INFLUENCES ON PERCEIVED
UNDERSTANDINGS AND HELP SEEKING IN
THE MATHEMATICS CLASSROOM

Jennifer Palisse

Submitted in fulfilment of the requirements for the degree of
Masters of Arts

Deakin University
December, 2016
I am the author of the thesis entitled

_Influences on perceived understandings and help seeking in the mathematics classroom_

submitted for the degree of Masters of Education (Research)

This thesis may be made available for consultation, loan and limited copying in accordance with the Copyright Act 1968.

'I certify that I am the student named below and that the information provided in the form is correct'

**Full Name:** Jennifer Palisse

**Signed:** Signature Redacted by Library

**Date:** 6/01/2017
DEAKIN UNIVERSITY
CANDIDATE DECLARATION

I certify the following about the thesis entitled (10 word maximum)

Influences on perceived understandings and help seeking in the mathematics classroom

submitted for the degree of Masters of Education (Research)

a. I am the creator of all or part of the whole work(s) (including content and layout) and that where reference is made to the work of others, due acknowledgment is given.

b. The work(s) are not in any way a violation or infringement of any copyright, trademark, patent, or other rights whatsoever of any person.

c. That if the work(s) have been commissioned, sponsored or supported by any organisation, I have fulfilled all of the obligations required by such contract or agreement.

d. That any material in the thesis which has been accepted for a degree or diploma by any university or institution is identified in the text.

e. All research integrity requirements have been complied with.

'I certify that I am the student named below and that the information provided in the form is correct'

Full Name: Jennifer Palisse

Signed: [Signature Redacted by Library]

Date: 06/01/2017
Abstract

What students might think it means to understand mathematics may not necessarily match what their teachers think it means to understand. A greater level of insight is needed so that teachers can better recognise what students might be thinking when they go about ‘understanding’ their work. To address this issue, this study explored whether help seeking might be an observable learning behaviour from which underlying student interpretations of understanding could be inferred.

A literature review was conducted which identified several ways students identify understanding. These were drawn from literature on mathematical beliefs when problem solving, conceptions of learning, conceptions of understanding, and literature on epistemological beliefs related to mathematics. Common themes regarding how students might recognise understanding were identified: a) solving a problem quickly; b) memorising all the steps; c) making connections between ideas; d) explaining ideas to others; e) finishing a problem; and f) understanding as equivalent to learning. An additional two student perceptions of understanding were identified from the data: g) understanding is an emotion; and h) understanding occurs by recognising patterns.

Help seeking behaviours were differentiated into two general groups (Newman, 2008): a) adaptive help seeking which involves asking for help when necessary, asking for hints and explanations, and applying the help in such a way that it improves independent problem solving; b) non-adaptive help seeking which involves asking for help when unnecessary, and directly asking either for answers or the next step needed to solve the problem. The study then sought to explore associations between help seeking and what students think it means to understand.
Data was collected from a Year 10 advanced Mathematics class. I acted as both the teacher and researcher. Data was collected for four students and detailed case studies were developed for two of the students. Activity from the other two students was reported on when it contributed to the exploration of relationships between understanding and help seeking.

Students took part in four tasks that were filmed in the classroom with a follow-up individual interview. This provided data on students’ help seeking behaviours where it was noted whether the help they sought was necessary (from the teacher/researcher perspective), whether they asked for explanations or answers, and how they applied the help they received. Interviews also provided data on students’ perceptions of understanding where students were asked to discuss their ideas on what it means to understand.

It was found that the nature of the relationship between students’ perceptions of understanding and help seeking behaviour were not directly related. Rather, it was found that the two constructs were found to be mediated via student learning goals. Because the original focus of this study did not lead to a direct relationship between help seeking and perception of understanding, other relationships were also explored throughout this study and the focus was shifted to also investigate a possible relationship between students’ level of understanding as identified by the researcher and help seeking behaviours (e.g., relational or instrumental understanding; Skemp, 1976). A relationship was found suggesting that students who use adaptive seeking are more likely to demonstrate relational understanding while students who use non-adaptive help seeking are more likely to use instrumental understanding.
Acknowledgements

I wish to express my sincere gratitude and appreciation to my supervisor Dr. Gaye Williams for the continual support of my Masters study. Your patience, ongoing support, and helpful suggestions have been a great motivation to go on and continue even more research.

Besides my supervisor, I would like to also thank my team-teaching partner who so willingly agreed for the research to take place in our mathematics classroom. Your calming nature and patience during times of filming were immensely appreciated. Thank you to my school principal who was irrefutably supportive throughout my research journey.

Grateful thanks are due to all the students who took part in this study. Their enthusiasm to take part in interviews was infectious and they graciously accepted being filmed and recorded while working on challenging tasks. Thank you all.

Lastly, my sincere thanks to my partner, who had the joy of living with me throughout this process. He was nothing but supportive throughout my entire research journey, and for that, I thank him.
# Table of Contents

Abstract.................................................................................................................................i

Acknowledgements ..................................................................................................................iii

Table of Figures ..........................................................................................................................vii

Table of Tables ..........................................................................................................................ix

Chapter 1  Introduction .............................................................................................................. 1

1.1  INTRODUCTION .................................................................................................................. 1

1.2  RATIONALE AND RESEARCH QUESTION ................................................................ 1

1.3  FORMAT OF THESIS .......................................................................................................... 6

Chapter 2  Literature Review ...................................................................................................... 7

2.1  HELP SEEKING .................................................................................................................. 7

2.1.1  Necessary ....................................................................................................................... 8

2.1.2  Non-adaptive help seeking .......................................................................................... 9

2.1.3  Adaptive help seeking ................................................................................................. 10

2.1.4  Non-adaptive help avoidance ...................................................................................... 11

2.1.5  Adaptive help avoidance ............................................................................................ 12

2.1.6  Summary ....................................................................................................................... 12

2.2  STUDENT PERCEPTIONS OF UNDERSTANDING ....................................................... 15

2.2.1  Identifying different ways students perceive understanding ........................................ 16

2.2.2  Summary of different perceptions of understanding .................................................. 29

2.3  SUMMARY ......................................................................................................................... 30

Chapter 3  Methodology ........................................................................................................... 31

3.1  INTRODUCTION ................................................................................................................. 31

3.2  RATIONALE ....................................................................................................................... 31

3.3  THE RESEARCH SETTING ................................................................................................. 35
3.3.1 Subjects .................................................................................................................. 36
3.3.2 Case studies ........................................................................................................... 37
3.3.3 The lesson context ................................................................................................. 38

3.4 DATA COLLECTION STRATEGIES ........................................................................ 39
3.4.1 Timeline .................................................................................................................. 39
3.4.2 Questionnaire ......................................................................................................... 40
3.4.3 Case studies ........................................................................................................... 42
3.4.4 Classroom observations ........................................................................................ 43
3.4.5 Tasks ....................................................................................................................... 44
3.4.6 Interviews .............................................................................................................. 48

3.5 OPERATIONALISING HELP SEEKING ...................................................................... 49
3.6 OPERATIONALISING STUDENT PERCEPTIONS OF UNDERSTANDING .................. 51
3.7 ETHICAL CONSIDERATIONS .................................................................................. 51

Chapter 4 Results............................................................................................................. 53

4.1 ADDITIONAL STUDENTS ....................................................................................... 53
4.2 TRANSCRIPT NUMBERING CONVENTION .............................................................. 57
4.3 CASE-STUDY: PIPPA ............................................................................................... 58
  4.3.1 Pippa is a hesitant student ................................................................................... 58
  4.3.2 Pippa’s perceptions of understanding ................................................................. 60
  4.3.3 Pippa’s help seeking behaviours ....................................................................... 80
4.4 CASE-STUDY: LILLY ............................................................................................... 97
  4.4.1 Lilly’s perceptions of understanding .................................................................... 97
  4.4.2 Lilly’s help seeking behaviours ........................................................................... 120
4.5 ADDITIONAL STUDENTS CONSIDERED FOR THIS STUDY ................................ 131
  4.5.1 Aaron .................................................................................................................. 132
  4.5.2 Kipp .................................................................................................................... 133

Chapter 5 Comparisons between cases and discussions .............................................. 135

5.1 STUDENT PERCEPTIONS OF UNDERSTANDING .................................................. 136
5.2 RESEARCHER IDENTIFIED UNDERSTANDING AND CORRESPONDING HELP SEEKING BEHAVIOURS … 142

5.2.1 Pippa’s help seeking and researcher identified understanding …………………… 142

5.2.2 Lilly’s help seeking and researcher identified level of understanding ……………… 149

5.2.3 Researcher identified understanding and help seeking for all four students ………… 153

5.3 STUDENT PERCEPTIONS OF UNDERSTANDING, HELP SEEKING, AND GOALS ……… 157

5.3.1 Self-regulated learning ……………………………………………………………………… 158

5.3.2 Pippa’s perceptions of understanding, help seeking, and goals ……………………… 159

5.3.3 Lilly’s Perceptions of understanding, help seeking, and goals ……………………… 165

5.3.4 Comparing Pippa’s and Lilly’s perceptions of understanding, help seeking, and goals ……… 173

Chapter 6 Conclusions ………………………………………………………………………………… 180

6.1 OVERALL CONCLUSION …………………………………………………………………………… 180

6.2 LIMITATIONS OF STUDY ………………………………………………………………………… 182

6.3 IMPLICATIONS FOR TEACHING AND RESEARCH …………………………………………… 183

6.4 FUTURE RESEARCH QUESTIONS ………………………………………………………………… 184

References …………………………………………………………………………………………… 186

Appendices …………………………………………………………………………………………… 197
## Table of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Decision tree identifying different forms of help seeking that will be used in this study.</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>Open-lid problem: Make the largest rectangular box by removing four shaded corner squares.</td>
<td>69</td>
</tr>
<tr>
<td>3</td>
<td>Position-time graph for which Pippa is trying to draw the corresponding velocity-time graph.</td>
<td>72</td>
</tr>
<tr>
<td>4</td>
<td>Pippa’s researcher-identified level understanding and influences effecting her level of understanding.</td>
<td>77</td>
</tr>
<tr>
<td>5</td>
<td>Velocity-time graph Pippa and Ruby worked on together.</td>
<td>90</td>
</tr>
<tr>
<td>6</td>
<td>Pippa's help seeking behaviours and influences on her behaviour.</td>
<td>96</td>
</tr>
<tr>
<td>7</td>
<td>Unit circle with right-angled triangle used to help find the values of sine and cosine by Lilly.</td>
<td>109</td>
</tr>
<tr>
<td>8</td>
<td>Lilly’s researcher-identified level understanding.</td>
<td>117</td>
</tr>
<tr>
<td>9</td>
<td>Lilly’s help seeking behaviours.</td>
<td>131</td>
</tr>
<tr>
<td>10</td>
<td>Relationship between Pippa’s help seeking and researcher identified understanding.</td>
<td>145</td>
</tr>
<tr>
<td>11</td>
<td>Relationship between Lilly’s help seeking and researcher identified understanding.</td>
<td>150</td>
</tr>
<tr>
<td>12</td>
<td>Comparison of all four case-study students and their observed understanding with help seeking.</td>
<td>155</td>
</tr>
<tr>
<td>13</td>
<td>Relationship between Pippa’s help seeking, perceptions of understanding, goals, and means.</td>
<td>165</td>
</tr>
</tbody>
</table>
Figure 14 Relationship between Lilly's help seeking, perceptions of understanding, goals, and means ................................................................. 171

Figure 15 How Lilly's choice of help seeking behaviours are influenced ......................................................... 172

Figure 16 Perceptions of understanding, emotions, and their relationship with self-regulated learning ................................................................. 178

Figure 17 Summary of how perceptions of understanding are associated with help seeking behaviours ................................................................. 179
Table of Tables

Table 1  Four quadrants of help seeking summarising terms used by different researchers .......................................................... 13

Table 2  Summary of the activities each student took part in .......................................................... 43

Table 3  Summary of Pippa’s views on understanding including what counts as understanding, what does not count as understanding, and indications that understanding has occurred .......................................................... 62

Table 4  Summary of Lilly’s reported views on understanding and observed understanding .......................................................... 100

Table 5  Comparison of student perceptions of understanding identified in the Literature Review .......................................................... 138

Table 6  Student perceptions of understanding emerging from the data not identified in the Literature Review .......................................................... 139

Table 7  Summarising Pippa’s researcher identified understanding and corresponding help seeking behaviour during three tasks .......................................................... 143

Table 8  Summarising influences on Pippa’s help seeking behaviours and researcher identified level of understanding .......................................................... 148

Table 9  Summary of influences on Lilly’s researcher identified level of understanding, help seeking behaviours .......................................................... 152

Table 10  Summary of Pippa’s perceptions of understanding, help seeking behaviours, and corresponding learning goal .......................................................... 160

Table 11  Summary of influences on Pippa’s choice of learning goal .......................................................... 163

Table 12  Summary of Lilly’s perceptions of understanding, help seeking behaviours, and means to achieve her learning goal .......................................................... 168
Chapter 1  Introduction

1.1  Introduction

This study investigates the relationship between students’ help seeking behaviours and how they perceive what it means to understand while engaged in problem solving activities in the mathematics classroom. Data was collected in a coeducational class of Year 10 students studying the Year 11 VCE subject Mathematical Methods Units 1/2 in Australia. The subject includes the study of polynomials functions and associated algebraic techniques (e.g., finding axial intercepts and turning points), an introduction to calculus, and the study of probability and associated counting methods. I acted as both the classroom teacher and researcher. A second teacher was also present in the classroom because this particular class utilised a team teaching model. We taught in a team-teaching arrangement, that is, two teachers teaching a class of 48 students at the same time.

1.2  Rationale and research question

My decision to undertake formal research in the area of student help seeking and their views on understanding came from my own observations as a teacher. As a teacher, my focus was often on building resilience and perseverance with my students. I achieved this by providing students with challenging tasks that would require an entire lesson to solve (75 minutes). Tasks were generally designed in such a way as to allow students to develop their own understanding or insight for certain concepts, or allow students
to generate their own algorithms and formulas for particular problems. For example, rather than stand at the front of my class and tell my students how to do long division by showing them a worked example, I might instead ask them to solve the following problem:

A chocolate factory needs to evenly divide a quantity of chocolate to be shipped off to 5 different supermarkets. Blocks of chocolate are packed into boxes and boxes are then packed into crates. Each crate contains 10 boxes and each box contains 10 blocks of chocolate.

If the factory has a total of 6 crates, 8 boxes and 7 blocks of chocolate to evenly distribute, how many blocks of chocolate will each of the 5 supermarkets receive?

Students would be given this problem to solve without having been shown the traditional algorithms used for long division. Instead, the intention of the problem was for students to work collaboratively on the task so that they were able to generate their own algorithm for long division (For further detail on this task, see Palisse, 2012).

By using problems in this way, I found many of my students developed a sense of pride in their mathematical abilities because they developed a sense of ownership in their knowledge as they discovered their own algorithms to solve the problem. It was the students themselves who worked out and developed their own algorithms for long division rather than having to have the teacher tell them the algorithm and then show them how to use it.

As I continued using tasks in this way, I noticed how much students enjoyed being given the opportunity to work things out for themselves. Often if I was about to put an answer up on the board, students in my class would scream and yell at me to stop and give them more time because they wanted to solve it for themselves.

However, this was not the case for all students. Some did not enjoy this style of learning. There were often two or three students in my class who would say to me they wanted structured notes and worked
examples to better ‘understand’. They were sometimes resentful that I never just put up the formula on the board and showed them how to use it.

After some time, I noticed that those students who did not enjoy working on challenging problems all seemed to display similar styles of help seeking. Help seeking behaviours denote what students do in the classroom if they feel they need assistance and support. This may take the form of a verbal question, either to another student or the teacher, or may be non-verbal (e.g., looking over someone’s shoulder at their exercise book). Help seeking that is considered beneficial to learning might involve a student asking for help after they have truly struggled with a problem for some time, and asking questions with the intention of making sense of the problem. Help seeking that would not be considered appropriate might involve a student asking for help the second they start working on a problem, and asking questions with the intent of either being told the answers or the next step.

Students who did not enjoy my approach to teaching tended to display help seeking that would not be considered appropriate. They sometimes asked for help the minute they started working on a problem telling me “I don’t get it. How do I start?” or “What do I do?” When I provided these students with my version of help, that is asking scaffolded questions that would provide the student with the minimum amount of help necessary so that they could continue working independently without further assistance, they were often frustrated with me as I would never provide them directly with the steps or formulas they wanted and never tell them what they explicitly needed to do. Some might roll their eyes at me and ask with a sigh “Can’t you just tell me what to do?” or they might instead ask “But what’s the formula I need to solve this?”

I began to question why some students might not enjoy learning in my class. I began to ponder whether these students held a different definition for what it means to understand mathematics than I had. Might students who use appropriate help seeking see understanding in terms of sense making and therefore ask questions that help in solidifying their comprehension of the problem while students who use
inappropriate help seeking instead view understanding as related only to being able to answer questions correctly? These questions formed the impetus for this study.

There exists a phenomenon known as “the illusion of knowing, thinking you know when you really don’t know” (Schommer, 1998, p. 117). Students may think they understand mathematics when it does not necessarily match what I, as their teacher, might mean by ‘understand’ (Skemp, 1979). Should a teacher tell their students ‘It is really important that you understand this’, some students, for example, may interpret this to mean ‘I need to memorise this’ or ‘I need to get the answer right’. The issue here is that if such a student could answer the question correctly then they would claim that they have indeed wholeheartedly understood as they have satisfied their definition of understanding. I, as their teacher, on the other hand, would likely claim they have not demonstrated any understanding at all.

Trying to change students’ minds on what it means to understand can be challenging. In the words of Skemp (1979):

> if pupils can get the right answers by the kind of thinking they are used to, they will not take kindly to suggestions that they should try for something beyond this (p. 10).

If we, as teachers, feel that it is important for students to develop a meaningful understanding of the mathematics they encounter, how might we go about changing their perception of what it means to understand? A greater level of insight is needed so that teachers can better recognise what students might actually be thinking when they go about ‘understanding’ their work.

The idea that some students attach a different meaning of understanding from their teachers is by no means new. For example, Skemp (1978) differentiates between two types of understanding: *relational* and *instrumental* understanding. Relational understanding is “knowing both what to do and why”, while instrumental understanding is using “rules without reason” (p. 9). A student with instrumental understanding might be able to follow the steps and procedures to obtain the correct answer and do the question. A student with relational understanding who can also correctly follow the steps and procedures,
but knows why it works. In both cases, students would argue that from their perspective they do understand mathematics. Yet Skemp claims that only relational understanding signifies real understanding.

A greater awareness of how students define understanding is needed. Identifying different ways in which students might identify what it means to understanding in the context of mathematics is one of the aims of this study. This will provide teachers with greater awareness into how students might potentially interpret the phrase ‘It’s important you understand your work’. Studies on students’ perceptions of understanding have identified that some students view understanding as drawing out connections between ideas (Burns, Clift, & Duncan, 1991), as equivalent to memorisation (Marton, Dall’Alba, & Tse, 1993), and equivalent to learning (Purdie, Hattie, & Douglas, 1996). However, such studies did not take place in the mathematics classroom leaving a gap in our understanding of how students perceive understanding within mathematics. This is a gap this study hopes to address.

Secondly, it would be useful to be aware of different behaviours that may act as an indication for different perceptions of understanding. For example, if a student uses memorisation as a revision strategy for a test, can it be automatically assumed that this student thinks ‘If I have memorised all the steps, then I understand’? Additionally, are there specific behaviours that teachers can use as indicators that students hold certain views of understanding? For this study, only one specific behaviour will be explored in detail: help seeking.

As such, the following research questions are proposed:

Is there a relationship between students’ help seeking behaviour and their perception of what it means to understand in the mathematics classroom?

If a relationship is found, what is the nature of the relationship between help seeking and perceptions of understanding?
1.3 Format of thesis

The following sections provides a Literature Review which situates the current study in terms of existing literature by unpacking different forms of help seeking behaviours as well as identifying different ways students might identify what it means to understand. Chapter 3 provides the methodology used for this study. Chapter 4 discusses the results of this study by examining two case-study students in detail, Pippa and Lilly. A cross case comparison is then provided in Chapter 5 along with a discussion of the results. Finally, the formation of the overall conclusions and future research questions for this study are provided in Chapter 6.
Chapter 2  Literature Review

This study explores whether there exists a relationship between students’ help seeking behaviour and how they perceive understanding within the context of mathematics. What follows is a review of relevant literature which is segmented into two sections. The first section will provide an overview of help seeking and identifies different types of help seeking behaviours. The second section provides examples of ways in which students might view and identify understanding within the context of the mathematics classroom.

2.1 Help Seeking

In this section, different types of help seeking are identified. The categories drawn out in this section will be used to help identify different help seeking behaviours in the students who took part in this study.

Newman (2008) differentiates between four types of help seeking:

a) Adaptive help seeking: asking for help, and the help is necessary;

b) Non-adaptive help seeking: asking for help, when it is not necessary;

c) Non-adaptive avoidance of help seeking: not asking for help, when it is necessary;

d) Adaptive help-avoidance: not asking for help and it is not necessary.

Newman argues that all research on help seeking is based on these four paradigms which forms the layout for this section. Different interpretations from the literature are provided for each type of help seeking,
after which a framework is provided which will be used to help identify different types of help seeking in this study. Before identifying each of the four types of help seeking, the term *necessary* is defined, as this term will be significant when classifying different forms of help seeking.

### 2.1.1 Necessary

Sometimes students may seek help when it is not necessary. For example, students may ask questions simply to get attention from their teacher or peers while others may seek help from their teacher without attempting the question on their own (Newman, 2008).

Whether help seeking is considered necessary or not will depend on one’s perspective. Sometimes a student may ask for help when they feel it is necessary, but not when the teacher feels it is necessary (Ryan, Patrick, & Shim, 2005). For example, sometimes students may worry about looking ‘stupid’ in front of other students. Such students may feel reluctant to attempt a problem in case they answer it incorrectly and consequently ask questions immediately upon starting the problem (Webb & Mastergeorge, 2003). This type of help seeking is considered unnecessary from a teacher/researcher perspective because such students are not considered to have been sufficiently stuck and might likely have been able to continue working on the problem without immediate help. On the other hand, lack of progress after “diligent attempts” to solve a problem, or an unsuccessful attempt with an incorrect answer, would justify seeking help, and be considered necessary from the help seeker’s perspective (Nelson-Le Gall, Kratzer, Jones, & DeCooke, 1990, p. 2).

In line with Nelson-Le Gall (1987), help seeking for this study will be considered necessary if a student is unable to continue working on a problem on their own. For the purposes of this study, it will be the
teacher’s/researcher’s perspective rather than the student’s perspective that will be used to judge what counts as necessary/unnecessary help seeking.

2.1.2 Non-adaptive help seeking

Students who ask for help the minute they encounter difficulties is called dependent help seeking (Ryan et al., 2005). In some cases, students may ask for help with the goal of obtaining the answer or the steps required to complete a problem so that it may be finished either more easily or faster. Such students tend to be more interested in completing the problem, rather than understanding the process (Nelson-Le Gall, 1986).

Nelson-Le Gall (1986) identified a similar help seeking style which she called executive help seeking. Executive help seeking involves asking for help, even when the student may not need it, asking directly for answers rather than hints, and in general preferring to have someone else solve the problem for them. Newman and Schwager (1995) argue that asking for hints can be considered inappropriate if the student has sufficient prior knowledge related to the task, but directly asking for answers can be considered appropriate if there is a lack of knowledge. Therefore, what might be considered adaptive help seeking for one student may be considered non-adaptive for a different student asking the exact same question.

For this reason, the context will need to be taken into consideration for this study.

Webb, Farivar and Mastergeorge (2002) define two types of questions: specific and general. General questions include vague statements, such as ‘I don’t get it’ or ‘How do you do it?’, while specific questions identify exactly what the student needs help with, such as ‘Why does the graph move to the left?’ Webb et al. found that non-adaptive help seekers tend to ask only general questions. For this study, asking directly for answers and asking general questions are considered non-adaptive help seeking behaviours.
Goos (2002) considers help seeking to be non-adaptive if the student passively accepts unhelpful suggestions or ignores possibly useful suggestions from other students. Webb et al. (2002) argue for the help seeking to be considered adaptive, students must appropriately apply the help they receive. The student must firstly understand the explanation given and secondly must be given an opportunity to apply the given explanation to the problem at hand. Secondly, they need to make an attempt at solving the problem themselves. For example, a student who is given an explanation and replies with ‘Oh, I get it now’, and does not then complete the problem themselves, is considered to be a passive participant in the learning process and considered to be a non-adaptive help seeker. Therefore, it is not just the types of requests for help that are of significance for this study but also the way in which students choose to either use or ignore offered help. If a student chooses not to use appropriate suggestions in a way that improves their problem solving, then the help seeking will be considered non-adaptive, even if such a student asked for help when necessary and asked for hints rather than answers.

