Charles Sanders Peirce on necessity

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1. Introduction

Necessity is a touchstone issue in the thought of Charles Peirce, not least because his pragmatist account of meaning relies upon modal terms. In contrast to the highly influential positivist currents in 20th century philosophy in which 'fact-stating' is paramount and put in the indicative mode, Peirce famously advised us to clarify the meaning of our terms using "would-be's", not "will-be's" (CP 5.453; 5.457, 1905). Peirce's pragmatic maxim is that we must look to the practical effects of our concepts, and the practical effects he is concerned with are those that would occur under certain circumstances, not those that will actually occur. This effectively translates every meaningful term into a set of hypothetical conditionals. Thus for instance, the fact that a table is *hard* means (amongst other things) that if I were to rest a dinner plate on it, the plate would not fall through to the floor. (Misak 2013, 29ff ; Hookway 1985)

Peirce thought deeply about necessity in all its aspects, and this is one of the areas of his work with as yet untapped insights. Discussions of necessity show up from his earliest philosophical writings based on Kant's logic and its table of 12 categories (one row of which is, of course, explicitly modal) to the end of his life where he was working out a forest of distinctions in semiotics. His thoughts on modality also span a multitude of philosophical areas – informing his work on mathematics, logic, metaphysics, philosophy of science and ethics.

His views on the topic have some intriguing features. His modal epistemology understands logical form as essentially structural, thus requiring representation by iconic or diagrammatic signs. Peirce made good on this commitment, developing the Existential Graphs, a diagrammatic logical notation, which is still under-studied by mainstream logicians. His metaphysics differs from many 19th Century peers in flatly denying 'necessitarianism' (determinism) and arguing for real chance. He also distinguished between a kind of necessity which corresponds to the compulsion given by one billiard ball to another (which we might refer to in Peircean terms as a 'Secondness Necessity') and a kind of necessity which corresponds to a real universal ('Thirdness Necessity'), whereas in contemporary philosophy the two are routinely conflated.

Peirce's views on necessity also evolved through his life. It has been argued (Fisch 1967; Lane 2007) that although Peirce was always firmly opposed to nominalism, his scholastic realism became more 'extreme', leading him to commit to at least some non-actual reality. In what follows, we shall give a snapshot of Peirce's

important and complex thoughts about necessity.

2. Pragmatism, Modality and Truth

Peirce is best known for his pragmatism and the view of truth he derives from it. Truth, for Peirce, is linked to what human inquirers would believe – truth is what is "indefeasible" or would best stand up to all experience and reasoning, were we to investigate the matter as far as we fruitfully could. Peirce says: "if Truth consists in satisfaction, it cannot be any actual satisfaction, but must be the satisfaction which would ultimately be found if the inquiry were pushed to its ultimate and indefeasible issue" (CP 5.569, 1901; 6.485, 1908). Peirce was a scientist as well as a philosopher and logician, and this experimental spirit led him to wholeheartedly embrace *fallibilism*: the idea that a thinker must be "at all times ready to dump his whole cart-load of beliefs, the moment experience is against them" (CP 1.55, 1896). He held that all of our beliefs form a corrigible whole, with none of them ear-marked as certainly true, and all of them subject in principle to overthrow by experience, if we are willing to make sufficiently far-reaching revisions through our system of belief. C.I. Lewis is an explicit follower of Peirce on this point, and Quine is an implicit follower.

Yet how does this model fit with the idea of *necessary* truth, which is generally thought of as constituting certain and unrevisable knowledge? How can one 'experiment' in logic and mathematics? Surely these sciences must be *a priori*? The concern deepens when it is noted that Peirce argued that there is real possibility and real necessity in the world (see §5), and that he had sophisticated modal notions at work in his exact logic. One might wonder how Peirce can hold together such metaphysical and logical researches and a pragmatist epistemology.

This challenge can be answered on a number of levels. The first is (somewhat ironically) *pragmatic*. For Peirce, as for Lewis and Quine, the necessity found in mathematical or logical truths is just the extent to which we find no reason to revise them. He explicitly addressed one supposed necessary truth of logic: the principle of bivalence (for any p, p is either true or false). Peirce argues that this is something we have to assume if we are to go on with a practice that is of great importance to us. To say that bivalence is a regulative assumption of inquiry is not a claim about special logical status (as a 'logical truth'). Rather, the principle is taken to be a law of logic by a "saltus"— an unjustified leap (NE 4: xiii). Our reason for making such assumptions is that if we do not make them, we will "be quite unable to know anything of positive fact" (CP 5.603, 1903). It appears that here Peirce is arguing for bivalence as a kind of necessary presupposition for other beliefs we wish to hold. But this is an unusually weak kind of necessary presupposition, for at the same time he distinguishes his approach from that of the *transcendentalist*:

> A transcendentalist would claim that it is an indispensible "presupposition" that there is an ascertainable true answer to every intelligible question. I used to talk like that, myself; for when I was a babe in philosophy my bottle was filled from the udders of Kant. But by this time I have come to want something more substantial.
>
> [CP 2.13, 1902]

A second level on which we might address the challenge of consistency within Peirce's thinking is a *semiotic* one. Viewed from the perspective of inquiry, our beliefs are all fallible, but the *expression* of any given belief can be decomposed and analysed in terms of Peirce's theory of signs. According to this theory, different kinds of signs with different functional roles – and thereby different modal purport – work together to compose any given proposition. Peirce believed that the necessarily true dimension of our thinking is conveyed by a particular kind of sign: the *icon*, one example of which is the *diagram.* Peirce addresses the question about fallibilism and necessary inference as follows:

