping side in the third dimension. Their study showed that a 3D analysis is probably more realistic than a 2D analysis as it is able to account properly for the fixity and geometry of abutments in the third dimension. Additionally, Zhang et al. (2013) who used the finite difference method, also found that slope stability can be influenced by the 3D geometries of the slope.

In recent years, many investigators have utilized the finite element method (FEM) to perform stochastic slope stability analyses to account for the influence of heterogeneity in soil properties (Griffiths and Fenton 2004; Hicks and Samy 2004; Griffiths et al. 2009; Hicks and Spencer 2010; Hicks et al. 2014). Particularly, Hicks and Samy (2004) show that simplified probabilistic analyses are problem independent and may result in an underestimation of slope reliability while Griffiths et al. (2009) highlighted that spatial variability is not properly accounted for in simplified probabilistic analyses, therefore can result in non-conservative estimates of probability of failure in certain cases. Additionally, the FEM has also been used to analyse embankment failures (Chai and Carter 2009; Chai et al. 2013). Despite the various studies performed using the FEM, no 3D chart solutions have been proposed to date due to the high amount of time required by a finite element analysis.

In recent years, the limit analysis method has also gained much popularity among researchers (Michalowski 1989, 1995; Donald and Chen 1997; Farzaneh and Askari 2003; Chen et al. 2004; Michalowski and Nadukuru 2012). Limit-analysis-based 3D slope stability analyses have also been performed (Chen et al. 2001a;
Chen et al. 2001b; Michalowski 2010; Michalowski and Martel 2011; Gao et al. 2012b). In fact, various 2D and 3D chart solutions based on limit analysis method have also been produced, such as the studies by Michalowski (2002, 2010) and Viratjandr and Michalowski (2006). Recently, Kumar and Samui (2006) investigated slopes with cohesive frictional soil and presented their studies in the form of stability charts. However, the above mentioned studies have only used an upper bound analysis and it is well known that using upper bound alone is unable to truly bracket the solutions.

Fortunately, Lyamin and Sloan (2002a, 2002b) and Krabbenhoft et al. (2005) have developed the finite element upper and lower bound limit analysis methods, which can be used to investigate stability of slopes restricted by 3D boundaries. Moreover, the actual solutions can be bounded by the numerical upper and lower bound methods from above and below. As highlighted by Sloan (2013), these finite element limit analysis methods do not require prior assumptions on the mode of failure and can be applied to various slope stability problems.

Some of the analyses that utilized the methods include those by Yu et al. (1998), Kim et al. (1999), Loukidis et al. (2003), Li et al. (2009, 2010), Lim et al. (2015b) and Qian et al. (2014). Particularly, Kim et al. (1999) and Loukidis et al. (2003), respectively investigated the effect of seismic loading and pore pressure on slope stability. Li et al. (2009, 2010) who applied a 3D analysis to investigate cut and natural slopes obtained higher factors of safety than those obtained by Yu et al. (1998) who used a 2D analysis. 3D slope stability assessments for frictional fill materials placed on undrained clay were investigated by Lim et al. (2015b). Additionally, Qian et al. (2014) used the finite element limit theorem and investigated slopes with two-layered purely cohesive soils. However, only 2D solutions were provided. In light of the methods’ successful application, the numerical upper and lower bound limit analysis methods (Lyamin and Sloan 2002a, 2002b; Krabbenhoft et al. 2005) are employed herein to investigate the stability of two-layered undrained clay (fill) slopes considering 3D effects.

### 3.3 PROBLEM DEFINITION

Fig. 3-1 shows the typical 2D and 3D slope geometry and boundary conditions for the problem considered in this chapter. The fill materials and foundation, which
have been divided into Region 1 and Region 2 have an undrained strength of $c_u$. The undrained shear strength of fill materials (Region 1) is assigned $c_{u1}$ while the pre-existing soil (Region 2) has undrained shear strength of $c_{u2}$. The strength difference between both layers is considered using $c_{u1}/c_{u2}$ ratios, which range from 0.2 to 5.

To study the effect of a 3D analysis, the slope geometry is extended from the 2D slope profile (x-z plane) in the y direction by a distance of $L/2$ ($L =$ length into the page of soil profile). The extension is done symmetrically to reduce the mesh size and computation time. In this study, a range of slope angles of $\beta = 15^\circ$, $30^\circ$, $45^\circ$, $60^\circ$ and $75^\circ$ is investigated. Furthermore, various magnitudes of $L/H$ and $d/H$ ratios are also investigated. In this study, $L/H$ ratios range from 1 to 10 and $d/H$ ratios are between 1.5 and 5.

For comparison purposes, the boundary conditions adopted in this thesis follow the studies by Li et al. (2009, 2010). For instance, the remote vertical boundary (Fig. 3-1(b)) is considered as a rough boundary and the symmetry face is a smooth vertical boundary. Similarly, these boundary conditions can also be found in several other recent slope stability investigations (Griffiths and Marquez 2007; Nian et al. 2012; Zhang et al. 2013). The investigation on the effects of the types of boundary conditions used can be found in the studies by Chugh (2003) and Zhang et al. (2013). Therefore, they will not be investigated in this study. It should be noted that this study is only applicable to slope applications with similar boundary conditions.

The final results of this study are presented in the form of stability number as shown below. This stability number has been modified from the stability number proposed by Taylor (1937). A similar form of stability number has also been applied in the study of Qian et al. (2014).

$$N_{2c} = \frac{c_{u1}}{\gamma HF}$$ (3-1)

where $c_{u1}$ is the undrained shear strength of Region 1, $H$ is the slope height, $\gamma$ is the unit weight and $F$ is the factor of safety.

The typical mesh configurations of the upper and lower bound limit analysis for this chapter are shown in Fig. 3-2. For numerical limit analysis modelling, the
mesh generation must follow two important guidelines, which are (1) The overall mesh dimensions are adequate to contain the computed stress field (lower bound) or velocity/plastic field (upper bound); and (2) There must be an adequate concentration of elements within critical regions. More detailed information of finite element limit analysis methods and their applications can be found in Lyamin and Sloan (2002a, 2002b), Krabbenhoft et al. (2005) and Li et al. (2009, 2010). It should be noted that the final element mesh arrangements (both upper and lower bound) are selected only after considerable refinements had been made. Additionally, it should be noted that tension crack has not been considered in this study. This is because tension crack requires time to develop and this study is only concerned with the immediate stability of the fill slope after construction.

3.4 RESULTS AND DISCUSSION

Due to similar obtained trends in the stability charts for various slope inclinations and \( L/H \) ratios, the stability charts in Fig. 3-3 - Fig. 3-5 only present the solutions for \( \beta = 15^\circ, 45^\circ \) and \( 75^\circ \) with \( L/H = 1, 3 \) and 5. For simplicity and demonstration purposes, only lower bound numerical solutions from the 3D analyses are shown (the full results, including upper bound results, can be found in the tables in appendix). Additionally, lower bound solutions are more conservative and thus the results can be utilized in design directly. By using upper and lower bound limit analysis methods, the failure loads of the slope stability analyses can be bracketed to within ± 10%.

Fig. 3-3 - Fig. 3-7 show that the stability number \( (N_{2c}) \) increases with an increase in \( c_u^1/c_u^2 \) ratio, slope angle or \( L/H \) ratio. In other words, the safety factor of a slope is reduced when the above parameters increases. For instance, for a given \( c_u^1 \), the increase in strength ratio of upper layer material over the bottom layer \( (c_u^1/c_u^2) \) indicates a decrease in \( c_u^2 \), which leads to a reduction in the safety of the slope. It is also significant to note that when the \( c_u^1/c_u^2 \) ratio is below 1, the change in the \( c_u^1/c_u^2 \) ratio or \( d/H \) is observed to have no effect on the stability numbers especially for \( \beta \geq 45^\circ \). In contrast, for \( c_u^1/c_u^2 \) ratio above 1, an increase in the stability number as \( d/H \) ratio increases is observed. However, the effect of \( d/H \) is less significant towards larger \( d/H \) as the stability number becomes constant. The results for homogeneous undrained clay obtained by Li et al. (2009) have also
been shown in Fig. 3-3. As expected, the homogeneous undrained clay results fit perfectly in between $c_{u1}/c_{u2} = 0.8$ and $c_{u1}/c_{u2} = 1.25$. This is because homogeneous undrained clay simply means $c_{u1}/c_{u2} = 1$.

Fig. 3-6 shows the results obtained from the 2D finite element limit analysis by Qian et al. (2014) plotted against different $L/H$ from 3D analyses. These figures clearly show the effects of 3D boundary on slope stability. Particularly, it can be seen that as $L/H$ increases, the stability number increases towards plane strain condition. It is worthwhile to note that when $\beta = 75^\circ$ (Fig. 3-6 (c)), the results from 2D LEM (Qian et al. 2014) are observed to be lower than the results from 3D lower bound method for $L/H = 10$ ($d/H<4$). This finding implies that 2D LEM could overestimate the predicted safety factor which can give an inaccurate indication of slope safety.

The effects of $d/H$ and $c_{u1}/c_{u2}$ ratios on $N_{2c}$ are also observed to be more apparent when $L/H$ is higher (Fig. 3-3 - Fig. 3-6). For example when $\beta = 45^\circ$ and $d/H = 5$ (Fig. 3-4), the increase in $c_{u1}/c_{u2}$ ratio from 2 to 5 results in an increase of 43% and 48% in stability numbers for $L/H = 3$ and $L/H = 5$, respectively. In Fig. 3-6 (b), the increase of $d/H = 1.5$ to $d/H = 5$ results in an increase of 32% in stability number for 2D analysis (LB) while the stability numbers of $L/H = 1$ are seen to be constant. However, the effects observed above are less apparent when $c_{u1}/c_{u2}$ ratio $\leq 0.8$, especially for $\beta \geq 45^\circ$ where the stability numbers are constant regardless of the change in $d/H$ or $c_{u1}/c_{u2}$ ratio. Fig. 3-7 shows that $N_{2c}$ increases when the slope becomes steeper (increase in $\beta$). As expected, steeper slopes are less safe, hence resulting in higher stability number.

Fig. 3-8 shows the results presented in the form of $F_{3D}/F_{2D}$ ratio for $c_{u1}/c_{u2} = 5$. In Fig. 3-8, as $L/H$ increases, the ratio of $F_{3D}/F_{2D}$ decreases towards unity. This is because a slope with lower $L/H$ (higher boundary restrictions) yields a higher factor of safety compared to that of higher $L/H$. In fact, when $L/H = 10$, the factors of safety from 3D analyses appears to be only a maximum of 20% more than those from 2D analyses. Hence, this implies that 2D solutions could be used directly due to insignificant 3D effects. From Fig. 3-8, it can also be observed that the ratio of $F_{3D}/F_{2D}$ is higher for lower slope angles. This means that 3D effect is less significant for steeper slopes. This could be due to the fact that gentler slopes
have deeper and larger failure mechanisms, hence the additional resistance
provided by the 3D boundary results in a higher $F_{3D}/F_{2D}$ ratio. Additionally, the
figures also show that the $F_{3D}/F_{2D}$ ratios are higher when $d/H$ is higher. This
finding is in good agreement with the results presented by Li et al. (2009).

Fig. 3-9 - Fig. 3-11 show the upper bound plastic zones obtained from this study.
These plastic zones can be seen as the failure mechanisms of the slopes. The
numbers next to the colour scale in the figures represent the plastic strain rates.
The plastic zones can generally be divided into two categories: $c_{u1}/c_{u2} \geq 1.25$ and
$c_{u1}/c_{u2} \leq 0.8$. Further details and discussions are given below.

### 3.4.1 Failure mechanisms for $c_{u1}/c_{u2} \leq 0.8$

The plastic zones from this study show that the failure mechanisms are closer to a
toe failure mode when $c_{u1}/c_{u2}$ ratio is less than 1. Hence, the constant stability
number obtained regardless of the $d/H$. Further detailed observation reveals that
the failure mechanisms are unaffected when $\beta = 45^\circ - 75^\circ$ regardless of the
change in $d/H$ or $c_{u1}/c_{u2}$ ratio; the failure mechanisms are always in the top layer.
However, it should be noted that $c_{u1}/c_{u2}$ ratio does influence the failure
mechanisms and stability number for $\beta = 15^\circ$ and $30^\circ$ slopes. For example, as
shown in Fig. 3-9, for $\beta = 30^\circ$, a comparison between $c_{u1}/c_{u2} = 0.8$ and $c_{u1}/c_{u2} = 0.2$
shows that the plastic zones are shallower when $c_{u1}/c_{u2} = 0.2$. Additionally, Fig.
3-9 reveals that $L/H$ ratios also affect the failure mechanisms. For example, when
$c_{u1}/c_{u2} = 0.8$, the plastic zones when $L/H = 5$ are deeper than that of $L/H = 2$. It can
be seen in Fig. 3-9 that the former one touches the bottom rigid layer.

### 3.4.2 Failure mechanisms for $c_{u1}/c_{u2} \geq 1.25$

Fig. 3-10 shows the plastic zones considering various $c_{u1}/c_{u2}$ ratios for $\beta = 30^\circ$. It
can be seen that as the $c_{u1}/c_{u2}$ ratio decreases, the plastic zones become smaller.
Specifically, this means that as the bottom layer becomes stronger relative to the
top layer, the failure mechanisms get smaller (i.e. for a given $c_{u1}$ and increasing
$c_{u2}$). This effect also explains the results where the stability numbers decreases
when $c_{u1}/c_{u2}$ ratio decreases. Fig. 3-11 shows that the depth of the failure
mechanisms will also increase with increasing $d/H$ ratio if the zones are restricted
by the bottom rigid layer for smaller $d/H$ ratio.
3.5 ASSUMED APPLICATION EXAMPLES

A case study is used to demonstrate the convenience of using the stability charts, consisting of a fill slope with the following parameters: \( c_u/c_d = 4.5 \), a soil unit weight of \( \gamma = 18\text{kN/m}^3 \), a depth factor of \( d/H = 2 \), and a fill material undrained shear strength of \( c_u = 50\text{kN/m}^2 \).

Example 1 – Determine the ideal slope angle with \( H = 5\text{m} \)

From the information above and using the stability numbers in the appendix, the results of \( F\) corresponding to \( \beta \) are obtained and tabulated in Table 3-1. Based on a desired \( F = 1.1 \), for 2D evaluations it can be seen that the maximum \( \beta \) for design would be 30°. However, when 3D effects are considered, the maximum \( \beta \) for design is 75° and 45° with \( L/H = 5 \) and \( L/H = 10 \), respectively. This again shows that the 3D effects are less significant in a slope with \( L/H = 10 \) and above. Additionally, for the case of \( \beta = 15° \), the factor of safety reaches a maximum of 3.56 for \( L/H = 1 \) while a mere 1.31 is obtained using a 2D analysis. That is a staggering 172% increase in factor of safety when a 3D analysis is used instead of the traditional 2D analysis. Therefore, this investigation clearly shows that when 3D conditions are part of a slope’s geometry, a steeper slope can be designed thus reducing the base area required. Effectively this translates into less land area required for construction of the slopes, which directly reduces project cost.

Example 2 – Determine the ideal slope height with \( \beta = 30° \)

The same slope parameters from the previous example are used in this example. The design height is now required to be calculated with \( \beta \) readily given. From Table 3-2, and again based on a design \( F = 1.1 \), it can clearly be seen that the ideal design height using a 2D analysis is \( H < 5\text{m} \). However, when 3D effects are taken into consideration, the maximum \( H \) that can be designed is \( H \leq 12\text{m} \) when \( L/H = 1 \). That is double the ideal design height using a 2D analysis. From the table it can clearly be seen that as \( L/H \) decreases, the allowable design height increases. From these two examples, it can clearly be concluded that using a 3D analysis for slope design where 3D conditions prevail will allow for a design with higher slope height or steeper slope angle.