2.1.3 Adaptive help seeking

Help seeking is considered to be adaptive if the student both asks for help and the help is necessary. It includes asking for help in order to learn independently, not simply to obtain the correct answer (Butler & Neuman, 1995; Newman, 2002; Ryan & Pintrich, 1997). Ryan, Patrick, & Shim (2005) define appropriate help seeking as asking for help only when truly necessary, but without being excessively reliant on the help giver. For help seeking to be considered appropriate, questions are limited to the minimum amount needed to allow the student to solve the problem themselves and might involve asking for hints about the problem, additional examples of similar problems, or clarification.
Butler (1998) instead uses the term *autonomous* help seeking. Like appropriate help seeking, autonomous help seeking includes asking for help when necessary and asking for hints and clarifications rather than answers. Butler then includes as part of her definition of autonomous help seeking that help seeking then results in an improved ability to solve subsequent problems independently.

With regards to types of question asked, Webb et al. (2002) found that adaptive help seekers were able to form specific questions that clearly identified for the help giver what they did not understand. They also found that adaptive help seekers often use general questions as a starting point. As they persist in asking for help, their style of questions became more specific.

2.1.4 Non-adaptive help avoidance

Behaviour exhibited by students who are in need of help, but choose not to seek it, is labelled *non-adaptive avoidance* (Newman, 2008). These are students who could benefit from receiving help but may avoid it for reasons such as the fear of looking ‘dumb’ in front of their friends (Ryan & Pintrich, 1997). Some students avoid asking for public help during whole class discussion to avoid embarrassment or mask incompetence (Butler, 1998). Other students may interpret asking for help as a sign of weakness (Spray, Scevak, & Cantwell, 2013). Butler (1998) labels students that may avoid asking for help publicly, such as asking the teacher during whole-class discussion for help or asking peers for help, but may still seek out help covertly, such as copying answers off a peer or from the back of a textbook, as *avoidant-covert* help seeking.
2.1.5 Adaptive help avoidance

Students who do not ask for help, and do not require it, is called *adaptive other* (Newman, 2008). The term *adaptive help avoidance* is used here instead so that it aligns more naturally in a four-quadrant help seeking table provided in Table 1 (p. 13). This might include students who are able to complete tasks successfully while working independently. Such students are likely autonomous help seekers, who strive for independent mastery, will spend longer periods of time working on a problem, and only ask for help if they are unable to solve it on their own (Butler, 1998).

2.1.6 Summary

A summary is provided in Table 1 showing the various types of help seeking behaviours that have been outlined. The table uses the same four quadrant layout as that used by Newman (2008), which differentiates between different forms of help seeking by whether the student did or did not ask for help, and whether the help was or was not necessary.

From the literature, a decision tree was constructed (Figure 1). The decision tree will help guide the analysis of this study. Namely, the following three questions will be used to guide the exploration of the data on help seeking:

a) Is the help seeking necessary?

b) Does the student ask for hints or answers? Does the student form specific or general questions?

c) Does the help improve later independent problem solving?
## Table 1: Four quadrants of help seeking summarising terms used by different researchers

<table>
<thead>
<tr>
<th>Seeks help</th>
<th>Avoids seeking help</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Help is necessary</strong></td>
<td><strong>Help is not necessary</strong></td>
</tr>
<tr>
<td>• Asks for help when necessary</td>
<td>• Avoid asking for help out of fear of looking dumb</td>
</tr>
<tr>
<td><strong>Appropriate help seeking</strong> (Ryan et al., 2005)</td>
<td><strong>Executive help seeking</strong> (Nelson-Le Gall, 1986)</td>
</tr>
<tr>
<td>• Asks for help when necessary, but not overly dependent</td>
<td>• Avoidant-covert help seeking (Butler, 1998)</td>
</tr>
<tr>
<td><strong>Autonomous help seeking</strong> (Butler, 1998)</td>
<td>• Avoids publicly asking for help, and instead may copy answers from a peer, or back of textbook</td>
</tr>
<tr>
<td>• Asks for help when necessary, asks for hints, and improve subsequent independent problem solving</td>
<td></td>
</tr>
</tbody>
</table>
Is the help seeking necessary?

Does the student ask for help because they are unable to continue working independently?

**NO**

**YES**

**Dependent help seeker**
(Ryan et al., 2005)

Does the student ask for hints/clarifications rather than answers?

Does the student form specific questions that clearly identify what they are finding confusing rather than form general ‘I don’t get it’ statements?

**NO**

**YES**

**Executive help seeking**
(Nelson-Le Gall, 1986)

**Appropriate help seeking**
(Ryan et al., 2005)

Does the help received result in an improved ability to solve problems independently?

**NO**

**YES**

**Autonomous help seeking**
(Butler, 1998)

Figure 1 Decision tree identifying different forms of help seeking that will be used in this study
To summarise the decision tree, the following four types of help seeking are defined as follows and will be the terms used in this study to differentiate between different forms of help seeking identified in this study.

a) Dependent help seeking: asking for help the minute one encounters difficulty.

b) Executive help seeking: asking for hints rather than answers as well as asking general questions which do not improve subsequent problem solving. Executive help seeking may or may not be necessary.

c) Appropriate help seeking: asking for help when necessary, asking for hints rather than answers, and asking specific questions.

d) Autonomous help seeking: asking for help when necessary, asking for hints rather than answers, asking specific questions, and help is used to improve subsequent problem solving.

By answering these three questions, students’ help seeking behaviours can be categorised into dependent, executive, appropriate, or autonomous.

### 2.2 Student perceptions of understanding

The notion of understanding is problematic. There is no unified definition for understanding and in the words of Tall (1978), “there must be nearly as many views of what constitutes ‘understanding’ as there are mathematical educators” (p. 50). The purpose of this section is to identify some, but not necessarily all, ways in which students might identify what it means to understand.

An overview is first provided outlining the way in which categories were constructed related to how students might perceive what it means to understanding. These categories emerged from literature on conceptions of learning, mathematical beliefs, and epistemological beliefs. A detailed explanation is then
provided for each identified student perception of understanding. Included, but not limited to, as part of
the umbrella of student perceptions of understanding are what counts as understanding, what does not
count as understanding, what signifies understanding has or has not occurred, and ways in which to
achieve understanding. Examples will be provided from the literature showing how students talk about
understanding. It is not types of understanding as identified by the researcher but rather the language that
students themselves have used to describe understanding that will be used to develop categories for
perceptions of understanding in this study.

2.2.1 Identifying different ways students perceive understanding

This section discusses the decisions that were made when forming categories for the different ways
students might view what it means to understand mathematics. Studies on perceptions of understanding
was underrepresented in the literature. Therefore, in order to explore student perceptions of
understanding, literature was included from areas in conceptions of learning, mathematical beliefs, and
epistemological beliefs as all three areas of literature contained examples where students identified
eamples of what did and did not count as understanding. Only few studies were found explicitly
exploring students’ conceptions of understanding (Burns et al., 1991; Entwistle, 1991).

Conceptions of learning

Säljö (1979) asked students ‘What do you actually mean by learning?’ and ‘How do you usually set about
learning?’ From their responses, he identified five different conceptions. Students identified learning as:
a) the increase of knowledge;
b) memorisation;
c) the acquisition of facts and steps which can be retained or used in practice;
d) the abstraction of meaning;
e) an interpretive process aimed at understanding reality.

These conceptions of learning were confirmed by Marton et al. (1993), who also added a sixth conception:

f) changing as a person.

Säljö (1979) argued that the first three conceptions of learning are associated with a surface approach to learning, while the last three are associated with a deep approach to learning. Deep learning is associated with intentions of understanding and gaining meaning when learning while surface learning is associated with intentions of reproduction.

Studies exploring student conceptions of learning (e.g., Boulton-Lewis, Lewis, & Wilss, 2003; Marton et al., 1993; Mugler & Landbeck, 2000) are included as part of this study as some students described learning as related to understanding. For example, one student provides the following description about learning:

I was very lazy in primary school. I just learned things by rote for the tests. There was no need to understand as there was nothing to be understood (Marton, Watkins, & Tang, 1997, p. 28).

Here the student identifies that they use rote memorisation in order to learn and that this did not require understanding. Therefore, while the description is about learning, it can be deduced that this student does not see memorisation as a type of understanding. This will be included in this study as part of student perceptions of understanding as for this student, memorisation relates to what does not count as understanding.
**Mathematical and epistemological beliefs**

Research in the area of mathematical and epistemological beliefs was also used to provide further examples of how students might identify what it means to understand. As of yet, there is no single agreed upon definition for beliefs in the literature (Sumpter, 2013). Schoenfeld (1985) refers to mathematical beliefs as “mathematical worldviews” which act as the perspective one takes when approaching mathematics (p. 186). Op’t Eynde, De Corte, and Verschaffel (2002) see mathematical beliefs as a “subjective concepts students hold to be true that influence their mathematical learning and problem solving” (p. 24).

Epistemology is a branch of philosophy concerned with the nature of knowledge and the justification of belief (Muis, 2004). Educational psychologists have defined epistemological beliefs as individuals’ beliefs about the nature of knowledge and how it is learnt (Bråten, Stromso, & Samuelstuen, 2008; Muis, 2007), rather than epistemology in the strict philosophical sense.

Schoenfeld (1989) explored students’ mathematical beliefs while engaged in problem solving activities. He found that several students believed that if they could not solve a problem quickly, then they had not understood the problem. Here is an example of a student with this belief from Higgins (1997):

> If you can’t do it too quickly, you don’t understand so much (p. 18).

Here, the student identifies a belief that problems should be solved quickly, and, from their perspective, associates it explicitly with understanding. Because some students associate this belief with understanding, the view that problems should be solved quickly is included in this study as a potential perception of understanding. In this example, the student’s perception of understanding relates to what does not count as understanding.

From the literature on mathematical beliefs and conceptions of learning, it was found that when students talk about understanding, they discuss it in the following ways which the following section will reveal:

1. If I can solve a problem quickly, then I understand (Higgins, 1997; Schoenfeld, 1988).
2. If I can memorise all the steps and rules, then I understand (Marton et al., 1997; Schoenfeld, 1988).

3. If I can make connections between ideas, then I understand (Crawford, Gordon, Nicholas, & Prosser, 1994; Reid, Petocz, Smith, Wood, & Dortins, 2003).

4. If I have learnt something, then I understand (Boulton-Lewis et al., 2003; Mugler & Landbeck, 2000).

5. If I can explain my ideas to others, then I understand (Mugler & Landbeck, 2000; Purdie et al., 1996).

There was one view to emerge from a review of additional literature which did not emerge from the set of mathematical beliefs or conceptions of learning:

6. If I have finished the problem, then I understand (Anthony, 1996b; Barnes, 2001).

What follows is a detailed explanation of each of these six student perceptions related to understanding. The purpose is to provide categories for identifying different ways students might identify what it means to understand. More may emerge from the data in this study.

2.2.1.1 **If I can solve a problem quickly, then I understand**

Here examples are provided where students feel that understanding is related either to how quickly they are able to solve a problem or whether they can complete a problem without ‘getting stuck’ or needing to stop and think.

Schoenfeld (1989) found that many students hold the belief that if one understands a mathematics problem, then it should be completed quickly. If one cannot finish the problem within a reasonable amount of time, then it means they do not understand. This belief was also confirmed by Frank (1988).
Students with similar views were identified by Higgins (1997). When students in Higgins’ study were asked “How can you tell if you understand something in mathematics”, some made the following comments:

1. If a problem is easy, you know and get it done and understand – if you can’t do it too quickly, you don’t understand so much.

2. I can just look at a problem and know how to do it – I’ve seen it before.

3. I can figure out a problem faster than anyone else. (p. 18)

From Statement 1 there is a direct association between quick learning and understanding: If you cannot solve the problem quickly, then you do not understand. Statement 2 provides the contrapositive: if you understand the problem, you should be able to start it immediately without needing to think. The final comment differs slightly from the others. Here understanding is not necessarily linked to how quickly the student felt they finished the problem. Instead understanding is linked with being able to finish a problem faster than other students.

Other research suggests an alternate interpretation for quick learning. While Schoenfeld (1989), Frank (1988), and Higgins (1997) each associate a belief in quick learning to be related to the total time taken to finish a problem, others have associated it with being able to finish a problem fluently. For example, Callejo and Vila (2009), Hannula (2006), and Furinghetti and Morselli (2009) all provide examples of students who view success as being able to proceed through a problem smoothly by advancing directly towards the solution without getting stuck. If the student does not know how to immediately proceed to the next step of the solution without needing to stop and think, then they reported that they did not understand the question. This need for fluency was present in one student, Lucy, reported by Barnes (2001). Here is Lucy’s response when asked how she knew she understood something:
If I can look at a problem and just know how to do it straightaway, don’t need to look it up, or think about how to do it or anything (p. 3).

Here, Lucy’s concern is not with the amount of time it takes her to solve a problem, but rather whether she can solve it without needing to stop and think. This suggests another perception of understanding related to what indicates a lack of understanding: If I need to stop and think, I don’t understand the problem. ¹

These examples demonstrate that quick learning can be interpreted in different ways by students. These include

a) If I can finish quickly, then I have understood.

b) If I cannot finish quickly, then I have not understood.

c) If I am the first to finish, then I have understood.

d) If I don’t immediately know what to do next, then I don’t understand the problem.

2.2.1.2 If I can memorise all the steps and rules, then I understand it

From the literature, it was found that some students feel memorisation:

a) is a learning strategy not necessarily related to understanding;

b) does not count as understanding;

c) is a type of understanding.

What the following examples will show is that a strategy such as memorisation might be viewed explicitly as a type of understanding by one student, yet seen as a strategy that does not count as understanding by...

¹ The use of italics will be used throughout this thesis to signify a particular perception of understanding.
another student. This demonstrates that perceptions of understanding are subjective to the individual student.

**Memorisation is a learning strategy**

Students interviewed in Schoenfeld's (1988) study placed importance on memorisation as a successful strategy they can use to get the answer. Similarly, Martino and Zan (2011) found some students view memorisation as part of what is required in order to be successful in mathematics. Spray et al. (2013) found that some students similarly recognised that memorisation could be a useful strategy to help get the answers but also considered memorisation to be a poor choice of strategy as the following student quotes demonstrate:

I find that repetition is the only way that I retain anything, but I’m sure that someone said it is the worse way to learn. (Paula)

I am guilty of rote learning. (Natasha, p. 51)

These studies demonstrate that some students identify memorisation as a useful strategy that helps in getting the answers but is not necessarily related to their views on understanding.

**Memorisation is not understanding**

Reid et al. (2003) interviewed one student who reported that memorisation could produce good academic results:

(Why would you want to rote learn things?) People do, and they do really well. (Well why is that?) Because if you are doing a lot of maths stuff and you have to reproduce proofs they just
learn it all and write it all out. (And you think that is superior to your attempt to understand
the stuff?) No it’s not superior: I’d rather understand it, but you can get better marks for rote
learning. (p. 170-1).2

Here, memorisation is not equated with understanding. This student instead sees value in memorisation
as a tool for achieving better marks.

As another example, Goos (2004) found evidence that some students considered understanding to not
involve memorisation as the following conversation between students demonstrates:

Duncan: You can work it out yourself, amongst the group, without having to be told.

Belinda: With groups, if you’ve actually worked it through yourself and not just learned it off
by heart then you’re more likely to -

Rob: - understand it. (p. 272)

For students in this class, understanding involves working things out for yourself and specifically does not
involve learning things off by heart.

These examples demonstrate that some students included memorisation as part of their perception of
understanding, but that it relates to their views on what does not count as understanding.

**Memorising is understanding**

There is an underlying assumption in Western education that memorisation and understanding are
conflicting and opposite (Marton et al., 1997; Mugler & Landbeck, 2000).3 Chinese educators instead see

---

2 The use of brackets as interviewer comments was a notation convention used by Reid et al. (2003).
3 Note that these studies set out to explore conceptions of learning but did so in the context of literature rather than mathematics.
understanding and memorisation as flowing together rather than distinct processes (Marton et al., 1993). Memorisation without meaning is instead seen as the act of committing things to memory only, often labelled rote memorisation.

Examples from Mugler and Landbeck (2000) show that some students view memorisation as a route to understanding:

There’s no point in memorising if you don’t understand… to me, you memorise to understand

(p. 197).

Others from the same study reported that understanding was useful to help in memorisation:

I find it difficult to memorise something without understanding it (p. 197).

For these students, memorisation forms part of their perceptions of what counts as understanding. Memorisation is not understanding per se, but a way to achieve understanding.

Purdie, Hattie, and Douglas (1996) also found examples where students incorporate memorisation and understanding as part of learning. Here a student claims learning has occurred:

when I memorized something and understood it perfectly (p. 94).

From this student’s perspective, understanding is something that goes hand-in-hand with memorisation.

Marton et al., (1997) argue that meaningful understanding can occur through repetition. This is supported by Sfard (1991) who identifies two types of understanding: operational and structural. Operational refers to knowing procedures and algorithms, while structural refers to being able to recognise an idea at a glance. She argues that learning procedures (operational understanding) forms a basis from which structural understanding can occur. Using a different frame-work, Rittle-Johnson and Alibali (1999) propose that understanding should be considered as lying on a continuum, with conceptual knowledge at one end, and procedural knowledge at the other. Conceptual knowledge refers to having an implicit
understanding of the relationships between various pieces of knowledge. Procedural knowledge refers to knowing the steps used to solve a problem. They found that improvement in procedural understanding can lead to improved conceptual understanding, but that such gains would be limited. Rittle-Johnson, Siegler and Alibali (2001) argue that one reason gains in procedure knowledge result in improved conceptual knowledge is that it frees up mental resources needed to solve problems. As students improve their procedural knowledge, perhaps through memorisation and repetition they become more fluent, thereby providing more mental resources for observing relationships between problems, generating new procedures or reflecting on the underlying concepts of the problem. This in turns allows an increase in conceptual understanding of the problem. This is in agreement with Kalyuga (2007) who argues that repetition can help in reducing the cognitive load needed to answer a problem. That is, if certain mathematical procedures become automated, the student’s cognitive load is reduced, allowing the student to focus on more sophisticated reasoning, such as developing comprehension. On the contrary, Vincent and Stacey (2008) found that having procedural knowledge did not help improved conceptual knowledge.

In summary, the examples here show that some students do not associate memorisation with understanding, but instead see it is a learning strategy. Others see memorisation as equivalent with not understanding their work. While others see memorisation as a route towards understanding.

### 2.2.1.3 If I can make connections between different ideas, then I understand

Another category for perceptions of understanding related to what counts as understanding is: *If I can make connections, then I understand*. Some students view mathematics as a set of independent and fragmented rules that do not relate to each other while others might view mathematics as a set of interconnected ideas
and concepts (Crawford et al., 1994; Reid et al., 2003). A cohesive view of mathematics was identified by Martino and Zan (2011), Young-loveridge et al. (2006), and Boaler (1998). These studies found that some students think mathematics is about making connections between ideas and knowing how different rules related to each other. Burns et al. (1991) and Entwistle and Entwistle (1992) found that the most common conception of understanding was related to making connections. That is, many students reported they felt they understood if they were able to identify the relationship between pieces of information.

2.2.1.4 If I have finished the problem, then I understand

Some students equate finishing a task with understanding, that is, finishing a problem signals understanding has occurred. For example, Anthony (1994, 1996a, 1996c) observed a Year 11 student, Gareth, who equated completing a problem with having understood or learnt the problem, regardless of whether he had answered it correctly or not. Barnes (2001) similarly found that Lucy equated understanding with task completion, but her definition of understanding included the requirement that the task was completed correctly.

In contrast, some students believe understanding is not about completing a problem. For example:

Sometimes I get the answer but I don’t know if I understand (Higgins, 1997, p. 18).

Here the student has an answer, but it does not equate to having understood the problem.

Additional examples come from Boaler (1998), who interviewed one student, Ian, at Phoenix Park, a school which emphasised learning mathematics independently through projects. Ian comments on the differences between Phoenix Park and his previous school:
Ian: It’s an easier way to learn, because you’re actually finding things out for yourself, not looking up things in the textbook.

Boaler: Was that the same in your last school, do you think?

Ian: No, like if we got an answer, they would say, “You got it right.” Here you have to explain how to get it.

Boaler: What do you think about that – explaining how you got it?

Ian: I think it helps you. (Ian, Year 10 student, p. 50)

Comments like “You got it right” from teachers at Ian’s previous school may suggest that at Ian’s previous school, once he had the right answer, the problem was considered finished. At Phoenix Park, more is required of Ian. Once he has the right answer, he needs to explain his reasoning. His comment “I think it helps you” is suggestive that explaining his reasoning helps him make sense of the mathematics, and helps him gain a better understanding. With this interpretation, Ian does not consider task completion as signifying understanding.

These examples show that some students may equate task completion with understanding. Others believe they have understood only if they have finished the problem and it is correct. In contrast, other students believe that task completion does not directly imply they have understood the problem.

2.2.1.5 If I can explain to others, then I understand

Purdie et al. (1996) found some students associate being able to explain ideas to others as signalling they have understood:

you understood the concepts well enough to be able to teach or explain it to someone else (p. 93)
Mugler and Landbeck (2000) found students with similar views of requirements for understanding:

> If I understand something, then I can explain that thing to somebody, just like my group mate,

and if he understands that, that means that I, you know... I’ve understood (p. 190).

This particular student feels they understand if they are firstly able to explain their ideas to someone else, and they are able to understand their explanation. For these students, the process of explaining something to someone else is an indication that understanding has occurred.

Skemp (1979) included explaining to others as part of understanding, and termed it *logical understanding*. Logical understanding involves students developing a generalisation that connects ideas together, as well as being able to explain this generalisation to others.

### 2.2.1.6 If I have understood something, then I have learnt it

Understanding is sometimes interpreted by students as being equivalent with learning (Boulton-Lewis et al., 2003; Mugler & Landbeck, 2000). For example, for the following student, learning involves the acquisition of knowledge, an understanding of the knowledge, and then the ability to use and apply the knowledge.

> Learning is to know something, understand it and then be able to keep it and share it (Mugler & Landbeck, 2000, p. 186).

Mugler and Landbeck (2000) found that students who discussed learning and understanding as a pair tended to fall into two groups, differentiated by which of understanding and learning occur first. Some students felt understanding follows after learning:

> When you say you’ve learned something, you’ve understood. Learning is a process and the end would be understanding that (p. 187).
Others felt that one must understand before learning can occur:

To learn something we have to understand it. They are not the same, for example, in order to learn, we have to understand something. When we are able to understand and use it, then only can we think we have learnt it (p. 188).

Like the students who viewed memorisation as a route to understanding (Marton et al., 1993), some students view learning as a route to understanding, while others see understanding as a route to learning. Thus, learning forms part of some students’ perceptions of understanding related to how understanding is achieved.

2.2.2 Summary of different perceptions of understanding

For this study, perceptions of understanding will be considered as a broad term which include what it means to understand, what it means to not understand, what signifies understanding has or has not occurred, what does not count as understanding, and what is necessary to achieve understanding. Additionally, it is the student’s perception of understanding that this study will focus on, rather than the teacher’s or researcher’s perceptions.

From the literature provided in this section, student perceptions of understanding may potentially be revealed in the following ways:

a) Finishing a problem quickly;

b) Memorisation;

c) Drawing connections between ideas;

d) Finishing a problem;

e) Explaining ideas to others;
f) Learning.

From the literature, it was also shown that each category identified here can be interpreted in different ways by students. For example, memorisation was seen by some students as equivalent to understanding, while other students felt memorisation did not imply understanding had occurred.

2.3 Summary

This chapter has outlined the different ways in which help seeking behaviours will be identified in this study which include dependent, executive, appropriate, and autonomous. Various ways in which students perceive understanding was also discussed which provides potential categories that may be recognised within this study. In the next chapter, the research method that were used through this study is outlined.
Chapter 3  Methodology

3.1 Introduction

This section outlines the choice of methodology used for this study. First, a rationale is provided for the choice of case-study method along with justification for choices regarding the use of instruments including questionnaires, classroom observations, and interviews. An outline is then given for the data collection strategies. Lastly, the way in which help seeking and perceptions of understanding will be operationalised throughout this study is discussed.

3.2 Rationale

This study investigates whether there exists a relationship between students’ help seeking behaviours and their perception of what it means to understand within the context of mathematics. This study follows recommendations that students’ perceptions should be examined while they are working in an authentic learning environment and engaged in problem solving activities (Bromme et al., 2010; Hofer & Pintrich, 1997; Muis, 2008). As such, the use of the natural classroom was seen as an appropriate setting and classroom observations was included as part of this study. This is in agreement with Anthony (1996b) who argues that
because learning strategies are not applied in a vacuum, but are influenced by multitude of variables, the use of the authentic learning setting [the classroom] was seen as crucial to the research process (p. 39).