> …when we talk of deductive reasoning being necessary, we do not mean, of course, that it is infallible. But precisely what we do mean is that the conclusion follows from the form of the relations set forth in the premiss. Now since a diagram…is…in the main an Icon of the forms of relations in the constitution of its Object, the appropriateness of it for the representation of necessary inference is easily seen.
>
> [CP 4.531, 1906]

But in a radically original move, Peirce claims that iconic signs can be experimented on, just as the physical world can be. The objects of these experiments should not be understood as objects in Plato's heaven, which would be rather mysterious, but merely the most general structural features of ordinary real world objects:

> Although mathematics deals with ideas and not the world of sensory experience, its discoveries are not arbitrary dreams but something to which our minds are forced...
>
> [Peirce 2010, 41]

> …all deductive reasoning, even simple syllogism, involves an element of observation; namely, deduction consists in constructing an icon or diagram the relations of whose parts shall present a complete analogy with those of the parts of the object of reasoning, of experimenting upon this image in the imagination, and of observing the result so as to discover unnoticed and hidden relations among the parts.
>
> [CP 3.363]

In the next section we say more about Peirce's modally-inflected theory of signs, first presenting its grounding in his philosophical categories.

3. Peirce's Modalized Semantics

3.1 Three Categories

A full understanding of any area of Peirce's thought requires familiarity with Peirce's three categories, which (in line with their derivation and ordering) he dubbed Firstness, Secondness, and Thirdness. Each of these categories, Peirce argued, is present in everything that comes before the mind or is experienced. In his early career he derived them – in the spirit of Kantian logic – from the structure of the proposition, whereby a predicate (First) and its subject (Second) are united by a copula (Third). The key paper here is "On a New List of Categories", in which Peirce was proud that he had reduced Kant's table of 12 by a factor of 4. Later in life he kept the same categories but pushed the derivation of them deeper, into phenomenology and mathematics, the latter being for Peirce, the most fundamental science of all.

It is important to note that Peirce's categories are not properties but *modes of being*. What this means is that we cannot actually pull them apart or imagine a situation in which one or two stand alone. They are distinguished from one another by the Aristotelian and then Scholastic method of abstraction or "prescission", in which we may distinguish different elements of a concept by attending to those elements and neglecting others. Postulating such high-level structural features of being is a project somewhat unfamiliar in post-Quinean analytic philosophy, where one is encouraged to believe that reality is exhausted by a list of existent entities and their intrinsic properties.

A First is something simple, monadic: a quality of feeling, image, or mere possibility. It is indescribable: "It cannot be articulately thought: assert it, and it has already lost its characteristic innocence . . . Stop to think of it, and it has flown!" (CP 1.357, 1890). The difficulty of pinning down a category that is "a special suchness…" is not lost on Peirce (CP 1.303, 1894). It is perhaps helpful to focus on the idea that Firstness 'comes first': it is "predominant" in being, in feeling, and in the ideas of life and freedom (CP 1.302, 1894). Even more importantly, given the continuity of Peirce's metaphysics with his logic of relations, a First is singular: "a pure nature . . . in itself without parts or features, and without embodiment" (CP 1.303, 1894). The First is a relatum prior to any relation, that which makes relations possible.

A Second is a dyadic element: the duality of action and reaction. It is the category that "the rough and tumble of life renders most familiarly prominent. We are continually bumping up against hard fact" (CP 1.324, 1903) and hence have revealed to us "something within and another something without" (CP 2.84, 1902). But as with the first category, this is all we can say of our encounters with hard fact. Any articulation or thought of what we experience takes us into the third category—

the triadic realm of experience proper, which involves interpretation, signification, intention, and purpose realized across time. As soon as we try to describe our encounters with the world, thought and signs are involved. Any perception that we can think of ourselves as having is a perceptual *judgment*, something that requires, in Peirce's terms, "mediation" by some general concept. For instance, when I see a yellow chair, I see it *as* yellow, *as* a chair (CP 7.619-630, 1903).

These categories infuse and inform Peirce's thinking about modality in a host of ways. Indeed, the categories are full of modal notions – the description of Firstness invokes possibility, the description of Secondness mentions mechanical necessity as a key example, and the description of Thirdness corresponds to a kind of *semantic* or *intentional* necessity that unites all instances 'with the same meaning' (so that, for example, if we understand the concept 'white', we know that certain other things must also receive the label).

3.2 Triadic Sign Theory

Peirce puts his triadic categories to work in structuring his theory of signs, along a number of dimensions. Firstly the structure of representation itself is triadic, always involving a sign, an object, and an interpreter. By including both an object and an interpretation as a vital component of every sign, Peirce unites insights from traditions currently opposed under the labels 'realism' and 'anti-realism'.

Secondly the relation between a sign and its object can take three forms, corresponding to *icons*, *indices*, and *symbols*. Icons are signs that represent their objects by virtue of a similarity or resemblance—a portrait is an icon of the person it portrays, a map is an icon of a certain geographic area and, in exact logic, a diagram is an icon of validity's key forms. Peirce notes of the icon that it "represent[s] relations in the fact by analogous relations in the representation" (CP 3.641, 1902) thus, representing *structure* is one of the icon's greatest strengths (Stjernfelt 2007, 49-116, Legg 2008, 207). Indices are signs that point to their object by "being really connected with it" (W 2, 56, 1867)—a pointing finger, a demonstrative pronoun such as "this" draws the interpreter's attention to the object. Smoke, for instance, is an index of fire. A symbol is a word, proposition, or argument that depends on conventional or habitual rules; a symbol is a sign "because it is used and understood as such" (CP 2.307, 1901). It has the same effect from interpreter to interpreter.