3.6 CONCLUSIONS
This study uses the finite element upper and lower bound limit analysis methods to investigate the stability of slopes specifically for two-layered undrained clay slopes. The results obtained are bracketed to within ± 10 % or better. In fact, this study has shown that as slope 3D boundary \((L/H)\) increases, the stability number increases, hence a slope with lower safety. Furthermore, \(c_{u1}/c_{u2}\) ratio is also found to have an effect on the stability number. For instance, when \(c_{u1}/c_{u2} \geq 1.25\), the stability of the slope is found to reduce with an increase in the \(c_{u1}/c_{u2}\) ratio (increase in stability number). This is further supported by observing the plastic zones of the slopes. As \(c_{u1}/c_{u2}\) ratio increases, the plastic zones become larger and deeper. It is also significant to note that the plastic zones are also controlled by the 3D boundaries of the slope \((L/H)\). From observations, larger \(L/H\) generally results in larger and deeper plastic zones. However, when \(c_{u1}/c_{u2}\) ratio is \(\leq 0.8\), steep slopes \((\beta \geq 45^\circ)\) yield a toe failure mode and therefore stability numbers maintain as a constant regardless the change of \(c_{u1}/c_{u2}\) ratio. The same cannot be said for gentle slopes \((\beta = 15^\circ \text{ and } 30^\circ)\). For these slopes, the plastic zones can be observed to be of a toe or base failure depending on the \(c_{u1}/c_{u2}\) ratio when the ratio is \(\leq 0.8\). Having said that, it is important to note that more thorough investigations would be dedicated towards future work to better investigate the different boundary types and its influence on fill slope stability.
3.7 REFERENCES


### 3.8 TABLES

#### Table 3-1 Values of $F$ for Example 1

<table>
<thead>
<tr>
<th>$L/H$</th>
<th>$\beta = 15^\circ$</th>
<th>$\beta = 30^\circ$</th>
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<th>$\beta = 60^\circ$</th>
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<td>$N_{2c}$ (LB) $F$</td>
<td>$N_{2c}$ (LB) $F$</td>
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#### Table 3-2 Values of $H$ for Example 2

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3.9 FIGURES

(a) 2D slope geometry

(b) 3D slope geometry

Fig. 3-1. Problem Configuration
Fig. 3-2. Typical Three-Dimensional Mesh Configuration
Fig. 3-3. Lower bound solutions for $\beta = 15^\circ$
Fig. 3-4. Lower bound solutions $\beta = 45^\circ$
Slope Stability Assessment of Layered Soil

Fig. 3-5. Lower bound solutions $\beta = 75^\circ$
Fig. 3-6. 2D and 3D results for various $\beta$ and $c_u/c_{u2}$
Fig. 3-7. Stability number for $c_{u1}/c_{u2} = 5$ for various slope angle
Fig. 3-8. $F_{3D}/F_{2D}$ for $c_{u1}/c_{u2} = 5$ and various $\beta$
Slope Stability Assessment of Layered Soil

(a) $c_{u1}/c_{u2} = 0.8$ and $L/H = 2$

(b) $c_{u1}/c_{u2} = 0.2$ and $L/H = 2$

(c) $c_{u1}/c_{u2} = 0.8$ and $L/H = 5$

(d) $c_{u1}/c_{u2} = 0.2$ and $L/H = 5$

Fig. 3-9. Upper bound plastic zones for $\beta = 30^\circ$ and $d/H = 2$ for various $c_{u1}/c_{u2}$ and $L/H$ ratio.
Slope Stability Assessment of Layered Soil

Fig. 3-10. Upper bound plastic zones for $\beta = 30^\circ$; $L/H = 3$ and $d/H = 5$ for various $c_{u1}/c_{u2}$ ratio.
Fig. 3-11. Upper bound plastic zones for $\beta = 45^\circ$; $L/H = 5$, $c_{u1}/c_{u2} = 3$. 

(a) $d/H = 2$  

(b) $d/H = 5$
Chapter 4 THREE-DIMENSIONAL SLOPE STABILITY CHARTS FOR FRICTIONAL FILL MATERIALS PLACED ON PURELY COHESIVE CLAY

K. Lim, A. V. Lyamin, M. J. Cassidy and A. J. Li

4.1 INTRODUCTION

Slope stability is a common problem in geotechnical engineering and thus has drawn the attention of many investigators for decades (Morgenstern and Price 1965; Azzouz and Baligh 1978; Leshchinsky and Baker 1986; Gens et al. 1988; Michalowski 1995; Jiang and Magnan 1997; Chang 2002; Li et al. 2010; Gao et al. 2012b; Sun and Zhao 2013; Zhang et al. 2013). In general, slope stability is influenced by the slope’s physical properties, the strength parameters of the soil and the slope geometry. In addition, the construction approach also affects the slope’s soil profile. In general, a slope can be classified as a cut slope, natural slope or fill slope based on the method of construction. Fill slopes are commonly encountered in the construction of embankments (Indraratna et al. 1992; Al-Homoud et al. 1997).

The limit equilibrium method (LEM) is one of the more conventional methods for assessing slope stability due to its simplicity. However, this method has limitations, such as assumptions regarding the inter-slice shear force and critical slip surface (Duncan 1996; Krahn 2003). Additionally, although slope stability problems can be analysed using two- or three-dimensional (3D) methods, it is found that utilising a two-dimensional (2D) analysis on slopes where 3D conditions prevail will lead to an underestimation of the slope’s degree of safety (Baligh and Azzouz 1975; Azzouz and Baligh 1978; Ugai 1985). This demonstrates the significance of accounting for 3D effects in slope stability analyses.

Slope stability charts were first produced by Taylor (1937) for purely cohesive soils. These stability charts have the advantage of being able to be used as tools for quick preliminary slope design. Stability chart solutions have been presented by several investigators (Gens et al. 1988; Kim et al. 1999; Lane and Griffiths...
2000; Michalowski 2002; Baker et al. 2006; Li et al. 2009, 2010; Shiau et al. 2011; Li et al. 2014; Qian et al. 2014). Recent studies by Michalowski (2002), Kumar and Samui (2006) and Viratjand and Michalowski (2006), have, respectively investigated homogeneous undrained and drained slopes, layered slopes with cohesive-frictional materials and submerged slopes subjected to water drawdown. However, the above studies only used an upper bound analysis. As is well-known, upper bound solutions are not conservative, and thus engineers must be cautious with their application.

It is worthwhile to note that the chart solutions produced by Kumar and Samui (2006) considered only cohesive-frictional materials for the soil in the layered slopes. This indicates that chart solutions for slopes with frictional fill materials placed on purely cohesive clay are not available. Qian et al. (2014) also developed slope stability charts for two-layered, purely cohesive soils. Their study contained a conventional LEM analysis as well as finite element upper and lower bound limit analyses; however, the study did not investigate 3D effects.

From the review of previous studies, it is apparent that existing chart solutions cannot be applied to certain types of slopes. For example, there is no stability chart for fill slopes where frictional fill materials are placed on purely cohesive clay. In this study, the finite element lower bound limit analysis method (Lyamin 2002a) is employed to investigate 2D and 3D fill slopes where frictional fill materials are placed on purely cohesive clay, which is a slope type that is commonly encountered in the embankment construction. As highlighted by Sloan (2013), the numerical limit analysis methods can be applied to various geotechnical problems and this is proven by the number of studies performed to date (Yu and Sloan 1997; Yu et al. 1998; Kim et al. 1999; Loukidis et al. 2003; Shiau et al. 2003; Sutcliffe et al. 2004; Merifield et al. 2006; Li et al. 2008, 2009, 2010, 2011). The results obtained in this study are also presented in the form of stability charts. It should be noted that the numerical lower bound method can provide conservative solutions that can be directly utilised in practice for preliminary designs.

4.2 PREVIOUS STUDIES
Several previous investigators have focused on addressing embankment stability problems (Almeida et al. 1986; Low 1989; Jiang and Magnan 1997), some accounting for soil reinforcements in embankments (Low et al. 1990; Kaniraj 1994; Long et al. 1996). Although other analysis methods have also been developed, the majority of the above embankment stability studies were based on LEM because of its simplicity.

The main benefit of LEM is that it can be used to determine the slope stability in terms of the factor of safety. Slope stability assessments were originally 2D analyses; however, it was later discovered that the assessments of slopes with significant 3D geometries should include the physical boundaries (Cavounidis 1987; Gens et al. 1988; Leshchinsky and Huang 1992b) because the results (factor of safety) from 2D analyses are more conservative than those from 3D analyses. In general, LEM—which utilises a moment or force equilibrium to assess the slope stability—has been the preferred analysis method for many researchers (Bishop 1955; Morgenstern and Price 1965; Spencer 1967; Fredlund and Krahn 1977; Leshchinsky and Huang 1992a). It is well known that certain assumptions must be made and that if those assumptions are valid, an LEM analysis is adequate. These limitations and assumptions include the inter-slice force, the satisfaction of the moment or force equilibrium and the initial slip surface of the slope stability investigation.

One of the most notable contributions to the LEM literature was Spencer (1967), who used LEM to investigate embankment problems; his study accounted for the parallel inter-slice shear forces. In addition, Low (1989) used his own derived semi-analytical method to analyse embankment stability. The results obtained are quite close to those obtained from the ordinary method of slices (Fellenius 1936). Additionally, Jiang and Magnan (1997) compared the results from the limit analysis method with LEM solutions and found good agreement for purely cohesive soil. However, the factor of safety for embankment soil with a low friction angle obtained by Jiang and Magnan (1997) using the limit analysis method was slightly lower than that obtained by Low (1989) with the ordinary method of slices.
The finite element method (FEM) is widely employed to capture the displacement and movement of soil within a slope (Potts et al. 1997; Troncone 2005). Many investigators have utilised FEM for slope stability analyses, such as Griffiths and Lane (1999), Lane and Griffiths (2000), Manzari and Nour (2000) and Zheng et al. (2005). FEM is also able to capture progressive failure in a slope. Griffiths and Lane (1999) adopted the FEM strength reduction method to investigate the stability of an embankment in which both the fill and foundation layer consist of purely cohesive clay. Their results indicate that FEM is suitable for embankment stability analyses.

Chai and Carter (2009) and Chai et al. (2013) also utilised FEM to analyse embankment failures. In particular, Chai and Carter (2009) investigated and discussed the progressive failure and soil softening effects, stating that they would be significant in slope stability assessments. It is, however, unfortunate that no comprehensive slope stability charts were provided.

Limit analysis methods for slope stability assessments have also been developed in the past few decades. The upper bound method is fundamentally based on kinematically admissible velocity fields, just as the lower bound method is based on statically admissible stress fields. Based on the limit theorems, Michalowski (1995), Donald and Chen (1997), Chen et al. (2003) and Chen et al. (2004) have utilised the upper bound method to investigate various slope stability problems. However, because only the upper bound method is used in their studies, the solutions are not conservative. As mentioned previously, Kumar and Samui (2006) also investigated the stability of layered soil slopes using the limit analysis method and proposed chart solutions for cohesive-frictional soils based on only the upper bound method. Considering the brief summary above, it can be observed that the currently developed slope stability methods are capable of solving several slope stability problems, including embankment cases. However, the authors are not aware of any slope stability charts that have been produced based on the lower bound method for fill slopes where frictional materials are placed on purely cohesive clay.

Despite the number of studies and investigations of slope stability that have been published, the slope stability results generated with a 2D analysis are conservative.
in terms of the factor of safety. In fact, as noted by Gens et al. (1988), the obtained shear strength parameters would then be overestimated. Furthermore, the authors have also discovered that neglecting edge effects could produce differences up to 30% in magnitude. This argument is further supported by Cavounidis (1987) who investigated the ratio of the factors of safety and determined that the factor of safety from a 3D analysis is always higher than that of 2D analysis, producing a ratio that is larger than unity. Many studies utilising LEM and considering 3D effects have been done (Leshchinsky and Baker 1986; Hungr 1987; Huang et al. 2002; Xie et al. 2006). However, it is worthwhile to note that these studies are an extension of the pre-existing 2D analysis method; therefore, the inherent limitations of LEM are still present.

Several authors, such as Michalowski (1989), Chen et al. (2001a), Chen et al. (2001b), Farzaneh and Askari (2003), Chen et al. (2005) and Michalowski (2010) have also used the limit theorems to investigate slope stability while incorporating 3D effects. In particular, Michalowski (2010) shows that a 3D analysis yields a higher factor of safety than a plane strain analysis. In addition, the studies by Griffiths and Marquez (2007) and Wei et al. (2009) used FEM and included 3D effects, such as boundary conditions, geometry and external loading.

Recently, finite element upper and lower bound limit analysis methods have been applied to various 3D slope stability problems by Li et al. (2009, 2010) and Li et al. (2014). The results again demonstrated that the factor of safety obtained with a 3D analysis is higher than that obtained from a 2D analysis. Additionally, their findings were presented in the form of stability charts. From the above discussion, the significance of 3D effects and convenience of stability charts can be observed. Therefore, the purpose of this study is to utilise the finite element lower bound limit analysis method to investigate 2D and 3D fill slopes. The fill slopes are composed by placing frictional material on purely cohesive clay. The results will be presented in the form of stability charts.

4.3 PROBLEM DEFINITION

Fig. 4-1 shows the typical problem configuration and boundary conditions for the slope studied in this chapter. To study the effect of a 3D analysis, the typical 2D configuration (x-z plane) will be extended by a distance of $L/2$ (the extension is
done symmetrically to reduce the size of the mesh) in the y-axis direction. A range of $L/H$ from 1 to 5 is investigated in this chapter.

This study will investigate the stability of fill slopes where frictional materials are placed on purely cohesive clay. Therefore, as shown in Fig. 4-1(a), the frictional fill materials have been assigned to Region 1 and the pre-existing soil (purely cohesive clay) has been assigned to Region 2. The frictional materials in this study will have a range of friction angles that are typically encountered in practice, $\phi' = 15^\circ - 45^\circ$, which covers the majority of frictional soils (Navy 1982). The pre-existing soil (purely cohesive clay) has an undrained strength of $c_u$. A range of different slope angles, $\beta$, and depth factors, $d/H$, are also considered in this study.

The mesh generation for the numerical lower bound limit analysis modelling must follow two important guidelines: (1) The overall mesh dimensions must be adequate to contain the computed stress field; and (2) There must be an adequate concentration of elements within the critical region. The final finite element mesh was selected only after considerable refinements had been made. A typical finite element mesh is shown in Fig. 4-2. Based on the study by Bouassida et al. (2014), the plastic points obtained from the numerical lower bound method could reveal potential failure mechanisms. These points will be adopted in this study to discuss the potential slope failure mode.

In this study, the results will be presented utilising the dimensionless stability number shown in Eq. (4-1).

$$N_{sc} = \frac{c_u}{\gamma HF}$$  \hspace{1cm} (4-1)

where $c_u$ is the undrained shear strength in Region 2 (Fig. 4-1(a)), $\gamma$ is the unit weight of the soil and $H$ and $F$ are the slope height and factor of safety, respectively. In this study, the unit weight of sand is assumed to be the same as that of clay. It should be noted that the $F$ obtained using Eq. (4-1) will be different from the conventional $F$ calculated with LEM due to the form of the stability number in the equation. This pattern was also discovered and highlighted in the study by Li et al. (2012). Although the friction angle ($\phi'$) is not included in Eq.
The chart solutions will be generated by considering various $\phi'$. In addition, the slope failure mode is found to be a base failure. This indicates that the clay material is always involved in the failure of the fill slope.

In the numerical limit analyses, for a given slope geometry and soil strength, the optimised solutions of the lower bound (LB) program can be determined as a function of the unit weight ($\gamma$). It should be noted that it is not necessary to determine the failure mechanism prior to computing any numerical LB results. In addition, the form of Eq. (4-1) is the same as the stability number adopted by Taylor (1948). However, the obtained stability numbers are completely different from those obtained by Taylor because the solutions in this study include the effects from top layer (fill).

4.4 RESULTS AND DISCUSSION

The obtained results are presented in Fig. 4-3 - Fig. 4-6. These figures display the results from 3D and 2D analyses for the various slope angles ($\beta$), friction angles ($\phi'$) and depth factors ($d/H$) considered in this study. The results have been presented using the dimensionless stability number defined in Eq. (4-1). A range of $L/H$ ($L/H = 1 – 5$) was taken into account for the 3D analyses, and various slope angles ($\beta = 15^\circ, 22.5^\circ, 30^\circ$ and $40^\circ$) were investigated for both the 2D and 3D analyses. As mentioned previously, the results are calculated as a function of the unit weight ($\gamma$). Therefore, a larger stability number implies that the slope is less stable for a given condition.

There is an obvious trend that can be observed in all of the plots in Fig. 4-3 through Fig. 4-6. Specifically, the stability number increases as $L/H$ increases; hence, the slope would be less safe due to the increased stability number. On the other hand, the stability number increases with geometries that are more similar to plane strain conditions (2D). This indicates that a 2D analysis of slopes will underestimate the factor of safety of the slope if 3D conditions prevail. In fact, the reduction of boundary effects and restrictions produces higher stability numbers and, hence, less safe slopes as they tend to plane strain conditions.

A distinct trend can also be observed for the change of $d/H$: the stability number increases as $d/H$ increases. However, it should be noted that this trend is more
obvious for higher values of $L/H$. Regardless of the change in $d/H$, the stability numbers are almost constant for low values of $L/H$. Additionally, it was found that the variation in the stability number with $d/H$ is much more apparent for higher friction angles. For example, this effect can be observed by comparing $\phi' = 45^\circ$ and $15^\circ$ for $\beta = 15^\circ$ and $L/H = 5$. For $\phi' = 45^\circ$, the stability number increases by up to 35.5% as $d/H$ changes from 1.5 to 5. However, this increment is approximately 10% for $\phi' = 15^\circ$.

One of the conventional limit equilibrium methods, Bishop’s simplified method (Bishop 1955), is also employed in this study for comparison purposes, and the results are also shown in Fig. 4-3 - Fig. 4-6. It can be observed from Fig. 4-3 - Fig. 4-6 that the results of LEM can fall below or above the LB solutions. This implies that an overestimated assessment would be obtained for a slope design. As indicated by Li et al. (2012), the solution obtained from LEM is not rigorous as neither static nor kinematic admissibility conditions are satisfied. Therefore, engineers should design with caution when utilising LEM. In particular, the differences in $N_{sc}$ between LEM and limit analysis solutions are found to be especially significant for lower $\beta$ and $d/H$ (i.e., $\beta = 15^\circ$ and $d/H = 1.5$). The difference can be as large as 23% for $\phi' = 45^\circ$. Fig. 4-7 shows the comparison between different $\phi'$ for varying $\beta$. From the charts, it can be concluded that the stability number decreases as the friction angle increases. This is reasonable because an increase in the friction angle implies an increase in the strength of the material, resulting in a safer slope. Therefore, in practice, a use of soil material with a higher friction angle is very desirable due to its higher strength and resulting higher factor of safety. Fig. 4-7 also reveals that the obtained stability numbers will change as $\phi'$ changes. Thus, the presented chart solutions reflects the change of $\phi'$ although it is not shown in Eq. (4-1). The effects of the slope angle can clearly be observed from Fig. 4-8: as the slope angle increases, the stability number increases. This is valid because an increase in the slope angle results in a more critical slope. Based on Fig. 4-7 and Fig. 4-8, the slope can be designed according to the desired slope and friction angles.