The choice of the classroom setting aims to reduce what Schoenfeld (2000) refers to as artificial behaviour, that is behaviour that is significantly different from what would be expected in the classroom setting due to pressure that may exist by having an observer present. The tasks that students worked on were problem solving tasks as such tasks provide a means of assessing students’ thinking and attitudes (Hino, 2007).

Of note, the use of classroom observations runs the risk of not focusing on research data. Potari (2002) argues that novice teachers/researches are often more focused on the teaching and learning that occurs between teacher and student rather than the focus of the research itself. Therefore, as a novice teacher/researcher, during classroom observations, my focus might rest more so on ensuring that students are engaged in the problem at hand, understand the task, and are learning from task, rather than being focused on how students working on the task might be perceiving what it means to understand in the moment. This made follow-up interviews necessary to ensure that I could elicit the information that I required from students in this study such as whether students felt they did or did not understand, or whether they felt help seeking was warranted.

An alternative to classroom observations is to observe students working individually on a problem in a separate room with an experimenter present (e.g., Butler & Neuman, 1995; Newman & Schwager, 1995). This was not seen as appropriate due to the possibility of artificial behaviour (Schoenfeld, 2000).

Due to the choice of the classroom setting, a case-study methodology was selected as

A case study is an empirical inquiry that investigates a contemporary phenomenon within its real life context (Yin, 2003, p. 13).

This is in agreement with other recent studies exploring students’ views which have chosen to use a case-study approach (e.g., see Pritchard, 2009; Spray et al., 2013).
Furthermore, a case-study allows for the individual nature of students’ views of understanding. Previously, it was shown that students’ views of understanding were personal and individual. For example, where one student might feel memorisation is equivalent to understanding, another might feel that memorisation is equivalent to a lack of understanding (2.2.1.2 If I can memorise all the steps and rules, then I understand it, p. 21). Because of the personalised nature of perceptions of understanding, a case-study methodology was selected to obtain an in-depth description of the phenomena in question and allow for explorations of differences between individuals.

In order to tap into students’ personal points of view, some researchers have called for the use of think-aloud protocols where students are asked to voice their thought processes out loud while engaged in problem solving (Muis & Franco, 2009). Due to the value placed on gathering data within the classroom setting, think-aloud techniques were not seen as appropriate. Additionally, think-aloud protocols may not be able to produce a complete and authentic record of each student’s thinking, and run the risk of affecting problem solving ability as students need to slow down while working in order to verbalise their thinking (Ericsson & Simon, 1993). This thereby reduces the authentic nature of problem solving activity.

Instead, stimulated recall interviews were seen as an appropriate alternative to think-aloud protocols (Anthony, 1994a), and were used for this study. This involves playing a section of a recording of a lesson to stimulate student recall of their behaviours and what they were thinking during the task. Stimulated recall interviews were chosen as the major source of data for information regarding how students view understanding as this is a difficult phenomenon to gather data from classroom observations alone (Schommer-Aikins, Duell, & Hutter, 2005). The main advantage of the interview process compared to alternatives such as questionnaires is that they provide opportunities to probe students’ thinking further should a question fail to produce a detailed response (Kloosterman, 2002). Additionally, because this study is focused on how the student rather than researcher identifies understanding, stimulated recall interviews were seen as appropriate for capturing the students’ points of view rather than the researcher’s (Clarke, 1997).
Students participated in a maximum of three interviews. The small number of interviews were intentionally chosen to limit the risk of students becoming ‘sick’ of being interviewed, and thereby possibly distorting the data (Anthony, 1994a). It was also felt that three interviews provided sufficient data on students’ perceptions of understanding.

There are limitations to the use of stimulated recall reports to explore students’ views on understanding. Some researchers who have conducted interviews have found that students often find it difficult to articulate how they view mathematics (Kloosterman, 2002; Young-Loveridge et al., 2006) or clearly articulate what they mean by understanding (Mugler & Landbeck, 2000). Furthermore, much of students’ thinking may be unconscious and therefore difficult to convey verbally (Hannula, 2006). Students may also only be able to describe types of understandings they are aware of, rather than how they actually view understanding (Spray et al., 2013). Lastly, there is also the risk that students may describe what the interviewer wants to hear (Richardson, 2013). However, other studies using multiple interviews found that students did not appear to be wanting to please the interviewer, but rather were able to express their actual points of view (McDonough & Sullivan, 2014).

An alternative to classroom observations with follow-up interviews is the use of large-scale questionnaires. Often studies exploring students’ perceptions and help seeking have made use of large scale questionnaires (e.g., Ryan et al., 2005; Schommer-Aikins et al., 2005). Large scale questionnaires have been criticised for validity issues due to the vagueness of some questions (Hofer & Pintrich, 1997). Furthermore, they have been criticised for their limiting nature in that students are unable to provide their own personal answers to questions (Richardson, 2013). Questionnaires including open questions (requiring students to provide written responses) have also been used with caution as students may not be articulate enough to provide accurate self-reporting concerning how they view and interpret what it means to understand (Muis, 2004, 2008). Lastly, questionnaires risk distorting the data through ‘led’ questions (Spray et al., 2013). While questionnaires were used as a data source for this study, they were
not used in isolation and instead results were triangulated with those from classroom observations and interviews.

As a final note, there exist difficulties with regards to the methodology investigating how students view and interpret what it means to do and understand mathematics and how this might influence their learning behaviours such as help seeking. Lester (2002) comments on the difficulty related to research that relies on students’ actions and behaviours as an indication of their points of view, while at the same time asserting that their behaviours are influenced by their points of views. This leads to a circular argument. In order to avoid this pitfall, data on help seeking will, where possible, not be used to inform interpretations of students’ views of understanding, and vice versa. The transcripts of students’ own words will also be provided to enable the reader to judge for themselves whether they justify the interpreted meaning (Leder, Pehkonen, & Torner, 2003). Furthermore, data from this study has been shown to another researcher who looked at transcripts associated with some of the decisions made throughout this study to check their reliability.

3.3 The research setting

This study was conducted in a medium sized co-educational secondary school, located in the inner suburbs of a capital city in Australia. The majority of students were focussed on achieving high ATAR scores, a score that denotes a student’s ranking relative to their peers and used for university entry requirements. There was generally a high level of student anxiety associated with pressures of achieving high marks coming sometimes from parents.

The study took place within a mathematics classroom which I team taught with a second teacher. This teacher was male and had had previous experience with research projects that required being filmed. The teacher’s previous educational research experience meant that he was likely to be tolerant of any
interfering effects the research process might have. He also approached teaching and learning with similar pedagogy to my own so that students had a similar experience in the classroom regardless of whether they received help or instruction from myself or from the second teacher. In general, he avoided approaching groups being filmed during class unless they specifically asked for his help. Thus, he had little direct impact on results for this study as his minimal interactions with students during classroom observations did not greatly affect their problem-solving approaches.

Both this teacher and the school principal were aware that the purpose of the research was to explore student help seeking behaviours and their perceptions of understanding. The students were informed about the project in a more general way rather than told the specific focus. They were told that the project was about gathering more information on how students learn mathematics to help develop better teaching practices. This was done to maintain an authentic learning environment and limit students behaving in a way that they believed I wanted to see.

### 3.3.1 Subjects

The selected class was a class of Year 10 students who were studying the VCE Year 11 subject, Mathematical Methods Units 1/2. This subject includes the study of polynomials, an introduction to calculus, and an introduction to counting methods relevant to probability. It is seen as a challenging and difficult subject by students. There were 48 students in total in the class with two teachers. 30 of these students were male, the other 18, female. The majority of students were of an Asian or South Asian background. Approximately half the students were born overseas, with families migrating to Australia within the last five–ten years.

The 48 students were considered by the school to be high-achievers and selected to participate in the accelerated mathematics stream. In general, they were extrinsically motivated to perform, with many
students attending after school and weekend tutoring (out of school programs). Many students in this class experienced pressures from parents to perform which generally became apparent during parent/student/teacher interviews where parents would often be dissatisfied with their child’s results, even if their results were exemplary. Marks were especially important to both parents and students.

Out of the 32 students that returned their consent forms, only 21 students (10 boys and 11 girls) gave full permission to take part in this study. This included permission to complete the questionnaire, appear on film, and take part in interviews. Other students gave partial permission (e.g., would complete a questionnaire, but did not want to appear on film). Only students who gave full permission were considered as potential case studies.

### 3.3.2 Case studies

Six students were identified as target students for case studies. Early classroom observations and questionnaire responses were taken into consideration when selecting these students. They were selected with the intention that they represented a cross-selection of the class in terms of help seeking behaviours and how they perceived understanding. For example, the selected students represented a range of adaptive and non-adaptive help seeking behaviours as evident from their questionnaire responses and early classroom observations. The target students then participated in individual stimulated-recall interviews.

During these interviews, it became apparent that one student was particularly nervous during the interview process. This student did not take part in any further interviews so as to minimise any further discomfort. Their data was not used for this study and they were not pursued as a case-study. A second student found it particularly difficult to articulate their thoughts during their interview. Because their interview comments did not add significant value to the study, they were not used as a case-study.
The remaining four students include two girls, Pippa and Lilly, and two boys, Aaron and Kipp (pseudonyms given). Given the scope of a Master’s thesis, a detailed case-study of all four students was unable to be included. A detailed account is provided for Pippa and Lilly in Chapter 4, while a brief summarised account is provided for Aaron and Kipp in the same chapter. That said, part of the findings from Aaron and Kipp were used to compare, where appropriate, with the findings for Pippa and Lilly.

### 3.3.3 The lesson context

Both teachers in the classroom used similar teaching styles. Lessons would typically begin with a ‘warm-up’ question which was designed to introduce the new ideas for the lesson. Through whole-class discussions and questioning from both teachers, the class would come to a consensus regarding the skills and ideas required to answer the warm-up question and the students would generally develop their own solution to the problem, rather than expecting the teacher to provide it for them.

The rest of the lesson was typically spent allowing students to work through a given list of questions from the textbook. During this time, both teachers would move about the classroom checking with individual students if they had any difficulties.

Most students worked with peers. Very few students worked individually. Desks were arranged in such a way as to encourage group work. Students would sit in a circular arrangement rather than seated facing the front. Table groups usually seated six to ten students.

If students experienced difficulties, neither teacher would directly provide answers, but rather ask a series of probing questions that would either highlight the mistake the student had made, or allow the student to develop their own insight into the problem. For example, a student might incorrectly simplify $2^x \times 2^3$ and ask the teacher for help. Both teachers would likely ask the student to show their working out where both teachers might notice the student has written the common mistake of $4^x3$. Rather than simply telling
the student that the base number should not change when simplifying indices, both teachers might ask the student to instead simplify $a^2 \times a^3$ where the student likely correctly writes $a^5$. Both teachers might then ask the student why the base number did not change in one example, yet changed in another. This is usually enough of a prompt for the student to recognise their own error and correct answer $2^x \times 2^3$ as $2^{x+3}$.

3.4 Data collection strategies

As there are no particular data collection techniques associated exclusively with case study, multiple instruments were used, including questionnaires, individual student interviews, and classroom observations. The following section provides an overview of the data collection process, followed by a more detailed discussion on each of the individual instruments.

3.4.1 Timeline

The research study was completed during Terms 1 – 3 of the school year. The following gives an outline of the times for which different data collection was implemented.

Term 1

- Questionnaire completed by students.
- Six potential case study students selected.
Term 2

- First Task: Ferris wheel task.
- Stimulated recall interview about Ferris wheel task.
- Second Task: position/velocity graphs.
- Stimulated recall interview about position/velocity graphs task.

Term 3

- Third Task: largest triangle task.
- Fourth Task: coke can task.
- Stimulated recall interview about both triangle and coke can task.

3.4.2 Questionnaire

Students were asked to complete a questionnaire (Appendix 1). This was delivered electronically and students could complete it in their own time. 32 students completed the questionnaire.

Included in the questionnaire were questions adapted from the Manual for the Patterns of Adaptive Learning Scales (Midgley et al., 2000) the Motivated Strategies for Learning Questionnaire (Pintrich, Smith, Garcia, & McKeachie, 1991) and the Indiana Mathematics Belief Scales (Kloosterman & Stage, 1992). These were written as a Likert scale response, where students were asked to select from five options ranging from strongly disagree to strongly agree.

Some questions asked about help seeking behaviours (e.g., *When I can’t understand something in maths, I ask another student in my class for help*), while others probed students’ mathematical beliefs (e.g., *Memorising steps is not that useful for learning how to solve worded problems*). It became apparent that the majority of the
class was greatly concerned with achieving high results, with 31 of the 32 students strongly agreeing with the statement *Getting a good mark in maths is the most satisfying thing for me right now.*

A statistical analysis was not conducted. Rather individual responses to specific questions were examined closely. Of interest were responses to the statement *It’s important to me that I thoroughly understand my work.* 21 students strongly agreed with this statement, 10 agreed and only one neither disagreed nor agreed suggesting that understanding was valued by the majority of students in this class. This lead to the question ‘How were students interpreting this question?’ All students seemed to be reporting that understanding was important, yet there were doubts that they all held the same definitions for understanding. For example, one student might believe understanding is memorisation, while another might instead believe understanding is quick learning. When they respond on the questionnaire, they report that *their* understanding is important, not ‘understanding’ as may have been intended by the authors of the survey. This raised some issues with regards to the how the statements had been worded in the questionnaire.

Responses to questionnaire statements then functioned as a starting point for selecting potential case-study students. Of interest was selecting students which valued memorisation and students who did not and also ensure that the selected potential case-studies represented a variety of help seeking behaviours from adaptive to non-adaptive.

In short, the questionnaire as a self-reporting instrument was used in this study to help identify case study participants and served only as a starting point for data collection on help seeking behaviours and perceptions of understanding.
3.4.3 Case studies

The target case-study students took part in a questionnaire, four tasks, and interviews. The tasks that each student took part in is summarised in Table 2. Tasks that students took part in are represented with an \( \times \). Tasks that students did not take part are shaded in grey.

Pippa and Lilly participated in all activities. However, Pippa was unwell with a cold during the largest triangle task. Although she was present, she took little part in the activity itself. Lilly took part in the velocity graphs task, but audio recordings failed to work during this task. Lilly did take part in the follow-up interview where she described in as much detail as she could how she and her partner had worked through the task.

Pippa and Lilly were selected as case-studies as they represented typical students in this class. Both students were willing to ask for help, though not excessively. Their views on mathematics also represented the two common types of students in this class that emerged from the questionnaire results where students tended to value either comprehension or high marks. Pippa valued high marks over understanding while Lilly valued comprehension over high marks.

Aaron and Kipp did not take part in all tasks due to absences. However, they were still included for this study as they represented atypical students in the class. Aaron was adamant that understanding was related only to memorisation and was also one of the few highly dependent help seekers in the class who continually asked for help when unnecessary. Kipp on the other hand was the only identified adaptive help avoider in the class. Results for Aaron and Kipp are not discussed in detail in this study but were included in order to corroborate findings from Pippa and Lilly, who are discussed in detail.
### Activity Participation

<table>
<thead>
<tr>
<th>Activity</th>
<th>Pippa</th>
<th>Lilly</th>
<th>Aaron</th>
<th>Kipp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Questionnaire</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>First Task: Ferris wheel problem</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>Interview following Ferris wheel problem</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second Task: Velocity graphs</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interview following velocity graphs task</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Third Task: Largest triangle problem</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Fourth Task: Best sized coke can problem</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Interview following the triangle/coke tasks</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>

*Table 2 Summary of the activities each student took part in*

### 3.4.4 Classroom observations

In total, four lessons were filmed and audio-recorded. This formed the main data source for help seeking. Four lessons were considered a sufficient number of activities for this study as the tasks varied in nature, style, and content, allowing for the potential for students to approach each task in a multitude of ways.
Each of these four lessons involved students working on an unfamiliar task described in the next section. Observations focused on how students responded to difficulties or breakthroughs through verbal queues, body language, and behaviours in the classroom (e.g., copying answers, consulting notes/textbook, task avoidance, panic or excitement).

For the four lessons filmed, two video recording devices were used. These were placed at the back of the classroom and focused on two groups for all four activities. The students that made up these two groups changed for each activity but always included Pippa and Lilly. The classroom was set up in such a way so that the students had their backs to the video recording devices to keep the recording devices as discrete and unobtrusive as possible and hence minimise any artificial behaviour. For each group an audio recording device was also placed on the desk. This was the size of a large USB stick and was also intended to be discrete to minimise self-conscious behaviour. In the case where groups were larger than two students, two audio recording devices were used per group, one placed at each end of the desk so that all students in the group could be heard.

Because help seeking may not always take verbal form, video as well as audio data was captured for each of the four tasks included in this study. This allowed for behaviours such as checking another student’s book for clarification to be captured.

After each lesson, I reviewed the video and selected a variety of learning episodes which were to be used during the stimulated recall interviews.

3.4.5 Tasks

Four tasks were used and filmed for this study. Each task ran for a period of one lesson (75 minutes) and was chosen to encourage students to construct new insights and knowledge. The construction of knowledge was stimulated by a problem. A problem is defined here...
as any task or activity for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific correct solution method.

(van de Walle, 2006, p. 11)

Students solve such problems “not to apply mathematics but to learn new mathematics” (Ibid.). In other words, by working through the problem, students should develop and learn new skills they previously did not have prior to the task.

A description of each problem is provided below. For the first three tasks, worksheets were provided to students. These are provided in the Appendices. For the final task, students were not provided with any worksheets, but instead provided with one empty can of soft drink.

Ferris wheel task – Week three of Term 2 (Appendix 2)

Students worked on this task as an introduction to trigonometric graphs. Prior to this lesson, students had spent time practicing finding exact values for special angles, but had not yet been introduced to the graph of \( y = \sin(x) \). The task itself involved matching a set of three cards together. These include a picture of a Ferris wheel (with radius and period of revolution included), a sine or cosine graph, and an equation. Before students worked on matching their cards together, they were asked to complete an introductory warm-up sheet which encouraged them to think about transformations applied to the graph of \( y = \sin(x) \) and \( y = \cos(x) \).

Students worked in pairs with only one pen/pencil per pair. This encouraged them to work as a team and vocalise more of their thinking (Williams, 2002). Two pairs were filmed for this activity. One pair included the two case-study students, Pippa and Lilly. The second pair included two boys, Aaron and Xander. The two pairs occasionally came together throughout the task to help each other.
The intention of the task was to allow students to work out for themselves how transformations affect the shape of sine and cosine graphs so that the next lesson, they could be introduced to terms ‘amplitude’ and ‘period’ and understand how and why the amplitude and period affect the shape of the graph.

**Position/Velocity graphs – Week nine Term 2 (Appendix 4)**

This lesson formed the introductory lesson to calculus. Students were given a set of graphs, organised in pairs. For each pair, either the position-time graph was given and students had to draw the corresponding velocity-time graph, or the velocity-time graph was given and students had to draw the position-time graph. Students worked in pairs with only one pen/pencil per group again for this activity.

Two pairs were filmed. One pair included Pippa and Ruby. The other pair included Clara and Lilly. All four students then took part in a follow-up interview. The audio for Clara and Lilly did not work and data for this pair from this activity was not captured but rather reconstructed. The pair was informed of this during their follow-up interview, where I instead asked them to explain to me, in as much detail as possible, how they went about solving the task. The reconstruction of the lesson through verbal reports can be considered justified as suitable data for this study (Ericsson & Herbert, 1980), though it is recognised that such an account is likely to lack the sort of depth that might have been expected had the students been able to watch the video footage (Clarke, 1997).

The purpose of this task was to introduce students to the gradient function and in particular, allow them to work out that a turning point will occur when the gradient function has a zero value.
Largest Triangle – Week 7 Term 3 (Appendix 5)

This task involved folding a piece of A4 paper to form a right-angled triangle. Students needed to decide how to fold the paper in order to make the triangle with the largest area.

Students worked in groups of four for this task. One group included Lilly, Phillis, Pippa and Kiara. This group broke off into two pairs, with Lilly and Phillis working together, Pippa and Kiara working as another pair. Pippa was unwell during this lesson and there is limited data for her during this activity. The second group included Aaron, Kipp, and two other students not reported on in this study.

The purpose of this task was to have students practice generating difficult formulas, including having to work out what information they need to gather to solve the problem. It was also intended as their first optimisation problem requiring calculus. The use of calculus to find turning points had not yet been shown to the students in this class.

Best sized coke can – Week 8 Term 3

This task involved another optimisation problem. Students were given cans of soft drink and asked to work out new measurements of a cylindrical can that would minimise the amount of aluminium needed, while ensuring that it still held the same amount of liquid (375 ml). This lesson occurred two lessons after the largest triangle task. A whole class discussion took place during the lesson following the triangle task where students presented to the class different solutions. Ideas were drawn out about how to generate an equation that could be used to draw a graph so that a maximum or minimum turning point could be found. It was discussed that calculus could be used to find the turning points by setting the derivative to zero.

Again, two groups were filmed for the coke can task. The first group included Lilly, Pippa, Kiara, Ruby, and Phillis. The second group included Aaron, Kipp, and two other students not reported on in this study.
The purpose of this task was to give students additional practice generating difficult formulas as well as further practice finding turning points by finding when the derivative equals zero.

3.4.6 Interviews

Interviews were the main source of data from which students’ perceptions of understanding were drawn from. Individual stimulated recall interviews, which involves playing back sections of a previously recorded lesson to the student to stimulate their recollection of the activity, were used after the Ferris wheel task and the velocity graphs task. Students only took part in one interview following the triangle and coke problems. This was done because it was felt that firstly, both problems were fairly similar in nature, and secondly, both tasks occurred only one lesson apart and it was felt that conducting two separate interviews would impact on students’ personal time too much.

After each filmed lesson, video/audio and recorded transcripts of the lesson were reviewed and parts of interest were chosen to show students during their follow-up interviews. Students were interviewed within two days following the filmed lesson. This allowed students to remember as much detail as possible. Interviews were kept short (20 minutes maximum) and there was a maximum of only three interviews per student throughout the study.

At the beginning of each interview, the student was asked to describe anything that might have happened during the lesson that was memorable, even if it was not mathematical. Following this, I asked each student to watch a segment of the lesson. In order to initiate student self-reporting, I would ask “What were you thinking there?” or “How were you feeling during this moment?” I made requests for clarification or confirmation where necessary, though in such a way as to avoid asking leading questions or making evaluative comments. At the end of every student’s first interview, they were also asked to give their own personal definition of what it means to understand. If they were unable to verbalise this, I
would ask them to given an example of when they either did or did not understood something in mathematics and try to explain how they knew they did/did not understand.

All stimulated interviews were audio-taped and then transcribed. A pen and paper was also provided to students if they wished to elaborate visually by writing or drawing diagrams.

3.5 Operationalising help seeking

Some of the statements from the questionnaire were used to infer help seeking behaviours when selecting potential case-study students. For example, the statement *I ask my teacher to help clarify concepts I don’t understand well* is indicative of adaptive help seeking as it implies the intention of comprehension rather than finding answers. The statement *Even if I have trouble learning in this class, I try to do the work on my own, without help from anyone* was instead indicative of help avoidance. Additional factors such as classroom observations were then needed to decide whether the help avoidance could be considered adaptive or non-adaptive.

My own observations and interpretations were used to operationalise help seeking in the classroom (Ryan et al., 2005). Considering the types of questions students asked in isolation was not sufficient and the context with which help seeking occurred needed to be considered. Examples from the study are provided here for elaboration. The following provides one comment from each of the four key students:


Lilly: Oh my god! I need help. Bad… We don’t know how to do the dilation.

Aaron: This has to be cos. Which one is that? Man I’m confused.

Kipp: Wait, what is the graph? I don’t get it.
On the surface, each statement is similar. They all express confusion suggesting a perceived need for help, and all state precisely what it is that they do not understand (i.e., what to sub, how to dilate, whether the graph is sine or cos, or how to graph). Yet, when the context is considered, each question indicates different help seeking behaviours. Prior to asking her question, Pippa had not made a significant attempt on the problem. Because of this, Pippa’s question was considered non-adaptive help seeking. Lilly on the other hand had spent a considerable amount of time working on the problem before asking for help, hence this was considered adaptive help seeking. In contrast, Aaron had spent quite some time also working on his task, yet when he received useful help from his peers, he failed to apply it appropriately and did not continue working on the task. This was considered non-adaptive help seeking. Kipp had also spent a considerable amount of time working quietly and independently. When Kipp asked for help, he did not understand the explanation given, so he continued to ask more and more questions until he felt he understood. He then went back to try the problem on his own. Because Kipp made appropriate use of the help given and applied it directly to the problem he was working on, his behaviour was considered to be adaptive help seeking.