This set of sign-types plays an important role in structuring living thought. Symbols in their role as habits established by convention pick out *general concepts*. Indices in their role as unmediated pointers pick out *specific individuals* (and locate us in the real world at a particular place and time). Icons in their role of conveying structure might be said to represent *logical form*. In "What Is a Sign" (EP 2, 4-10;

1894), Peirce explained these unique and mutually enabling roles as follows:

> Suppose a man to reason as follows: The Bible says that Enoch and Elijah were caught up into heaven; then, either the Bible errs, or else it is not strictly true that all men are mortal. What the Bible is, and what the historic world of men is, to which this reasoning relates, must be shown by indices. The reasoner makes some sort of mental diagram by which he sees that his alternative conclusion must be true, if the premise is so; and this diagram is an icon or likeness. The rest is symbols; and the whole may be considered as a modified symbol…The art of reasoning is the art of marshalling such signs, and of finding out the truth.

4. Peirce's Modal Epistemology

So far we have explored a little of how for Peirce all representation has modal purport. We now discuss Peirce's ideas about how philosophers should gather and evaluate modal knowledge in a deliberately focused manner.

4.1 Information-Relative Account

From his first published papers in 1867, Peirce analyzed modality in terms of states of information, or what is known (Morgan 1979; Lane 2007). A proposition is necessarily true if contained in – and possibly true if consistent with – some subject's state of information. This epistemic approach contrasts with contemporary modal semantics according to which a proposition is necessarily true if true in all possible worlds and possibly true if true in at least one possible world. It is worth noting the way in which the latter account decouples entailment from provability in principle. This allows a greater externalism (and thus, arguably, realism) into modal epistemology (Legg 2012, 6); but at the same time it challenges our ability to know modal truths (since if we do not know them by proving them, how might we know them?)

4.2 Kinds of Modality: Designated States of Information

Peirce used his information-relative approach to define different kinds of modality in terms of the kind of information the subject is postulated to have (their 'designated state of information'). For instance, in 1893 he distinguished:

- *essential* or *logical* modality, where what is known is "…*nothing*, except the meanings of words, and their consequences"
- *substantial* modality, where what is known is "…everything now existing, whether particular fact or law, together with all their consequences"
- *physical* modality, where the subject is "…thoroughly acquainted with all the laws of nature and their consequences, but…ignorant of all particular facts".
- *practical* modality, which involves "…[i]magining ourselves to know what the

resources of men are, but not what their dispositions and desires are"

- *mathematical* and *metaphysical* modality, which involves imagining perfect knowledge in those areas.

The first two, he notes, "are of special interest to the logician" (CP 4.66-67, 1893). The first appears to correspond to what we would regard today as logical necessity (i.e. tautology) plus analyticity, while the second corresponds to the actual, at a given time.

4.3 The "Modal Shift"

There are indications that from about 1896 onwards, Peirce was working towards a greater realism in his modal epistemology. Lane (2007) has argued that his thinking evolved from a pure information-relative account towards an account in which we do not discern modal facts merely via inference from a given state of information. However Peirce does not replace the information-relative account with the kind of externalism (frequently accompanied by metaphysical realism) seen in contemporary modal semantics. Rather, he thinks that the certain necessary truths are accessed through a kind of *direct perception*:

> It is not that certain things are possible because they are not known not to be true, but that they are not known not to be true because they are, more or less clearly, seen to be possible.
>
> [CP 6.367, 1902]

According to this account, we discern modal facts via some kind of unmediated intellectual vision of a "world of ideas" which is "but a fragment of the ideal world" (Lane 2007, 563). Peirce writes that with respect to this ideal world we are "virtually omniscient", and that by inspecting it one might be able to determine that a given proposition is impossible although the laws of logic do not show that it contains any contradiction. Peirce was here pursuing a particular example: whether two collections might both be sufficiently large that neither could be mapped into the other (CP 3.526, 1896; cited in Lane 2007, 561). He ended up deciding that the answer to this question could not be proven, but ultimately only 'seen'.

This new development in Peirce's modal epistemology separates out a 'positive' sense of possibility from his previous 'negative' sense of 'not provable that not *p* in information state *S*'. Lane argues that this shift was accompanied by an embrace of a new ("Aristotelian") conception of real possibility. Peirce calls this "the great step that was needed to render pragmaticism an intelligible doctrine" (MS 288, 129, 1897; cited in Lane 2007, 551-552). How does real possibility render pragmaticism intelligible? We noted in our introduction that the pragmatic maxim may be understood to translate every meaningful term into a set of hypothetical

conditionals. But in order to obtain a coherent account of meaning, these conditionals must be *subjunctive*.