Fig. 4-9 and Fig. 4-10 compare the factor of safety obtained from 3D analysis with that obtained from the 2D analysis. The ratio of $F_{3D}/F_{2D}$ is simply the inverse
ratio of the stability numbers \((N_{sc})_{2D}/(N_{sc})_{3D}\). One of the most notable outcomes that can be observed in these graphs is that the ratio of \(F_{3D}/F_{2D}\) decreases as \(L/H\) increases. This phenomenon is similar to that presented and discussed above when the stability number increases as the geometries approach plane strain conditions. Through these \(F_{3D}/F_{2D}\) ratios, the effect of \(L/H\) can be couched in a framework that is more common in slope stability, viz. the factor of safety rather than the stability number. The argument from Cavounidis (1987) that the factor of safety from a 3D analysis will never be less than that from a 2D analysis is supported.

In Fig. 4-9 and Fig. 4-10, another similar trend to that observed in the 3D analysis above can be observed: \(F_{3D}/F_{2D}\) increases as \(d/H\) increases. This finding clearly indicates that the 3D effect is much higher for larger values of \(d/H\). A similar trend was also observed in the study of Li et al. (2009) for homogeneous purely cohesive clay slopes. To explain this further, a detailed examination of Fig. 4-11 is required. Fig. 4-11 shows the 2D and 3D lower bound plastic points for \(\beta = 22.5^\circ\) and \(\phi' = 35^\circ\). It can be observed from the figure that the plastic points of the 2D analysis are much wider and deeper than those from the 3D analysis. This clearly demonstrates the influence of the end effects of the 3D boundary. Thus, the increased \(F_{3D}/F_{2D}\) ratio for larger \(d/H\) is likely due to the difference in the depth of the plastic points. In fact, the plastic points from both analyses (2D and 3D) clearly indicate base failures for the slopes.

It is also important to note that careful observation reveals an increase in \(F_{3D}/F_{2D}\) as the friction angle increases. In fact, \(F_{3D}/F_{2D}\) is as large as 7 or 6 for the cases where \(\beta = 15^\circ\), Fig. 4-9(a), or \(\beta = 22.5^\circ\), Fig. 4-9(b), when \(\phi' = 45^\circ\). However, the ratio then drops to approximately 3.3 when \(\beta = 22.5^\circ\) and \(\phi' = 25^\circ\), as observed in Fig. 4-10(a). Therefore, from these observations, it can be concluded that a larger difference between the slope angle, \(\beta\), and friction angle, \(\phi'\), will produce a higher \(F_{3D}/F_{2D}\) ratio.

### 4.5 APPLICATION EXAMPLES

A case study will be used to demonstrate the convenience of using the stability charts, consisting of a fill slope with the following parameters: \(\beta = 22.5^\circ\), a sand layer friction angle of \(\phi' = 35^\circ\), a clay layer cohesion of \(c_u = 20\) kN/m², a soil unit
weight of $\gamma = 18\text{kN/m}^3$, a slope height of $H = 5\text{ m}$ and a depth factor of $d/H = 2$. Based on the given parameters, the chart solutions (Fig. 4-4(c)) yield a stability number of 0.159 for a 2D analysis. Hence, using this obtained stability number and the above parameters, the factor of safety is calculated to be 1.4 (2D analysis), as shown in Table 4-1. If 3D effects are taken into consideration, the stability numbers obtained from a 3D analysis for $L/H = 1$ and 5 would be 0.038 and 0.123, respectively (Fig. 4-4). With the slope parameters given above, these values yield respective factors of safety of 5.8 and 1.8. Obviously, it can be observed that these obtained factors of safety are clearly higher than those obtained from the 2D analysis. Therefore, this indicates that the obtained factors of safety for cases where 3D conditions prevail are higher and that using a 2D analysis for the slope stability assessment would highly underestimate the factor of safety. It should be noted that the larger stability number (i.e., the 2D solutions) actually yields a smaller factor of safety, $F$. This is because $F$ is part of the denominator of the stability number, as shown in Eq. (4-1). Therefore, a higher stability number will give a lower $F$ for a given set of parameters.

In addition to using the charts to obtain the factor of safety, the following example shows the required slope height ($H$) for the slope design. The same slope parameters from the previous example are used, except that the height of the slope $H$ is now unknown. From Table 4-1, it can be observed that, for 2D analysis, if a factor of safety $F > 1.1$ is desired, the maximum allowable height for the design would be $H \leq 6\text{ m}$. However, if 3D boundary effects are taken into consideration, the allowable height for the design changes to $8\text{ m}$ for $L/H = 5$. For extreme cases where $L/H = 1$ and $L/H = 2$, the respective values for $F$ with $H = 8\text{ m}$ are 3.67 and 1.89. Therefore, a conclusion can be drawn that is similar to the above example: when 3D conditions prevail in a slope, a 3D slope stability assessment produces a more accurate result.

**4.6 CONCLUSIONS**

This study has investigated fill slopes consisting of frictional fill materials placed on purely cohesive clay. The analyses have been conducted using the finite element lower bound limit analysis methods. The results obtained are presented using chart solutions, which are convenient for practicing engineers. For
comparison purposes, results from LEM have also been shown. It was found that the results obtained from a 2D LEM would overestimate the results compared to the 2D lower bound analysis. Therefore, engineers should be careful when using results generated with LEM in slope stability assessments.

This study also investigated 3D effects on slope stability analyses. The results from the 3D analysis show an increase in the stability number as $d/H$ or $L/H$ increases. In addition, the effects of the slope and friction angles were also discussed. It was observed that an increase in the slope angle produces an increase in the stability number and that an increase in the friction angle results in a decrease in the stability number.

For comparison purposes, the ratio of $F_{3D}/F_{2D}$ was also presented. One notable outcome is the increase in $F_{3D}/F_{2D}$ as $d/H$ or the friction angle increases or as $L/H$ decreases. In addition to the usual effects of $d/H$, $L/H$ and the friction angle that were discussed, the most significant outcome from this study is that the ratio of the factor of safety ($F_{3D}/F_{2D}$) is always larger than unity. It was shown that as $L/H$ increases, the value of $F_{3D}/F_{2D}$ decreases to unity. In addition, for cases with a low slope angle and a high friction angle (i.e., $\beta = 15^\circ$, $\phi' = 45^\circ$), the factor of safety from the 3D analysis differs from that calculated from the 2D analysis by as much as sevenfold. This clearly shows that for such cases, if a 2D analysis was performed for the slope stability analysis, the obtained results will heavily underestimate the stability of the given slope.
4.7 REFERENCES


Slope Stability Assessment of Layered Soil


Slope Stability Assessment of Layered Soil


### 4.8 TABLES

**Table 4-1 Calculation of $F$ for different $L/H$ and $H$**

<table>
<thead>
<tr>
<th>$L/H$</th>
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<th>$H$ (m)</th>
<th>$F$</th>
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4.9 FIGURES

(a) Typical 2D problem definition

(b) 3D configuration and boundary conditions

Fig. 4-1. Problem configuration and boundary conditions
Fig. 4-2. 3D mesh configuration
Fig. 4-3. 3D and 2D analysis results for $\beta = 15^\circ$
Fig. 4-4. 3D and 2D analysis results for $\beta = 22.5^\circ$.
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Fig. 4-5. 3D and 2D analysis results for $\beta = 30^\circ$

Fig. 4-6. 3D and 2D analysis results for $\beta = 40^\circ; \phi' = 45^\circ$
Fig. 4-7. Comparison of friction angle for various $\beta$
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Fig. 4-11. Plastic points for $\beta = 22.5^\circ$; $\phi' = 35^\circ$; $d/H = 5$
Chapter 5 STABILITY CHARTS FOR FILL SLOPES BASED ON FINITE ELEMENT LIMIT ANALYSIS METHOD

K. Lim, A. J. Li, A. V. Lyamin and M. J. Cassidy

5.1 INTRODUCTION

Traditionally, slope stability analyses have generally been performed using a two-dimensional (2D) analysis. However, the studies by Baligh and Azzouz (1975) and Leshchinsky and Huang (1992) showed that the physical boundaries of a slope do influence the end results. Similarly Gens et al. (1988) showed that the factor of safety obtained from a 3D analysis is always higher than that of a 2D analysis. In fact, Gens et al. (1988) discovered that the difference between the factor of safety obtained from a 3D analysis and that from a 2D analysis can be as much as 30%. Additionally, Cavounidis (1987) suggested that the factor of safety obtained from a 3D analysis will always be larger than that of a 2D analysis. Therefore, using a 2D analysis to deal with a 3D slope problem will produce results that are conservative and underestimated and may lead to unnecessary slope reinforcement. Hence, a slope that is restricted by 3D geometry should be designed and analysed using a 3D analysis.

Apart from the above mentioned advancement in slope stability, chart solutions have also been proposed to provide quick first estimates of slope stability. For instance, Taylor (1937) who investigated undrained clay slopes was the first investigator to develop chart solutions for slope stability assessments. Since then, stability charts have been produced for various slope stability problems because of the convenience of the chart solutions for preliminary design (Yu et al. 1998; Michalowski 2002; Li et al. 2008, 2009, 2010; Michalowski 2010). Some of the chart solutions that considered 3D effects are (Taylor 1937; Gens et al. 1988; Michalowski 2002; Kumar and Samui 2006; Michalowski and Martel 2011; Michalowski and Nadukuru 2012; Sun and Zhao 2013) while more recent chart solutions that have been produced include (Gao et al. 2012b; Gaopeng et al. 2014; Qian et al. 2014; Lim et al. 2015a; Lim et al. 2015b; Tang et al. 2015). However,
investigations and convenient solutions on the slope in the present study (i.e. fill slopes) are limited.

Further to the methods mentioned above, in recent years, the finite element limit analysis methods (Lyamin and Sloan 2002a, 2002b; Krabbenhoft et al. 2005) have also been utilized to solve slope stability problems. In fact, Sloan (2013) highlighted that the methods can be applied across various geotechnical engineering problems. Particularly, the studies by Li et al. (2009, 2010) showed that the methods are suitable for 3D slope stability analysis.

Therefore, this chapter aims to utilise the lower bound finite element limit analysis method developed by Lyamin and Sloan (2002a) to perform a 3D slope stability analysis and produce a set of stability charts for fill slopes. More specifically, slopes with frictional fill materials placed on undrained clay with increasing strength. For comparison purposes, the results based on 2D analyses are also presented. It should be noted that the numerical lower bound method provides conservative solutions that can be directly used in practice for preliminary designs. The novelty of this chapter can be summarized as follows:

- Better understanding of slopes with frictional fill materials placed on undrained clay with increasing strength is achieved (i.e. failure mechanism).
- Quick 2D and 3D solutions of the slope can be obtained without the need to construct and simulate the slope in numerical software, which may be time-consuming and undesirable during discussions.
- The parameters affecting the stability of this type of slope are investigated, thus enabling engineers to select the ideal material strength or slope geometry during design.

5.2 PREVIOUS STUDIES

Embankment problems are commonly encountered in slope stability investigations and various approaches have been used in those investigations. For instance, Low (1989) derived his own semi-analytical method to analyse embankment stability and obtain close-matched results with those from the ordinary method of slices (Fellenius 1936). Additionally, Spencer (1967) who
used the LEM to investigate embankment problems accounted for parallel inter-
slice shear forces his study. In fact, due to its simplicity, the limit equilibrium
analysis is often used in practice to evaluate slope stability (Bishop 1955;
Morgenstern and Price 1965; Fredlund and Krah 1977; Leshchinsky 1990;
Duncan 1996; Krah 2003). However, it is to be noted that the LEM requires prior
assumptions regarding the inter-slice shear force and the failure surface.

Conventional displacement finite element method (Matsui and San 1992; Potts et
al. 1997; Manzari and Nour 2000; Cala and Flisiak 2003; Hammah et al. 2005)
has also been employed to investigate slope stability problems. Particularly,
Griffiths and Lane (1999) who used the strength reduction method (SRM) showed
that the FEM is also suitable for embankment stability analyses. In fact, the stress
and displacement in the slope can also be computed using a finite element
analysis (Potts et al. 1997; Chai and Carter 2009). However, it is well known that,
for slope stability assessments, the FEM is generally more time consuming than
the LEM and limit analysis, particularly for parametric studies that have to be
done in three-dimensional (3D) models. Therefore, it would be the reason that
there have been no FEM based comprehensive chart solutions produced to date.

In the past few decades, limit analysis methods have also been rigorously applied
to solve slope stability problems (Donald and Chen 1997; Jiang and Magnan
1997; Michalowski 1997, 2002; Chen et al. 2004; Viratjandr and Michalowski
2006). Particularly, Michalowski (2002) also considered pore water pressure
effects and seismic loading in his study. Additionally, Jiang and Magnan (1997)
used the upper bound limit analysis method to analyse embankment stability and
compared the results obtained with LEM solutions. Their results show that the
factors of safety obtained for undrained clay embankment using the different
methods are similar. However, it was shown that, the factor of safety for
embankment soil with a low friction was slightly more conservative using the
limit analysis than that obtained by Low (1989) with the ordinary method of
slices.

5.2.1 3D analysis
In recent years, many slope stability analyses considering 3D effects have been
investigated (Leshchinsky et al. 1985; Chen et al. 2005; Cheng and Yip 2007;
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Griffiths and Marquez 2007; Li et al. 2009, 2010; Michalowski 2010; Michalowski and Nadukuru 2012). Although many LEM based 3D analyses have been performed, most of these studies are extensions of the existing 2D methods (Hungr 1987; Leshchinsky and Huang 1992b; Lam and Fredlund 1993; Huang et al. 2002; Xie et al. 2006). Therefore the inherent limitations of the pre-existing LEM methods remain. Won et al. (2005), Griffiths and Marquez (2007) and Wei et al. (2009) who used the FEM, have also investigated 3D effects in their respective studies. For instance, Griffiths and Marquez (2007) considered inclined sloping side in the third dimension. Wei et al. (2009) indicated that 3D strength reduction method can be very sensitive to the convergence criterion, boundary conditions and the design of mesh.

Chen et al. (2001a), Chen et al. (2001b), Chen et al. (2005) and Michalowski (2010) have all utilized the limit analysis method to perform 3D slope stability analyses. The results presented by Michalowski (2010) verified that the factor of safety obtained from a 3D analysis is higher than that of a 2D analysis. This fact was further supported by Li et al. (2009, 2010) who showed that the ratio of 3D factor of safety over 2D is higher than unity. Hence, the discussion above clearly highlights that various methods have been used to analyse 3D boundary effects and one common conclusion can be drawn from them: a slope restricted by 3D geometry should be investigated using a 3D analysis to avoid underestimation of the slope safety factor.

As mentioned earlier, the finite element upper and lower bound limit analysis methods (Lyamin and Sloan 2002a, 2002b; Krabbenhoft et al. 2005) which were developed recently have been rigorously applied in slope stability analysis (Yu et al. 1998; Loukidis et al. 2003; Li et al. 2009, 2010; Lim et al. 2015b). Additionally, stability charts have also been proposed in all of the studies. It should be noted that Lim et al. (2015b) investigated fill slopes where frictional fill materials are placed on undrained clay. However, they considered only uniform undrained shear strength ($c_u$) for the clay layer (foundation). In fact, for normally consolidated soil, undrained shear strength can increase with depth (Gibson and Morgenstern 1962). Hence, this study will adopt the numerical lower bound limit analysis method (Lyamin and Sloan 2002a) to investigate this type of slope. The results obtained will be presented in the form of stability charts.
5.3 **PROBLEM DEFINITION**

Fig. 5-1 shows the typical slope geometry and boundary conditions for the slope problem considered in this study. The slope is divided into two regions, specifically Region 1 and Region 2. Region 1 is the frictional fill materials (sand) and the soil in Region 2 is undrained clay with increasing strength (pre-existing layer). For Region 1, a range of friction angles are investigated ($\phi' = 25° – 45°$). This would cover most problems of practical interest (Navy 1982). On the other hand the soil in Region 2 has an undrained shear strength ($c_u$) that increases with depth. The undrained shear strength profile can be expressed in Eq. (5-1).

$$c_u(z) = c_{u0} + \rho z$$

(5-1)

where $c_{u0}$ is the undrained shear strength (kPa) at the surface of Region 2 (shown in Fig. 5-1(a)), $\rho$ is the rate of increment in kPa/m (mentioned above) of the undrained shear strength with depth, and $z$ is the depth (m) from the surface of Region 2. It should also be noted that the strength of the soil is assumed to increase linearly with depth as is the case in normally consolidated clays (Gibson and Morgenstern 1962).

For 3D investigation, the slope geometry in Fig. 5-1(a) has been extended by $(L/2)$ in y-axis direction from the x-z plane. This extension has been done symmetrically to reduce the number of mesh elements required. A range of slope angles ($\beta$), depth factors ($d/H$), and $L/H$ are also considered in this study. The remote vertical boundary in Fig. 5-1(b) is considered as a rough boundary and the symmetry face is a smooth vertical boundary. This follows the study by Lim et al. (2015b). Similar boundary conditions were also adopted in other recent slope stability studies (Chugh 2003; Griffiths and Marquez 2007; Zhang et al. 2013). That said, the influence of different boundary conditions can be found in the literature (Chugh 2003; Zhang et al. 2013), hence will not be investigated herein.

Finally, the mesh generation for the numerical lower bound limit analysis modelling must follow two important guidelines: (1) The overall mesh dimensions must be adequate to contain the computed stress field; and (2) There must be an adequate concentration of elements within critical region. Further detailed information of the lower bound finite element limit analysis method can be found...
in Lyamin and Sloan (2002a). It should also be noted that the final element mesh was selected only after considerable refinements had been made and a typical finite element meshes can be seen Fig. 5-2.