In short, questions students asked as a stand-alone statement were not used to help decide whether the behaviour was adaptive or non-adaptive. Rather it was the context with which it was said that helped my interpretation of the students’ behaviour.

If a student did not ask for help, they were considered adaptive/non-adaptive help avoiders if their body language and / or interview comments were consistent with such a decision. For example, students who frowned, or chewed quietly on a pen, may be in a pensive state and could be considered adaptive help avoidance. If the same body language was coupled with agitation or embarrassment, such as hiding their work with their arm, then this was considered non-adaptive help avoidance. Student interviews were particularly useful to help decide between adaptive/non-adaptive help avoidance. Sometimes when asked ‘What did you learn that was new in this lesson?’ and a student responded with ‘Nothing. It was so confusing!’ Further probing about why they didn’t seek out help was useful for determining the nature of
the help seeking. Students who stated they were too scared or embarrassed were categorised as non-adaptive help avoiders. Students who commented that they felt they were on the brink of making progress if they had just a little more time to keep trying without help were considered adaptive help-avoiders.

### 3.6 Operationalising student perceptions of understanding

Follow-up interviews were used as the main data source to identify perceptions of understanding that students hold. During their first interview, participants were asked to give their own definition for what it means to understand mathematics. Their reported views on understanding were then compared with their behaviour observed in the classroom. If classroom observations matched the reported behaviour, it was assumed that the student’s view of understanding really did match their reported views. For example, Lilly reported that understanding required being able to explain ideas to others. She was observed during one activity explaining her thinking and reasoning to other students to help clarify her own thinking. It was therefore assumed that Lilly really did think that understanding involves being able to explain to others. With this in mind, students will not be considered to hold a particular perception of understanding unless they are seen to also enact it. This was done to reduce the risk of students identifying perceptions that the interviewer might want to hear.

### 3.7 Ethical considerations

Ethical considerations in regards to the participants’ welfare were taken into account. Firstly, I acted as both teacher and researcher. To decrease the risk that students may have felt pressured to participate in the study and may have felt that they would not have access to my help and attention in the classroom if they did not consent, they were assured that their participation was voluntary. Whether they choose to
participate or not, they were assured they would have access to the same learning opportunities. Students were also told that the tasks that would be used as part of this study would have been used regardless of whether formal research was taking place. In this way, students were more likely to view the tasks used in this study as relevant to their actual learning and more likely to genuinely engage with the tasks.

Students may initially have felt uncomfortable about having video cameras in the classroom. Hence the cameras used were small devices and placed at the back of the classroom in a discrete manner. Students who did not consent to being filmed, were seated away from the cameras. All students were given assigned seating for the filmed lessons so that those who did not consent to be on film were not singled out. This had the potential to disrupt the authentic learning environment as students were not able to sit in their usual seats. To compensate, students were, where possible, seated with their friends they usually sat with.

The following steps were taken to address the risk associated with reporting of data in publications. Comments made in class or during interviews, which were unlikely to cause harm to either the student, or others, was considered low risk and the use of a pseudonym considered sufficient. In cases where comments were at risk of causing harm but would make a significant contribution to the study, precautions such as grouping comments from different sources such as classroom interactions or interview, or including a comment without linking it to a particular pseudonym was used. Data that could not be reported in a way that kept confidential the identity of a student, teacher, or school, was omitted.
Chapter 4  Results

In this chapter results relating to two case-study students, Pippa and Lilly, are presented. For each case-study the data is separated into two sections. The first discusses data relevant to how each student views understanding. The second section discusses data used to identify help seeking behaviours for each student by identifying whether the help seeking was necessary, whether each student asked predominantly for hints, answers, or the next step, and how the help was received and applied. A brief account of the second two key students, Aaron and Kipp, is then provided. A short background for each student who took part in this study, who also appear in the provided transcripts, is provided first.

4.1 Additional students

A short background on each student relevant for this study is provided first. This includes a short background on the case study students, as well as other students who worked alongside the case study students within the same group. This gives the reader some background information about each student who participated in this study. The following descriptions are based on my observations as both classroom teacher and researcher. I draw on what I have learnt as a teacher working with these students over time, and my observations as a teacher and a researcher while students were working on the tasks for this study. My observations as a researcher during student interviews have also contributed to these descriptions.
Pippa

Pippa is a 15-year-old student who feels she is not very good at mathematics. She is a high-achieving student and places a great amount of pressure on herself to obtain high grades. Pippa moved from Asia to Australia and has been here since she was in Year 7, having completed her primary education in China (See below, 4.3.2.3 Relationship between the purpose of the task and Pippa’s reported understanding, p 65).

She was a hesitant student and highly anxious about failing. She often procrastinated in class and spent as much time as possible setting herself up for work rather than doing her work. As a result, she often fell behind in her homework and assignments across all subjects and ‘overwhelmed’ was the word she tended to use during these times.

Lilly

Lilly is an energetic and enthusiastic 15-year-old student. She enjoyed working with peers and worked well collaboratively. She was however self-conscious when she did not know how to answer a question but did achieve excellent results.

Ruby

Ruby is an enthusiastic student and enjoys working things out for herself. She is a high achieving student and focuses more on understanding her work rather than the marks she obtains. She valued understanding as evidenced in her comment during an interview:

Probably more important than the answer is understanding how to get the answer is more important to me.
Ruby was supportive of her group members, offering help when they were stuck. Here is one example from the coke problem:

Pippa: I’m still lost. I don’t get it.

Ruby: You’re still lost? Do you want me to explain it to you?

**Phillis**

Phillis is an enthusiastic and high achieving student. She enjoyed working on the triangle and coke problems:

Phillis: I enjoyed doing those types of like problem solving questions. Because I guess it really challenges me more to think. And other than just basic workbook questions. It’s like applying workbook questions to real life.

She valued understanding her work and she often went back over her work if she did not fully understand how she had found her answer:

Phillis: I guess if I see something and it doesn’t really make sense I guess. Just kind of confuses me a little. And I’m like ‘How did I get that?’ why did I divide it by this or times it by that. So I kind of have to go back to the question and do the whole thing again to find out why that actually happened.

**Aaron**

Aaron is popular amongst other students. He focused on obtaining high marks and felt that this was achieved through memorisation rather than understanding:

Aaron: So like rote learning has always been something that has worked really well for me.
He obtained poor tests results in this class but maintained a positive attitude. He asked for help regularly though tended to explicitly ask the teacher what he needed to do next rather than hints or explanations and generally asked for help when unnecessary:

Aaron: Can you help us do this as well? Cause like we were just about to go on to this.

Here Aaron asks for help from the teacher before he has even started or attempted the question, deeming this help seeking unnecessary.

Kipp

Kipp was an eager student who strove to always work things out for himself:

Kipp: If I’m going to do it, I will do it with no help.

He valued effort over ability and attributed not being able to finish a problem to not trying hard enough. Here is his reaction when I asked him during an interview how he felt when he was unable to finish the triangle problem:

Kipp: Maybe I wasn’t trying hard enough. I felt like I wasn’t trying hard enough. So long as you try as hard as you can then you can get it.

Xander

Xander was also focused on achieving high test scores and, like Aaron, struggled to do so. He often wasted class time and completed little work. Xander relied on memorisation and was unable to provide any justification for the methods he used suggesting he did not comprehend the work he was completing. He
also tended to accept explanations at face value without asking himself if he understood the content or not:

Xander: I trust other people more [than myself]. When someone is just explaining it to you, you probably just believe what they are saying.

For Xander, understanding was related to repetition:

Xander: Well, when you get something and then you do lots of work on them, and it seems pointless. So if they’re not a challenge, then you understand.

Xander likely used of repetition in order to memorise his work as he was observed often in class repeating formulas he had learned elsewhere (such as private tuition outside of school) rather than trying to think through the problem himself.

### 4.2 Transcript numbering convention

Student quotations will be numbered throughout this chapter so that key parts can be easily identified and referred to for analysis and discussion. Statements that are lengthy and contain multiple ideas are separated into single lines, each line with its own number. They are not separate statements but form part of the same block of speech which has been separated to allow for easier referencing. Quotations relevant to Pippa will have numbers starting with P. Quotations relevant to Lilly will have numbers starting with L. Comments made by myself as the classroom teacher are labelled ‘Miss P’. Comments I made during interviews are labelled ‘Int’.
4.3  Case-study: Pippa

Data relevant to Pippa is presented in this section. It is separated into two large sections: her perceptions of understanding and her help seeking behaviours. Before exploring these, a short introduction is provided outlining Pippa’s general demeanour, showing that in general, Pippa was a hesitant student in class.

4.3.1  Pippa is a hesitant student

This section is not intended to demonstrate any types of understanding or help seeking behaviours, but rather to help paint a picture of the type of student Pippa is in the classroom.

In general, Pippa lacked confidence and was a hesitant student as shown by the separate quotes below which are drawn from all four filmed tasks:

P1.1  Pippa: Yeah actually I don’t know.

P2.1  Pippa: I actually have no idea.

P3.1  Pippa: But I’m not sure if that’s right or not. I don’t think it is.

P4.1  Pippa: Except I don’t have any idea of what I’m doing. I dunno. Maybe I did.4

Statements such as “I don’t know” were common for Pippa. I asked Pippa during the interview following the velocity-graph tasks why she felt she needed to use phrases like ‘I don’t know’, even during times when she knew her answer was correct. She responded by commenting on a time when she worked with her partner, Ruby, during the velocity-graphs task.

4 Here each comment comes from a different activity or interview and do not form part of the one continuous comment. Thus they are labelled individual as P1, P2, P3, and P4. The subsequent .1 represents the first line of transcript from the respective quote.
Pippa: P5.1 So like Ruby says something but I was like scared to disagree because I wasn’t sure either.

P5.2 So I didn’t want to say oh no it obviously isn’t this answer.

Int: P5.3 Why were you scared to disagree with Ruby?

Pippa: P5.4 Because I, I’m not confident.

P5.5 I felt like oh I don’t know it that well.

P5.6 But if like I actually know it well, I wouldn’t be scared.

P5.7 So I was too afraid to strongly disagree even though I knew it was wrong.

P5.8 But I just told her I think it is wrong cause I was too scared.  

Pippa reports herself as fearful of disagreeing with someone because she is not sure herself (P5.1) and indicates that this guides her behaviour (P5.2). She seems to be so scared of getting the answer wrong, that she would prefer to say she does not know how to solve it rather than risk getting it wrong even if she is fairly certain that an answer is incorrect (P5.7). Pippa reports that if she really knows the material well, then she would no longer be scared (P5.6) and would presumably might be more confident offering suggestions to other students.

The idea that Pippa is a hesitant student who lacked confidences will be drawn on throughout the remainder of this chapter.

---

5 Here each line begins with P5 (representing example 5 for Pippa) and belongs to the one conversation from the same interview. The first two rows, P5.1 and P5.2 represent the one comment from Pippa that has been written as two separate lines. This has been done so that specific comments may be referred to more easily during analysis. Line P5.3 then represents what I have said as the interviewer. Lines P5.4-P5.8 then represent a single comment from Pippa, again separated into separate lines for easy referencing.
4.3.2 Pippa’s perceptions of understanding

This section outlines ways in which Pippa characterises understanding and a summary of the findings that will be drawn out in this section is provided in Table 3. These include comments about what counts as understanding, what does not count as understanding, whether understanding is valued, and what counts as an indication that understanding has occurred. This is listed in the second column of the table. The first column outlines how Pippa reported a particular perception of understanding while the last two columns provide a sample quote for the related perception, as well as the source it came from. The third column identifies, where appropriate, how Pippa’s particular perception of understanding relates to theoretical models of understanding, such as Skemp’s (1976) instrumental (using rules without knowing why) and relational (know both how to use rules and why they work) understanding.

The following sections then provide illustrative examples from the tasks and interviews that demonstrate each of the ways Pippa has identified what it means to understand or not understand.

4.3.2.1 Understanding is ‘knowing why’

When Pippa was asked what ‘understanding’ means to her during the interview following the Ferris-wheel task (the first task included as part of this study exploring sine and cosine graphs), she identified two types of understanding:

Pippa:  
P6.1 Because I used to memorise it [mathematics].  
P6.2 Like I don’t know why.  
P6.3 But it’s just like that because someone told me it’s just like that.  
P6.4 Oh because someone told me so you think it’s right.  
P6.5 But now I think, oh I get why it’s like that.
<table>
<thead>
<tr>
<th>Pippa’s perceptions of understanding</th>
<th>Identified as understanding, lack of understanding or not a type of understanding</th>
<th>Relationship with theoretical understanding</th>
<th>Illustrative statement from Pippa</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understanding is ‘knowing why’</td>
<td>What counts as understanding</td>
<td>Meaningful understanding (not quite relational)</td>
<td>You fully know what happens and why it happens.</td>
<td>Interview following Ferris wheel task</td>
</tr>
<tr>
<td>Knowing how to get the final answer without comprehending each step</td>
<td>Does not count as understanding</td>
<td>Instrumental understanding (Skemp, 1976)</td>
<td>But I don’t think that’s called understanding if you know the end results.</td>
<td>Interview following Ferris wheel task</td>
</tr>
<tr>
<td>Getting high grades is important</td>
<td>Requirement for success in mathematics</td>
<td>Instrumental understanding</td>
<td>In a test I think it’s better to know just how to get the answer.</td>
<td>Interview following triangle and coke problems</td>
</tr>
<tr>
<td>Understanding is not valued or important</td>
<td>Not required for success in mathematics</td>
<td>Instrumental understanding</td>
<td>But if it’s for an exam, then I don’t really care (about understanding). I just want to get the answers.</td>
<td>Interview following triangle and coke problems</td>
</tr>
</tbody>
</table>
Table 3 Summary of Pippa’s views on understanding including what counts as understanding, what does not count as understanding, and indications that understanding has occurred
Pippa previously perceived understanding as memorisation (Error! Reference source not found.), implying she no longer sees memorisation as a type of understanding. A later quote below (P10.2, p. 65) further confirms that Pippa does not equate memorisation with understanding. Instead she now perceives understanding as knowing why (Error! Reference source not found.).

During the same interview, Pippa expanded on what she meant by ‘knowing why’:

Pippa:  

P7.1 Understanding I think is more like knowing why it happens.

P7.2 Rather… like, you may know the end results.

P7.3 But I don’t think that’s called understanding if you know the end results.

P7.4 But if you know why [her emphasis] the results happen.

P7.5 You fully know what happens and why it happens.

From this, understanding is a combination of both knowing the final answer is (P7.2), and ‘knowing why’.

Evidence for what Pippa means by ‘knowing why’ is unclear at this point. This emerged throughout the research period. I will progressively build on what has been found to this point in the analysis as more results are presented, and justify the position I take at the end of this section that ‘knowing why’ is related to an understanding that is more meaningful than instrumental understanding, but not quite equivalent with relational understanding.

Of interest is that both Pippa and Lilly provided almost identical descriptions for understanding when asked the question ‘What do you think it means to understand?’ (For Lilly’s description, see below, 4.4.1.1 Understanding is more than knowing facts, p. 98). Both students mention that just knowing how to get the answer does not count as understanding and both used the phrase ‘knowing why’, meaning that understanding requires ‘knowing why’ the answer works. However, on closer inspection, it was found that Pippa approached her work with instrumental understanding while Lilly approached her work with relational understanding. Because both students provided almost identical descriptions of understanding, yet demonstrated different levels of understanding, it was decided to include not just students’
perceptions of understanding in this study, but to also include the types of understanding students were using such as instrumental or relational.

4.3.2.2 If I comprehend all the steps, then I understand

During the interview following the triangle and coke tasks, I asked Pippa if she felt she had understood the coke problem given she had worked in a large group. Her response highlights the difference Pippa sees between solving a problem individually and solving as a group.

Pippa reports that if you do not know every step of the problem then you do not understand (P8.1). In order to count as understanding, Pippa feels she should be able to solve the task individually (P8.1). Furthermore, Pippa feels that not knowing ‘what’s going on’ implies a lack of understanding (P8.7-P8.8). From this, one perception related to understanding is If I don’t know all the steps, then I do not understand.

What might ‘knowing all the steps’ mean for Pippa? Pippa sees understanding as requiring an explanation that she can comprehend (P8.7). This might mean ‘My friends didn’t tell me the steps’ but it might also be interpreted as ‘The steps didn’t make sense when my friends told me’. At P8.4, Pippa states that listening to other students without participating would not count as understanding. Yet, at P8.7, she requires an explanation from her group members to count as understanding. This suggests that listening
for the answer and listening to an ‘explanation’ hold different meanings for Pippa. If listening for the answer does not count as understanding, and an ‘explanation’ does count as understanding, then it seems likely that what Pippa expects from an explanation is more than just being told the steps. It suggests she expects to develop comprehension. If this is the case, then the perception *If I do not know all the steps, then I do not understand* has overtones of comprehending every step rather than memorising every step. From this it appears Pippa might instead view understanding as *If I do not comprehend all the steps, then I do not understand*.

### 4.3.2.3 Relationship between the purpose of the task and Pippa’s reported understanding

I asked Pippa during the interview following the triangle and coke problems whether she felt understanding or getting the answer was more important. Here are two of her comments:

Pippa:

<table>
<thead>
<tr>
<th>P9.1</th>
<th>In a test I think it’s better to know just how to get the answer.</th>
</tr>
</thead>
<tbody>
<tr>
<td>P9.2</td>
<td>Because you want to know how to get good marks.</td>
</tr>
<tr>
<td>P9.3</td>
<td>But if it’s normal learning, then I would rather focus on understanding why that happens.</td>
</tr>
<tr>
<td>P9.4</td>
<td>But if it’s for an exam, then I don’t really care.</td>
</tr>
<tr>
<td>P9.5</td>
<td>I just want to get the answers.</td>
</tr>
</tbody>
</table>

Pippa:

<table>
<thead>
<tr>
<th>P10.1</th>
<th>I don’t think it’s [understanding] necessarily completely important.</th>
</tr>
</thead>
<tbody>
<tr>
<td>P10.2</td>
<td>It’s just I find it easier to remember during a test if I understand something.</td>
</tr>
</tbody>
</table>

Pippa’s views of understanding, as well as goals for learning, appear dependent on the purpose of the task. During tests, Pippa’s goal for learning is getting high marks (P9.2). During ‘normal learning’, Pippa’s goal is to “understand” or comprehend her work (P9.3). By ‘normal learning’, Pippa appears to mean regular classroom activities such as the problem-solving activities, discussions, or textbook work.
Two themes emerge with regards to Pippa’s views on understanding. The first relates to finding the answer. Pippa identifies a difference between tests (P9.1) and ‘normal learning’ (P9.3). During normal learning, Pippa’s goal is to comprehend. When the purpose of the task is an assessment, Pippa’s goal is to get the right answer. Because Pippa links comprehension, or “knowing why”, with ‘normal learning’, then it is likely that she associates a lack of comprehension with tests and exams. That is, for Pippa, getting the answer does not require understanding. This forms another perception of understanding: Knowing how to get the answer does not count as understanding.

The second theme relates to the tension between understanding and getting high marks. For Pippa, achieving high marks is important (P9.2). She would rather get the right answer on a test than understand (P9.4-P9.5). Students with similar views were identified by Crawford et al. (1994). If the task is an assessment, then understanding is not important for Pippa (P10.1). Instead, Pippa sees understanding as helpful in achieving her goal of high grades (P10.2).

Because Pippa identifies being able to get the answer without understanding during tests, then it can be assumed that Pippa uses predominantly instrumental understanding when completing tests. There is insufficient information here to decide whether Pippa uses relational understanding during ‘normal learning’, but subsequent examples will show that it is unlikely that Pippa demonstrated relational understanding during the activities observed for this study.

In short, knowing how to get the answer forms part of Pippa’s overall views on understanding, relating to her views on what does not count as understanding. Furthermore, during tests and exams, Pippa does not feel that understanding is valuable.
4.3.2.4 Understanding is related to repetition

In the following example, Pippa discusses how she now understands why cosine graphs have the shape they do, while before completing the Ferris wheel activity, she did not. I asked her to explain why she now felt she understood:

Pippa: P11.1 You draw it out and you know, you kind of understand it.

Int: P11.2 What do you mean by that?

Pippa: P11.3 I feel that it’s right.

P11.4 What I did was right.

P11.5 It’s kind of hard to say.

P11.6 But then because you draw it out [graphs of sine and cosine functions].

P11.7 And there’s so many exercises in class.

P11.8 So over time you just kind of get the pattern.

Int: P11.9 So you got familiar with it?

Pippa: P11.10 Yeah, I got familiar with it.

P11.11 At the start I’ve only seen this once.

P11.12 So you can’t have a pattern with it.

P11.13 But because you’ve done this so many times, you’re like oh, I get this because repeating the same thing.

P11.14 And then you kind of just get it.

Pippa reports developing an understanding of sine and cosine graphs through repetition (P11.13. See below P15.2, p. 72). She associates not recognising a pattern with lack of understanding (P11.12). When she first started working on questions related to sine and cosine graphs, she felt the questions were
unfamiliar or was unable to recognise a pattern, and interprets this as a lack of understanding (P11.11).

Of note, she makes no mention about understanding why the pattern exists.

Pippa’s view that understanding and lack of understanding can be related to recognition of patterns is supported by her behaviour during the problem-solving activities. Two examples are provided here, one in which Pippa believes the problem is familiar, another unfamiliar.

During the Ferris wheel activity, Pippa and her partner, Lilly, have finished the first side of the ‘warm up’ sheet (Appendix 2), and have moved onto the second side. Here is Pippa’s response:

P12.1  Pippa: This is pretty easy. It’s just like the other one [the other side of the sheet].

Both sides of the worksheet have identical layouts. Questions on both sides were identical, where sine had been replaced by cosine in each case. For example, the first side of the warm-up sheet asks students to explain why \( \sin(\pi) = 0 \) while the second side asks students to explain why \( \cos(\pi) = -1 \). Based on Pippa’s initial response (P12.1), it appears that Pippa recognised the similarity between the two sides. The recognition of the similarity between the two sides is reported as being easy by Pippa. Because Pippa identifies understanding something as making things easier (See P15.15, p. 73), it is likely the recognition of something as familiar is seen as counting as understanding.

The second example comes from the coke can activity. When I realised that Pippa’s group had managed to solve the coke problem using more than one method, I set them a new problem in the final 20 minutes of class: the open-lid box problem. The problem is as follows:

Take an A4 sheet of paper. Cut out the same sized squares from each of the four corners. Now fold to form an open-lid box. What length square should be cut from the A4 paper to make a box with the largest volume?

The problem is represented in Figure 2. The left-hand side represents a sheet of A4 paper. The shaded corner squares represent which parts of the paper to remove before folding. The right-hand side then represents the box that is to be folded from the sheet of paper.
From these examples, the following two views emerge: If the problem is familiar, then I understand it and the inverse If the problem is unfamiliar, then I will not understand it.

How might this relate to a theoretical perspective of understanding? Pippa reports recognising patterns after repeating similar questions, but does not comment on knowing why the patterns work. This fits with Simon’s (2003) definition of empirical activity. Simon (2003) defines two types of mathematical activities that students can engage in: empirical activity and logico-mathematical activity. Empirical activity involves students generalising formulas from patterns. In empirical activity, students recognise that a formula is appropriate, but not why it is appropriate. Logico-mathematical activity involves recognising why a generalised formula is appropriate. For example, a student may measure the areas of several triangles and recognise a pattern: that all areas can be found by multiplying the base and altitude and dividing by two. Such a student would be engaged in empirical activity. A student engaged in logico-mathematical activity may then recognise that the formula works because two triangles can be arranged to make a rectangle, from which the area of a triangle must be half that of the area of a rectangle. With regards to Pippa, she reports recognising patterns after repeating similar questions, but does not comment on knowing why the patterns work. Therefore, Pippa is more likely engaged in empirical rather than logico-mathematical activity.

Figure 2 Open-lid problem: Make the largest rectangular box by removing four shaded corner squares
How might this relate to relational and instrumental understanding? For this study, Pippa is considered to have displayed a more meaningful level of understanding than instrumental alone. Although Pippa applies steps and formulas without recognising the “logical necessity” of the underlying pattern, suggesting she is using instrumental understanding, Pippa goes one step further than instrumental understanding, which is seen as an activity in memory alone and nothing more (Tall, 1978). Pippa is able to construct new knowledge when repeating problems, as the patterns she recognises are her own. She was not told the patterns by other students or the teacher, and drew her own conclusions on what was similar between each question. The act of recognising her own patterns moves beyond an exercise in memory and is therefore interpreted as a more meaningful level of understanding than instrumental alone. According to Simon (2003), Pippa is not considered to be demonstrating understanding, as Simon defines understanding as requiring the recognition of “the logical necessity of a particular pattern” (p. 185). Therefore, Pippa is considered to display a level of understanding that sits somewhere between instrumental and relational.