In the early (1878) "How to Make Our Ideas Clear", Peirce had suggested that the meaning of 'This diamond is hard' is in part 'If scratched, it will resist'. This entails that "There is absolutely no difference between a hard thing and a soft thing so long as they are not brought to the test." If we imagine, then, "…that a diamond could be crystallized in the midst of a cushion of soft cotton, and should remain there until it was finally burned up" (W 3: 266, 1878), there is no truth of the matter about that diamond's hardness. Peirce was later anxious to correct his mistake:

> But I desire to express…that in those original papers I fell into a grievous error. Looking into the future and thinking of what would take place, I overlooked the great truth that the future, as living, always is more or less general, and I thought that what *would be* could be resolved into what *will be* …
>
> [MS 289, 17–18]

Peirce's modal shift provides the epistemology for us to know 'would-bes'.

5. Peirce's Logic

Peirce was not only the most logically proficient of the founders of pragmatism, but one of the great logicians of the 19th century. Amongst the enormous number and variety of his contributions, he invented a quantified, sound and complete first-order logic in algebraic form independently of and at the same time as Frege[1] (a development which grew from more than a decade's preparatory work in the logic of relations[2]), discovered the Sheffer stroke decades before Sheffer, experimented with many-valued logics, and made significant advances in the logic of statistical reasoning. Possibly his most original contribution, and the one of which he was most proud, was his Existential Graphs (EG).

5.1. Overview of The Existential Graphs

In the EG Peirce sought a logical notation which would:

> …afford a method (1) as *simple* as possible (that is to say, with as small a number of arbitrary conventions as possible), for representing propositions (2) as *iconically,* or diagrammatically and (3) as *analytically* as possible.…
>
> [CP 4.561n]

They were presented to the world in 1896, when Peirce was 57, although he had been working on them for the previous decade and a half. The Alpha graphs cover propositional logic, the Beta extension covers first-order predicate logic with identity, and the Gamma extension covers modal notions, abstractions, and a meta-

language for talking about the graphs themselves.

The EG have yet to receive significant attention in mainstream logic, for a range of reasons. Peirce was almost entirely locked out of academic philosophy, under-read and underappreciated in his lifetime. The EG have been seen as one of the least accessible areas of his work[3]: indeed, his penchant for expressing his thoughts in technical language was the source of considerable tension between Peirce and his fellow pragmatists, James and Dewey (Misak 2013). And finally, once an algebraic, stepwise[4] approach became dominant in the new logic, Peirce's diagrammatic system appeared unfamiliar and cumbersome[5]. It thus remained buried, except to a few select logicians, during and after Peirce's lifetime. It is only since Don Roberts published his book on the EG in 1973 that they have been available to a wider (if still specialized) audience[6]. John Sowa also deserves mention for introducing Artificial Intelligence researchers to "Conceptual Graphs", which derive from the EG.

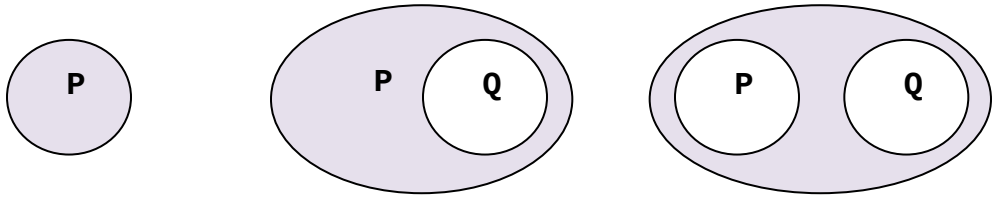
Peirce asks us to think of the graphs as an inquiry between two individuals, or between one's current and future self (MS 450, 3; MS 650). The inquiry starts with much that is taken for granted – a Sheet of Assertion that represents the universe of discourse. The Graphist then inscribes the graph, and the Interpreter draws inferences from what is inscribed, according to the rules of the system. They both see the results. Sometimes Peirce says that the Graphist is an ordinary logician or inquirer, and Grapheus is the creator of the universe of discourse, who also makes continuous additions to it from time to time (CP 4.431). Such ideas are now being worked out in Dialogical Logic, which understands a proof as equivalent to a 'winning strategy' in a two-person, zero-sum game[7].

Thus, Peirce's way of thinking about his Existential Graphs and their status in inquiry is of a piece with his pragmatist view of inquiry as conducted against a body of background belief, with an aim to settle belief by learning from the surprise of experience, leading to more successful action and thought.

5.2 Alpha

All Alpha Graphs consist merely of the *Sheet of Assertion* (a blank page), *proposition letters*, and two operators: *conjunction*, represented implicitly by writing proposition letters side by side, and *negation*, represented by a simple closed curve called a *cut*. Any well-formed part of a graph is a subgraph. The conditional is represented by two nested cuts in a '*scroll'.* Thus:

Figure 1:



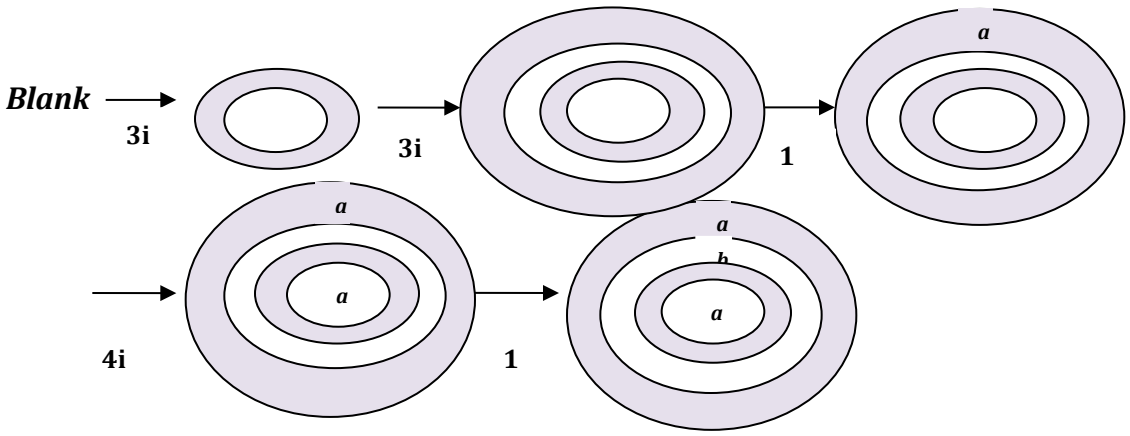
~P **P ⊃ Q** **P ∨ Q**

The proof rules depend crucially on whether the area in question is surrounded by an odd or even number of cuts (linking with 'turn-taking' in game-theoretic semantics and dialogical logic):

1. **Insertion.** In an odd-numbered depth, any subgraph may be inserted.
2. **Erasure.** In an even-numbered depth, any subgraph may be erased.
3. **Double Cut.** A pair of cuts with nothing in between may be: **i)** inserted in, or **ii)** erased from, any subgraph.
4. **Iteration / Deiteration.** Any subgraph may be: **i)** 'iterated inwards', into a cut within the scope of the area in which it appears, or **ii)** if such a subgraph might have been obtained by such iteration, it may be erased.

John Sowa argues that Peirce's system is especially elegant, by proving Frege's first axiom, a ⊃ (b ⊃ a), in five steps:

[Figure 2:]



[Sowa 2011, 368-369]

5.3 Beta

The beta graphs add only *lines of identity* which attach to the alphabetic portion of the graphs (at as many argument slots as are required), rendering them predicates rather than propositional variables:

Figure 3:

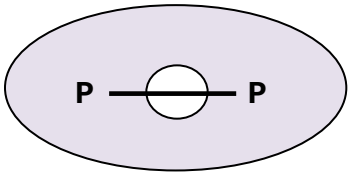
Someone loves someone

Someone loves themself

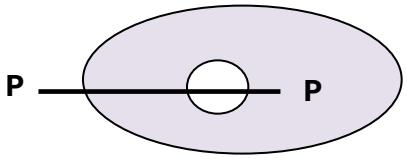
In this way the Alpha graphs are revealed as a special case of the Beta Graphs (and propositional as a special case of predicate logic) where the 'relations' have zero adicity (Peirce called these *medads*) (Sowa 2011).

Unattached ends of lines of identity serve as quantifiers, with their interactions with the cuts allowing scope distinctions. The lines of identity may themselves be negated, producing a more elegant treatment of exact number statements than standard approaches using '=':

Figure 5:



There are not two Ps



There is exactly one P

The proof rules are those of Alpha with some extra permissions concerning the line of identity.

5.4. Gamma

What is today referred to as "The Gamma Graphs" consists of a wealth of fragments of a number of systems. Peirce never finished these, although his letters to Lady Welby[8] show he was pushing hard to do so towards the end of his life. With regard to what we have now, as Roberts says, "it is occasionally difficult to be sure of just what Peirce was up to" (1973, 64). The rather cumbersome machinery he worked with includes sheets of assertion, tints, cut-outs, and bits of metal and fur. We therefore confine our discussion to a few simple steps Peirce took.

Change in Sheet of Assertion:

The biggest shift in Gamma concerns reconceptualizing the Sheet of Assertion from "….simply a universe of actual existent individuals, and…facts or true assertions concerning that universe" (CP 4.512, 1903). Now Peirce planned to use two sides of the Sheet (*recto* and *verso*), also tacking sheets together. A top sheet might represent "a universe of existent individuals" and true propositions about them. The sheets

underneath, which he cut out parts of the top sheet to expose, "represent altogether different universes with which our discourse has to do…" (CP 4.514, 1903). These shifts may themselves give rise to further shifts:

> At the cuts we pass into other areas, areas of conceived propositions which are not realized. In these areas there may be cuts where we pass into worlds which, in the imaginary worlds of the outer cuts, are themselves represented to be imaginary and false, but which may, for all that, be true.
>
> [CP 4.512, 1903]

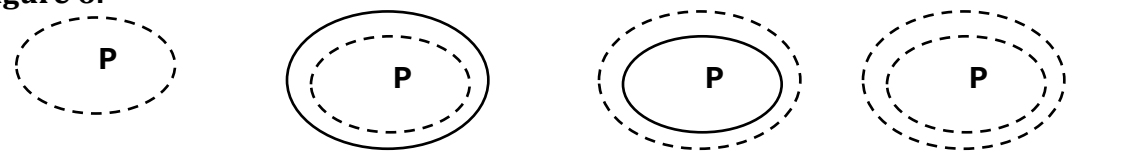
It seems that Peirce began to struggle with the two-dimensional limitations of the standard sheet of paper:

> … the entire universe of logical possibilities might be conceived to be mapped upon a surface. Nevertheless, if we are going to represent to our minds the relation between the universe of possibilities and the universe of actual existent facts, if we are going to think of the latter as a surface, we must think of the former as three-dimensional space in which any surface would represent all the facts that might exist in one existential universe.
>
> [CP 4.514, 1903]

Modal Context:

One key new symbol Peirce introduced on the Gamma graphs was the modal context, signified by a broken cut, which corresponds to "possibly not *P*"[9]:

Figure 6:



Possibly not P ◊~P | Necessarily P □P | Possibly P ◊P | Possibly necessarily P ◊□P

Peirce developed rules for this new operator which allow derivation of "*g* is true" from "*g* is necessarily true"; "*g* is possible" from "*g* is necessary"; and "Every catholic must adore some woman" from "There is a woman, whom every catholic must adore" (CP 4.566)[10]. However, the manipulations Peirce envisioned for modal logic are problematic in the state in which he left them[11].