To produce results which are applicable to different input of soil and slope parameters, a dimensionless stability number shown in Eq. (5-2) is utilized.

\[
N_{ci} = \frac{c_{uo}}{\gamma HF}
\]

where \( \gamma \) is the unit weight of the soil and \( H \) and \( F \) are the slope height and factor of safety, respectively. For simplicity, the unit weight of soil in Region 1 and 2 are assumed to be the same in this study. Although the stability number in Eq. (5-2) has similar form to that adopted by Taylor (1937), the definition of the numbers are entirely different because this study includes the effects from the frictional materials in Region 1. In addition, the undrained shear strength considered in Taylor’s chart solutions was constant.

As highlighted by Li et al. (2012), \( F \) obtained using Eq. (5-2) can differ from the conventionally used \( F \) obtained from a limit equilibrium analysis or finite element analysis. It is due to different definition of \( F \) adopted in various methods. In this study, stability charts are presented by considering various friction angle (\( \phi' \)), therefore the parameter (\( \phi' \)) is not included in Eq. (5-2). A base failure mode is found in this study which indicates that the clay layer is always encompassed in the failure of the fill slope.

5.4 RESULTS AND DISCUSSION

This study investigated the stability of slopes with frictional fill materials placed on undrained clay with increasing strength and the results obtained are shown in Fig. 5-3 - Fig. 5-6. The results include both 2D and 3D analyses for various slope angle (\( \beta \)), friction angle (\( \phi' \)) and \( \rho \). A range of \( L/H \) ratios (\( L/H = 1 \text{ – } 5 \)) were investigated to consider 3D boundary effects. Although depth factors of \( d/H = 1.5 \text{ – } 5 \) were investigated, only the results for \( d/H = 1.5 \text{ and } 2 \) have been shown due to the constant results for most of the cases when \( d/H \geq 2 \). Although Yu et al. (1998) and Li et al. (2009, 2010) adopted different soil profiles, their studies also reveal
that the stability numbers are almost unchanged for $d/H \geq 2$ if the soil undrained shear strength in Region 2 increases with depth.

In this study, slope angles of $\beta = 15^\circ$, $22.5^\circ$, $30^\circ$ and $40^\circ$ were also investigated. As previously mentioned, a range of friction angles were also studied for each slope angle. Therefore, the stability number can be observed according to the desired known slope and friction angles. It is also essential to note that only friction angles of $\phi' > \beta$ were investigated due to the rules of repose angle of frictional materials where $F \leq 1$ will be deemed unsafe and hence would not be of interest to design engineers.

From Fig. 5-3 - Fig. 5-6, it can be observed that the stability number decreases as $\rho$ increases. As expected, for a given $c_{ud0}$, higher $\rho$ implies higher strength in the undrained clay layer, thus increasing the safety of the slope (lower stability number). This trend exists regardless of the change in friction angle, slope angle or $L/H$ ratio. In fact, this effect is more apparent with geometries closer to plane strain conditions (2D). For example in Fig. 5-3(b), for the case of $\beta = 15^\circ$, $d/H = 2$ and $\phi' = 45^\circ$, the stability number reduction ($\rho = 0.25$ kPa/m to $\rho = 1$ kPa/m) can be as much as 30% and 12% for plane strain condition and $L/H = 1$ respectively. This shows that $\rho$ has less influence when a slope is highly restricted by the 3D geometries ($L/H = 1$).

The results from this study also show the effects of $L/H$ ratio (i.e. 3D effects). It can be observed from Fig. 5-3 - Fig. 5-6 that the stability number increases as $L/H$ increases. This is because as $L/H$ gets larger, the slope becomes less safe (increase in stability number) due to the smaller 3D boundary effects. Therefore this shows that an underestimated factor of safety would be obtained by using a 2D analysis for slopes highly restricted by 3D geometries. The stability numbers for clay with uniform strength ($\rho = 0$ kPa/m) obtained by Lim et al. (2015b) are also shown in Fig. 5-3 - Fig. 5-6. The figures show that slopes with frictional fill materials placed on undrained clay with uniform strength would be less safe compared to when the materials are placed on a foundation with increasing strength.

Fig. 5-7 shows the comparison of friction angle for $\beta = 15^\circ$ and $22.5^\circ$ and it can be observed that the stability number decreases (increased safety factor) as the friction angle increases. This proves that materials with higher friction angle
should be used to produce a safer slope design. In Fig. 5-8, the effect of slope angle on the stability numbers can be observed when $\phi' = 45$ and $\phi' = 35$. An increase in stability number (less safe) can be seen when the slope angle increases. Based on Fig. 5-7 and Fig. 5-8, the slope can be designed according to the desired slope and friction angle.

Fig. 5-9 and Fig. 5-10 compare the factors of safety obtained from 3D analyses with those from the 2D analyses ($F_{3D}/F_{2D}$ ratio), for various slope and friction angles. The figures clearly demonstrate the significance of 3D effects on slope stability analysis. For example, it can be observed that $F_{3D}/F_{2D}$ are the highest when $L/H = 1$ and decrease towards unity as $L/H$ increases. Additionally, it can be seen that the factors of safety from 3D analyses are fairly close to those from 2D analyses for large $L/H$. For instance, Fig. 5-10 shows that the factor of safety obtained using a 3D analysis is only approximately 10% more than that of 2D analysis when $L/H = 5$. Hence, this implies that 2D solutions could be used directly due to the insignificant 3D effects.

In addition, Fig. 5-9 and Fig. 5-10 show that $F_{3D}/F_{2D}$ ratios are much lower when the friction angle is closer to the slope angle. Closer observation reveals that the ratio of $F_{3D}/F_{2D}$ can increase to as much as 5 (i.e. $L/H = 1; \beta = 15^\circ$) when $\phi' = 35^\circ$ (Fig. 5-9) while the maximum $F_{3D}/F_{2D}$ ratio when $\phi' = 25^\circ$ is approximately 3 (Fig. 5-10). This clearly indicates that, the 3D boundary effects are more significant when the difference between the friction and slope angles is higher.

Based on the discussions above, it should be highlighted that the 3D boundary effects are more significant for lower $L/H$ and higher friction angle. In light of that, a slope should be designed more carefully especially for small $L/H$ and high friction angle because for such cases, a 2D analysis will lead to inaccurate results (conservative factor of safety).

### 5.4.1 Failure mechanisms

Based on the study by Bouassida et al. (2014), the plastic points obtained from the numerical LB method could reveal potential failure mechanisms. Fig. 5-11 - Fig. 5-15 show the lower bound plastic points for various cases investigated in this study. For instance, Fig. 5-11 shows the plastic points for $d/H = 2$ and 3. It can clearly be seen that the plastic points remain unaffected although the $d/H$ has...
increased. Therefore this could be the reason for the constant stability number obtained when \( d/H \geq 2 \). Additionally, it can also be seen from Fig. 5-11 that as \( \rho \) increases, the failure mechanisms become shallower. A more apparent influence of \( \rho \) can be seen in Fig. 5-12 where the lower bound plastic points are observed to be shallower when \( \rho = 1 \text{ kPa/m} \) compared to those when \( \rho = 0.5 \text{ kPa/m} \). Therefore, this clearly shows the effect of \( \rho \) in the foundation, where a larger \( \rho \) leads to a decrease in the stability number and thus an increase in the stability of the slope.

The end effects of 3D boundary can clearly be seen in Fig. 5-13. The plastic points obtained from 2D analysis are slightly deeper than those obtained from a 3D analysis (\( L/H = 3 \)). Hence, since the failure mechanisms are already restricted by the 3D boundaries, the strength increase with depth would become a secondary factor in the slope stability. Therefore, this may explain the finding that the effect of \( \rho \) is less significant when a slope is highly restricted by 3D boundaries. From Fig. 5-13, shallower and smaller failure mechanisms are also observed for larger \( \beta \). On the other hand, the influence of friction angle on the failure mechanism can be observed in Fig. 5-14. The figure shows a more concentrated region of plastic points near the slope surface for \( \phi' = 25^\circ \) compared to that for \( \phi' = 45^\circ \). Likewise, the plastic points are seen to be less pronounced at the slope surface as the friction angle increases. Instead, the plastic points for \( \phi' = 45^\circ \) are more pronounced in the clay layer where they are slightly wider and deeper compared to the plastic points obtained for \( \phi' = 25^\circ \). Therefore, this would explain the higher \( F_{3D}/F_{2D} \) ratio obtained when the difference between the friction and slope angles is higher. This phenomenon is found to exist based on detailed observation of plastic points for other cases obtained in this study. For example, from Fig. 5-15, a similar phenomenon is also observed for \( \beta = 15^\circ \) where the plastic points are more pronounced near the slope surface for lower friction angle. Likewise, based on the failure mechanisms in Fig. 5-12(b) and Fig. 5-13(d), which are obtained using a 2D analysis, the failure surface is deeper for higher friction angle. It should be noted that, the plastic points from all cases (both 2D and 3D analyses) clearly indicate base failures for the slopes.

The above discussion shows that the failure mechanisms of this type of slope can be affected by various factors. Particularly, by having a better understanding of
the failure mechanisms, the location of reinforcement can be determined. Additionally, future planning of nearby geotechnical structures can also be made in accordance to the influence of the slopes or vice versa. For instance, proper materials can be used or appropriate geometry of slopes can be designed to minimize the impact on adjacent structures.

**5.5 APPLICATION EXAMPLES**

A case study is shown below to demonstrate the use of the dimensionless stability number and the capability of the stability charts. The case study is assumed to have the following parameters: a slope angle of $\beta = 30^\circ$, a sand layer friction angle of $\phi'$ (Region 1) = $35^\circ$, a clay layer cohesion of $c_{u0} = 15$ kN/m$^2$ with $\rho = 0.5$ kPa/m, a soil unit weight, $\gamma = 18$ kN/m$^3$, a slope height of $H = 5$ m and a depth factor of $d/H = 2$.

From the above parameters, the chart solutions yield a stability number of $Nci = 0.125$ (2D analysis). Based on Eq. (5-2), the factor of safety can be calculated as $F = c_{u0} / \gamma H Nci$ and thus a factor of safety of $F = 1.25$ (2D analysis) can be obtained. On the other hand, if 3D boundary effects are considered, a 3D analysis would produce stability numbers of $Nci = 0.042$ and $0.116$ for $L/H = 1$ and $5$, respectively. The stability numbers can then be calculated to give factors of safety of 4.01 and 1.43, respectively. As shown by the calculation in Table 5-1, the difference between the factors of safety (2D and 3D analysis) is fairly marginal when $L/H = 5$ (approximately 12.7 % underestimation using a 2D analysis). However, if the slope is influenced by significant 3D geometries such as $L/H = 1$, a factor of safety of 4.01 is obtained which is a staggering 69% underestimation if a 2D analysis is used. This clearly shows that using a 2D analysis for slopes restricted by 3D boundaries will results in an underestimation of the factor of safety and could potentially result in cost increase due to unnecessary reinforcement of the slopes.

Additionally, the following example further shows the significance of considering 3D effects in slope stability analysis particularly in the selection of material’s strength. The same slope parameters from previous example are used, except that slope angle is now $\beta = 22.5^\circ$ and the friction angle, $\phi'$ of the fill material is now unknown. Also, a factor of safety of $F = 1.5$ is required. Based on 2D analysis, the
required friction angle, $\phi'$ to satisfy an $F = 1.5$ is $45^\circ$. However, if the slope is restricted by 3D boundary of $L/H = 3$ or $L/H = 5$, the friction angle required would be $\phi' = 25^\circ$ and $30^\circ$, respectively. This example shows that if the local material has a maximum friction angle of $\phi' < 45^\circ$, the cost of the project will increase because the fill material will be required to be sourced elsewhere or the slope will need to be reinforced.

5.6 CONCLUSIONS

Many slope stability problems in the past have traditionally been investigated using a 2D analysis which has been shown to be conservative. In this study, 3D fill slopes consisting of frictional fill materials placed on undrained clay with increasing strength have been investigated and chart solutions which are convenient tools for engineers in practice have also been proposed. The analyses have been conducted using the lower bound finite element limit analysis method and for comparison purposes, the results from 2D analyses have also been discussed. The findings obtained in this study can be summarized as follows:

- The results from this study show that the factor of safety increases as $\rho$ increases. Similarly, this study also quantifies a decrease in factor of safety as slope angle increases or friction angle decreases.
- It has been found that the stability numbers are almost unchanged when $d/H \geq 2$. This phenomenon is different from that obtained for uniform undrained clay foundation. A closer observation on the plastic points (failure mechanisms) reveals that the plastic points are relatively unaffected by the change in $d/H$ when $d/H \geq 2$.
- Regarding 3D effects on this type of slope, a closer observation reveals that the influence of 3D boundary effects is more significant for cases with low slope angle and high friction angle. In other words, 3D boundary effects are more significant when the difference between the friction and slope angles is larger.
- The plastic points obtained from a 2D analysis have been found to be deeper and wider than those from a 3D analysis. Therefore it should be stressed that inaccurate factors of safety and failure mechanisms can be
obtained if 3D boundary effects are ignored when investigating slopes restricted by 3D geometries.

- The influence of \( \rho \) has been found to be less significant when the 3D boundary is smaller. For instance, the change in the stability numbers when \( \rho \) changes has been found to be less apparent when \( L/H = 1 \) compared to plane strain conditions.

From this study, it can be seen that the failure mechanisms and stability of the slope are influenced by the increment in soil strength with depth. Therefore, based on this theory, future work can be dedicated towards investigating the influence of multi-layered soil foundation with different soil strength on slope stability.
5.7 REFERENCES


### 5.8 TABLES

**Table 5-1** Factors of safety for 2D and various $L/H$

<table>
<thead>
<tr>
<th>Boundary Conditions</th>
<th>$L/H = 1$</th>
<th>$L/H = 2$</th>
<th>$L/H = 3$</th>
<th>$L/H = 4$</th>
<th>$L/H = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stability number, $N_{ci}$</td>
<td>0.133</td>
<td>0.042</td>
<td>0.073</td>
<td>0.092</td>
<td>0.110</td>
</tr>
<tr>
<td>Factor of safety, $F$</td>
<td>1.25</td>
<td>4.01</td>
<td>2.28</td>
<td>1.82</td>
<td>1.52</td>
</tr>
<tr>
<td>% underestimation using 2D result</td>
<td>N/A</td>
<td>68.8</td>
<td>45.2</td>
<td>31.1</td>
<td>17.5</td>
</tr>
</tbody>
</table>
5.9 FIGURES

(a) 2D configuration of slope problem

(b) 3D configuration of slope geometry and boundary conditions

Fig. 5-1. Problem configuration and slope geometry
Fig. 5-2. Typical mesh configuration
Fig. 5-3. Lower bound solutions of stability numbers for $\beta = 15^\circ$
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(a) $\beta = 15^\circ$ 

(b) $\beta = 22.5^\circ$ 

(c) $\beta = 30^\circ$
Fig. 5-10. Factor of safety ratio of $F_{3D}/F_{2D}$ for $\phi' = 25^\circ$ ($d/H = 2$)
Slope Stability Assessment of Layered Soil

Fig. 5-11. Lower bound plastic points for $\beta = 22.5^\circ$, $\phi' = 35^\circ$ and $L/H = 3$

- (a) $d/H = 2$ and $\rho = 0.25$ kPa/m
- (b) $d/H = 3$ and $\rho = 0.25$ kPa/m
- (c) $d/H = 2$ and $\rho = 0.75$ kPa/m
- (d) $d/H = 3$ and $\rho = 0.75$ kPa/m
Slope Stability Assessment of Layered Soil

Fig. 5-12. Lower bound plastic points for various $\beta$ (2D and $d/H = 2$)

(a) $\beta = 30^\circ$, $\phi' = 35^\circ$ and $\rho = 0.5$ kPa/m

(b) $\beta = 30^\circ$, $\phi' = 35^\circ$ and $\rho = 1$ kPa/m

(c) $\beta = 40^\circ$, $\phi' = 45^\circ$ and $\rho = 0.5$ kPa/m

(d) $\beta = 40^\circ$, $\phi' = 45^\circ$ and $\rho = 1$ kPa/m
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Fig. 5-14. Lower bound plastic points for $\beta = 22.5^\circ$, $\rho = 0.25$ kPa/m, $L/H = 2$, $d/H = 2$ and various $\phi'$.
Fig. 5-15. Lower bound plastic points for $\beta = 15^\circ$, $\rho = 1$ kPa/m, $L/H = 2$, $d/H = 1.5$ and various $\phi'$.
Chapter 6 PARAMETRIC MONTE CARLO STUDY OF FILL SLOPES

K. Lim, M. Cassidy, A. J. Li and A. V. Lyamin

6.1 INTRODUCTION

Slope stability is a common problem in geotechnical engineering and thus has been extensively investigated. Generally, either the limit equilibrium method (LEM) or finite element method (FEM) is used to investigate the problem. Over the years, various stability chart solutions, which can provide quick first estimates of slope stability, have been developed (Taylor 1937; Gens et al. 1988; Yu et al. 1998; Kim et al. 1999; Michalowski 2002; Kumar and Samui 2006; Li et al. 2009, 2010; Michalowski 2010; Qian et al. 2014). These charts have proven to be convenient tools for geotechnical engineers for preliminary design (Li et al. 2010). However, these charts do not consider the effects of uncertainties in soil parameters.