The recognition of new patterns that can then be applied with new problems can also be seen as horizontal mathematizing. Treffers and Goffree (1985) separate mathematical activity into horizontal and vertical mathematizing. Horizontal mathematizing refers to students strengthening connections between mathematics they already know and the mathematics they are working on in the moment. While vertical mathematizing means working within the mathematics itself by forming new mathematical structures. Because Pippa is only strengthening ideas between questions, she is considered to be engaged in horizontal mathematizing.

To further explore the nature of this understanding, Pippa’s cultural background is taken into consideration. Pippa has completed her primary education in China. Leung (2001) describes Chinese mathematics education as involving memorisation. Repetition and practice is emphasised and curriculum is heavily content driven and orientated towards the goal of examinations. In Chinese education, memorisation is an accepted way of learning, even when what is being committed to memory is not
understood as the act of memorising is seen as a route to meaningful understanding (Marton et al., 1993). These aspects are reflected in Pippa’s behaviours and beliefs. She has a preference for repetition and a strong focus on achieving a high performance in exams at the expense of understanding.

Marton et al., (1993) argued that memorisation forms a route to meaningful understanding. Pippa’s reports of identifying patterns fit with this. Repetition allows the process of answering questions to become automated, thereby likely reducing the cognitive load, resulting in some gain in conceptual knowledge where conceptual knowledge refers to an understanding of the relationship between various pieces of knowledge (Kalyuga, 2007; Rittle-Johnson & Alibali, 1999). Therefore, being engaged in horizontal or empirical activity, should not be considered strictly instrumental understanding, that is, understanding with no meaning. Using the idea of a continuum from Rittle-Johnson and Alibali, repetition can be seen as a tool to move along the continuum from instrumental towards relational understanding. Taken together, Pippa is seen to have used a more meaningful level of understanding than instrumental understanding, though did not quite reach relational understanding.

4.3.2.5 When I make connections, I understand

The last type of understanding to explore is Pippa’s association between making connections and understanding. An example is provided from Pippa’s interview related to the velocity-graphs task. In this example, Pippa talks about the difference between learning about velocity in Physics and in Mathematics. In Physics, she felt she was told what to do without being told why, while in Mathematics, she started to think visually about physically ‘walking’ a graph which allowed her to begin to make connections between ideas.

During the activity, students were given four position-time graphs where each graph included sections with positive, negative and zero velocities. Pippa connected a concept that was new to her (velocity) to an idea already she was already familiar with (speed graphs):
Chapter 4

Results

Pippa: Oh! So if velocity, if you turn it around, it’s like speed. So if it’s going lower, that means it’s going faster.

Pippa made this comment while she was working on the first position-time graph, shown in Figure 3 (See also Appendix 4). The position-time graph includes only motion in one direction (negative direction). Pippa was unsure whether the corresponding velocity graph should have positive or negative values given that she had only seen graphs in Physics with positive values for speed. It was during this task that she came to the realisation that velocity is speed with direction, and that the larger the negative velocity, the greater the speed.

Figure 3 Position-time graph for which Pippa is trying to draw the corresponding velocity-time graph

Here are her comments about this realisation from her interview:

Pippa: P15.1 Suddenly everything just links together.

P15.2 And like after doing like two questions, I start to look back at the question.

P15.3 And everything starts to make sense.

Int: P15.4 Do you know what it was that linked together?
Pippa: P15.5 Like at the start, I never really thought of picturing the idea.

P15.6 I just look at the graph and just do it.

P15.7 But as the lesson moves on, I thought of the car actually moving.

Int: P15.8 Now I remember when you wanted to hand up your sheet, you said something about Physics. That you didn’t understand in Physics. But then you said that this helped you with Physics. How has this activity helped?

Pippa: P15.9 Because it’s Maths. I dunno. In Physics, it is Maths, but not Maths. Like they’re trying to take out the maths component by explaining it.

P15.10 But it doesn’t link together.

P15.11 So you can see a bunch of information just falling apart.

P15.12 But they don’t link together.

P15.13 Like I remember it, but I just don’t get why it is like that.

P15.14 So I find it hard.

P15.15 But when it links together, it makes it a lot easier for me.

Int: P15.16 Can you tell me more about things that link together?

Pippa: P15.17 Like so umm in Physics, we did something like that.

P15.18 But like it doesn’t make sense

P15.19 because they say it ‘oh you stop so you just do nothing’.

P15.20 Yeah, that statement, they didn’t say how to get it.

P15.21 Whereas in maths, you think in more picture and mathematical way.

P15.22 So I’m the start I’m just like oh the car has just stopped moving.

P15.23 But after I get used to picturing it, I find it a little bit easier to picture it.
So it felt a little bit easier.

P15.6 provides an example of instrumental understanding. Pippa has yet to make any connections between ideas, and she reports answering questions without any understanding. She ‘just does it’. She reports this as a lack of understanding, reinforcing the argument that one of Pippa’s views related to lack of understanding is *Knowing how to get the answer does not count as understanding*.

There are two views of understanding related to making connections here, one relating to what counts as understanding, the other to what does not count as understanding. Firstly, understanding occurs when connections are made (P15.1, P15.3). There is also the inverse: understanding has not occurred when connections have not been made (P15.10-P15.13). Understanding is also reported as being ‘easy’ (P15.15, P15.23, P15.24), that is, from Pippa’s perspective, the outcome of understanding is that things become easier.

Pippa’s demeanour differed during the velocity graphs activity when compared with the other tasks. During the Ferris wheel task and the coke problem, Pippa regularly used phrases such as ‘I don’t know’ or ‘I don’t get it’. She was also unlikely to disagree with other members of her group and thus was not an active member of her group (See 4.3.1 *Pippa is a hesitant student*, p. 58). In contrast, during the velocity graphs task, Pippa appeared far more active. There were significantly fewer ‘I don’t know’ comments and she appeared more willing to disagree with her partner and even explain why some of her partner’s answers were incorrect (For examples, see below, *Help seeking during the velocity graphs task focused on comprehension*, p. 88).

Why might her demeanour have differed during the velocity graphs task? Pippa had already learnt about position and speed graphs in Physics before working on the problem in this class. Her partner, Ruby, who was not studying Physics, had not yet learnt about kinematics and this was her first experience working with position-time and velocity-time graphs. Pippa therefore had more experience working with velocity graphs than Ruby. It is possible that her change in demeanour resulted from her view that she had more
experience or expertise working with velocity graphs, making her more willing to contribute to her group’s discussion. This will have further significance when exploring Pippa’s help seeking, as it will be shown that her help seeking behaviour also changed when Pippa felt she had more experience working with velocity graphs than her partner suggesting an underlying fear of lagging behind the rest of her group.

4.3.2.6 Summary of Pippa’s understanding

A summary is now provided of the findings. This is separated into two sections. The first summarises Pippa’s level of understanding from a theoretical perspective (i.e., instrumental or relational understanding). The second section summarises Pippa’s perceptions of understanding. Included within the umbrella of her perceptions of understanding are what counts as understanding, what does not count as understanding, and indications that understanding has occurred.

Pippa’s researcher identified level of understanding

In general, Pippa approached her work using instrumental understanding. However, when Pippa was able to identify patterns in questions or able to make connections between ideas, she developed a more meaningful understanding than instrumental, though not quite relational (Horizontal mathematizing, Treffers & Goffree, 1985).

Two factors were found that influenced Pippa’s level of understanding, though there may be more that were not identified within this study. The first influence related to the purpose of the task. When the task was an assessment, Pippa approached it using instrumental understanding, that is, she was only interested in finding answers regardless of whether she felt she did or did not understand what she was doing. When the task was normal learning, she tried to comprehend her work by searching for patterns or making
connections which resulted in a more meaningful understanding than instrumental, resulting in horizontal mathematizing.

The second influence related to Pippa’s prior task experience comparative with other students. When Pippa’s prior task-experience was less than or equal to the students she was working with, she approached the task with instrumental understanding. When her prior task-experience was comparatively higher than those around her, her level of understanding improved.

A summary of Pippa’s researcher-identified level of understanding is provided in Figure 4. Using the work of Rittle-Johnson et al. (2001), who argue that understanding forms a continuum, different levels of understanding are placed along a continuum. Where Rittle-Johnson et al. place procedural (knowing the steps used to solve a problem) and conceptual (implicit understanding of the relationship between pieces of knowledge) understanding at either end of their continuum, Skemp’s (1979) instrumental and logical understanding are positioned at either end of the continuum provided in Figure 4. Logical understanding refers to forming generalisations between ideas and being able to explain these ideas to others. Relational understanding lies at the centre. The use of a continuum is not intended to imply that one type of understanding is more important or desirable than another. Rather it is used here to allow for comparisons between different case-study students where different students might sit at different ends of the continuum.

The most frequently observed form of understanding Pippa was found to use during the four tasks was instrumental understanding. This is represented in Figure 4 as a shading with grey diagonal stripes. This was found to occur when the purpose of the task was an assessment, but could also occur if Pippa felt she did not have more prior experience working on the problem than the students she was working with.
Pippa was found to approach her work with a level of understanding between instrumental and relational. This is represented in Figure 4 as shaded in green stripes. This occurred only when both the purpose of the task was an assessment and Pippa felt she more prior experience working on the problem than other students. The frequency with which Pippa approached her work with different levels of understanding is represented by the width of the shading. The width of the shading is representative of how often Pippa approached her work with the corresponding level of understanding. Thus, it is widest for instrumental understanding and narrows towards, but does not quite reach, relational understanding.
Pippa’s perceptions of understanding

Pippa held two general perceptions of understanding. One emerged during tests: *Knowing how to get the answer does not count as understanding*. During tests, understanding was not valued. Rather it was answering questions correctly that was valued. When this occurred, Pippa approached the task with instrumental understanding and reported it as such, that is, she reported being able to find answers without knowing what she was doing. Similar students who feel that mathematics does not need to make sense in order to be a successful mathematics student have been identified in other studies (Anthony, 1994a; Di Martino & Zan, 2011; Frank, 1988).

The second perception of understanding emerged during ‘normal learning’ and was related to comprehension. This included the following three views of understanding related to what counts as understanding, and what signifies a lack of understanding:

- *If I do not comprehend all the steps, then I do not understand*;
- *Understanding involves recognising patterns*;
- *If I can make connections between ideas, then I understand*.

These views were generally reported when Pippa was able to generate her own knowledge by either forming her own connections between ideas, or recognising her own patterns in questions. The key point here is that it was Pippa, rather than an external source such as the teacher, who recognised either the patterns or connections. Similar findings were found by Cavanagh (2011) who found that when students were given the opportunity to work collaboratively in groups, make connections between ideas, and actively contribute to the learning of the group, they perceived an improvement in their level of understanding. For Pippa, generating her own insight while working with others was also perceived as an improvement in understanding.
Learning goals evident from Pippa’s perceptions of understanding

Pippa’s perceptions of understanding (both what counts and does not count as understanding) revealed some of her underlying goals for learning. During tests, Pippa viewed lack of understanding as Knowing how to get the answer does not count as understanding. This was associated with her goals of wanting to achieve high marks (P9.2). The means with which she could achieve this goal was by knowing how to get the answers (P9.1). This was not observed explicitly as part of this study as none of the four tasks used were included as assessment and these assertions are instead reliant on Pippa’s self-reported views of understanding. For a similar case-study student, see Ella, discussed in Sumpter (2013). Ella’s goal was also to achieve good grades which was achieved by knowing how to get the answers. Students with similar goals have been identified by Di Martino and Zan (2011) who argue that such students view knowing how to get the answers as a necessary requirement in order to succeed in mathematics.

During ‘normal learning’, Pippa’s views of understanding included making connections, recognising patterns, and comprehending all the steps. During normal learning, her learning goal was one of comprehension (P9.3). The means with which to achieve this was to be an active participant in the solution process, that is, she wanted to be able to recognise her own patterns and form her own connections between ideas. Pippa did not value this type of understanding, but did report that it made it easier for her to achieve her primary goal of answering question correctly in exams to achieve high marks (P10.2). For a similar student, who also aimed for high test scores and associated understanding with making this goal easier, see Hannula’s case-study of Rita (2002). Rita also held goals of being an active participant in the solution process.
4.3.3 Pippa’s help seeking behaviours

In this section, Pippa’s help seeking is explored by looking at two separate help seeking episodes from the coke can problem. The first illustrates that Pippa tends to ask for answers or the next step required rather than hints. The second episode is provided to offer insight into Pippa’s underlying intention when asking for help. The two episodes will be used, along with other selected interactions from other tasks and Pippa’s interviews, to help identify Pippa’s help seeking style.

In terms of context, Pippa worked in a group of five during the coke task. Group members included four other girls: Lilly, Phillis, Ruby and Kiara. All members of the group had been present during the previous optimisation problem: the triangle problem. Pippa had been particularly unwell with a cold during the triangle problem and did not actively engaged with the task, though did stay in class through the whole lesson:

P16.1 Pippa: But I was really sick so I was… I couldn’t really think.

On the other hand, Lilly and Phillis were some of the few students in the class who had managed to solve the triangle task. Therefore, Pippa was working with a group of students who had experienced success with the triangle task, while she had not yet spent any significant class time familiarising herself with optimisation problems.

The two episodes are provided below. An interpretation is not offered here, but rather both episodes will be unpacked in the three sections that follow: necessity of help, the types of questions asked, and whether it improved subsequent problem solving. These episodes were chosen because they are representative of Pippa’s typical help seeking behaviour observed through the research period.
Episode One

This exchange occurred approximately 45 minutes into the 75-minute lesson. Ruby, Lilly, and Phillis had already generated an equation for the surface area of the coke can in terms of the radius with a fixed volume of 375 ml. During this time, Pippa watched and observed her group derive the equation. Ruby and Phillis then used their CAS calculator to graph their equation and found the coordinate of a minimum turning point through graphical analysis. Using the coordinate of the turning point, Ruby substituted the value of the radius and surface area into her surface area equation, so that she could find the corresponding height of the can of coke.


P17.2 Ruby: We subbed it into our formula and I got 300.

P17.3 Pippa: Wait can I see? How did you work out the height?

P17.4 Ruby: You just sub it back in.

P17.5 Pippa: Back into what?

P17.6 Lilly: The surface area formula.

P17.7 Pippa: Oh. But you don’t even know the surface area.

P17.8 Lilly: You do. When we graphed it.

P17.9 Phillis: When we graph it, the y value was the surface area.

P17.10 Pippa: Oh. Does it tell you the y value? Let me see (calculator is passed over).

P17.11 Ruby: Yay! I feel smart!
Episode Two

This second exchange takes place after the first, and after the group has successfully solved the coke problem through a graphical approach. They then tried to use a different approach to solve the problem by using their newly learnt calculus skills. In this example, Phillis is explaining to Pippa that the group is now trying to find the derivative by solving $\frac{dy}{dx} = 0$ to find the turning point.

P18.1 Pippa: Wait. How did you do it?

P18.2 Lilly: So dy over dx equals zero.

P18.3 Phillis: You know the graph we did, just then?

P18.4 Pippa: Yeah.

P18.5 Phillis: Umm… found like the derivative of it.

P18.6 Pippa: yeah.

P18.7 Phillis: And then that’s the derivative.

P18.8 Pippa: Yeah.

P18.9 Phillis: And then you make

P18.10 Pippa: Equals zero yeah.

P18.11 Phillis: And you make the gradient

P18.12 Pippa: Equals zero yeah.

P18.13 Phillis: And solve for x.

P18.14 Pippa: Yeah.

P18.15 Phillis: And then you get this.

P18.16 Pippa: The answer yeah?
4.3.3.1 Necessity of help

This section discusses whether Pippa asks for help when necessary, that is, help seeking that occurs because Pippa is unable to continue making progress on the problem independently. In Episode One, Pippa admits she is confused (P17.1), suggesting a perceived need for help. However, can Pippa’s help seeking also be considered necessary from a teacher/researcher perspective? Part of the criteria for the necessity of help is whether the student has made a legitimate attempt at the problem before asking for help (2.1.1 Necessary, p. 8). Prior to the exchange given in Episode One, Pippa appears to sit and listen quietly to her group with limited verbal participation. Nor does she write anything in her book, make any use of her CAS, or take any measurements of the provided coke cans. In short, Pippa appears to be a passive member of her group. In addition, P17.7 implies that Pippa was not aware that her group had already come up with an equation for the surface area. This suggests she might not have been paying attention to what other members of her group were doing. If this is the case, then Pippa’s help seeking could be considered unnecessary as she did not appear to work on the problem herself prior to asking for help.

However, Pippa’s apparent passiveness may not be intentional. For example, towards the end of the coke task, I praised the group for their excellent work:

P19.1 Miss P: Nice job guys. Good team work.

P19.2 Pippa: But I made no contribution towards this.
Pippa appeared deflated at this point as evidenced by a furrowed brow and slouched shoulders. Her disheartened state suggests she held a personal expectation that she should be an active participant in the group’s solution process and that she wanted to be an active member of her group.

Furthermore, while Pippa may have appeared passive, she seemed to have still been actively engaged in the task as she was able to generate her own ideas. For example, early into the lesson her group started discussing how to approach the problem. Ruby, Phillis, and Lilly felt they needed to find a formula, while Pippa had an alternate idea:

P20.1 Ruby: Can’t we just use the formula?

P20.2 Phillis: Yeah we could use the formula.

P20.3 Lilly: What do you mean? What formula?

P20.4 Ruby: Well if you got the volume and surface area yeah? And then you do something else? And then you could graph it?

P20.5 Pippa: Isn’t there a ratio?

While the rest of the group talked about finding a formula, Pippa instead suggested there might be a ratio, by which she means trying to find a ratio between the surface area and volume. This was the first moment during this task that the suggestion of finding a ratio had been mentioned by any group member, indicating that this was Pippa’s own thought and that Pippa had come up with the idea independently of her group. This suggests Pippa was actively engaged with the task mentally. Therefore, it can be argued, from a teacher/researcher perspective, that Pippa had made some attempt at the problem to justify asking for help. Therefore, the help seeking can be considered necessary.

However, the next section will show that for this example, Pippa does not struggle for a significant period of time before asking for help and will immediately ask other students for answers or the next step, rather than hints, when she realises they have made a ‘breakthrough’.
4.3.3.2 Types of questions Pippa asks

The purpose of this section is to identify the types of questions Pippa asks. This will be done in two sections. The first explores the types of questions Pippa asked during the coke problem, in particular, the types of questions she asks immediately following moments when her group makes a ‘breakthrough’. The second section then explores the types of questions Pippa asks during a different activity: the velocity graphs task. Illustrative episodes from the velocity graphs task are provided as Pippa demonstrated different help seeking behaviours during this task as opposed to the coke task.

‘How’ questions immediately following a ‘breakthrough’

In Episodes One and Two, Pippa initiated help seeking with the following three questions which are provided here for convenience:

P17.1 Pippa: Wait, what do you sub? What did you sub?

P17.3 Pippa: How did you work out the height?

P18.1 Pippa: Wait. How did you do it?

What might the purpose of these questions be? To answer this, the context they occurred in needs to be considered. Both Episode One and Two take place immediately following a ‘breakthrough’ from the group. By breakthrough, I mean a moment of sudden progress after a lull in progress. Immediately preceding Episode 1, Lilly is excited because she has found the final measurements for the height and radius that will be the ‘best’ sized coke can:

P21.1 Lilly: So our height is going to be 7.384. And our radius is going to be 3.91. We got it guys!

P21.2 Phillis: Did you do it algebraically?
P21.3 Lilly: Huh?

P21.4 Phillis: Did you do it algebraically?

P21.5 Lilly: Yeah, we just found it. Yeah, you just sub in, you sub umm… the radius. Cause that’s your surface area, you that into your surface area.

P21.6 Phillis: Yeah, yeah.

Preceding Episode 2, Phillis becomes excited because Ruby has confirmed that an algebraic approach using calculus resulted in the same answer as using a graphical approach:

P22.1 Ruby: What was the answer we had before?

P22.2 Phillis: 3.9. Oh my god!

P22.3 Lilly: Did we get it?

P22.4 Phillis: Yeah! 3.9.

P22.5 Lilly: That’s awesome!

Immediately following these ‘breakthroughs’, Pippa asks questions P17.3 and P18.1. Both of these questions start with “Wait” and use the past tense to ask “How did you…?” The use of the past tense suggests that Pippa is aware that the students around her have experienced a breakthrough and the use of “Wait” suggests she wants them to stop to tell her something. The use of the word ‘how’ suggests that Pippa wants someone in her group to tell her what steps were just used to get to the answer. This has parallels with executive help seeking, where executive help seeking involves asking for direct answers or the next step rather than hints, and a general preference for having someone else provide a solution for them (Nelson-Le Gall, 1986).

However, it is important to take into account the context of this example. Pippa was unwell during the preceding triangle problem and has significantly less experience working with optimisation problems than the other students in her group. Considering Pippa’s lack of experience, asking for answers instead of
hints might be considered an appropriate form of help seeking (Newman & Schwager, 1995). There is the possibility that Pippa may have been asking directly for answers so that she can work at the same pace as her group.

In order to come to a decision regarding whether Pippa’s questions can be considered adaptive or non-adaptive, the types of questions she ask as she persists in asking for help are taken into consideration. As Pippa persists in asking questions, her questions continue to ask for the next steps (P17.3, P17.5). According to Webb et al. (2002), adaptive help seekers might begin by asking for hints rather than answers or the next steps but that their questions begin to shift from asking for answers to asking for explanations. Webb et al. found non-adaptive help seekers instead tended persist with asking for answers or the next steps required. Because Pippa’s questions continue to ask for the next step, it is reasonable to assume that Pippa’s help seeking is non-adaptive.

As well as asking directly for answers, questions P17.3 and P17.3 are specific in nature. “Wait, what do you sub?” and “How did you work out the height?” clearly identifies for the help giver what Pippa wants help with. She wants to know what to substitute and how to find the height. This is unexpected as Webb et al. (2002) found that it was adaptive rather than non-adaptive help seekers that were able to form specific questions.

As a final note, there appear to be two possible underlying intentions when asking questions. Because Pippa had been away during the previous task, it is possible that the purpose of this style of help seeking was to catch up and stay on par with the students she was working with. The students in Pippa’s group were likely working at a pace that was too fast for Pippa to keep up with and executive help seeking allowed Pippa to work at the same pace as her group. This suggests that incorporated in Pippa’s social needs is a fear of lagging behind the rest of her group. She resorts to executive help seeking when her social need is at risk.
Additionally, Pippa appears to be concerned with lagging behind the rest of her group. Pippa may resort to executive help seeking behaviours when in fear of falling behind as this form of help seeking allows her to work at the same pace as her peers.

The second likely intention when asking questions is likely to get “the answer” (P18.16). After being told by her group to find the derivative and make it equal zero, Pippa declared that “Ok that makes more sense” (P18.18). She stopped asking her group any more questions, suggesting she was satisfied with the help she was given. This behaviour corresponds with Nelson-Le Gall's (1986) description of executive help seeking where students see the purpose of seeking help as reaching a final answer rather than a learning strategy to help them build an understanding of how to solve the problem.

**Help seeking during the velocity graphs task focused on comprehension**

What about instances where Pippa initiates help seeking not because she realises other students know what to do, but because Pippa might be confused? From the four tasks, only one exchange was found. This occurred during the velocity-graphs tasks, the outcome of which resulted in Pippa connecting her knowledge of speed graphs with velocity graphs (See P15.1-P15.24 in Section 4.3.2.5 *When I make connections, I understand*, p. 71). During this task, Pippa worked with her partner, Ruby, who, unlike Pippa, has not studied velocity graphs previously. Throughout the task, Pippa was uncharacteristically confident with significantly fewer ‘I don’t know’ or ‘I’m not sure’ comments that were common throughout other tasks (See 4.3.1 *Pippa is a hesitant student*, p. 58).

Here is an example of Pippa’s help seeking during the velocity graphs task when asking the teacher for help. This example is typical of Pippa’s help seeking during the velocity task, but not during the other three tasks included in this study:

P23.1  Miss P:  You guys ok?
P23.2  Pippa: I don’t get it. Like if you… you are in the negative position, and you’re going back up, do you continue the velocity from the negative or from zero?

P23.3  Miss P: Sorry, to continue the what?

P23.4  Pippa: So do you continue the velocity from the negative cause you’re already going back down? Or do you start from the zero? Cause you’re going the opposite direction. I just don’t know where it starts.

Like the previous questions (P17.1, P17.3), Pippa starts by asking a specific question. She is able to identify that she is unsure whether velocity graphs should always start from the horizontal axis (P23.4). Pippa’s questions here might be interpreted as asking for the teacher to tell her what to do, in that she wants the teacher to provide her with a fact about where to start velocity graphs. However, it will instead be argued that the help seeking observed during this task had an underlying intention of comprehending her work.