5.4. Varieties of Modality

Systems:

A question currently of some interest amongst logicians is whether Peirce would have taken a stand on the many modal logic systems developed in the twentieth century, starting with the pioneering work of C.I. Lewis. Would he have had a preference? For instance, does the 'epistemic' nature of his understanding of modality lead him to develop a version of S4?

It seems likely that Peirce meant the EG to be more flexible than that. Just as we would not expect a proof system such as semantic tableaux to take a position on this question, neither should we expect the EG. Thus (Van Den Berg 1993) has worked out Gamma Graph systems for K, T, S4 and S5. (See also (Braüner 1998) for a simpler version of S5 which draws graph-based analogues of standard Gentzen rules). On the other hand (Pietarinen 2006a, 350) notes that Peirce's own description of the rules for the broken cut seems to rule out S5. Meanwhile, Zeman has devised a way to model accessibility relations between worlds in terms of Peirce's tinctures, using an R-G-B model of colours to place them in a partial ordering which enables S4 as its basic system (Zeman 1997b, 23; see also Zeman 1997c). Finally, it has been noted that at other times Peirce experimented with a specific sign (or 'selective') in arrow form: A  B, to represent a relation between two states of information that looks to be some form of accessibility (Pietarinen 2006a, 352; see also Roberts 1973).

Multi-Modality:

As well as its rich structure of interrelated modal systems, modal logic divides into 'kinds', such as temporal, epistemic, and deontic logic, the relationship between which is still far from well worked-out. It seems that Peirce was also planning to explore this issue in Gamma using his different colours and textures, but did not get far. Although Øhrstrøm has written, "there is…no indication in his works of a study of the ways in which various kinds of modality may interact with one another", (Øhrstrøm 1997), a beginning has been made in (Pietarinen 2006a, 350-351).

5.5. Peirce and Possible Worlds Semantics?

It has become popular in some circles to suggest that Peirce anticipates contemporary possible worlds semantics, even claiming that the latter "must be considered a rediscovery of something Peirce did a century ago" (Zeman 1997, 19; see also Pietarinen 2006a). Weak reasons to object to this include the fact that the connection of influence does not exist (logicians such as Saul Kripke and Ruth Barcan Marcus did not find inspiration in Peirce) and that Peirce hardly uses the expression "possible world", although it had been in use for a long time and he certainly knew his history of philosophy. On one of the very few occasions he does

use it, he says, "we only wildly gabble about such things" (CP 6.509).

A thorough treatment of this question might usefully separate a *logical* from a *metaphysical* issue. The former asks whether in any of his logical analyses Peirce saw fit to quantify over sets of propositions. This he surely did (as we will see in the next section). The latter asks whether he might have considered such analyses to have existential import, for instance in the sense of David Lewis (1986), who argues that a plenitude of individual possible worlds exists concretely. This is much less likely, because the individualized nature of Lewis' possibilia, and their in-principle spatio-temporal disconnection from the actual render the theory a rather extreme form of nominalism by Peirce's lights. Towards the end of his life Peirce made the intriguing remark that:

> Modality is not, properly speaking, conceivable at all, but the difference, for example, between possibility and actuality is only recognizable much in the same way as we recognize the difference between a dream and waking experience.
>
> [CP 4.553ff, c.1906]

5.6. Conditionals

How best to analyze the truth-conditions of the rich tapestry of 'if…then…' usage in ordinary language is a deep matter which continues to engage logicians and philosophers. Peirce's writings on this topic reflect its difficulty, raising a number of tangled issues which he only gradually clarified through his career. Two important and related issues[12] are whether to understand conditionals as *de inesse* or 'ordinary', and as Philonian or Diodoran.

Consequentia de Inesse

Peirce referred to material implication as a consequence *de inesse*. He notes that "[s]uch a proposition is altogether true or altogether false" (CP 4.376), essentially concerning only what happens here and now. So for instance, if I assert "If I eat ice-cream then grass is red", and I am not eating ice-cream, then my assertion is true. As noted by bewildered first-year logic students worldwide, this seems to miss an important problem. Peirce here can be seen as entering the fray that later would play out in the work of Russell, C.I. Lewis, Quine, and Barcan Marcus.

Peirce contrasted *de inesse* conditionals with what he sometimes called 'ordinary' conditionals, for their greater approximation to common usage, and sometimes called 'hypotheticals', in homage to Kant. The latter pertain to what happens not just here and now, but across some "range of possibilities". This allows the ordinary conditional to be "sometimes" or "possibly" true across the set. Such quantitative judgments require quantifiers, and as others have noted (Zeman 1997a), it was when Peirce developed his own quantification theory in 1885 that he

made progress defining the ordinary conditional[13].

Philonian or Diodoran?

At the same time Peirce was considering an issue which traces back to a dispute in Stoic logic between Philo and his teacher Diodorus over whether the antecedent of a conditional has existential import:

> Philo held that the proposition "if it is lightening it will thunder" was true if it is not lightening or if it will thunder and was only false if it is lightening but will not thunder. Diodorus objected to this.