Traditionally, the factor of safety found in a deterministic approach has often been used in slope stability analyses. However, it has long been recognized that such an approach does not explicitly account for the uncertainties in soil parameters; thus, a number of probabilistic slope stability analyses have been performed (Alonso 1976; Whitman 1984; Christian et al. 1994; Chowdhury and Xu 1995; Low et al. 1998; Duncan 2000; El-Ramly et al. 2002; Griffiths and Fenton 2004; Xu and Low 2006; Cassidy et al. 2008; Kanning and van Gelder 2008; Mbarka et al. 2010; Li et al. 2012; Huang et al. 2013; Hicks et al. 2014). Duncan (2000) highlighted that using the same factor of safety for all slope stability analyses is a “one size fits all” approach that could result in some degree of risk and inappropriate factors of safety in some cases. Furthermore, many fellow researchers have also expressed the same point of view as Duncan on this matter (Cherubini et al. 2001). Additionally, Liang et al. (1999) stated that slopes with the same factor of safety may pose different risks depending on the degree of variability of the soil parameters. By using probabilistic analysis, one can assess the chance of a slope failure by directly incorporating the uncertainties of the
material properties and the strength parameters. Results can also then be used to determine the appropriate factor of safety for a given slope stability design. As supported by Li et al. (2012), this approach allows a more rational and effective design to be achieved.

In recent years, the finite element limit analysis methods (2002a, 2002b; Krabbenhoft et al. 2005), has been utilized to solve slope stability and other geotechnical problems (Yu et al. 1998; Kim et al. 1999; Loukidis et al. 2003; Li et al. 2009, 2010; Shiau et al. 2011; Li et al. 2014; Qian et al. 2014). Two distinct solutions can be produced using the methods and these are the upper bound solution which is based on kinematically admissible velocity fields and the lower bound solution which is based on statically admissible stress fields. These methods have the advantages of application without pre-assuming the slip surface or any other assumptions of statics, unlike the LEM. Further details regarding the methods can be found in Lyamin and Sloan (2002a, 2002b) and Krabbenhoft et al. (2005). That said, apart from the studies by Li et al. (2012) and Huang et al. (2013), probabilistic slope stability studies utilizing finite element limit analysis have been fairly limited. Recently, Qian et al. (2014) and Lim et al. (2015) used the finite element limit analysis methods and developed a set of stability charts for slope design. Therefore, this study aims to extend the use of the stability number demonstrated in Qian et al. (2014) and Lim et al. (2015) as well as to provide guidance for its use in a probabilistic study. The novelty of this study lies in the following:

- A simple probabilistic-based reference work for the original study is presented.
- Based on the results presented, an appropriate and more rational factor of safety can be selected to achieve the targeted probability of failure.
- Provides a better understanding of the influence of uncertainties in soil properties on such a slope configuration.

6.2 REVIEW OF PROBABILITY ASSESSMENTS FOR SLOPE STABILITY

The first sets of chart solutions were produced by Taylor (1937), and many other charts have been developed since (Michalowski 2002; Baker et al. 2006;
Slope Stability Assessment of Layered Soil

Viratjandr and Michalowski 2006; Li et al. 2009, 2010; Michalowski 2010). The majority of these charts are based on the LEM or limit analysis, while those based on FEM are fairly limited due to its complexity and time consuming nature. In light of that, a number of charts based on finite element limit analysis have been rigorously developed for cut or natural slopes (Yu et al. 1998; Kim et al. 1999; Li et al. 2008, 2009, 2010) while a few chart solutions on fill slopes have been recently proposed (Qian et al. 2014; Lim et al. 2015). Fill slopes are often encountered in the construction of highways and dams; therefore, chart solutions based on fill slopes are very valuable to the engineering community.

However, although chart solutions can provide a quick first assessment of slope stability, there have been limited probabilistic assessments that can provide guidance regarding their use. While one may argue that a quick first assessment would be sufficient in slope stability, the studies by Phoon and Kulhawy (1999a, 1999b) and Duncan (2000) have shown otherwise – the coefficient of variance ($COV$) in certain soil parameters can be significantly high; for example, the undrained shear strength of clay can have a coefficient of variance of up to 0.4. In such an instance, as highlighted by El-Ramly et al. (2002), the factor of safety alone can mean very little in terms of slope safety. Additionally, Alonso (1976) advocated in his study that the relationship between the factor of safety and the probability of failure can be useful on a preliminary basis for design or the evaluation of risk. Furthermore, the study by Liang et al. (1999), who investigated embankment dams using a reliability-based approach, showed that even a slope with a factor of safety of $F > 1$ may have a probability of failure of approximately 0.4. This finding shows that a probabilistic assessment would be very useful in a slope stability analysis because it can provide a degree of confidence compared to the conventionally used factor of safety.

Christian et al. (1994) provided a plot of the nominal probability of failure versus the expected factor of safety for the construction of a dike on soft, sensitive clays. It was shown that different factors of safety could be selected for the desired probability of failure, especially for different stages in the life of a project. Just recently, Javankhoshdel and Bathurst (2014) extended the studies by Taylor (1937) and Steward et al. (2011) and developed a set of simplified probabilistic slope stability design charts for cohesive and cohesive-frictional soil.
Additionally, Alonso (1976) also presented a plot of the mean factor of safety versus the probability of failure for a case study. It was highlighted that the relationship between the mean factor of safety and the probability of failure is valid only for slopes with similar uncertainties, geometry and general conditions. Therefore, this fact shows that the same plot of the mean factor of safety versus the probability of failure cannot be used for slopes with different configurations and as of now such plots for fill slopes do not exist.

To date, many probabilistic studies on slope stability have been performed. However, majority have been performed using the LEM (Li and Lumb 1987; Christian et al. 1994; Low et al. 1998; Liang et al. 1999; Malkawi et al. 2000; Cho 2007; Low et al. 2007; Ching et al. 2009; Cho 2009a) or numerical models (Griffiths and Fenton 2000, 2004; Xu and Low 2006; Griffiths et al. 2009; Srivastava and Babu 2009; Griffiths et al. 2011). Some of these even consider the spatial variability of soil (Griffiths and Fenton 2000; Low 2003; Hicks and Samy 2004; Cho 2007; Low et al. 2007; Cho 2009; Griffiths et al. 2009; Hicks and Spencer 2010; Suchomel and Mašín 2010; Li and Hicks 2014), which will not be considered in this thesis. Although probabilistic analysis considering spatial variability is recognized to be a more realistic approach, probabilistic analysis based on point-to-point \( COV \) is still widely used (Xu and Low 2006; Mbarka et al. 2010; Wang et al. 2010; Li et al. 2012; Javankhoshdel and Bathurst 2014; Ma and Wang 2014; Reale et al. 2015). Particularly, as highlighted by Javankhoshdel and Bathurst (2014), using the latter approach, a more conservative design can be made. It is interesting to note that the studies by Low et al. (1998) and Low (2003) showed that reliability assessment problems can be easily solved using a spreadsheet optimization tool. Moreover, Wang et al. (2010) investigated the effects of the coefficient of variation of various soil properties on the reliability of slope stability and highlighted that the uncertainties of undrained shear strength can significantly affect slope reliability.

Recently, a few probabilistic assessment studies utilizing the finite element limit analysis methods have been conducted (Li et al. 2012; Huang et al. 2013). It should be noted that Huang et al. (2013) also considered spatial variability in his study. In fact, the present study is very similar to the study by Li et al. (2012), who also utilized the numerical limits analysis results and stability number in their
probabilistic study on rock slope stability. Particularly, the applicability of the newly proposed slope stability number was successfully demonstrated. However, it is to be noted that the study by Li et al. (2012) was on rock slopes; hence a different slope configuration from the present study. Additionally, rock-based Hoek Brown parameters were used in their study. Therefore, the results obtained are not applicable to the present study.

From the preceding discussion, it can be seen that probabilistic assessments can be useful for providing guidance to geotechnical engineers on the risk evaluation of slopes in addition to the conventional factor of safety. It is well known that chart solutions can provide a quick first assessment for preliminary slope design, and thus many have been developed (Michalowski 2002; Li et al. 2009; Sun and Zhao 2013; Qian et al. 2014; Lim et al. 2015). However, more sophisticated numerical modelling (Yao et al. 2007; Yao et al. 2009) is still needed during design, especially for large-scale complex projects.

In fact, some of these charts have implemented the use of dimensionless stability numbers, which allows geotechnical engineers to quickly assess the stability of a given slope. While it has been shown that the factors of safety can be obtained relatively easy using the proposed stability numbers, it would be interesting to study the influence of uncertainties in soil properties on such a slope in a probabilistic analysis.

6.3 PROBLEM DEFINITION

6.3.1 Stability number definition

Conventional LEM assesses slope stability in terms of the factor of safety based on the amount of shear strength available to resist the driving shear stress, while the FEM relies on the strength reduction method to achieve a similar form of results – known as the factor of safety. Slope stability analyses are also commonly performed using the various stability charts developed to date, and some of these charts even present their results in the form of stability numbers. In a recent study on fill slope, Qian et al. (2014) utilized the finite element limit analysis methods to investigate the stability of two-layered undrained clay slopes (Fig. 6-1) and presented a set of stability charts incorporating the use of a stability number, as shown in Eq. (6-1).
\[ N_{2c} = \frac{c_{u1}}{\gamma H F} \]  

(6-1)

where \( c_{u1} \) is the undrained shear strength of fill material, \( \gamma \) is the unit weight, \( H \) is the height of the slope and \( F \) is the factor of safety.

Qian et al. (2014) showed in the study that Eq. (6-1) can be used to conveniently obtain the factor of safety, \( F \), of a slope or the maximum allowable height for design. For example, for a given \( c_{u1} \), unit weight \( \gamma \) and slope height \( H \), the factor of safety \( F \) can be calculated as \( F = c_{u1} / \gamma H N_{2c} \). More details on the application of \( N_{2c} \) can be found in Qian et al. (2014). However, as mentioned by Qian et al. (2014), the stability numbers, although in similar form, are not applicable to the study by Taylor (1948) because the stability numbers used by Qian et al. (2014) include the effects of \( c_{u1}/c_{u2} \) ratios. Although the factors of safety obtained from Eq. (6-1) and those using the conventional LEM represent a failure when they equal unity, they are generally not equal because of their different definitions. Thus, this study aims to provide guidance on the use of the factor of safety obtained using Eq. (6-1) for different probabilities of failure.

### 6.3.2 Coefficient of variation, COV, consideration

The slope configuration used in this study and the degree of uncertainties of the soil properties are shown in Fig. 6-1. The coefficient of variation, \( COV \), as defined in Eq. (6-2), will be adopted in this study.

\[ COV = \frac{\text{standard deviation}}{\text{mean value}} = \frac{\sigma}{\bar{x}} \]  

(6-2)

Hence, based on the study by Duncan (2000), a range of \( COV \) of 0.1 – 0.4 applies for undrained shear strength, while a range of \( COV \) of 0.03 – 0.07 is valid for the unit weight. Based on these recommendations, this study investigated the full range of \( COV \) of 0.1 to 0.4 for the undrained shear strength but kept the unit weight \( COV \) at 0.07 (reflecting the higher variability condition with the most conservative result).

Although Baecher and Christian (2005) stated in their study that soil properties can follow a normal or log-normal distribution, this study was investigated based on soil properties that follow a normal distribution. In fact, soil properties following a normal distribution have commonly been used in probabilistic studies.
(Lumb 1966; Matsuo and Kuroda 1974; Tobutt 1982; Christian et al. 1994; Sivakumar Babu and Murthy 2005; Xu and Low 2006; Ma and Wang 2014) due to their simplicity. Furthermore, it was highlighted by Hicks and Samy (2002) that for the range of $0.1 < COV < 0.3$ of undrained shear strength, $c_u$, the probability of obtaining negative values is negligible.

### 6.4 RESULTS AND DISCUSSION

The results from this study are obtained using the numerical limit analysis methods (Lyamin and Sloan 2002a, 2002b; Krabbenhof et al. 2005) coupled with basic Monte Carlo simulations in which the distributions of undrained shear strength and unit weight are simulated. Therefore, for every simulated undrained shear strength and unit weight, the factor of safety $F$ was calculated using the average stability numbers of numerical lower and upper bound limit analysis solutions. Further details of the upper and lower bound limit analysis methods can be found in (Lyamin and Sloan 2002a, 2002b; Krabbenhof et al. 2005). Then the probability of failure was obtained based on the frequency of $F < 1$. In all of the results presented in this study, a total of 1500101 Monte Carlo trials were performed for each case. The high number of trials was adopted so that the change in the probability of failure per trial would be sufficiently small for accuracy purposes (two or more significant digits) and to satisfy convergence. In fact, the higher the Monte Carlo trials, the better the accuracy of a Monte Carlo simulation.

#### 6.4.1 Effects of slope angle and slope height

This section of results investigated the effects of different slope angles and slope heights. The parameters used for this part of study are provided in Table 6-1. Using Eq. (6-2), the standard deviation of the individual parameters can be calculated. The results are tabulated in Table 6-2 and Table 6-3. As expected, Table 6-2 shows a decrease in the mean factor of safety as the slope angle or slope height increases while Table 6-3 shows that the probability of failure increases as the mean factor of safety decreases. The mean factors of safety were calculated based on the average of upper and lower bound solutions. In fact, these outcomes are reasonable because the increase in slope angle or slope height results in a lower mean factor of safety. Therefore, this finding shows that the probability of failure and the mean factor of safety are correlated. A similar point was also
expressed by Alonso (1976), Christian et al. (1994) and Li et al. (2012). Hence, to further investigate this relation and the effects of the \( COV \) of undrained shear strength, more analyses were performed using similar parameters with different \( COVs \) of undrained shear strength. The results are presented and discussed below.

### 6.4.2 Different \( COVs \)

Fig. 6-2 shows the results obtained in this study for the \( COVs \) of undrained shear strength investigated. It should be noted that the \( COVs \) of \( c_u1 \) and \( c_u2 \) are identical. In other words, the variation in \( c_u1 \) and \( c_u2 \) was proportionate to maintain a fixed \( c_u1/c_u2 \) ratio of 2 in this case. The figure clearly shows the relationship between the mean \( F \) obtained and the corresponding probability of failures for the various \( COVs \). In fact, it can be observed that a slope with soil strength that has a lower \( COV \) will have a steeper gradient than with one that has a higher \( COV \). Also, it can be observed from Fig. 6-2(b) that the influence of different mean \( c_u \) values is insignificant. For instance, the relationship lines of the probability of failure versus the mean factor of safety for different mean \( c_u \) values can be seen to be similar.

Additionally, the effect of \( COVs \) can also be seen in Fig. 6-3. From Fig. 6-3, the distribution of the factor of safety obtained from the Monte Carlo simulations using the numerical limit analysis methods can be observed. In fact, the figure shows that the distribution of the factors of safety increases in width as \( COV \) increases. Therefore, the accumulation of factors of safety, \( F < 1 \), is larger for higher \( COVs \), hence the higher probability of failure.

As previously mentioned, the \( F \) obtained using the stability number is generally different from the \( F \) obtained using the LEM due to their different definitions. However, their applications are still similar. For instance, the chart of Fig. 6-2 can be used to obtain a probability of failure for a desired factor of safety or vice versa. For example, for a given factor of safety of \( F = 1.5 \) – commonly used in slope design (Australian Geomechanics 2007; Council 2007; Los Angeles Department of Building and Safety 2011; Ottawa 2012) – the probability of failure for undrained clay with \( COV = 0.4 \) (extreme cases) is approximately 0.12, while that of undrained clay with \( COV = 0.1 \) is less than 0.0001. In other words, a slope that consists of undrained clay with \( COV = 0.4 \) would have at least a 1 in 10
chance of failure, while that with $COV = 0.1$ would have a failure rate of 1 in 10000.

Thus, the above discussion clearly highlights the contribution of a probabilistic assessment whereby a slope with a similar factor of safety would pose different probability of failure depending on the $COV$ of the soil properties. Additionally, the convenience of the presented chart solutions is shown. For instance, based on the charts, the appropriate factor of safety corresponding to the probability of failure or vice versa can be obtained.

6.4.3 COVs of $c_{u1}$ and $c_{u2}$ varied

The discussion above has clearly highlighted the effects of different $COV$s. However, it is perhaps more practical to encounter different $COV$s for fill materials and underlying foundations instead of them just varying proportionately (as was investigated above). Even if both the materials have the same $COV$, it is more practical for the properties to vary independently. Fig. 6-4 shows the results of the simulations based on independently varied $c_{u1}$ and $c_{u2}$. The results have been presented for different combinations of the $COV$s of $c_{u1}$ and $c_{u2}$. For example, Fig. 6-4(a) shows the results for various $COV$s of $c_{u2}$, while the $COV$ of $c_{u1}$ remains at 0.1. The rest of the figures are the results for the $COV$s of $c_{u1} = 0.2$, 0.3 and 0.4, respectively. It is noted that because $c_{u1}$ and $c_{u2}$ now independently vary, the $c_{u1}/c_{u2}$ ratio will also vary as a result. Therefore, a limit of the $c_{u1}/c_{u2}$ ratio = 0.2 and 5 has been applied in the simulation, which covers most problems of practical interest (Merifield et al. 1999). Similar to the section above, results of different mean $c_u$ values have been presented in Fig. 6-5. It can again be observed that the relations between the probability of failure and the mean factor of safety are identical for the different mean $c_u$ values.

Overall, the probability of failure is again observed to decrease as the mean factor of safety increases. Moreover, collectively as a group, $c_{u1}$ with a lower $COV$ has a lower probability of failure for a given factor of safety. This observation is reasonable because a lower $COV$ represents lower variance in soil properties and hence higher reliability. Additionally, for larger $COV$s, such as $COV = 0.3$ and 0.4, a slope would pose higher probability of failure compared to smaller $COV$s; hence, the conventional factor of safety ($F = 1.5$) would be inappropriate for
design if a relatively small probability of failure is required. For example, a dam
design may require a probability of failure of lower than 0.0001 (Whitman 1984).
Hence, when $c_{u1} COV = 0.3$, the factors of safety required to satisfy this
requirement would be nearly 5 and 6 for $c_{u2} COV = 0.3$ and 0.4, respectively.