Ruby was often confused during this task and made a number of errors when trying to draw the velocity/position graphs. Here is one graph that Ruby was experiencing difficulties with (Figure 5). The shaded area on the left represents the specific part of the graph that Pippa and Ruby were trying to interpret.

As the pair try to draw the corresponding position-time graph, rather than Pippa simply correct Ruby by telling her what she should have drawn, she stopped and explained to Ruby how to read and interpret the first section of the velocity-time graph:
Figure 5 Velocity-time graph Pippa and Ruby worked on together

Pippa:  P24.1  So let’s say I’m here.

P24.2  Positive velocity has to be in front [meaning moving forwards].

P24.3  You go in front.

P24.4  Negative velocity means going backwards.

P24.5  So if it’s slowing down [Pippa points to graph indicating the section where the value of the velocity is decreasing] but it’s still positive.

P24.6  So slowing down when walking to the front.

P24.7  Get it or nah?

Pippa concludes by telling Ruby the final answer: that the velocity graph represents forward motion with decreasing speed (P24.6). During this interaction, Pippa acted out the motion for Ruby that was implied by the velocity graph by using her fingers to mimic walking forwards and backwards on her desk. Throughout this exchange, Pippa provides Ruby with explanations for her final answer. She justifies that the motion is forwards as the velocity has a positive value (P24.2) and that the speed is slowing down
because the value of the velocity is decreasing (P24.5). Lastly, she checks to see if Ruby has understood her explanation (P24.7).

This interaction suggests that Pippa is concerned with comprehension: she wants to make sure her partner comprehends why the velocity graph represents forward motion with decreasing speed. In order to do so, she provides justifications and reasons for why the graph represents this type of motion. Because Pippa’s help giving behaviour involved providing justifications and checking for comprehension, it is likely that when Pippa asked the teacher whether velocity graphs always start from the origin (P23.2) also held an underlying intention of obtaining an explanation so that Pippa could comprehend her work.

**Summary**

These two illustrative examples show that Pippa can ask different forms of questions. During the coke problem, she asked for the next steps rather than hints (both when initiating and persisting in help seeking) with an underlying intention of obtaining the final answer. During the velocity-graphs task, she did not ask for answers, but instead asked for explanations as well as provided explanations when providing help with an underlying intention of comprehension. During both tasks, Pippa was able to form specific questions that clearly identified for the target help giver what she needed help with.

**4.3.3.3 Subsequent problem solving**

This section discusses how Pippa applied and used help she received. Help seeking is considered adaptive if students use the help they are given to improve subsequent problem solving while working independently (2.1.3 *Adaptive help seeking*, p. 10) and non-adaptive if the student makes no attempt to try to solve the problem on their own after they have received help (2.1.2 *Non-adaptive help seeking*, p. 9).
Illustrative examples are provided here suggesting Pippa appeared a passive recipient of the given help. Despite this, she did show signs of improvement when she started working on an addition problem.

**Applying received help on the current problem**

Immediately following both Episodes One and Two, Pippa did not appear to apply the help she was offered by doing any working out in her exercise book, or make any use of her CAS calculator. Thus, from observations, it appears Pippa did not apply the help she was offered.

On the other hand, Pippa’s comment “Ok that makes a bit more sense” (P18.18) suggests that the help was perceived as useful. From a teacher/researcher perspective, this is not considered an indication that the help has improved Pippa’s understanding or subsequent problem solving skills as comments such as ‘I get it now’ without any evidence of applying the help received by retrying the problem are considered non-adaptive help seeking behaviours (Webb et al., 2002).

Therefore, Pippa’s help seeking here is considered non-adaptive in that she did not use the help she received to reattempt the coke problem on her own. The next section will discuss moments when Pippa was able to use the help she received during the coke problem and apply it to new tasks.

**Applying received help on later problems**

To determine whether Pippa was able to improve her problem solving by being able to solve a problem on her own, an additional example from the same lesson is explored. When I realised that her group had managed to solve the coke problem using more than one method, I set them a new problem in the final 20 minutes of class: the open-lid box problem (For description of task, see p. 69). As the group worked on this new problem, Pippa’s demeanour changed. There was a significant reduction in hesitant ‘I don’t
know’ comments during the open-lid problem when compared with the coke problem. Pippa was also far more active during the open-lid problem than the coke problem. She took several measurements of cut up pieces of paper, constructed a few boxes of different sizes, made calculations, and worked on deriving a formula in her exercise book. She also made significantly more contributions to her group’s discussion.

While Pippa was not observed applying any help she was given directly to the coke problem itself, she was observed applying the help she received during the coke problem to the open-lid problem instead. Once the group had been given the problem, Pippa immediately suggested:

P25.1 Pippa: We need to find a formula for the side.

Finding a suitable formula was the approach initially used her group during the coke task (P17.1-P17.11). Recall that Pippa wanted to approach the coke problem using a ratio (P20.5). Yet here, she chooses not to use a ratio approach, but rather follows what she had observed her group using earlier.

As Pippa continued with the task, she continued applying steps she had watched her group used:

P26.1 Pippa: So do you sub it in?

This question did not appear to have any relevance to her group’s conversation, nor was it appropriate given that the group had no equations or values yet that could be substituted. Nor does it seem likely that Pippa had generated her own equation on her own and was speaking to herself as it is not until a few minutes later that Pippa formed an equation. It seems more likely that Pippa believed something needed to be substituted as this was what she was told to do during the coke problem (P17.2, P17.4).

Together, these two examples suggest that Pippa’s approach to solving the open-lid task involved replicating the solution process she observed her group use during the coke task. Furthermore, because substituting at this point had little mathematical relevance, it also suggests that Pippa did not understand why or what her group had substituted in the first place.
Pippa’s problem solving approach is consistent with findings from Webb and Mastergeorge (2003). Webb and Mastergeorge suggest that some students who observe other students solving a problem correctly falsely assume that they also know how to do the problem, particularly if they have not attempted to solve the problem themselves, and assume that the problem can be solved with identical methods to those observed previously.

### 4.3.3.4 Summary of Pippa’s help seeking

The previous sections have demonstrated that Pippa’s help seeking behaviours differed during the coke problem and the velocity graphs task.

During the coke problem, Pippa asked for help when necessary, tended to ask for answers or the next step rather than hints, and asked specific rather than general questions. Pippa was satisfied when she was given the final answer and told what to do (P18.16, p. 82). When Pippa received help on the coke task, she did not use it to attempt the problem on her own. However, she did show improvement in subsequent problem solving when working on the secondary task, the open-lid problem. She did so by replicating the method she observed her group use during the coke problem, though it was not always appropriately applied.

Because of the high frequency of questions asking for answers rather than hints during the coke problem, it is reasonable to assume that Pippa is, in general, an executive help seeker (a similar help seeking approach was also observed during the Ferris wheel task).

However, the way in which Pippa applied help she was given during the coke task suggests some adaptive help seeking behaviours. Webb, Troper and Fall (1995) devised a ranking of different responses observed when students received help. If a student received help and they showed no response, they were given a rank of 0. If a student explains or reworks the problem in order to produce the correct answer, they were
given a rank of 6. According to this ranking system, Pippa would be placed at level 5, that is, a student who applies another student’s method and completes the problem. According to Web et al., a student at level 5 or 6 is considered to be adaptive.

Although Pippa was able to apply another students’ method to help her complete a new problem, her help seeking is not considered adaptive during the coke task as it was evident that in some instances, she did not understand the method she was applying. This is a similar position to Webb (1993) who found that some students, while working in groups, were able to acquire the necessary techniques from the group in order to solve the problem on their own. Webb concludes that while such students may still be able to perform well in assessments, as they knew how to get the answers, they were not necessarily competent as they did not understand. Similarly, Pippa was able to perform well during the largest open-lid box problem by knowing how to get the answer, but she did not show competence as she did not understand the steps she was using. Therefore, Pippa’s help seeking is considered to be non-adaptive.

In contrast, during the velocity graphs task, Pippa tended to instead ask for explanations rather than direct answers, which were also specific in nature as with the coke problem. Pippa’s underlying intention when seeking help during this task was geared towards comprehension. This suggests a more adaptive style of help seeking.

Why might Pippa’s behaviour differ between the velocity-graphs task and the coke can task? It likely relates to Pippa’s prior experience working with tasks. During the coke problem, Pippa had been unwell during the preceding optimisation problem and therefore lacked the same level of experience that the other students in her group had. Instead, during the velocity graphs task, Pippa had already learnt some kinematics during Physics and therefore had significantly more experience working with velocity-time graphs while her partner she worked with had not seen velocity-graphs before. It therefore appears that during tasks when Pippa’s prior experience was greater than the students she worked with, she displayed more adaptive styles of help seeking. When her prior experience was less than those she worked with,
she displayed executive help seeking. Executive help seeking can be seen as behaviour Pippa resorts to when in fear of lagging behind the rest of the group.

Pippa’s help seeking is summarised in Figure 6. Help seeking behaviours are placed along a continuum. Studies have found that some students can initiate help seeking by asking general questions, or asking directly for answers or the next steps, suggesting executive help seeking. As their gaps in knowledge become filled, their executive questions can turn to requests for hints and/or justification, suggesting adaptive help seeking (Muis & Franco, 2009; Webb & Mastergeorge, 2003). This suggests a continuum because the same student can display different forms of help seeking during the one episode. Dependent help seeking (asking for help the minute one encounters difficulty) and autonomous help seeking (asking for help when necessary, asking for hints rather than answers, and the help improves independent problem solving) are placed at either end of the continuums.

**Key**

- Type of help seeking
- Pippa’s help seeking when she was aware she **did not** have more previous task experience than other students
- Pippa’s help seeking when she was aware she **did** have more previous task experience than other students

**Figure 6** Pippa’s help seeking behaviours and influences on her behaviour
Figure 6 shows that the most commonly observed form of help seeking for Pippa was executive help seeking. This is represented with grey horizontal stripes. Pippa was then found to use appropriate help seeking in only some circumstances. This is shaded in Figure 6 in green stripes. This only occurred if Pippa felt she had more experience with the problem than the students she was working with, and the purpose of the task was not an assessment. The width of the shaded area is representative of how often Pippa approached each task with the corresponding help seeking. Thus, it is widest for executive help seeking and narrows towards appropriate, as this type of help seeking was less frequently observed.

4.4 Case-study: Lilly

Data relevant to Lilly is presented in this section. Like the previous section for Pippa, the data is presented in two sections: her perceptions of understanding and her help seeking behaviours.

4.4.1 Lilly’s perceptions of understanding

This section outlines ways in which Lilly perceives understanding. A summary of the findings that will be drawn out in this section is provided in Table 4. It summarises the various ways in which Lilly identifies understanding including what counts as types of understanding, what does not count as understanding, and reactions that occur when understanding either has or has not occurred. Also included are strategies such as making connection and recognising patterns which were identified as related to understanding by Pippa but do not form part of Lilly’s perceptions of understanding. They are included here for comparative purposes between the two case-studies. The table follows the same format that was used for Pippa (Table 3, p. 62).
4.4.1.1 Understanding is more than knowing facts

I asked Lilly during the interview following the Ferris wheel task what she felt understanding mathematics meant to her:

Lilly:  
L1.1 It’s more than just understanding like a formula.  
L1.2 I have to have evidence to back it up.  
L1.3 For me, to understand something I have to know why this is.  
L1.4 So understanding something is like, like you have to be able to explain something and you have to be able for them to…  
L1.5 they have to be able to do it on their own without just knowing facts.  
L1.6 It’s more than knowing facts.  
L1.7 It’s like why the facts are true.

For Lilly, being able to finish a problem independently is a requirement for understanding (L1.5). This requirement for understanding was not observed during the tasks for this study, nor did Lilly repeat it again during additional interviews. Lilly also associates understanding with being able to explain to others (L1.4). This idea will be discussed at greater detail in Section 4.4.1.4 Making connections is a helpful strategy (p. 104).

Additionally, for Lilly, there is a difference between knowing facts and understanding. Applying formulas (L1.1) without knowing why (L1.3) does not count as understanding. Understanding is instead knowing “why the facts are true” (L1.7). From the current data, what “why the facts are true” means is unclear but suggests that for Lilly, understanding is something different to memorisation (L1.2, L1.7). Lilly’s use of the word “evidence” (L1.2) suggests an expectation that the use of a formula goes hand in hand with a justification or validation for why it works. Additional examples will be provided throughout this section where it will be argued that “why the facts are true” is likely related to relational understanding for Lilly.
### Lilly’s perceptions of understanding

<table>
<thead>
<tr>
<th>Identified as understanding, lack of understanding</th>
<th>Relationship with theoretical understanding</th>
<th>Illustrative statement from Lilly</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowing facts is not understanding</td>
<td>Instrumental (only reported but not demonstrated while working on tasks)</td>
<td>It’s more than knowing facts.</td>
<td>Interview following Ferris wheel task</td>
</tr>
<tr>
<td>Understanding is knowing why</td>
<td>Understanding</td>
<td>Relational</td>
<td>Interview following Ferris wheel task</td>
</tr>
<tr>
<td>Memorisation is not understanding</td>
<td>Learning strategy</td>
<td>Instrumental</td>
<td>Interview following triangle and coke problems</td>
</tr>
<tr>
<td>If I can explain to others, then I understand</td>
<td>Understanding</td>
<td>Logical understanding (Skemp, 1979)</td>
<td>Interview following Ferris wheel task</td>
</tr>
<tr>
<td>Recognising patterns is a helpful strategy</td>
<td>Learning strategy</td>
<td>Consolidation (Dreyfus &amp; Tsamir, 2004)</td>
<td>Interview following triangle and coke problems</td>
</tr>
<tr>
<td>Making connections between ideas</td>
<td>Learning strategy</td>
<td>Building-with (Hershkowitz, Schwarz, &amp; Dreyfus, 2001)</td>
<td>It was that moment when it’s kind of like that this linked in with all the graphs we’d done in the past.</td>
</tr>
<tr>
<td>---------------------------------</td>
<td>-------------------</td>
<td>---------------------------------------------------</td>
<td>--------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>I need to finish questions quickly</td>
<td>A requirement for success</td>
<td></td>
<td>Like I see a question and I’m like I’ve been on this for too long and I can’t do this.</td>
</tr>
<tr>
<td>If I don’t understand, and no one else does either</td>
<td>Lack of understanding</td>
<td>Assurance</td>
<td>So if anyone else isn’t getting it, it kind of makes you feel like I’m not the minority here.</td>
</tr>
<tr>
<td>If I don’t understand, but others do</td>
<td>Lack of understanding</td>
<td>Frustration or panic</td>
<td>And then there’s a point when I’m still trying to find the answer and I’m stuck here. How is everyone else getting it but I’m not?</td>
</tr>
<tr>
<td>Finishing the problem without experiencing frustration</td>
<td>Joy and excitement</td>
<td></td>
<td>Oh, ok. I actually get this!</td>
</tr>
<tr>
<td>Finishing the problem after a period of frustration</td>
<td>Relief</td>
<td></td>
<td>We did it! And it was like that eureka moment when we were just like ok so we can do this one and probably do the rest.</td>
</tr>
</tbody>
</table>

*Table 4 Summary of Lilly’s reported views on understanding and observed understanding*
4.4.1.2 Understanding is not memorisation

During the interview following the triangle and coke problems, I asked Lilly whether she felt memorisation was a useful learning strategy.

Lilly:

L2.1 Memorising is definitely useful for certain things let’s say.
L2.2 For certain things.
L2.3 I feel like once you understand something like really well, you need to definitely understand it without memorising.
L2.4 Cause you can’t just memorise equations and memorise all these things without knowing how to get them.
L2.5 cause when you get a worded problem you can’t straight away implement your equation.
L2.6 But I feel like in some tests memorising definitely does help,
L2.7 but you do need that, you sort of need to understand it,
L2.8 but it’s definitely a short cut.

Int:

L2.9 Ok.

Lilly:

L2.10 If you understand it and (her emphasis) you get this memorised equation it’s so much easier to like sub in things, it’s so much easier.
L2.11 But then I feel like it’s risky.
L2.12 Cause if you like rely too much on it, you forget all the knowledge behind it.

The usefulness of memorisation is related to the purpose of the task. If the task is a test, memorisation is helpful because it allows her to save time (L2.8). Memorisation forms part of Lilly’s perceptions of understanding in that it forms part of her views on what does not count as understanding (L2.3). Instead, Lilly identifies memorisation as a strategy that is helpful (L2.6) and makes answering questions easier
(L2.10). The idea that Lilly identifies certain learning strategies as helpful, but does not identify them as types of understanding, will re-emerge in subsequent sections.

4.4.1.3 Recognising patterns in similar questions is a helpful strategy

This section provides examples where Lilly looks for patterns in similar questions. While recognising patterns is not included as part of Lilly’s perceptions of understanding, it is included here because it was significant for Pippa for whom it did relate to understanding. Furthermore, it is included as it demonstrates that Lilly approaches her work with relational understanding.

During the interview following the triangle and coke tasks, I asked Lilly to comment on the similarities and differences between both tasks.

Lilly: L3.1 From just using the exact steps from the triangle one,

L3.2 there’s no way you could have got the coke can one I feel like.

L3.3 But it definitely helped.

L3.4 The two were quite similar.

L3.5 In that you had to find the turning points and to find the max value.

L3.6 And keep one thing the same.

Lilly is capable of recognising similarities between questions (L3.4). She recognises that both the triangle and coke problems involve combining two equations to reduce the number of variables from three to two so that it can be graphed (L3.6) from which the turning point can be found (L3.5). While she recognises that the two problems were similar (L3.4) she also acknowledges that they were not identical (L3.1). This suggests that Lilly is capable of recognising which parts of the questions are essential and which are
superficial and recognises that the coke problem could not be solved by copying the “exact steps” from the triangle problem (L3.1).

As the interview continued, Lilly described how she approached the coke problem by looking for similarities in questions:

Lilly: L4.1 I thought about finding patterns in like the types of questions that you do.

L4.2 If you find that oh, if you kind of get the vibe that this question is going to be similar to something you’ve done before,

L4.3 I think it helps.

L4.4 Because then you can think back to oh what did I do to solve this thing?

L4.5 I could apply some of the steps to this one.

Lilly reports actively looking for patterns (L4.1) as a strategy she uses when problem solving. Of note there is no mention made in either this or the previous example of understanding. No phrases are used such as ‘I get it’ or ‘That makes sense’ to suggest that recognising patterns forms part of Lilly’s perceptions of understanding.

Instead of viewing the recognition of patterns as part of understanding, Lilly sees recognising patterns as a tool that she can use to help her solve problems more easily. By recognising similarities between two questions, she can directly apply the set of steps she used previously on the new problem (L4.5). She refers to this as helpful (L4.3) suggesting Lilly likes to know what steps are needed to solve a problem.

What type of understanding might the recognition of patterns represent? Lilly is able to move beyond recognising simple similarities between questions. She is able to also recognise what is similar between question structures and what is superficial and irrelevant between questions. This is a level beyond recognising simple patterns between questions. A similar student, Ben, was identified by Dreyfus and Tsamir (2004). Ben was also able to recognise which parts of similar questions were superficial and which
parts of the questions related to important concepts. Dreyfus and Tsamir describe this process as part of *consolidation* which they define as occurring

when a previously constructed mathematical knowledge structure becomes so familiar that it

is available to the learner for flexible problem solving and conscious reflection (p. 297).

Because consolidation occurs, recognising key similarities between questions is seen as relational rather than instrumental understanding.

### 4.4.1.4 Making connections is a helpful strategy

This section shows that Lilly likes to learn by making connections, but that it does not necessarily relate to how she views understanding. As with the previous section, it is included here because making connections formed part of Pippa’s views of what counted as understanding. Lilly’s preference for making connections is supported by her questionnaire responses, strongly agreeing with the statement *While in class, I try to relate the ideas I am learning to what I already know.*

Two illustrations are provided demonstrating Lilly connects ideas from the various tasks back to ideas and topics we had studied earlier in the year.

During the interview following the velocity graphs activity, I asked Lilly to describe how she went about solving the task. She discussed how she realised that a velocity of zero would correspond to a stationary point on the position-time graph. Next, she realised how this could help her graph polynomials, a topic we had studied earlier in the year where we only focused on finding *x*-intercepts, not turning points.

Lilly:

1.5.1 Cause if you looked at the velocity time graphs you’re like what does… what’s the velocity doing?

1.5.2 Then what really helped me was the intercepts.

1.5.3 So when I thought about it, it helped set something up for me.
During the velocity-graphs task, Lilly realises that a point of zero velocity corresponds to a gradient of zero on the position-time graph (L5.5). She then makes the connection to a topic studied earlier in the year: graphing polynomials. She realises that a zero gradient corresponds to a turning point (L5.16).

Lilly made similar comments during the interview following the Ferris wheel task where Lilly linked the graphs of sine and cosine back to the topic of transformations, a topic we had studied earlier in the year.

Lilly:  

L6.1  It was that moment when it’s kind of like that this linked in with all the graphs we’d done in the past.

L6.2  It’s just like transformations basically.
L6.3 But now that I know that this changes how high it is, and this
changes how long,

L6.4 I suppose then, it makes it a lot easier.

Lilly recognises that the trigonometric graphs required her to apply her knowledge of transformations (L6.2). She refers to making connections as ‘linking with the past’ (L6.1).

At no point during either interview does Lilly use any terms usually associated with understanding such as ‘I get it’ or ‘That makes sense’. This suggests making connection does not form part of Lilly’s perceptions of understanding. Instead, Lilly refers regularly to ‘linking ideas’ as being helpful or easier (L5.2, L5.3, L5.13, L6.4). Making connections is a strategy Lilly can use to identify how to go about finding the answer, that is, the act of making a connection has given her a process she can now apply directly to a new problem (L5.12).

What type of level of understanding might this present? Hershkowitz, Schwarz, and Dreyfus (2001) describe understanding in terms of a process called abstraction which is defined as

a process in which students vertically reorganize previously constructed mathematics into a new mathematical structure (p. 195).

Abstraction involves three processes:

- **Constructing**: integrating abstracted knowledge and piecing it together to form new knowledge;
- **Recognising**: identifying the mathematics relevant to the problem;
- **Building-with**: using mathematical procedures in a new context.

Lilly constructs new knowledge: a turning point will have a zero gradient. She then realises that this can be applied to a different context: finding turning points of polynomials. The first instance for which she made this realisation can also be considered constructing as this process involves the organisation of new
mathematical structures, rather than the strengthening of already known structures. Because construction occurs, this type of understanding can be considered as relational rather than instrumental.

4.4.1.5 If I can explain it to others, then I understand

Previously, it was suggested that Lilly felt understanding requires being able to explain ideas to others. The quote is included here to enable discussion in this context:

Lilly: So understanding something is like, like you have to be able to explain something
and they have to be able to do it on their own without just knowing the facts (L1.4–L1.5, p. 98).

For Lilly, not only does understanding require being able to explain her thinking to someone else, but in order for it to count as understanding, the other person needs to then be able to complete the problem themselves. Her use of “without just knowing the facts” suggests she feels the second person must be able to finish the problem with comprehension. This has similarities with some students identified in Mugler and Landbeck (2000) who felt that understanding had occurred if they were able to not only explain their thinking to a friend, but that their friend understood as well.

Examples from the Ferris wheel task are now provided to demonstrate that Lilly’s behaviour while problem solving matches her reported views on understanding. In this example, Lilly is having trouble finding the values of \( \sin(0) \) and \( \sin(\pi) \). Her partner, Pippa, tries to explain the graph by referring to its \( x \)-intercepts. Lilly does not feel she understands until she has explained it out loud Pippa.

L7.1 Pippa: So basically, \( \pi \) is the \( y \) intercept. No the \( x \) intercept of that graph. Is that right?

L7.2 Lilly: Oh I kind of get it now. So when sine equals like \( \pi \) full circle, that means that like, cause you know how the sine is like, is this part here (height)?
Pippa begins by providing an explanation that $x = \pi$ will correspond to an $x$-intercept on the unit circle (L7.1). Lilly listens to this idea and subsequently provides her own alternative explanation (L7.2). Instead of thinking of the values on the unit circle in terms of $x$- and $y$-intercepts, Lilly visualises a right-angled triangle in the unit circle and considers its height and base (Figure 7). For example, a triangle at $\frac{\pi}{2}$ would result in a ‘triangle’ with a base length of zero and height of one unit (L7.6). This was an idea that had been discussed in class two lessons prior to this activity.

During this process, Lilly is constructing. She integrates her knowledge of $x$- and $y$-intercepts with triangles within a unit circle in order to form new knowledge about special values of trigonometric functions. Lilly does not claim that she finally understands until she has finished explained her thinking to Pippa (L7.8).
During the follow-up interview, I asked Lilly about the explanation she gave Pippa:

L8.1 Lilly: When I was trying to explain it, I was kind of at the stage where I was trying
to explain it to myself cause I didn’t completely understand.