Peirce notes his own preferences:

> Most of the strong logicians have been Philonians, and most of the weak ones have been Diodorans. For my part, I am a Philonian; but I do not think that justice has ever been done to the Diodoran side of the question.
>
> [NE 4,169, 1898]

In order to "fit him [the Diodoran] out with a better defense than he has ever been able to construct for himself….", Once again, Peirce draws on a quantified analysis of the conditional: "…namely, that in our ordinary use of language we always understand the range of possibility in such a sense that in some possible case the antecedent shall be true" (NE 4, 169).

Peirce's mature theory of the conditional thereby resolves both issues at once. It goes beyond the material conditional in considering not just the here and now but a range of possible scenarios. It answers the Diodoran worry insofar as in order to decide the truth of a conditional it evaluates only possible scenarios where the antecedent is *true*. This would seem to have anticipated to some degree the truth-functions for the counterfactual conditional in for example (D. Lewis 1973), although what Peirce would have had to say about the problem faced by Lewis of defining 'nearness' across the evaluation set is an interesting question.

6. Peirce's Modal Metaphysics

Peirce was a naturalist in the broad sense that he held that the only way to find out what there is in the world is scientific inquiry (in a broad sense of science which means any inquiry that takes experience seriously). But he rejects any naturalism that gives ontological priority to matter or physicality—like all the pragmatists, he wants to consider whether value, generality, necessity, chance, etc. might be part of the natural world. His answer is 'yes' to all of the above.

In phenomenology and logic Peirce showed the *irreducibility* of the three categories (as concepts, to one another). In metaphysics he considers their *reality*.

Since the three have been shown to be mutually irreducible, it is possible to affirm or deny the reality of all of them independently. Peirce dubbed the doctrine of real Firstness Tychism, and real Thirdness Synechism. He devoted much less time to explicitly defending real Secondness – presumably because it is generally taken for granted by philosophers – but at one point refers to it as Anancism (CP 6.202, 1893).

6.1 Tychism

Recall that Firstness represents 'what comes first'. Peirce's Tychism holds that there "is an element of real chance in the universe." A key paper here is "The Doctrine of Necessity Examined", published in 1892. Here Peirce examines and rejects "the common belief that every single fact in the universe is precisely determined by law" (CP 6.36, 1892), which he calls *necessitarianism* (today's 'determinism'). He does not deny that there are natural laws, but holds that they are violated in infinitesimal degree on sporadic occasions.

He considers a series of arguments for necessitarianism and rejects them all. First is the claim that it is a "postulate" of scientific reasoning. This is incorrect because in science, to 'postulate' a proposition "is no more than to hope it is true". He then considers whether necessitarianism may be established empirically ("by observation of nature"). This is dubious because of the claim's apparent exactitude (deviations from natural law are *zero*), whereas scientists know that the strictest measurements "…yet fall behind the accuracy of bank accounts" (CP 6.44, 1892). He also considers the argument that chance is *inconceivable*, to which he answers, essentially, 'Try harder'. Or perhaps chance is *unintelligible,* since the hypothesis of indeterminism is not explanatory. Here Peirce argues that on the contrary, real chance explains the growth of variety in the Universe in a way mechanical law never could:

> The ordinary view has to admit the inexhaustible multitudinous variety of the world, has to admit that its mechanical law cannot account for this in the least, that variety can spring only from spontaneity, and yet denies without any evidence or reason the existence of this spontaneity, or else shoves it back to the beginning of time and supposes it dead ever since.
>
> [CP 6.59, 1892]

It is worth noting a certain subtlety of this view within Peirce's information-state analysis of modality. Recall that an important sense of modality for Peirce is *substantial modality*: what is necessary or possible according to "everything now existing…together with all their consequences". Peirce's Tychism holds that there is no *present* substantial contingency, but there *is future* substantial contingency, as laws of nature 'swerve' across time (Lane 2007, 559). This differs interestingly from

today's 4D, 'tenseless' metaphysics of contingency.

5.2 Anancism

In §3.1 Secondness was defined in terms of 'action and reaction'. This category gives rise to a kind of necessity that is manifested in brute forcefulness (for instance between billiard balls, pace Hume). Peirce tosses brickbats at other philosophers – most notably Hegel – for not recognizing real Secondness – which he sometimes refers to as "the outward clash".[14]

5.3 Synechism

We saw that Thirdness pertains to our seeing a yellow chair *as yellow*: a general concept that in order to honor its meaning *must* be extended to all relevantly similar instances. Peirce's argument for real Thirdness attempts to show that we live in a universe which is *genuinely intelligible* – that the ways in which we extend our predicates give us at least some predictive power. In his 1903 Harvard lectures, Peirce made a particularly vivid and dramatic case for this. He held a stone in the air in front of his audience, and challenged them to admit that they knew that when he dropped it the stone would fall, rather than flying upwards:

> Suppose we attack the question experimentally. Here is a stone. Now I place that stone where there will be no obstacle between it and the floor, and I will predict with confidence that as soon as I let go my hold upon the stone it will fall to the floor. I will prove that I can make a correct prediction by actual trial if you like. But I can see by your faces that you all think it will be a very silly experiment.
>
> [CP 5.93, 1903]

He pointed out that, "It would be quite absurd to say that...I can so peer into the future merely on the strength of any acquaintance with any pure fiction" (CP 5.94, 1903). He concluded that his audience possessed knowledge of an "active general principle", exerting influence on physical objects. Peirce thought that such a commitment to general principles should be understood as a form of scholastic realism or realism about universals, and that the kind of influence exerted by such universals was different from the 'efficient causal biff' manifested by one individual billiard ball striking another (although of course the two co-exist: both real Secondness and real Thirdness are required to build a full picture of reality).