Additionally, Fig. 6-4 further demonstrates the advantage of risk assessment of a
standalone factor of safety, particularly when there is a large uncertainty in soil
properties. For instance, the commonly used factor of safety in slope stability is $F$
= 1.5 (Australian Geomechanics 2007). Therefore, based on this recommended $F$,
the clay COVs and their corresponding probabilities of failure can be seen in
Table 6-4. It can be seen from the table that for extreme cases when $COV = 0.4$, a
slope may have a risk of failure of 1 in 10. Therefore, the above discussions show
that a smaller $COV$ would be better for slope design, and as highlighted by Ching
and Phoon (2014), a smaller $COV$ (lower than 0.1) is achievable if more soil tests
are performed. It is to be noted that the relationship between the probability of
failure and the factor of safety obtained in this section is more realistic compared
to those in the section above (identical COVs of $c_{u1}$ and $c_{u2}$) because of the
independently varied COVs of the two soil layers. Additionally, the application of
the $c_{u1}/c_{u2}$ ratio limit also prevents extreme values from being obtained, which
may be the case for large COVs. Furthermore, to ensure that no negative values of
shear strength are obtained, a left truncation of 0 has been applied to the Monte
Carlo simulated shear strength. Similar approach was also adopted by Hicks and
Samy (2002) and Hicks and Spencer (2010).

6.4.4 Different $c_{u1}/c_{u2}$ ratios
In this section, the effect of the mean $c_{u1}/c_{u2}$ ratio has been investigated. The mean
$c_{u1}/c_{u2}$ ratio can be obtained as mean $c_{u1}/$mean $c_{u2}$. In this section, a range of the
mean ratios of $c_u$ have been investigated; the results are presented in Fig. 6-6.
Because soil consistency can be controlled relatively well for a fill material, only
the COVs of $c_{u1} = 0.1$ and 0.2 are investigated here.

Fig. 6-6 shows that when the COVs of $c_{u1}$ and $c_{u2}$ are equal, the plot resembles a
band of lines with an approximately similar trend. However, for all other plots, a
very different trend among the plots can be seen. For instance, plots for a mean
$c_{u1}/c_{u2}$ ratio $> 1$, such as $c_{u1}/c_{u2}$ ratio $= 2$ and 4, have trends that closely resemble
each other, while plots for a mean $c_{u1}/c_{u2}$ ratio < 1, such as $c_{u1}/c_{u2}$ ratio = 0.2 and 0.5, are slightly more identical to one another. Thus, Fig. 6-6 clearly shows that the mean $c_{u1}/c_{u2}$ ratio will affect the results of the risk evaluation investigated in this study. However, there is no obvious trend that can quantify the effects of the mean $c_{u1}/c_{u2}$ ratio on the probability of failure. Therefore, it is advised that the charts provided be used with proper engineering judgement, and if required, further simulations may be performed.

### 6.5 APPLICATION EXAMPLES

A similar case study to that by Qian et al. (2014) is shown below to further illustrate the application of the stability number in a probabilistic manner. The parameters for the case study of a fill slope are as follows: $\beta = 30^\circ$, a height $H = 5$ m, a unit weight $\gamma = 18$ kN/m$^3$, a depth factor $d/H = 2$, a fill material undrained shear strength $c_{u1} = 50$ kN/m$^2$ and a $c_{u1}/c_{u2}$ ratio of 4.0.

An average stability number – of the lower and upper bound – of 0.455 was obtained by Qian et al. (2014) and was calculated to produce a factor of safety $F = 1.2$. Based on Fig. 6-6, the probabilities of failure have been calculated for different COVs and are presented in Table 6-5. It can be seen that even with $COV = 0.1$ for both $c_{u1}$ and $c_{u2}$, the probability of failure is still 5 in 1000. In fact, a probability of failure of 2 in 10 can be obtained if $c_{u1}$ has a $COV = 0.2$ and $c_{u2}$ has a $COV = 0.4$. Thus, this finding shows that an obtained factor of safety $F = 1.2$ may still pose a high risk, as shown by the probabilistic assessment, depending on the degree of uncertainties in the soil properties.

To design a safer slope, a typical 0.0001 failure rate for dam design is used in the following example. Similar parameters to those above are used, except that the design height, $H$, is now unknown. The study by Ching and Phoon (2014) showed that a $COV$ of 0.3 or more can be obtained if no prior shear strength test for soil is conducted and a smaller $COV$ of 0.1 can be obtained if more tests on the soil shear strength are performed.

Thus, based on the information, Fig. 6-6 can be used to obtain the required mean $F$ versus the targeted probability of failure. The results have been tabulated in Table 6-5, and the table shows that if more soil investigations are performed, then
the allowable design height is almost double that of little or no soil investigation. For instance, when \( c_{u1} \ COV = 0.2 \) and \( c_{u2} \ COV = 0.4 \), the allowable design height is 2.3 m. However, when the \( COV \)s of \( c_{u1} \) and \( c_{u2} \) are 0.1, the allowable design height increases to 4.4 m.

Thus, the examples above clearly show that soil with very high variability (\( COV \)) is very undesirable in slope design because the risk of slope failure increases as the uncertainties in the soil properties increase. Additionally, the examples show that more investigations of the soil shear strength would be highly beneficial because a smaller \( COV \) can then be obtained. In addition to a lower risk slope, the benefits of smaller \( COV \)s can also be seen directly in practical terms, such as the increase in design height.

### 6.6 CONCLUSIONS

In this study, a probabilistic assessment using the finite element limit analysis methods is performed. Although stability charts for two-layered undrained clay have been proposed recently, this study shows that the factor of safety obtained using the charts may not be sufficient to fully define the stability of the slope. As shown in this study, different relationships of the probability of failure versus the mean factor of safety can be obtained for this type of slope. Hence, this study proposes several convenient probabilistic charts that are able to provide guidance for selecting the appropriate factor of safety – when using the stability numbers presented in Eq. (6-1) – with a targeted probability of failure in a slope design. Various values of \( COV \)s for undrained shear strength are considered in those charts. The probabilistic charts proposed can be used as a supplement to the recently proposed chart solutions for fill slopes with two layers of undrained clay. However, it must be highlighted that the charts presented in this study are only applicable to slopes with a similar configuration. Nonetheless, this study shows that in addition to providing a quick first assessment for slope design, finite element limit analysis methods can also be used in a risk assessment.

Additionally, this study also investigated the effects of different values of the mean \( c_{u1}/c_{u2} \) ratio. However, there is no obvious trend that can quantify the effect of the mean \( c_{u1}/c_{u2} \) ratio on the probability of failure. Therefore, to better assess the probability of failure of two-layered undrained clay slopes, proper engineering
judgement is warranted. Because this study aims to provide guidance, only simple probabilistic analysis without considering soil heterogeneity has been performed. In fact, such investigations are still widely performed. However, it is understood that the consideration of soil heterogeneity and spatial variability of soil properties although more complex may be a more realistic approach. Therefore, further works have been dedicated to study these effects.
6.7 REFERENCES


6.8 TABLES

**Table 6-1** Input parameters for the study of different slope angles and slope heights

<table>
<thead>
<tr>
<th>Mean $c_{u1}$ (kN/m²)</th>
<th>Mean $c_{u2}$ (kN/m²)</th>
<th>Mean $\gamma$ (kN/m³)</th>
<th>d/H</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 (COV = 0.1)</td>
<td>30 (COV = 0.1)</td>
<td>16 (COV = 0.07)</td>
<td>3</td>
</tr>
</tbody>
</table>

**Table 6-2** The mean factor of safety for various slope angles and slope heights

<table>
<thead>
<tr>
<th>Height $H$ (m)</th>
<th>Slope Angle β</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>15°</td>
</tr>
<tr>
<td></td>
<td>30°</td>
</tr>
<tr>
<td></td>
<td>45°</td>
</tr>
<tr>
<td></td>
<td>60°</td>
</tr>
<tr>
<td></td>
<td>75°</td>
</tr>
<tr>
<td>1.76</td>
<td>1.65</td>
</tr>
<tr>
<td>1.37</td>
<td>1.28</td>
</tr>
<tr>
<td>1.12</td>
<td>1.05</td>
</tr>
<tr>
<td>0.95</td>
<td>0.89</td>
</tr>
<tr>
<td>0.82</td>
<td>0.77</td>
</tr>
</tbody>
</table>

**Table 6-3** The probability of failure for various slope angles and slope heights

<table>
<thead>
<tr>
<th>Height $H$ (m)</th>
<th>Slope Angle β</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>15°</td>
</tr>
<tr>
<td>0.009</td>
<td>0.00022</td>
</tr>
<tr>
<td>0.188</td>
<td>0.368</td>
</tr>
<tr>
<td>0.679</td>
<td>0.851</td>
</tr>
<tr>
<td>0.954</td>
<td>0.988</td>
</tr>
<tr>
<td>9</td>
<td>0.00024</td>
</tr>
<tr>
<td>0.029</td>
<td>0.450</td>
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<tr>
<td>0.81</td>
<td>0.575</td>
</tr>
<tr>
<td>0.988</td>
<td>0.994</td>
</tr>
<tr>
<td>11</td>
<td>0.000479</td>
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<tr>
<td>0.045</td>
<td>0.511</td>
</tr>
<tr>
<td>0.922</td>
<td>0.996</td>
</tr>
<tr>
<td>0.998</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>0.000758</td>
</tr>
<tr>
<td>0.061</td>
<td>0.575</td>
</tr>
<tr>
<td>0.946</td>
<td></td>
</tr>
<tr>
<td>0.998</td>
<td></td>
</tr>
</tbody>
</table>
Table 6-4 The corresponding probability of failure for various COVs based on $F = 1.5$

<table>
<thead>
<tr>
<th>COV</th>
<th>COV</th>
<th>Pf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.11</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>COV</th>
<th>COV</th>
<th>Pf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.13</td>
<td></td>
</tr>
</tbody>
</table>

* Note: Pf = Probability of failure

Table 6-5 The results for various COVs for the application example

<table>
<thead>
<tr>
<th>COV</th>
<th>COV</th>
<th>Pf (Based on $F = 1.2$)</th>
<th>F (Based on $P_f = 0.0001$)</th>
<th>Allowable H (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.005</td>
<td>1.4</td>
<td>4.4</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.08</td>
<td>1.6</td>
<td>3.8</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>0.1</td>
<td>1.7</td>
<td>3.6</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.18</td>
<td>1.8</td>
<td>3.4</td>
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<tr>
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</tr>
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<td>0.2</td>
<td>0.1</td>
<td>2</td>
<td>3.1</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>0.18</td>
<td>2.4</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.23</td>
<td>2.7</td>
<td>2.3</td>
<td></td>
</tr>
</tbody>
</table>

* Note: Pf = Probability of failure
6.9 FIGURES

Fig. 6-1. Slope configuration for a two-layered undrained clay slope and the COVs considered
Fig. 6-2. The relation between the probability of failure and the mean factor of safety for Case 1 – the COVs of $c_{ul}$ and $c_{u2}$ are identical (dependently varied)
Fig. 6-3. Distributions of safety factors for different COVs of $c_u$
Fig. 6-4. The relation between the probability of failure and the mean factor of safety for Case 2 – the COVs of $c_{u1}$ and $c_{u2}$ are independent.
Fig. 6-5. The relation between the probability of failure and the mean factor of safety for different mean $c_u$ values and COVs.
Fig. 6-6. The relation between the probability of failure and the mean factor of safety for different $c_{u1}/c_{u2}$ ratios.
Chapter 7 FILL SLOPE STABILITY ASSESSMENT USING ARTIFICIAL NEURAL NETWORK

K. Lim, A. J. Li, S. Y. Khoo and A. V. Lyamin

7.1 INTRODUCTION

In recent years many chart solutions have been produced to provide quick first estimates of slope stability (Michalowski 2002; Baker et al. 2006; Michalowski 2010; Michalowski and Martel 2011; Gao et al. 2012b; Sun and Zhao 2013; Qian et al. 2014). These quick preliminary tools are extremely useful in slope design as they offer a simple yet time efficient way to analyse slope stability. However, many of these charts are problem dependent and thus only applicable for a prescribed range of parameters. For instance, cases with parameters that do not exactly match those in chart solutions may require the solutions to be inter/extrapolated which may result in an accuracy issue. Additionally, the users are also required to manually read the charts and thus may also result in inaccuracies. In light of these issues, an artificial neural network (ANN) may prove to be very suitable. In fact, as highlighted by Shahin et al. (2001), ANNs have been successfully applied in solving various geotechnical engineering problems such as pile capacity, modelling soil behaviour, site characterisation, earth retaining structures and others. The success of ANNs is due to the fact that they can be trained using a set of training data and once trained – due to their robustness and extrapolation capabilities, they can be used to predict or provide an output from a set of inputs. In fact, it has been shown that in some applications, an ANN is able to predict the outcome excellently and may even outperform the traditional methods (Shahin et al. 2001).

It is well known that conventional stability chart solutions are still based on the conventional deterministic approach where a safety factor is used to determine the slope stability. However, such approach does not consider the uncertainties in the soil properties (Liang et al. 1999; Duncan 2000). Particularly, Duncan (2000) stated that using the same factor of safety for all slope stability analyses is a “one size fits all” approach that will lead to inappropriate factors of safety in some
cases. This is because slopes with the same factor of safety may pose different risks depending on the degree of variability of the soil properties (Liang et al. 1999). In light of this, a number of probabilistic-based slope stability analyses have been performed (Alonso 1976; Low 2003; Xu and Low 2006; Low et al. 2007; Cassidy et al. 2008; Hicks and Spencer 2010; Griffiths et al. 2011; Li et al. 2012; Huang et al. 2013; Hicks et al. 2014). That said, as highlighted by Silva et al. (2008), because of the difficulty in performing a probabilistic analysis by rigorous mathematical means, risk-based analyses are not as well adopted.

In this chapter, we aim to develop an accurate and quick solution to analyse the stability of two-layered undrained clay slopes and at the same time, provide a simple reliability assessment of the slopes. The novelty of this chapter lies in the following:

- The successful adoption and development of an ANN scheme to conveniently assess the stability of two-layered undrained clay slopes.
- Solutions can be obtained based on a deterministic and/or a probabilistic approach.
- Manual reading of chart solutions is not required and the successful adoption of an ANN to predict the probability of failure considering various influence factors.

### 7.2 PREVIOUS STUDIES

#### 7.2.1 Slope stability investigation

Slope stability problems are commonly encountered in geotechnical engineering. Due to its simplicity, the conventional limit equilibrium method (LEM) are commonly applied in slope stability assessments (Taylor 1937; Bishop 1955; Janbu et al. 1956; Morgenstern and Price 1965; Janbu 1973; Sarma 1973; Fredlund et al. 1981; Duncan 1996). However, it is well known that certain assumptions must be made and that if those assumptions are valid, a LEM analysis is adequate. In recent years, the finite element method (FEM) and the limit analysis method have also been rigorously used in slope stability investigations (Michalowski 1989; Matsui and San 1992; Michalowski 1995, 1997; Dawson et al. 1999; Griffiths and Lane 1999; Manzari and Nour 2000; Michalowski 2002; Chen et al. 2003; Farzaneh and Askari 2003; Chen et al. 2004;
Hammah et al. 2005; Troncone 2005; Zheng et al. 2005; Zheng et al. 2006; Wei et al. 2009; Gao et al. 2012a; Nian et al. 2012; Sun and Zhao 2013; Sun and Qin 2014). However, it is well known that the use of the finite element method can be very time-consuming (Li et al. 2009, 2010). Thus in light of such limitations, the limit analysis methods based on a kinematically admissible velocity field (upper bound) and/or a statically admissible stress field (lower bound) have been proposed. Unfortunately, due to the difficulty in computing the stress field in a slope, most studies performed to date have been based only on the upper bound and thus the solutions are not conservative.

While various stability charts have been produced for homogenous soil slopes (Gens et al. 1988; Michalowski 2002; Li et al. 2009, 2010; Gao et al. 2012b; Sun and Zhao 2013), there are very limited chart solutions available for layered soil slopes (Kumar and Samui 2006; Qian et al. 2014; Lim et al. 2015b). In fact, fill slopes are commonly encountered in the construction of embankments (Indraratna et al. 1992; Al-Homoud et al. 1997). It is important to highlight that the majority of chart solutions have been produced using the limit analysis methods. Speaking of which, in recent years, a new limit analysis-based formulation – the finite element limit analysis methods – have been developed (Lyamin and Sloan 2002a, 2002b; Krabbenhoft et al. 2005). As highlighted by Sloan (2013), these finite element limit analysis methods can be used to solve various geotechnical problems. Many chart solutions have been produced utilizing the finite element limit analysis methods (Yu et al. 1998; Kim et al. 1999; Loukidis et al. 2003; Li et al. 2008, 2009, 2010; Li et al. 2014; Qian et al. 2014). Unlike the conventional limit-analysis-based studies, these methods have the advantage of producing more conservative results using the lower bound limit analysis method. Additionally, the finite element limit analysis methods do not require any assumptions of statics or a pre-determined failure surface as required by the LEM and the conventional limit analysis. Hence, in light of these advantages, the finite element limit analysis methods are adopted in this chapter.