Here Lilly directly relates being able to explain to others as helping in her own understanding.

In short, one of Lilly’s perceptions of what counts as understanding is *If I can explain to others, and they also understand, then I understand*. In terms of level of understanding, explaining to others has parallels with *logical understanding* (Skemp, 1979). Logical understanding involves students developing a generalisation that connects ideas together, as well as being able to explain this generalisation to others.
4.4.1.6 If I cannot finish a problem quickly, then I will not be able to finish it

This section provides examples showing that Lilly feels she should be able to finish problems quickly or not at all. This view was not related to her perceptions of understanding but is included here as it provides evidence for her goals for learning.

During the interview following the velocity graphs task, Lilly spoke about how she felt when she was unable to answer a question from the textbook.

Lilly:  

L9.1 The thing is, if I spend too long on a maths problem I kind of get pulled into this vortex and it kind of like sucks me in.

L9.2 It feels really satisfying to get through something and then tick it off at the end.

L9.3 I don’t know. It feels like you accomplished something.

L9.4 So when you miss something and you can’t tick it off, and you’re just like why can’t I get this?

L9.5 And it gets really frustrating.

L9.6 And sometimes the more frustrated you get, the less, like you can’t think as clearly as you like to.

L9.7 So that’s why sometimes I give up a lot of the time.

L9.8 I’m not going to lie.

L9.9 Like I see a question and I’m like I’ve been on this for too long and I can’t do this.

Lilly reports frustration (L9.6) when a question takes too long to finish. Moreover, she interprets this as meaning that she will not be able to answer the question at all (L9.9) which results in the behaviour of giving up (L9.7). Her description of a vortex evokes feelings of panic (L9.1). This suggests the following perception, though not related to understanding: If I cannot finish a problem quickly, then I will not be able to
do it. This has been identified by others as the mathematical belief quick learning (Higgins, 1997; Schoenfeld, 1988).

In contrast, Lilly reports a sense of satisfaction when she is able to complete a set of questions (L9.2-L9.3). Students who experience a similar sense of satisfaction have been identified by Barnes (2001).

Lilly’s preference for finishing questions quickly was observed during the Ferris wheel activity. During this activity, Pippa and Lilly worked together on the first side of the warm up sheet. They then realised they were the last group in the class to be working on the warm up activity, and were yet to begin the second side:

Lilly:  L10.1  But how are they already done with that sheet? Help!

Pippa:  L10.2  I know.

Lilly:  L10.3  How are they already done?

L10.4  Oh my goodness.

L10.5  We’ve been stuck on this sheet for too long.

L10.6  Ok let’s just do it quickly.

L10.7  Oh my god.

Lilly is worried because other students have already finished the warm-up task and she has not (L10.3). There is a sense of panic at this point. Her tone of voice is of a higher pitch, but lower in volume and tempo. Her negative emotional response was indicated by the repeated use of ‘Oh my god’ (L10.6).

This is the only example from this study where Lilly exhibited signs of panic. In contrast, during the coke can activity, Lilly and her group took almost 60 minutes to solve the problem and there is no evidence of panic during this task. Yet in this Ferris wheel example, Lilly and Pippa have only spent approximately ten minutes working on the warm up activity and are feeling panicked. This suggests that it is not the total number of minutes needed to solve a problem that Lilly is concerned with. Instead, Lilly’s worry is
dependent on the progress other students are making. In other words, when she is aware that other students have finished (L10.3) she begins to worry regardless of the length of time she has spent on the problem. Similar case-study students have been reported by Hannula (2006) and Furinghetti and Morselli (2009) where such students also wanted to be able to finish a question quickly and reacted with panic when unable to do so.

Because L9.9 implies a concern for the total time taken, it is likely that Lilly’s concern for spending too long on a question is indeed based on the total time taken, but that the amount of time she considers appropriate is determined by how long other students in the class are taking to finish the same task.

While quick learning does not form part of Lilly’s views on understanding, it does highlight some of her views of what it means to be successful in mathematics (Di Martino & Zan, 2011). For Lilly, being successful involves finishing problems quickly.

### 4.4.1.7 Understanding is related to an emotional response

During the interview following the triangle and coke problems, I asked Lilly to talk about how she felt when Ruby and Phillis had made a ‘breakthrough’ during the coke problem, while she sounded confused.

Lilly:  
L11.1 When they were all like, when they had that moment of like yeah I get it now.

L11.2 This makes sense. Oh I’ve got the answer. It’s going to be so easy.

L11.3 And then there’s a point when I’m still trying to find the answer and I’m stuck here.

L11.4 How is everyone else getting it but I’m not?

L11.5 So definitely everyone else’s reactions play a big part in how I feel.
Int: L11.6 Oh?

Lilly: L11.7 So if anyone else isn’t getting it, it kind of makes you feel like I’m not the minority here.

Lilly reports that her emotional response is dependent on the reactions of others (L11.5). If other students react in a way that suggests they understand and Lilly feels she does not understand, then Lilly reports a sense of panic (L11.4). Her tone of voice at this point has a raised pitch and is said quickly. In contrast, there is a sense of reassurance when Lilly feels she does not understand but everyone else around is also having trouble understanding (L11.7). This gives an additional two perceptions for understanding:

a) If I don’t understand, and no one else does either, then I feel reassured.

b) If I don’t understand, but others do, then I feel panicked.

These relate not directly to Lilly’s views on what does not count as understanding but rather relate to responses associated with lack of understanding.

The next example from the interview following the velocity-graphs task demonstrates how Lilly feels when she has not answered a question correctly but another student has.

Lilly: L12.1 And then someone goes ‘ooh’

L12.2 and I’m like damn it.

L12.3 And it’s kind of embarrassing.

L12.4 And I’m like are you serious?

L12.5 That’s ok.

L12.6 I get it now.

L12.7 And then I move on.

L12.8 It doesn’t stick with you forever.

Lilly feels annoyed if someone else has answered the question correctly and she has not (L12.2). Additionally, ‘Ooh’ was said in the same excited tone of voice one might use when they finally realise or
understand something (L12.1) and suggests Lilly is most annoyed not by someone answering a question correctly but if they have managed to understand it as well. She reports feeling embarrassed (L12.3) suggesting she expects that she should be able to understand the question as well. However, the feelings of embarrassment do not last (L12.7-L12.8). Instead she feels satisfied (L12.5) because she feels she now understands too (L12.6). This implies that one of Lilly’s underlying goals is to understand the problem she is working on despite the potential to feel embarrassed. Therefore, part of Lilly’s perceptions of understanding includes the notion that understanding is valued and something to strive for.

The following examples demonstrate how Lilly responds when she has made her own ‘breakthrough’. These examples are drawn from across the Ferris wheel, triangle and coke tasks and all follow a moment where progress has been made after a period of struggle.

L13.1 Lilly: Ok wait. We’ve got this!
L14.1 Lilly: [gasp] I got it!
L15.1 Lilly: I’m getting somewhere!

All these comments were made in an excited tone, with a high-pitched voice, increased volume and speed. Often bouts of high fives were included. These positive emotional responses might be described as joy or excitement. For example, Lilly was particularly excited when she solved the triangle task. She danced around the room with her CAS calculator. Her joy and sense of triumph was further indicated by the way she went on to explain her entire solution process to me, not once, but twice.

I asked Lilly how she felt during the Ferris wheel task when she finally, after a long period of time, managed to match her first two cards together:

Lilly: L16.1 We did it!

L16.2 And it was like that eureka moment when we were just like ok,
L16.3 so we can do this one and probably do the rest.
Lilly describes these moments as “eureka” moments, a moment of clarity and feeling of making progress.

Of note, Lilly was not observed during the four filmed lessons using phrases normally associated with understanding, such as ‘I get it’, during moments of excitement/joy. This might suggest that Lilly does not associate such emotions with understanding. However, it is possible that during such moments, Lilly was focused more on the moment of excitement rather than the cause that had to excitement, resulting in ‘understanding’ not being reported. This in turn seems unlikely as Lilly commented on moments of excitement during her interviews when the heightened feelings of excitement would have passed. As she did not explicitly relate excitement/joy with understanding either during the task itself, or during her interviews, it is likely that Lilly did not perceive excitement/joy as an indication that understanding had occurred.

Instead L16.3 gives a sense of relief. Lilly has finally matched a pair of cards and now feels that she can do the rest. L16.3 also gives a sense of ‘you’re finally on your way again’, suggesting that some of Lilly’s excitement when finishing problems may be interpreted as relief rather than joy. This is supported by Barnes (2000), who reports that for one student, the emotional response of joy instead gave an impression of anxiety followed by relief when “you’re on your way again” (p. 34). Lumby (2011) identifies relief as the “cessation of anxiety” (p. 249). Relief should be interpreted not as a response that occurs after a positive event, but rather when a negative event has stopped (Pekrun, Goetz, Titz, & Perry, 2002). This has parallels with Lilly.

In short, Lilly experiences positive emotions when a breakthrough occurs. She does not make an association between ‘breakthroughs’ and understanding. Instead, they relate to comments about either making progress (L15.1, L16.3) or getting the answer (L14.1). Additionally, Lilly associates negative emotions with lack of understanding, but does not associate positive emotions with understanding.

The emergence of emotions as a factor was unexpected in this study. It is possible that Lilly may have considered reporting understanding in terms of emotions as I sometimes used the open-ended question
'How were you feeling during this moment?' I used this question, along with questions such as ‘What did you learn here?’ or ‘Tell me what was going on’, but found ‘How were you feeling during this moment?’ generated the greatest amount of student-led talk and allowed for rich data. The use of such a question may have led Lilly to therefore identify understanding in terms of emotions. Therefore, while Lilly may have indeed identified understanding as relating to emotions, the questions used during interviews may have resulted in this perception as being over reported.

### 4.4.1.8 Summary of Lilly’s understanding

A summary is now provided of the findings. This is separated into two sections. The first summarises Lilly’s level of understanding from a theoretical perspective (i.e., instrumental or relational understanding). The second section summarises Lilly’s perceptions of understanding. Included within the umbrella of her perceptions of understanding are what counts as understanding, what does not count as understanding, and responses when understanding either has or has not occurred.

### 4.4.1.9 Lilly’s researcher identified understanding

In general, Lilly approached her work with relational understanding. When she made connections between ideas, it was argued she was building-with (Hershkowitz et al., 2001). When she explained ideas to others, she was constructing (Hershkowitz et al., 2001), as well as demonstrating logical understanding (Skemp, 1979). Recognising patterns was shown to form part of the process of consolidation (Dreyfus & Tsamir, 2004). Because constructing and building with imply the formulation of new procedures that are mathematically appropriate and the ability to recognise how to apply what is already known in a variety of new contexts, Lilly is considered to be displaying relational understanding (Lampert, 1986).
From the tasks included in this study, no instances were found where Lilly approached her work without meaningful understanding. Therefore, she is not considered to have demonstrated any intentions to develop instrumental understanding.

Lilly’s researcher identified understanding is summarised in the following diagram (Figure 8). The same continuum scale that was used in Figure 4 (p. 77) for Pippa is used here, where instrumental understanding forms one end of the continuum, and logical the other end. The chequered shaded area represents the different types of understanding Lilly was observed using. Only one style of shading is used here as no factors were found to influence Lilly’s level of understanding. The width of the shaded area in Figure 4 is representative of how often Lilly approached her work with the corresponding level of understanding. Thus, it is the widest for relational understanding and narrows towards logical understanding. It does not pass towards the left of relational understanding as Lilly was not found to approach her work with instrumental understanding.

Figure 8 Lilly’s researcher-identified level understanding
4.4.1.10 Lilly’s perceptions of understanding

Lilly was more likely to report what counted as lack of understanding or what did not count as understanding rather than what counted as understanding. For example, memorisation was reported as not counting as understanding. A lack of understanding resulted in feelings of frustration or panic. Being able to explain ideas to others was the only perception Lilly associated with what counts as understanding.

Instead, moments when Lilly demonstrated relational understanding, and might be expected to be reported as understanding, were not reported as such. For example, Lilly demonstrated relational understanding when she was able to draw connections between ideas or recognise patterns between similar questions. Rather than reporting these as understanding, she reports these as helpful strategies.

Why might explaining to others have been reported as a type of understanding, while memorisation did not count as understanding, and making connections and recognising patterns were both reported as learning strategies rather than understanding? One interpretation is the difference between constructing and applying knowledge. The process of explaining ideas to others allowed Lilly to recognise and identify new concepts related to what sine and cosine represent on the unit circle. Lilly instead saw making connection and recognising patterns as a useful strategy to help her apply new knowledge to make solving problems an easier process. Memorisation was a strategy to help answer questions in a test more quickly and can again be seen as the application of knowledge to answer questions during tests as an easier process.

Thus, from Lilly’s perspective, making connections, recognising patterns, and memorisation were not related to generating new insights. Instead they all related to the application of knowledge. This suggests that it is only strategies associated with generating new insight that Lilly associated with understanding. Strategies that she associates with making finding answers easier she instead does not report as a type of understanding.

This is supported by the fact that Lilly associated ‘breakthroughs’ or finishing problems with feelings of joy and excitement, but not with her views on what counts as understanding. That is, getting the answer,
as well as strategies associated with finding answers, do not form part of Lilly’s perceptions of understanding.

In terms of validity, I have made the assumption that the lack of the use of phrases such as ‘I understand’, ‘I get it’, ‘That makes sense’, implies that Lilly did not associate these strategies with understanding. Hence, I did not consider these to form part of Lilly’s perceptions of understanding. However, the absence of reference to understanding does not conclusively mean that Lilly does not include such examples as part of her views on understanding.

4.4.1.11 Lilly’s perceptions of understanding and goal setting

Lastly, some of Lilly’s goals for learning can be inferred from the examples provided in this section. Overall, Lilly had a goal of comprehending her work (L12.5-L12.6). The means with which she achieved this goal, or her view of what counted as being successful in mathematics, was by being able to solve a problem fluently. Fluency is defined for the case of Lilly as being able to proceed smoothly through a problem without needing to stop and think or without getting stuck or working at the same pace as other students in the class.

Strategies such as making connections and recognising patterns allowed Lilly to achieve her means of fluency, that is, making connections or recognising patterns allowed Lilly to work out what steps would need to be applied to the problem at hand without needing to stop and think what to do next (L4.5). When she was unable to solve a problem fluently, she experienced negative emotions such as panic and frustration. When fluency returned, she experienced relief. This did not relate closely with her perceptions of understanding. Only some perceptions of understanding related to her need for fluency which include If I don’t understand but others do, then I feel panicked and If I don’t understand and neither does anyone else, then I feel reassured.
4.4.2 Lilly’s help seeking behaviours

This section explores Lilly’s help seeking behaviours. Help seeking episodes are provided showing that Lilly asked predominantly for hints rather than direct answers. One episode is provided from the Ferris wheel activity where Lilly displayed both adaptive and non-adaptive help seeking behaviours. This will be used to show that while Lilly exhibited some non-adaptive help seeking behaviours, she can be considered, in general, an adaptive help seeker. The episode chosen is atypical for Lilly’s behaviour and provided here so that of possible factors that influence her help seeking can be explored.

4.4.2.1 Help seeking episode from the Ferris wheel task

This help seeking episode comes from the Ferris wheel task. Lilly was working with her partner, Pippa, on the warm-up questions requiring them to apply a set of transformations to the graph \( y = \sin(x) \) (Appendix 2). An interpretation of Lilly’s help seeking is not immediately offered but rather unpacked through the following three sections: necessity or help, the types of questions asked, and whether the received help was used appropriately.

During the Ferris wheel task, Lilly received help from Pippa, though it was not requested. Lilly then requested help from the pair sitting next to them, Xander and Aaron. Xander then joined Lilly’s table to help.

L17.1 Lilly: Ah, this is confusing!

L17.2 Pippa: That means it’s two pi three over two sine.

L17.3 Lilly: What? You do it. [hands pen to Pippa]

L17.4 Pippa: Like for pi, wouldn’t it be like, so \( x \) is pi. Wouldn’t it be like two pi and three over two sine. Two pi over three sine.
L17.5 Lilly: What do you mean?

L17.6 Pippa: Cause you dilated it.

L17.7 Lilly: But what would it look like on a graph?

L17.8 Pippa: Do you want to like just put points on the graph and put it together?

L17.9 Lilly: We could try. Guys [to Xander and Aaron], how did you do this one?

L17.10 Xander: Uhh… that’s the dilation. So that reaches up to two. So it goes up to two.

L17.11 Pippa: Oh Ok! Cause that one, x is just x.

L17.12 Lilly: And then what does the three over two do?

L17.13 Xander: The 1.5? That’s just a dilation.

L17.14 Lilly: So how would you draw it?

L17.15 Pippa: So half pi…

L17.16 Lilly: Do the intercepts for the y axis change at all?

L17.17 Xander: Yep. No wait. For the y or the x? It’s like it does go up and down. And then it’s three pi. Three over two pi.

L17.18 Lilly: Oh!

L17.19 Pippa: I think I kind of get it.

L17.20 Lilly: It’s supposed to touch here. It’s supposed to touch here. [Places some coordinates on her graph]

L17.21 Pippa: Yeah. [to Xander] Thanks.

L17.22 Xander: I think you did it wrong.

L17.23 Lilly: Why?
Xander: You didn’t do the two $x$. The $1.5$ sine. Cause the way you drew it is $0.75$ sine. I think.

Pippa: Why does this suddenly become $y$ now? Cause it’s narrower? [Xander returns to his own desk]

Lilly: Help! Xander!

Xander: [Xander returns to Pippa and Lilly’s desk] I’ll give you a hint. [Draws a graph but cannot be seen on camera]

Lilly: Why is it going down? Ahh… is it doing this? [Draws a graph, but cannot be seen by camera] How do you know it intercepts here?

Xander: We just do.

Pippa: They’re geniuses [laughing].

Xander: You [Aaron] explain it to them.

Aaron: Think about transformations right. It’s saying the $y$-value is going to be two each time going up right? The dilation factor is going to be $1.5$.

Xander: Did we do it right?

Aaron: I think we did it right. So I think basically the spacing needs to be $1.5$ between each. Because $1.5$…

Xander: Did we do it right?

Aaron: I think we did it right.

Xander: Did we do it right?

Lilly: Yeah, let’s ask [the teacher].
Approximately four minutes passed, during which time, Lilly did not put up her hand to request help. Instead, she tried to continue working on the problem with her partner. After four minutes, the teacher walked by and she called for help.

L17.39  Lilly: Oh my god! I need help. Bad!

L17.40  Miss P: What’s wrong?

L17.41  Lilly: We don’t know how to do the dilation.

4.4.2.2  Necessity of help

This section explores whether Lilly’s help seeking can be considered necessary. It will be argued that Lilly demonstrates two types of help seeking. At L17.9, Lilly asks Aaron and Xander for help, which by the definitions of help seeking formulated for this study is considered unnecessary. At L17.39, Lilly asks the teacher for help, which is considered necessary. This section is broken into two: one section for question L17.9, another for question L17.39.

“Guys, how do you do this one?” (L17.9)

As Lilly begins to work on the problem, she admits she is confused (L17.1). From Lilly’s perspective, help seeking could be considered warranted. However, this help seeking episode is interpreted as unnecessary. First, when Lilly is considered to be asking for help is clarified. The initiation of help seeking is considered to occur at L17.9, where Lilly asks Xander and Aaron for help. Prior to this, Lilly receives help from Pippa. This had not been explicitly requested by Lilly.

Prior to L17.9, Pippa had been explaining to Lilly how to apply a dilation factor to the graph of \( y = \sin(x) \). Pippa suggested they could try applying the required transformations to individual coordinates rather than
the graph as a whole (L17.8). This is a potentially useful strategy from a teacher/researcher perspective. For example, take the coordinate \( \left( \frac{\pi}{2}, 1 \right) \), a point lying on the graph of \( y = \sin(x) \). If a dilation of a factor of 1.5 is applied from the \( x \)-axis, then the coordinate would become \( \left( \frac{\pi}{2}, 1.5 \right) \). It would have allowed the pair to plot significant points such as turning point and intercepts for their graph, enough to allow them to work out the general shape of their new graph. Lilly responded to Pippa’s strategy with the phrase “We could try” (L17.9). Lilly’s tone of voice here was slower in tempo, lower pitch and volume, but raised in pitch at the end, suggesting it was said in a somewhat submissive tone. Immediately after telling Pippa “We could try”, Lilly turned to Aaron and Xander to ask them how to sketch the graph. There was no pause between these two statements, suggesting she had not given Pippa’s suggestion much consideration. This implies that Lilly did not take up an opportunity to adopt a strategy that would have allowed her to continue working independently on the problem without need for additional help.

Because help seeking is considered necessary only when the help seeker asks for help when they can no longer continue making progress independently, L17.9 is considered unnecessary. According to Goos (2002), this would make Lilly an non-adaptive help seeker, as Lilly chose not to use Pippa’s potentially useful strategy.

“Oh my god! I need help. Bad!” (L17.39)

In this section, it is argued that while Lilly previously asked for help that was unnecessary, the help she asked at L17.39 is necessary. Firstly, Lilly was told by Xander that her answer is incorrect (L17.22). Because Lilly asked for help after being told her answer was incorrect, L17.39 is considered appropriate as Lilly had made a legitimate yet unsuccessful attempt at the problem before seeking help (Newman & Schwager, 1995).
Furthermore, Lilly heard Xander’ expressions of self-doubt asking Aaron on three occasions if they had correctly answered the question (L17.33, L17.35, L17.37). This might have caused Lilly to be doubtful of Xander’ explanation and in response, asked the teacher for help (L17.38).

Because Lilly was likely aware her answer was wrong, and that the help-giver was himself confused, it is reasonable to conclude that L17.39 can be considered necessary.

4.4.2.3 Types of questions asked

This section is divided into two sections. The first shows that Lilly asked for answers rather than hints when initiating help seeking during the Ferris wheel task. The second section discusses how the nature of these questions changed from asking for answers to asking hints as Lilly persisted in seeking help.

Asking for answers

Lilly initiated help seeking by asking for the required steps rather than hints. She initiated help seeking with a ‘how’ question: “Guys, how did you do this one?” (L17.9). At this point, Lilly was speaking faster than usual, suggesting panic. When Xander responded that \( \frac{3}{2} \) acts as a dilation factor (L17.13), Lilly immediately asked “So how would you draw it?” (L17.14). The speed at which she asked her next question suggests she did not spend time thinking about Xander’ comment about the dilation factor. This implies that Lilly’s aim is not to pause and think carefully over the problem but to continue asking questions until she finds out what she needs to do to solve the problem.

Furthermore, L17.14 is said in a higher pitch than L17.12, suggesting Lilly is becoming increasingly panicked. It is likely that the panic displayed at L17.12 and L17.14 are a result of being one of the last
pairs to finish drawing their graph (4.4.1.6 If I cannot finish a problem quickly, then I will not be able to finish it, p. 110).

**Asking for hints**

Throughout the remainder of this help seeking episode, there is a distinct change in demeanour from Lilly. Prior to L17.18, Lilly’s voice was fast and of a slightly high pitch. But at L17.18, Lilly exclaims “Oh!” using a louder voice, suggesting excitement. She also became more animated with rapid hand movements. It is here that the types of question Lilly asked changes. L17.20 indicates that Lilly believed she knows what she now needs to do in order to proceed. She no longer asked for the next steps but instead began to ask for justifications (“How do you know it intercepts here?” L17.28).

When asked about this episode during the follow-up interview, Lilly commented on not being satisfied by just being given the answer:

L18.1  Lilly:  Cause I think someone drew out the graph for us. But then we were like oh cool, but why [her emphasis] does it intersect here?

The phrase “oh cool” is not said in an excited tone. It is instead said at a slower speed, lower pitch, and slightly longer gap between ‘oh’ and ‘cool’ than would be expected, suggesting she was dismissive or unconvinced of the graph that had been drawn for her. In addition, she elongates and stresses the word ‘why’ suggesting that this is what she considers important suggesting that Lilly is not satisfied with just being given the answer but would rather have been provided with the underlying explanation with the answer. If this is the case, then this supports the argument that the intention of Lilly’s question at L17.28 was to obtain a justification.
Summary

In short, for the Ferris wheel task, Lilly initiated help seeking by asking for answers. As she persisted in asking for help, her questions shifted and she started to ask for justifications and explanations.

4.4.2.4 Use of help

It has already been discussed that Lilly did not make appropriate use of the suggestions offered to her by Pippa (L17.8-L17.9), implying she is a non-adaptive help seeker (Goos, 2002). However, there is additional evidence from the same activity where Lilly demonstrates the opposite: she is able to appropriately reject suggestions that are unhelpful.