6. Conclusion: Issues for Further Work

It should be clear from this snapshot of the many things Peirce said about the many aspects of necessity that there is grist for a number of mills. Epistemologists might want to explore Peirce's iconic approach to knowledge of necessity, since it offers an

interesting alternative to the current landscape. It might allow us to move beyond a certain tradition (deeply influenced by Hume, and then the logical positivists) of throwing one's hands up at modality as a major sceptical problem; and either taking a eliminativist or projectivist stance towards it, or embracing mysterious metaphysical realisms such as (D. Lewis 1986).

Formal logicians might to want to continue the work of excavating – using modern tools and conceptions – both Gamma graphs and Peirce's analysis of the conditional. The task is not easy because Peirce's logic is so finely sifted with the rest of his philosophy, but could be hugely rewarding. Metaphysicians might want to explore Peirce's denial of necessitarianism, especially in the light of certain recent developments in physics. Synechism, too, might be worth a new look, especially by those who are frustrated with the 20th's Century's founding of metaphysics on set theory, since Peirce claimed that true "generals" or universals are not reducible to the extension of any set.

It is unsurprising that the two early 20th century philosophers most heavily influenced by Peirce were also philosopher-logicians – C.I. Lewis and Frank Ramsey[15]. Ramsey's view of laws as rules with which we meet the future is explicitly drawn from Peirce. Lewis attributes to Peirce many ideas he used in founding modal logic for the twentieth century (e.g. C. I. Lewis and Langford, 1959). And like Peirce, Lewis is first and foremost a pragmatist. We have a certain amount of freedom in our choice of a set of logical rules – freedom within the constraints of hard facts about deducibility that need to be captured by a logic. Edwin Mares in this volume reminds us that Lewis took a priori truths to be true partly because of stipulations we make and that logical truths are both a priori and analytic. Necessity is thus to be interpreted as an epistemological operator.

But at the same time we have seen that Peirce is also a realist about modality. In this paper we offer a brief overview of how he sought to combine the two into one harmonious position. The attempt is complex, and we have had to merely gesture at it here. But it is important to see the magnitude of Peirce's ambition. He wanted to put forward an account of necessity on which our actual practices are taken seriously, which can be fully formalized and clarified, and which allows for real chance, direct forceful interactions between individuals, and intelligible patterns across reality as a whole. He wanted to combine ideas from current anti-realist and realist theories with the original thought that 'generals' are real, but not existent. Some may think that Peirce was reaching for the impossible, but here we have tried to go some distance towards showing how he might have succeeded.

A Note on Citations

In-text references to the works of Charles Sanders Peirce are in the following form:

The Collected Papers of Charles Sanders Peirce: CP n.m, date; where n is the volume number and m is the paragraph number.
The Writings of Charles S. Peirce: A Chronological Edition: W n, m, date; where n is the volume number and m is the page number.
The Essential Peirce: EP n, m, date; where n is the volume number and m is the page number
The New Elements of Mathematics: NE n, m, date; where n is the volume number and m is the page number.
References to the Charles S. Peirce Papers at Houghton Library, Harvard University are cited as MS n, m, date; where n is the manuscript number and m is the page number.

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[1] "On the Algebra of Logic: A Contribution to the Philosophy of Notation" (1885).

[2] See for instance, "Description of a Notation for the Logic of Relatives, Resulting from an Amplification of the Conceptions of Boole's Calculus" (1870).

[3] Although John Sowa writes, "Peirce's existential graphs…are the simplest, most elegant, and easiest-to-learn system of logic ever invented" (Sowa 2011).

[4] See Legg (2012 and 2014) for a discussion of how Peirce's diagrammatic proofs differ from "stepwise" proofs.

[5] See Quine (1934) for the expression of this opinion.

[6] The material still awaits full publication. The first volume of Peirce's chronological papers appeared in 1982. The most recent appeared in 2009, with a projected twenty-three more to come, including much of the Existential Graphs.

[7] For a lucid introduction to this area of logic, see (Rückert 2001). For a rich treatment of its philosophical implications, see (Marion 2009). One of the first to develop the idea that Peirce had covered this ground was (Hilpinen 1982).

[8] See the letters, compiled in Hardwick and Cook (1977).

[9] The exposition below is drawn from (Pietarinen 2006a, 349).

[10] These transformations are well explicated in (Øhrstrøm 1997).

[11] For one thing, they allow one to prove that for any *q*, it is necessary that *q*. He worried about the rule which allowed one to infer 'There is a man x, such that if x is bankrupt, then x must commit suicide' from 'There is a man x and a man y, such that if x is bankrupt, then y must commit suicide'. Peirce in 1906 called this permission "quite out of place and inacceptable" (CP 4.580).

[12] Some commentators run these issues together, e.g. (Zeman 1997a). However a good *prima facie* case for separating them is made in (Lane 2007, 556n).

[13] Peirce himself admits this at CP 2.349. See CP 3.374 for an early discussion drawing on algebraic notation.

[15] See Misak (forthcoming).