### 7.2.2 ANN in slope stability

In recent years, it has been shown that artificial intelligence (AI) can be used to solve various geotechnical problems (Sakellariou and Ferentinou 2005; He and Li 2009; Pradhan and Lee 2010b; Samui et al. 2011; Ahangar-Asr et al. 2014).
Particularly, among the available AI techniques, ANNs are the most commonly used technique in geotechnical engineering (Shahin 2016). The paper by Shahin et al. (2001) revealed that many applications in geotechnical engineering have found success using artificial neural networks (ANNs). In fact, the contribution of ANNs in slope stability assessment cannot be disregarded. In recent years, ANNs have been successfully applied in slope stability assessment particularly in landslide susceptibility mapping (Feng 1995; Lu and Rosenbaum 2003; Ermini et al. 2005; Sakellariou and Ferentinou 2005; Wang et al. 2005; Samui and Kumar 2006; Yilmaz 2009; Chauhan et al. 2010; Poudyal et al. 2010). For example, the studies by Abdalla et al. (2015) and Gelisli et al. (2015) showed that ANNs can be used to predict the factors of safety of slopes. However, only single layer cohesive-frictional soil slopes were investigated in those studies, therefore they are not applicable to fill slopes due to the different slope configurations. Additionally, probabilistic analysis wasn’t included in those studies.

Multi-Layered Perception (MLP) with back propagation is the most popular and widely used model of an ANN (Mayoraz et al. 1996; Chauhan et al. 2010; Pradhan and Lee 2010a). Additionally, Ermini et al. (2005) who applied two different ANN models – MLP and Probabilistic Neural Network (PNN) – also expressed his preference for the MLP model. Generally the MLP model consists of an input layer and an output layer with a hidden layer between the two layers. In fact, Yesilnacar and Topal (2005) and Yilmaz (2009) highlighted that the use of more than one hidden layer does not usually result in an advantage. Additionally, the MLP model has been commonly used with back propagation (Rumelhart et al. 1988). The use of the back propagation algorithm allows for the error between the target outputs and the network derived outputs to be minimized iteratively by adjusting the weights between the layers in response to the errors.

However, the use of ANNs in slope stability analyses has also received some mix results. For instance, studies using other model have shown to produce more accurate results compared to the use of an ANN. In particular, Samui (2008) and Pradhan and Lee (2010a) who respectively, used a support vector machine and a frequency ratio model yielded results with better accuracy than the use of an ANN. On the other hand, Yilmaz (2009) who used the frequency ratio, the logistic regression and an ANN in his landslide susceptibility mapping study found that
results using an ANN were more accurate compared to the two former models. Nonetheless, Goh (1994), Lee and Lee (1996) and Sivakugan et al. (1998) used ANNs to investigate various geotechnical engineering applications and found that the results predicted using ANN models were actually better than some of the conventional mathematical equations and model.

### 7.2.3 Probabilistic slope stability analysis

To date, many probabilistic-based slope stability studies have been performed (Lambe 1985; Li and Lumb 1987; Christian et al. 1994; Griffiths and Fenton 2000; Malkawi et al. 2000; Hicks and Samy 2004; Low et al. 2007; Cho 2009a; Griffiths et al. 2011). In fact, probabilistic analyses based on the finite element limit analysis have also been performed in recent years (Li et al. 2012; Huang et al. 2013; Lim et al. 2016). Particularly, Li et al. (2012) and Lim et al. (2016) respectively, provided a plot of the mean factor of safety versus the probability of failure for rock slopes and fill slopes. Additionally, a set of simplified probabilistic slope stability design charts for cohesive and cohesive-frictional soil has also been produced (Javankhoshdel and Bathurst 2014). These plots can be used to select the appropriate factor of safety for the desired probability of failure (Christian et al. 1994). However, as highlighted by Alonso (1976), the relationship between the mean factor of safety and the probability of failure is only valid for slopes with similar uncertainties, geometries and general conditions. Recently, Lim et al. (2016) investigated and showed that the uncertainties in soil properties may have a significant influence on the reliability of fill slopes. Particularly, interesting relationships between the probability of failure and factor of safety for two-layered undrained clay slopes were shown by the study.

Based on the above discussions, the following can be concluded: (1) ANNs can, and have been successfully applied to slope stability analysis, (2) many chart solutions, which can provide convenient and quick assessments of slope stability have been developed and (3) a probabilistic slope stability analysis is extremely useful in addition to the conventional deterministic approach. Specifically, regarding a probabilistic slope stability analysis, the relationship between the mean factor of safety and the probability of failure can allow for the appropriate factor of safety to be selected. However, this relationship has yet to be thoroughly investigated particularly for fill slope stability.
Thus, this chapter proposes to use an ANN to provide a comprehensive and convenient way of slope stability assessment of fill slopes, specifically targeting two layered undrained clay slopes. As previously mentioned, a simple probabilistic approach is also adopted in this study to account for the uncertainties in the soil properties.

### 7.3 METHODOLOGY

#### 7.3.1 Finite element upper and lower bound limit analysis methods
Recently, Qian et al. (2014) and Lim et al. (2015a) investigated the stability of fill slopes (shown in Fig. 7-1) and proposed a non-dimensional stability number as shown in Eq. (7-1)

\[
N_{2c} = \frac{c_{u1}}{\gamma HF}.
\]  

(7-1)

where \(c_{u1}\) is the undrained shear strength of Region 1, \(H\) is the slope height, \(\gamma\) is the unit weight and \(F\) is the factor of safety.

Hence, the stability number is also adopted in this study. In addition, a similar slope configuration to that in Fig. 7-1 is also used in this study. As discussed earlier, the finite element limit analysis methods can, and have been utilized to investigate various geotechnical engineering problems. Furthermore, the numerical methods do not require any prior assumptions regarding the inter slice shear forces or slip surfaces. Therefore, the numerical limit analysis methods are adopted in this chapter and the average of the upper and lower bound solutions are used as the training for the ANN network. Essentially, the upper bound method is based on kinematically admissible velocity fields while the lower bound is based on statically admissible stress fields. Further details on the finite element limit analysis methods can be found in (Lyamin and Sloan 2002a, 2002b) and (Krabbenhoft et al. 2005).

#### 7.3.2 Artificial Neural Network (ANN)
ANNs have been proven to be a universal approximator where the linear combinations of the non-linear neurons and weights, after proper training or selections, can approximate any linear or non-linear functions (Man et al. 2012). This capability has motivated us to choose a single hidden layer feed forward neural network to map the different inputs to the outputs for the respective
analyses. The trained ANN is treated as a continuous differentiable mapping of the inputs to the outputs.

Fig. 7-2 describes the single-hidden layer feed forward neural network. In this thesis, the extreme learning training machine (ELM) (Huang et al. 2006) is used to train our network. In fact, it was proven by Huang et al. (2006) that a single layer neural network (NN) is sufficient to be a universal approximator to estimate any continuous functions. Moreover, by employing a single hidden layer, the hassle of selecting the number of hidden layers can be minimized and the training algorithm can also be mathematically analysed and developed using the ELM concept.

It is to be noted that although the gradient-based back propagation (BP) algorithm has been commonly used in the area of neural computing, the process is very time consuming and the error convergence rate is slow. This is because the BP algorithm is computed based on the error between the ANN output and the desired output. For instance, the output weights are updated based on this error iteratively until the error converges to zero or a predefined sufficiently small number. In contrast, the ELM algorithm (Huang and Babri 1998; Huang et al. 2006; Man et al. 2012; Man et al. 2013) and the references herein have revolutionised the training of neural networks. Unlike the BP algorithm, the weights of an ANN in the ELM training algorithm can be randomly assigned and the ANN can be treated as a linear system. Additionally, the batch learning capability of the ELM algorithm, which can train the ANN in one operation allow for a significantly faster learning speed than the BP and all the BP-like algorithms. Therefore, because of this remarkable merit, the ELM algorithm has recently received a great deal of attention in computational intelligence, with application and extension to many other areas.

In Fig. 7-2, there are \( n \) inputs, \( x_1, \ldots, x_n \), and \( m \) output, \( y_1, \ldots, y_m \). The hidden layer has \( M \) linear nodes, \( \omega_{ij} \) (for \( i = 1, \ldots, M \) and \( j = 1, \ldots, n \)) are input weights, \( \alpha_{ij} \) (for \( i = 1, \ldots, m \) and \( j = 1, \ldots, M \)) are the output weights, \( o_i \) for \( i = 1, \ldots, M \) are outputs of hidden nodes. The input data vector \( x(k) \) and the output data vector \( y(k) \) can be expressed as follows:
\[ x(k) = [x_1(k) \ x_2(k) \ \ldots \ x_n(k)]^T \]  
(7-2)

\[ y(k) = [y_1(k) \ y_2(k) \ \ldots \ y_n(k)]^T. \]  
(7-3)

The output of the ith hidden neuron can be computed as

\[ o_i = \varphi \left( \sum_{j=1}^{n} w_{ij} x_j(k) \right) = \varphi \left( w_i^T x(k) \right) \text{ for } i = 1, \ldots, M \]  
(7-4)

with \( \varphi(\cdot) \), the activation function and

\[ w_i = [w_{i1}(k) \ w_{i2}(k) \ \ldots \ w_{in}(k)]^T \text{ for } i = 1, \ldots, M \]  
(7-5)

and the ith output of the neural network, \( y_i(k) \) can be expressed as:

\[ y_i(k) = \sum_{j=1}^{M} \alpha_{ij} o_j = \sum_{j=1}^{M} \alpha_{ij} \varphi \left( w_j^T x(k) \right) = \xi^T(k) \alpha_i \]  
(7-6)

where

\[ \alpha_i = [\alpha_{i1} \ \alpha_{i2} \ \ldots \ \alpha_{Mi}] \text{ for } i = 1, \ldots, m \]  
(7-7)

and

\[ \xi(k) = [\varphi(w_1^T x(k)) \ \varphi(w_2^T x(k)) \ \ldots \ \varphi(w_M^T x(k))]^T. \]  
(7-8)

Therefore, using Eq. (7-6) – Eq. (7-8), the vector \( y(k) \) can be expressed as

\[ y(k) = [y_1(k) \ y_2(k) \ \ldots \ y_m(k)]^T \]

\[ = [\xi^T(k) \alpha_1 \ \xi^T(k) \alpha_2 \ \ldots \ \xi^T(k) \alpha_m]^T \]  
(7-9)

\[ = [\xi^T(k) \alpha]^T \]

with

\[ \alpha = [\alpha_1 \ \alpha_2 \ \ldots \ \alpha_m]. \]  
(7-10)

Suppose we have \( N \) training input vectors \( x(1), x(2), \ldots, x(N) \) and \( N \) desired output data vectors \( y_d(1), y_d(2), \ldots, y_d(N) \) for training the ANN in Fig. 7-2. It is easy to get
\[
\begin{bmatrix}
y^T(1) \\
\vdots \\
y^T(N)
\end{bmatrix}
= 
\begin{bmatrix}
x^T(1) \\
\vdots \\
x^T(N)
\end{bmatrix}
\alpha = G\alpha
\] (7-11)

with

\[
G = \begin{bmatrix}
\varphi(w_1^T x(1)) & \cdots & \varphi(w_M^T x(1)) \\
\vdots & \ddots & \vdots \\
\varphi(w_1^T x(N)) & \cdots & \varphi(w_M^T x(N))
\end{bmatrix}
\] (7-12)

Matrix \( G \) is called the hidden layer output matrix. Based on the ELM training algorithm, the input weights and the biases of the hidden layer of the ANN can be randomly assigned. The output weight matrix, \( G \), of the ANN can be computed in a single iteration where

\[
\alpha = (G^T G)^{-1} G^T Y_d
\] (7-13)

and

\[
Y_d = \begin{bmatrix}
y_d^T(1) \\
\vdots \\
y_d^T(N)
\end{bmatrix}
\] (7-14)

### 7.3.3 ANN Training

As previously discussed, this study includes two different assessments namely a conventional slope stability assessment and a reliability analysis. Therefore, the training of the ANN is performed separately for the analyses. The training inputs and outputs of the respective assessment are as follows:

(i) **For the slope stability assessment,**
- **Training inputs:** slope angle \((\beta)\), \(d/H\) and \(c_u\) ratio \((c_{u1}/c_{u2})\).
- **Desired training output of the ANN:** \(N_{2c}\).

(ii) **For the reliability assessment,**
- **Training inputs:** slope angle \((\beta)\), coefficient of variation \((COV)\) of \(c_{u1}\), \(c_{u2}\) and unit weight \((\gamma)\), \(d/H\), \(c_u\) ratio \((c_{u1}/c_{u2})\) and mean \(F\).
- **Training outputs of the ANN:** probability of failure \((P_f)\).

For the conventional slope stability assessment, a total of 375 data are utilized for training. Each data used in the training is an average stability number generated
by the numerical upper and lower bound limit analysis methods. The ranges for
the input parameters are $15^\circ \leq \beta \leq 75^\circ$, $1.5 \leq d/H \leq 5$ and $0.2 \leq c_u/c_u \leq 5$.

On the other hand, a total of 133 000 data are used to train the ANN network for
the reliability assessment. It should be noted that Monte Carlo simulations are
used in this study to obtain the probability of failure $P_f$ and it is the aim of this
study to provide the results to an accuracy of two or more significant digits.
Similar parameters and ranges to the above training are considered with an
addition of the following ranges of parameters: $0.1 \leq c_{ul} COV \leq 0.4$, $0.1 \leq c_{u2} COV \leq 0.4$ and $0.04 \leq \gamma COV \leq 0.07$. These ranges of $COVs$ can be found in the
study by Duncan (2000).

For the construction of a continuous differentiable mapping of the inputs to the
outputs, we consider an ANN with 200 hidden nodes. The input weights are
generated randomly within $[-1,1]$, and the sigmoid function

$$\varphi(t) = \frac{1}{1 + \exp(-t)}.$$  \hspace{1cm} (7-15)

which is continuously differentiable, is used as the nonlinear activation function
of the hidden nodes.

The output weight matrix $\alpha$ is computed using the ELM algorithm (Eq. (7-15)).

7.4 RESULTS AND DISCUSSION

Fig. 7-3(a) compares the training results (ANN output) with the actual data from
the slope stability assessment. The results for the slope stability assessment are
presented in the form of stability number ($N_{sc}$). It can be seen that a highly
accurate continuous input-output mapping performance has been achieved.
Additionally, a set of validation data has also been used to verify the performance
of the trained ANN network. This set of validation data was not used in the
training of the network thus has not been seen by the network before. A set of 160
data has been used to validate the results from the ANN network trained for slope
stability assessment and the results are shown in Fig. 7-3(b). Again it can be
observed that the output from the trained ANN model highly matches the real
data.
On the other hand, 28000 unseen data (approximately 20% of trained data) has been used to validate the trained ANN model for reliability analysis. Fig. 7-4 shows the performance of the trained network. The reliability analysis results are presented as the ANN predicted probability of failure versus the actual calculated probability of failure ($P_f$). It can be seen that the data obtained scattered around the line, $y = x$ with a coefficient of correlation, $R = 0.9985$. Hence, this strongly validates the trained ANN.

In addition, this study has also proposed a number of categories in which the probability of failure can be classified into. The proposed categories are presented in Table 7-1. To establish this classification, it is appropriate to refer to Whitman (1984) who indicated that the probability of failure of $10^{-4}$ can be used for dam design. Silva et al. (2008) expressed similar view. This therefore shows that $P_f$ of $10^{-4}$ or lower is used for very important projects. As a result, detailed investigations are required and convenient preliminary solutions are not suitable for this kind of projects. Hence, a $P_f < 10^{-4}$ is assigned as Category 1 (very safe).

A total of 28000 unseen data has been used to validate the trained ANN for reliability analysis. It can be seen from Table 7-2 that the trained ANN correctly predicted the category for 98.3% of the data. Also, 0.9% of the data has a predicted probability of failure > the calculated probability of failure. Hence, this shows that the trained ANN can be used as a risk assessment tools with 99.2% of acceptance rate (including conservative estimates). It should be noted that the primary aim of this study is to provide quick preliminary estimates of slope stability and thus more thorough and detailed investigation would be required for the final or a more important slope design. This may include the consideration of other mechanical behaviour such as consolidation and spatial variability to provide a more precise risk assessment. Nonetheless, the results from this study suggest that an ANN can be used as a convenient and efficient tool in a slope stability assessment.

### 7.5 CASE STUDIES

A case study from Qian (2014) is used to verify the accuracy of the trained ANN. Slope parameters used are as follows: a $c_u/c_{u2}$ ratio = 4.5, $c_{u1} = 50$ kN/m², a slope height $H = 5$ m, a unit weight $\gamma = 18$ kN/m³, and a depth factor $d/H = 2$. It is to be
noted that the ANN will return the results in the form of the stability number shown in Eq. (7-1) and the factor of safety can be calculated using the stability number.

### 7.5.1 Conventional slope stability assessment

Table 7-3 shows the $N_{2c}$ obtained from the ANN for different values of $c_{u1}/c_{u2}$. The factors of safety shown in the table are calculated as $(c_{u1}/\gamma H)/N_{2c}$. $F$ can then be used as the input to the ANN when performing a reliability analysis. In general, the use of chart solutions will still need 5-10 minutes to assess a slope, but using the ANN tool will only need a few seconds. That said, it should be noted that this difference is not because of the inclusion of reliability analysis. Overall, it can be seen that the factors of safety obtained from the ANN are identical to those obtained by Qian et al. (2014) who computed $F$ based on chart solutions. In fact, this example further demonstrates the convenience of the ANN scheme by computing $F$ for $c_{u1}/c_{u2} = 0.8$ and $c_{u1}/c_{u2} = 3$ (various $d/H$). The results obtained using the ANN are again identical to those obtained from the charts. Also, it can be observed that the factor of safety decreases as $d/H$ ratio increases. The computing time is very short for each case and hence, this shows that the comprehensive ANN can be used to provide quick estimate of the slope stability.

In addition, the trained ANN can also be used to conveniently assess the slope stability of a stronger fill material. While the undrained shear strength in the foundation layer (Region 2) cannot be easily modified, the undrained shear strength in the fill material can vary depending on where it is sourced. Thus, Table 7-3 (Case 4) shows the effect of different strength of $c_{u1}$ in terms of factors of safety. Similar parameters to those above are used except that the foundation layer is now assumed to have an undrained shear strength $c_{u2} = 30$ kPa and the $c_{u1}/c_{u2}$ ratios now vary according to $c_{u1}$. With the undrained shear strength of the foundation layer remaining constant, as $c_{u1}$ increases, the $c_{u1}/c_{u2}$ ratio increases. As a result, it can be seen from Table 4 that the stability number, $N_{2c}$ increases. It should be noted that as the $c_{u1}/c_{u2}$ ratio increases, both the stability number and the factor of safety also increase. Therefore this seems to suggest that the increase in $c_{u1}$ actually results in a safer slope despite the increase in the stability number. This example once again shows the convenience of the ANN in providing quick solution for the different soil properties.
7.5.2 Probabilistic analysis

As mentioned earlier, a reliability analysis can also be performed using the ANN and the results are shown in Table 7-3. For demonstration purposes and based on the range of values found in the study by Duncan (2000), this study uses a coefficient of variation (COV) of 0.15 for the undrained shear strength, \( c_{u1} \) and \( c_{u2} \) as well as a COV of 0.04 for the unit weight, \( \gamma \) of the soil. From Table 7-3, the categories in which the probabilities of failure belong to are shown (the categories of the probability of failure are based on Table 7-1). For example, the probability of failure obtained for Case 1 increases from a Category 3 to Category 6 when the factor of safety decreases from 1.35 to 1.05. The change in the category implies an increase in the probability of failure as the factor of safety decreases. Hence, this trend is reasonable. Similar trend can also be observed for Case 3. Therefore, the application of the trained ANNs is clearly verified.

Additionally, a range of COV (Case 5) is also investigated to show the influence of uncertainties in soil properties on a slope’s probability of failure. Similar slope parameters to those above are used here. However, a slope angle of \( \beta = 30^\circ \) is used here. This is to demonstrate the effect of the COV on what would be a stable slope with \( F = 1.14 \). The results are shown in Table 7-3. It is shown that even with a small COV of 0.1, the probability of failure falls in the Category 4 which implies a fairly risky slope. Additionally, the study by Ching and Phoon (2014) showed that a COV of 0.25 can be obtained if no prior shear strength test for soil has been performed. Thus, using a COV of 0.25, the probability of failure is now classified as a Category 5 which has a chance of failure of at least 1 in 10. Hence, using this ANN scheme, a quick reliability analysis can be performed and thus providing vital information on the reliability of the slope. Otherwise, using only the conventional deterministic approach, which in this case gives an \( F = 1.14 \) would produce a misleading sense of safety. This was also highlighted by El-Ramly et al. (2002).

7.6 CONCLUSIONS

In this chapter, an artificial neural network with the extreme learning machine is used to develop a quick and convenient way of assessing the stability of two-layered undrained clay slopes both deterministically and probabilistically. The
training data for this study are obtained using the finite element upper and lower bound limit analysis methods. The developed tool, which can be used to provide a reliability indication of the slopes, offers an additional assurance to the conventional factor of safety. The outputs obtained from the trained ANN are shown to be very similar to the real (actual) data. Thus, this indicates that the ANN can be used directly.

Additionally, this study also shows that slopes are influenced by the uncertainties in soil properties. Particularly, it is shown that even though a slope may be stable using a deterministic approach, the slope is still susceptible to failure and is very likely to fail if the $COV$ of the soil properties is large or the deterministic factor of safety is very close to 1.

The case study also demonstrates the convenience of the ANN when investigating materials with different strength. As shown, the stability numbers can be obtained for the different $c_{u1}/c_{u2}$ ratios very quickly. Because the stability numbers are obtained automatically using the tool rather than looking up the charts manually, computation time is effectively reduced. Also human errors that may occur due to chart reading can be prevented.

In conclusion, this study shows that a trained ANN can be used as an efficient tool in slope design including risk assessment and while more thorough analysis would be required for the final design, an appropriate ANN scheme can be used in preliminary design to offer a direction in which the detailed investigation can focus on and therefore saving time and resources.
7.7 REFERENCES


7.8 TABLES

Table 7-1 Categories of the probability of failure

<table>
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<tr>
<th>Category</th>
<th>Probability of Failure</th>
<th>Risk Classification</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>&lt; 10^{-4}</td>
<td>Very Safe (Criterion for dams (Whitman 1984))</td>
</tr>
<tr>
<td>2</td>
<td>10^{-4} – 10^{-3}</td>
<td>Low risk</td>
</tr>
<tr>
<td>3</td>
<td>10^{-3} – 10^{-2}</td>
<td>Moderate risk</td>
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<td>4</td>
<td>10^{-2} – 0.1</td>
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<td>&gt; 0.3</td>
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Table 7-2 The performance of the trained ANN network using validation data

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<td>No. of ANN predicted &lt; calculated (Non-conservative)</td>
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Table 7-3 Parametric example

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<th>$c_u$ (kPa)</th>
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<th>COV2*</th>
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<th>Pf</th>
<th>Pf Category (ANN)**</th>
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*COV1 = COV of $c_u$, COV2 = COV of $c_u$ and COV3 = COV of $\gamma$. ** Refer to Category Table (Table 1)
7.9 FIGURES

**Fig. 7-1.** Slope geometry for a two-layered clay slope

**Fig. 7-2.** A single hidden layer neural network
(a) Training performance of the ANN using ELM algorithm with 375 data set.

(b) Performance of the trained ANN with 160 validation data.

**Fig. 7-3.** Slope stability assessment
Fig. 7-4. Performance of the trained ANN for reliability analysis using validation data

R² = 0.9985
Chapter 8 CONCLUDING REMARKS

In conclusion, this research utilizes the finite element limit analysis methods to investigate various slope stability problems encountered in fill slopes. The primary contribution of this research is the study of 3D effects on fill slope stability and the proposal of chart solutions, which can be conveniently used in practice by geotechnical engineers. Then to calibrate the newly proposed stability number, this research performs a simple probabilistic analysis that takes into consideration the uncertainties in the soil properties. The final topic of this thesis is on the use of an ANN to predict fill slope stability. In fact, functions of both deterministic and probabilistic assessment have been included in the ANN scheme. Therefore this research (thesis) can be summarized into the following topics:

1. Slope stability assessment of two-layered undrained clay
2. Slope stability assessment of frictional fill materials placed on undrained clay
3. Slope stability assessment of frictional fill materials placed on undrained clay with increasing strength
4. Parametric Monte Carlo study of fill slopes
5. Fill slope stability assessment using artificial neural network (ANN)

8.1 GENERAL OBSERVATIONS FOR FIRST THREE TOPICS (CHAPTER 3-5)

Three dimensional (3D) analyses were performed for the first three topics shown above and the results have been presented in the form of dimensionless stability numbers. Additionally, the results from the 3D analyses were compared with the results from the 2D analyses. The use of the dimensionless stability numbers allows for the factor of safety or slope height to be calculated very quickly. As expected, the comparisons showed that the results from 2D analyses were more conservative than those from 3D analyses. In other words, the factors of safety obtained using a 2D analysis were lower than that obtained using a 3D analysis. Hence, this finding shows that utilizing a 2D analysis to investigate slopes influenced by 3D geometry will lead to overdesign - uneconomical. On the other
hand, in such a situation, a 3D analysis would produce a more accurate result (more economical) and as shown in several parametric examples, a slope with a larger height or a steeper slope angle can be designed. Additionally, the effects of 3D boundary were also investigated in terms of $L/H$. Collectively for the three topics, the $F_{3D}/F_{2D}$ ratios were found to decrease as $L/H$ increased (lower 3D boundary restrictions). In other words, 3D boundary effects were observed to be less significant when $L/H$ increased.

Apart from these general observations discussed, the following discussion will be based on the findings from the individual topics. It is to be noted that only some of the key findings from each chapter are discussed below to avoid duplications of similar discussion in the chapters (papers).

8.2 TWO-LAYERED UNDRAINED CLAY SLOPE

In this chapter, the materials in both layers were governed by the undrained shear strength. A range of $c_u$ ratio was investigated. It was observed that when the $c_{u1}/c_{u2}$ ratio $\geq 1.25$, the safety of the slope was found to decrease with the increase in the $c_{u1}/c_{u2}$ ratio (increase in stability number). Further details to this finding were obtained by observing the failure mechanisms where an increase in the size of the failure mechanism was observed when the $c_{u1}/c_{u2}$ ratio increased. Additionally, the results also showed that when the $c_{u1}/c_{u2}$ ratio is $\leq 0.8$, steep slopes ($\beta \geq 45^\circ$) would yield a toe failure mode, resulting in a constant stability number regardless of the change in the $c_{u1}/c_{u2}$ ratio. However a different phenomenon was observed for gentle slopes – a mix of toe and base failure was observed depending on the $c_{u1}/c_{u2}$ ratio when the ratio is $\leq 0.8$. These findings imply that for steep slopes, as long as the fill materials are weaker than the foundation, the degree of the difference in the undrained shear strength of the two layers does not have an effect on the stability of the slopes and the failure mechanisms. That said, the stability of the slope will still be affected if there is a change in the undrained shear strength.

8.3 FRictional fill materials placed on undrained clay slope
This chapter investigated the fill slope stability of fill materials placed on a foundation with undrained shear strength, $c_u$. In addition to the use of the finite element limit analysis method, the results from the LEM have also been presented. It was concluded that the results obtained from the 2D LEM would overestimate those from the 2D lower bound limit analysis. While comparing the $F_{3D}/F_{2D}$ ratios, it was found that for cases of low slope angle and high friction angle (i.e. $\beta = 15^\circ$, $\phi' = 45^\circ$), the ratio of the factor of safety obtained from a 3D analysis and that from a 2D analysis can be as much as 7, which clearly indicates that for such cases, a 2D analysis would highly underestimate the stability of the slope and would be overly conservative. Hence, based on the findings, it can be concluded that the appropriate analysis should be performed for slopes restricted by physical boundaries to prevent overdesign. Also, engineers should be more cautious when using the LEM in slope design as it was shown to overestimate the results.

### 8.4 Frictional Fill Materials Placed on Undrained Clay with Increasing Strength Slope

It is common to encounter soil with undrained shear strength that increases with depth especially in normally consolidated soil. As a result, this chapter investigated the stability of fill slopes with an inhomogeneous foundation (i.e. frictional fill materials placed on undrained clay with increasing strength). In this chapter, a range of $\rho$ (rate of increment of strength) of 0.25 – 1 kPa/m was considered. Similar to the previous two cases, results from both the two and three-dimensional analyses have been presented for comparison purposes. Apart from the observed 3D boundary effects, the findings from this chapter showed that as $\rho$ increased, the factor of safety of the slope also increased (decrease in stability number). However, it was observed that the effect of $\rho$ was less significant as $L/H$ decreased. Additionally, it was also seen that the effect of the depth factor ($d/H$) was insignificant when $d/H$ was greater than 2. Similar findings were also presented by other studies on undrained clay with increasing strength slopes. The influence of several factors ($\rho$, $\phi'$ and $\beta$) on the failure mechanisms were also discussed. Based on this discussion, better understanding of this type of slope has been achieved.
8.5 PROBABILISTIC ANALYSIS OF TWO-LAYERED UNDRAINED CLAY SLOPE

As reported in the literature, the use of only the factor of safety can be misleading as it does not disclose any information on the risk or potential failure of the slope. Hence, this chapter utilized a probabilistic analysis and investigated the effects of uncertainties in soil properties on slope stability. In fact, the purpose of this study was to provide guidance on the use of the stability number proposed in the recent study of two-layered undrained clay slopes. The most significant contribution from this chapter is that the relationship between the mean factor of safety and the probability of failure has been found for two-layer undrained clay slope. In fact, it was also shown that the relationship was affected by various factors such as the COV of the soil properties and the mean \( \frac{c_{u1}}{c_{u2}} \) ratio. Therefore, this finding shows that proper engineering judgement or detail simulations may be required in the design of this type of slope. Nonetheless, from the newly developed relationships and the proposed stability number, the appropriate factor of safety can be selected to achieve the desired probability of failure hence providing a simple guidance for the use of the recently developed chart solutions.

8.6 FILL SLOPE STABILITY ASSESSMENT USING ARTIFICIAL NEURAL NETWORK (ANN)

From the previous chapter, it was found that the probability of failure of two-layered undrained clay slopes can be influenced by many factors. Hence, to solve this issue and encouraged by the robustness of artificial neural networks (ANNs), this chapter adopted an ANN to provide a robust and convenient method to predict the stability of two-layered undrained clay slopes. In fact, it was shown that once trained, the ANN scheme was able to accurately map the outputs from a given set of inputs. In this case, the stability number and probability of failure were obtained successfully through the use of the trained ANN scheme. In fact, the outputs can be obtained very quickly in a matter of a few seconds. Thus, the effectiveness of ANN in slope stability has been demonstrated.

8.7 SUMMARY
The above discussion shows that chart solutions for fill slopes considering 3D effects have now been developed. These quick and convenient solutions are the first of its kind and will provide geotechnical engineers with quick first estimate of fill slope stability. In fact, results from this thesis also identify and quantify the various factors affecting fill slope stability. Therefore, this will allow for a more efficient design to be produced given that more in depth knowledge of fill slopes is known. Additionally, this study also opens the door for future work on probabilistic analysis of fill slopes utilizing the limit theorems particularly for the other two types of fill slopes. Similarly, considering the interesting results obtained for the probabilistic analysis of two-layered undrained clay slopes, it would also be interesting to investigate slope stability considering spatial variability of the soil properties. It must be highlighted that the finite element limit analysis methods utilized in this study allow for the construction of the lower bound stress field, which are difficult to compute using the conventional limit analysis methods. Lastly, this thesis has also demonstrated the applicability of ANNs in slope stability. It was shown that an effective preliminary risk assessment can be performed using the newly developed ANN scheme. From this discussion, a number of future works have been identified and they are discussed below.

8.8 RECOMMENDATIONS FOR FURTHER WORK

8.8.1 Probabilistic/stochastic analysis

While this study has investigated the stability of two-layered undrained clay slopes using a probabilistic analysis, the effects of uncertainties were not considered for the other two types of fill slopes investigated. It was highlighted that the effect of uncertainties particularly on the relationship between the factor of safety and the probability of failure is different for different slope configuration. Additionally, it is known that soil heterogeneity can influence the stability of a slope. Therefore, it is worth investigating these effects in future work. Additionally, the finite element limit analysis methods which have not been often utilized in probabilistic analyses can also now be utilized for such analyses.
8.8.2 Pore pressure effects
Pore pressure effects can be considered to be a destabilizing factor in slope stability and has been extensively investigated to date. In fact, many landslides were caused by the rise in the water table which can be affected by rainfall infiltration. Therefore, considering the importance and significance of pore pressure effects within slope stability, fill slope stability analyses considering the influence of pore pressure effects are vital. Fortunately, the finite element limit analysis methods allow for the inclusion of pore pressure and thus the effect of pore water pressure on slope stability is definitely worth investigating.

8.8.3 Multi-layered soil slopes
Based on the knowledge obtained from the investigations in this thesis, particularly the suitability of an ANN to handle complex and multiple inputs, investigations of multi-layered soil slopes can be performed. In fact, using an ANN, the various inputs from multi-layered soil slopes can be considered without the use of stability charts, which in this case may not be suitable due to the large number of inputs. Additionally, it would also be interesting to study the failure mechanisms of slopes that have different layers of soil with different soil strength.

8.8.4 Seismic effects
Seismic force can be seen as an additional loading on a slope and is one of the key factors that can trigger a landslide in what would have been a safe slope. This is because the seismic force, which provides an additional loading – can significantly reduce the factor of safety of the slope. Therefore in places where earthquakes are common, slope investigations considering seismic force are necessary. In fact, the displacement of a slope due to the seismic load can provide a valuable insight towards the slope stability analysis. The study incorporating seismic effects can in fact also be presented in the form of stability charts (similar to those found in literatures) where geotechnical engineers can easily use to determine and assess the safety of a slope as well as the consequence (displacement) of the seismic loading corresponding to the different degree of seismic force.
8.8.5 Adoption of ANNs in stochastic and more complex analysis
While the study in this thesis shows that an ANN, once trained can be utilized to predict the output of given inputs, it is hoped that in the future such capability of ANNs can be harvested to perform more complex analysis such as to predict the stability of slopes using a stochastic analysis. This is because the majority of probabilistic analyses considering spatial variation performed to date are based on individual cases and quick solutions that are capable for such analysis are not currently available.

8.8.6 Fill slope stability considering different type of materials
Now that chart solutions for various types of fill slopes have been developed, a new direction in slope stability analysis has been created (i.e. vis-à-vis fill slopes stability). For instance, the following types of fill slopes can be investigated in future work:

1. Frictional fill materials placed on cohesive-frictional materials.
2. Two-layered cohesive-frictional materials slope.
APPENDIX A (Chapter 3)

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\( \beta = 30^\circ \)

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### Slope Stability Assessment of Layered Soil

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### Slope Stability Assessment of Layered Soil

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*Slope Stability Assessment of Layered Soil*
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