During the Ferris wheel task, Aaron offered Lilly help on identifying which graphs might be sine and which might be cosine. Lilly chose not to use the help and I asked her why during the follow-up interview.

Lilly: L19.1 The method that they [Aaron and Xander] were using, well it didn’t have much foundation to it.

L19.2 Cause like they were saying the sine graph starts here [at the origin].

L19.3 So therefore we know all the equations with sine graphs are going to be matching the graphs that look like this.

L19.4 But I wasn’t buying it

L19.5 because I was thinking what if you like translated it half a unit this way and then it looks like cos.

L19.6 And that wouldn’t make sense.

L19.7 Are you sure that this method worked?

L19.8 It was confusing because he was really sure his method worked.
Lilly’s comments confirm that she made a deliberate choice not to use and apply Aaron’s help (L19.4) because she felt it was mathematically incorrect (L19.1). Aaron had argued that all sine graphs start at (0,0), while all cosine graphs start at (0,1). As Lilly states, not all sine graphs will pass through the origin, as some can be translated left or right to look like \( y = \cos(x) \) (L19.5). This example is considered adaptive help seeking because Lilly made a deliberate choice not to use an unhelpful strategy (Goos, 2002).

Therefore, throughout the Ferris wheel activity, there were two contrasting behaviours. At the beginning of the Ferris wheel activity, Lilly chose to avoid useful help. Toward the end, she chose to ignore unhelpful suggestions. Reasons for the contradictory nature between the two behaviours will be discussed at the conclusion of this section.

Currently, only data related to how Lilly decided to either use or discard help she was offered has been discussed. No useful examples were found demonstrating how Lilly went on to apply the help she received. This is because much of the help she needed related to the use of the CAS calculator during the triangle and coke problems. The help that Lilly needed and received, was of a technical nature and related to what buttons to press, or how to use certain functionalities, rather than anything of a conceptual nature.

**4.4.2.5 Summary of help seeking**

The illustration from the Ferris wheel task shows that Lilly displayed contradictory help seeking behaviours during this task. At one point, Lilly asked for help that was necessary, at another, she asked when unnecessary. The types of questions varied from asking for answer to asking for hints. Lastly, there was evidence showing that Lilly chose to ignore potentially useful help, and later chose to ignore unhelpful help. In other words, in some circumstances, she displayed non-adaptive help seeking, in others, adaptive help seeking. The purpose of this section is to provide possible reasons for the contrasting behaviour.
The non-adaptive help seeking that was observed might be explained if the circumstances in which these help-seeking instances occurred are considered. Of note, all instances of non-adaptive help seeking occurred before Lilly displayed adaptive help seeking behaviours. One interpretation is that non-adaptive help seeking occurred during moments when Lilly lacked the necessary prerequisite skills for transformations to be able to complete the warm-up activity during the Ferris wheel task. Transformations of functions had been covered earlier in the year. Many students had found the topic challenging and it is possible that Lilly had either forgotten parts of the topic or never grasped them to begin with.

Nelson-Le Gall (1986) argues that question asking is akin to knowledge acquisition. Thus, when knowledge is low, the rate of question asking will be higher. As knowledge becomes filled, adaptive help seekers will begin to realise what is known and what is not known and begin to ask questions only to continue filling in gaps that are not known, thereby becoming progressively more adaptive in nature. Because there is evidence of adaptive help seeking behaviours occurring after non-adaptive help seeking, then according to Muis and Franco (2009), the non-adaptive help seeking can be considered a starting point only with the purpose of learning “foundational knowledge” (p. 309), rather than an indication of the student wanting to have someone else solve the problem for them. The role of Lilly’s help seeking might therefore function to fill in any gaps in her knowledge. This provides an explanation for the instance of unnecessary help seeking. When Lilly ignored Pippa’s suggestion, she may have lacked the required prior knowledge necessary at the start of the task to allow her to realise the potential Pippa’s idea had to offer. Lilly’s help seeking will therefore be considered, in general, as adaptive. This is in agreement with Webb and Mastergeorge (2003) who found that adaptive help seekers often initiate help seeking by asking for answers or the next steps needed to solve a problem, but shift their help seeking to start to ask for explanations as they persist in asking for help. That is, adaptive help seekers sometimes used non-adaptive help seeking as a starting point only.
An alternate explanation is to consider Lilly’s help seeking in the context of whether she felt she was spending too long on a problem. Moments when Lilly displayed non-adaptive forms of help seeking only occurred when she felt she was one of the last to finish a problem (i.e., she was stuck but others around her were not). For example, when she became aware she was spending too long on the warm-up activity, she ignored Pippa’s useful suggestions. Of note, the use of non-adaptive help seeking occurred after Lilly became aware she was one of the last to finish, suggesting a causal relationship between Lilly comparing herself with other students, and her help seeking behaviour.

Adaptive help seeking was observed when she felt she was stuck, but others around were too. For example, when she realised that Xander was unsure of his answers, Lilly called the teacher over to ask for help when necessary and formed a specific question that identified clearly what she did not understand.

It was unable to be determined whether Lilly could be considered an autonomous help seeker (Butler, 1998) as there were no occurrences in the data demonstrating whether the help Lilly received made any significant improvement in her approach to problem solving.

Using the same continuum scale for help seeking as was used in Figure 6 (p. 96) for Pippa, Lilly’s help seeking behaviours are represented in Figure 9. The diagram shows that the type of help seeking Lilly used most frequently was appropriate help seeking. This is represented as the grey chequered shading. Lilly was found to use executive help seeking in only some circumstances. This is shaded in Figure 9 in blue. This only occurred if Lilly felt she was spending too long on a problem (i.e., was aware she was spending longer on a problem than other students in the class). The width of the shading is representative of how often Lilly used the corresponding help seeking during the four tasks in this study. Thus, it is widest at appropriate help seeking and narrows towards adaptive and executive, as these two types of help seeking were less frequently observed.
4.5 Additional students considered for this study

Two additional students, Aaron and Kipp, were included in this study, but not reported on in detail. Because generalisations cannot be made from two case-study students alone, the additional two students are included so that associations between help seeking and understanding might be more readily seen. A similar analysis of classroom observations and interviews that was done for Pippa and Lilly was also conducted for Aaron and Kipp. A summary of findings is given here, but for the sake of brevity, quotations and explorations of their meaning are not included. As a generalisation, Aaron showed similarities with Pippa, while Kipp showed similarities with Lilly. The corroboration of additional cases strengthens the validity of the findings in this study.
4.5.1 Aaron

Like Pippa, Aaron’s primary goal was to achieve high grades. Unlike Pippa, Aaron reported that the best strategy to achieve such a goal was to use memorisation whereas Pippa liked to use the strategy of repetition. For Aaron, sense making was not necessary when memorising content. Thus, he likely used rote memorisation, that is, an action of memory only. Both Pippa and Aaron reported that understanding was not valued or useful in achieving their goal of high marks.

Aaron spoke about subjects he was performing well in and attributed the success of his high marks directly to being able to memorise the content. In some instances, he defined understanding as ‘conceptual understanding’ and reported that memorisation did not count as understanding. In other instances, he explained that memorisation was understanding. He was unable to expand on what he meant by conceptual understanding but his problem solving approach suggested that the primary way in which he learnt was through rote memorisation.

Like Pippa, Aaron’s language about understanding revolved around getting the answers. While they both held the same learning goal of high marks, they held different ways on how to achieve this. For Pippa, her means of achieving high marks was getting the answer. For Aaron, memorisation was his way of achieving high marks.

Aaron’s help seeking was dependent. He asked directly for answers or formulas, did not always ask for help when necessary, and generally was unable to use the help offered to him to correctly solve the problem (Ryan et al., 2005). Additionally, Aaron asked his friends for help with the intention of having them finish the problem for him. He was often observed initiating help seeking and then starting up an off-task conversation with another friend while his group solved the problem for him. He did not show any shift in help seeking behaviours and was not observed using adaptive help seeking at any point during the tasks used in this study.
In terms of researcher identified understanding, Aaron showed only instrumental understanding and there was no evidence for relational understanding.

### 4.5.2 Kipp

Kipp was a competitive student. He liked to be the first student to finish a problem. If he had managed to finish a problem but realised he was not the first to do so, he reported feeling irritated and angry. Unlike Lilly who was concerned with not being the last to finish, Kipp was concerned with being the first to finish. Kipp attributed not finishing first to not trying hard enough. Thus, effort was important for Kipp. When Kipp did solve a problem on his own, he described it as feeling happy or good. Like Lilly, Kipp reported taking too long on a problem with negative emotions but associated finishing problems with positive emotions.

Kipp liked to solve problems independently. He liked to work independently as long as he could before asking for help. As such, he was the only student observed to display adaptive help avoidance (Newman, 2008). He was also the only student observed to use help he received by then going back to rework the question on his own. Because of this, Kipp’s help seeking was considered *autonomous* (Butler, 1998) throughout this study.

If Kipp realised that students around him were making progress while he still did not know what the next steps were to the problem, he reacted with panic and displayed help seeking behaviour similar to Lilly. He began by first asking for answers rather than hints. As he persisted in asking questions, his questions changed from asking others what to do next to asking for explanations. If he felt the explanation he received did not help his understanding, he persisted in asking more questions until he felt he did understand. Thus, like Lilly, Kipp’s help seeking behaviour changed from non-adaptive to adaptive as
gaps in his knowledge became filled. Like Lilly, no influences were found to impact on either his perceptions of level of understanding.

Kipp described understanding as being able to answer any question, no matter how complicated, unfamiliar, or hard the question is. Unlike Lilly who tended to describe understanding in negative terms (lack of understanding), Kipp described understanding in positive terms, something that can be achieved. Like Lilly, Kipp also used emotions to describe understanding. Unlike Lilly who uses emotions to describe how she felt in response to realising she did or did not understand, Kipp reported that understanding is the feeling of being calm, that is, from Kipp’s perspective, understanding is equivalent to feeling calm, rather than an indication that understanding has happened. Feeling nervous meant he did not understand.

Kipp and Lilly also held similar goals for learning. Both held a goal of comprehending and mastering their work. The way in which to achieve this was through fluency. For both, getting stuck was a signal that their goal of comprehension was not being met.

Kipp was also a reflective student who liked to understand his work. Like Lilly, he too displayed relational understanding. He was also able to clearly articulate his thoughts to other students, suggesting he also developed logical understanding.
Chapter 5  Comparisons between cases and discussions

This chapter explores possible associations between help seeking behaviours and the ways in which students have identified or perceived what it means to understand mathematics.

At first, it was found that when both Pippa and Lilly were asked to define ‘understanding’ in mathematics during their first interview, they both gave very similar accounts of understanding. Both described understanding as being more than just using formulas (P7.3 “But I don’t think that’s called understanding if you know the end results.” p. 63; L1.1 “It’s more than just understanding like a formula.” p. 98). Understanding involved what both students phrase as ‘knowing why’ (P7.4 “But if you know why [her emphasis] the results happen.” p. 63; L1.7 “It’s like why the facts are true.” p. 98). Thus, both students were considered to hold the same reported perception of understanding. However, through classroom observations at first, and later confirmed through interviews, data showed that while both students reported the same perception of understanding, they were found to approach their work with different levels of understanding. Pippa was found to approach her work with predominantly instrumental understanding (Figure 4, p. 77), while Lilly was found to be using predominantly relational understanding (Figure 8, p. 117). Because Pippa and Lilly were found to report the same perception of understanding, yet demonstrated different levels of understanding, instrumental and relational understanding (Skemp, 1976) were also considered as part of this study to examine associations between these constructs and students’ help seeking behaviour where instrumental understanding refers to knowing how while relational understanding refers to knowing both how and why.
This chapter will therefore include both perceptions of understanding and researcher identified understanding, which refers not to how students view understanding, but rather students’ level of understanding as identified by myself as the teacher/researcher. In addition, students in this study often spoke about understanding in terms of their goals for learning, how to go about achieving their goals, or what they felt was necessary in order to be successful in mathematics. As will be argued in this chapter, it was found that a direct association between perceptions of understanding and help seeking did not exist. Because the original focus of this study did not lead to a direct association between help seeking and perceptions of understanding, other associations were also explored. The focus was shifted to also investigate a possible association between researcher identified understanding and help seeking and associations between perceptions of understanding, help seeking, and goals.

In order to investigate these possible associations, this chapter is separated into three sections. The first section argues that no associations were found between one specific perception of understanding and one specific type of help seeking. The second explores associations between researcher identified understanding and corresponding help seeking behaviours. The final section explores in more general terms how perceptions of understanding and help seeking relate through the model of self-regulated learning.

5.1 Student perceptions of understanding

From the literature review, it was revealed that students talk about and identify what it means to understand in several ways. These included the following:

a) Finishing a problem quickly;

b) Memorisation;

c) Drawing connections between ideas;
d) Finishing a problem;
e) Explaining ideas to others;
f) Learning.

These categories related to what students see as equivalent to understanding, what does and does not count as understanding, what signifies understanding has or has not occurred, or ways to achieve understanding.

This section will discuss the data from this study in light of these six categories. These categories will be drawn on but not all will be used, as not all perceptions of understanding were identified from the results in this study. The purpose is to identify whether one specific student perception of understanding is always associated with one specific help seeking behaviour.

Table 5 summarises the different views of understanding identified by students throughout this study that were also identified in the Literature Review (2.2.1 Identifying different ways students perceive understanding, p. 16). The first column lists the different views of understanding that were identified in the Literature Review. The second column lists whether the same view of understanding was also identified in this study. The last two columns then show whether the particular view of understanding was identified by Pippa or Lilly, and whether it related to understanding, lack of understanding, or whether it did not relate to understanding at all.

All descriptions of understanding identified in the Literature Review emerged from the data except for understanding as learning. An additional two themes emerged from the data that were not identified in the Literature Review: recognising patterns and understanding as an emotion. This has been summarised in Table 6.
## Perceptions of understanding identified in the Literature Review

<table>
<thead>
<tr>
<th>Description of understanding</th>
<th>Was it found in this study?</th>
<th>Pippa</th>
<th>Lilly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quick learning</td>
<td>Yes</td>
<td></td>
<td>Measurement of success</td>
</tr>
<tr>
<td>Memorisation</td>
<td>Yes</td>
<td>Used to be understanding but is now lack of understanding</td>
<td>A strategy that, when coupled with understanding, makes answering questions easier and quicker</td>
</tr>
<tr>
<td>Making connections</td>
<td>Yes</td>
<td>Understanding</td>
<td>A helpful strategy</td>
</tr>
<tr>
<td>Explaining to others</td>
<td>Yes</td>
<td></td>
<td>Understanding</td>
</tr>
<tr>
<td>Understanding is learning</td>
<td>No</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Task completion</td>
<td>Yes</td>
<td>Answering questions correctly does not necessarily mean understanding</td>
<td>Answering a question incorrectly means lack of understanding</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Solving problems not related to understanding</td>
</tr>
</tbody>
</table>

*Table 5 Comparison of student perceptions of understanding identified in the Literature Review*
Perceptions of understanding not identified in literature

<table>
<thead>
<tr>
<th>Description of understanding</th>
<th>Pippa</th>
<th>Lilly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recognising patterns</td>
<td>Understanding</td>
<td>A helpful strategy</td>
</tr>
<tr>
<td>Understanding as an emotion</td>
<td></td>
<td>Lack of understanding is related to negative emotions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Solving problems is related to positive emotions, but not to understanding</td>
</tr>
</tbody>
</table>

Table 6 Student perceptions of understanding emerging from the data not identified in the Literature Review

Additionally, Table 5 shows that some categories were not perceived as the same by Pippa and Lilly. For example, both students discussed the importance of recognising patterns. Pippa identified this as a type of understanding while Lilly identified this not as a type of understanding, but as a helpful learning strategy. The appearance of different points of view by different students for the same perception was not unexpected as it was shown in the Literature Review that not all students equated the same description of understanding with the same perception of understanding.

Next, some of the common themes of understanding to emerge from the data are expanded upon. This will be used to show that while two students may report the same view of understanding, it did not necessarily lead to the same learning behaviours. This will be used to show that a relationship between one particular view of understanding and one particular help seeking behaviour cannot be made.
Understanding as emotions

Emotions was an unexpected factor to emerge from this study. For Kipp, understanding was emotions (4.5.2 Kipp, p. 133). For Lilly, emotions signified that understanding either had or had not occurred (4.4.1.7 Understanding is related to an emotional response, p. 112).

Studies on perceptions of understanding have found that students often link understanding to feelings of satisfaction (Burns et al., 1991; Entwistle & Entwistle, 1992). In contrast, Lilly and Kipp associated satisfaction with finishing a question but not with understanding.

Making connections

Making connections was reported by Pippa, Lilly, and Aaron but differed for all three students. For Aaron, making connections formed part of his views on understanding and he felt making connections helped him better memorise formulas (4.5.1 Aaron, p. 132). For Aaron, making connections was also used in context of instrumental understanding. For Pippa and Lilly, making connections instead lead to more meaningful levels of understanding than instrumental. Only Pippa associated making connections with understanding (4.3.2.5 When I make connections, I understand, p. 71). Lilly did not associate making connections with understanding at all, and instead viewed making connections as a helpful strategy that would allow her to know what the required step was for a problem without needing to stop and think (4.4.1.4 Making connections is a helpful strategy, p. 104).

Explaining to others

Explaining to others was another commonly reported view of understanding. It was reported by Lilly and Aaron, but also reported by an additional three students who were interviewed as part of the study. Even
though Aaron and Lilly both reported that explaining to others was a type of understanding, their underlying intentions differed. Aaron enjoyed explaining to others so that it could help him better memorise rules and formulas (4.5.1 Aaron, p. 132). Lilly instead liked to explain to others so that she could better comprehend the underlying concepts (4.4.1.5 If I can explain it to others, then I understand, p. 107).

**Making things easier**

One final theme to emerge from both Pippa and Lilly was their frequent reference to things being ‘easy’ or ‘helpful’. For Pippa, understanding made doing mathematics easy (P10.2 “It’s just I find it easier to remember during a test if I understand something,” p. 65. See also P12.1, p. 68; P15.15, p. 73). Not understanding made doing mathematics hard (P15.13-P15.14 “Like I remember it, but I just don’t get why it is like that. So I find it hard.” p. 73). In contrast, easy and helpful was not associated with understanding for Lilly (L2.6-L2.7 “But I feel like in some tests memorising definitely does help, but you do need that, you sort of need to understand it,” p. 101. See also L3.3, p, 102; L5.13, p. 105). Lilly reported recognising patterns and making connections as helpful strategies that helped make finding answers easier and more efficient. Because Lilly associates these strategies with finding answers, ‘easy’ and ‘helpful’ are not associated with a type of understanding for Lily suggesting that answering questions does not form part of her views on understanding.

5.1.1.1 **Summary**

This section demonstrates that while two students may report the same view of understanding, they may hold very different underlying intentions and/or behaviours. For example, both Pippa and Aaron reported that understanding occurs when connections are made. When Pippa sought to make
connections, it was with the intention of generating meaning, and she tended to use a more adaptive form of help seeking by asking for explanations rather than answers (4.3.3.2 Types of questions Pippa asks, p. 85).

When Aaron wanted to make connections, it was with the intention of helping his memorisation skills, and was observed using only dependent help seeking through this study (4.5.1 Aaron, p. 132). Both students reported the same perceptions of understanding, yet displayed different help seeking behaviours. Therefore, it seems unlikely that one particular help seeking behaviour can be associated directly with one specific student perception of understanding.

5.2 Researcher identified understanding and corresponding help seeking behaviours

This section explores whether an association exists between students’ level of understanding as identified by the teacher/researcher (i.e., instrumental or relational understanding), and students’ corresponding help seeking behaviour. Results for Pippa and Lilly are discussed separately, followed by comparisons between Pippa and Lilly, as well as the two additional students, Aaron and Kipp.

5.2.1 Pippa’s help seeking and researcher identified understanding

Comparing Pippa’s help seeking with her observed understanding suggests instrumental understanding was more likely to be accompanied with executive help seeking (asking for answers or the next step rather than explanations, Nelson-Le Gall, 1986) while relational understanding was more likely to be
accompanied with appropriate help seeking (asking for help when necessary and asking for explanations rather than answers, Ryan et al., 2005).

For example, during the velocity graphs task, Pippa was able to draw together connections between ideas demonstrating meaningful understanding through horizontal mathematizing (Treffers & Goffree, 1985, section 4.3.2.5 When I make connections, I understand, p. 71). During the same task, she displayed appropriate help seeking behaviours by asking for explanations rather than answers (See Help seeking during the velocity graphs task focused on comprehension, p. 88). Thus, during the velocity graphs task, Pippa demonstrated more meaningful understanding than relational understanding and used adaptive help seeking. This information is summarised in the first row of Table 7.

<table>
<thead>
<tr>
<th>Row no.</th>
<th>Activity</th>
<th>Researcher identified level of understanding</th>
<th>Help seeking behaviour</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Velocity graphs task</td>
<td>More meaningful understanding than instrumental (Horizontal mathematizing, Treffers &amp; Goffree, 1985)</td>
<td>Appropriate help seeking</td>
</tr>
<tr>
<td>2</td>
<td>Coke can task</td>
<td>Instrumental understanding (Skemp, 1976)</td>
<td>Executive help seeking</td>
</tr>
<tr>
<td></td>
<td>Ferris wheel task</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7 Summarising Pippa’s researcher identified understanding and corresponding help seeking behaviour during three tasks

In contrast, during the coke problem, Pippa used executive help seeking by asking for answers rather than hints with the underlying intention of finding the answer (See 4.3.3.4 Summary of Pippa’s help seeking, p. 94). Similar behaviours were observed during the Ferris wheel task. In this context, Pippa approached her work using instrumental understanding (See 4.3.2.6 Summary of Pippa’s understanding, p. 75). She was able to complete her work by applying formulas without necessarily knowing why they worked (e.g., P15.6 “I just look at the graph and just do it.” p. 73. See also Applying received help on later problems, p. 92).
Thus, during the coke and Ferris wheel tasks, Pippa demonstrated instrumental understanding accompanied with executive help seeking. This information is summarised in the second row of Table 7.

Table 7 summarises Pippa’s help seeking behaviours and corresponding researcher identified understanding for three of tasks in which she participated in during this study. The largest triangle task is not included here because she was unwell during this lesson and did not participate in the task fully (P16.1 “Pippa: But I was really sick so I was… I couldn’t really think.” p. 80). The table suggests that for Pippa, an association exists between the researcher identified level of understanding and corresponding help seeking behaviours. Instrumental understanding was accompanied with non-adaptive help seeking while a more meaningful level of understanding than relational was accompanied with adaptive help seeking.

This association between understanding and help seeking is represented in Figure 10. The horizontal axis represents different possible types of help seeking behaviours while the vertical axis represents different levels of understanding. The diagram was constructed by merging Pippa’s level of understanding (Figure 4, p. 77) and her help seeking behaviours (Figure 6, p. 96) so that an overall picture of how help seeking and observed understanding were associated can be drawn.

A four quadrant grid results with the following combinations:

- Quadrant 1) Adaptive help seeking and relational/logical understanding
- Quadrant 2) Non-adaptive help seeking and relational/logical understanding
- Quadrant 3) Non-adaptive help seeking and instrumental understanding
- Quadrant 4) Adaptive help seeking and instrumental understanding
Figure 10 Relationship between Pippa’s help seeking and researcher identified understanding
The shaded area represents Pippa’s activity of combined help seeking and researcher identified level of understanding. This places Pippa predominantly in quadrant three, that is, Pippa was observed using executive help seeking accompanied with instrumental understanding most frequently. A small section of quadrant four is also shaded, showing that Pippa was observed using instrumental understanding with adaptive help seeking less often.

Influences on Pippa’s help seeking and understanding were also identified throughout this study, which have been represented in Figure 10. These are discussed in the next section.

5.2.1.1 Influences on Pippa’s researcher identified understanding and help seeking behaviours

Two factors were identified as possible influences on Pippa’s help seeking behaviors and level of understanding in this study, though others may also exist. It was found that Pippa demonstrated meaningful understanding when she felt she had more experience working on the problem than the students she was working (See Figure 4, p. 77). She also tended to use executive help seeking when worried about lagging behind the rest of her group. This information has been summarised in the first two rows of . Because both help seeking behaviours and level of understanding differed when the Pippa’s comparative task experience was higher than other students, it is likely that prior task experience, or concerns about not lagging behind, can be considered as an influential factor on both Pippa’s help seeking and level of understanding. This lends support to the argument that help seeking and level of understanding are associated as they appear to shift as a pair in association with Pippa’s comparative task experience.

The second possible influence identified in this study was the purpose of the task. When the purpose of the task was an assessment, Pippa reported that she was more interested in knowing how to answer the questions even if she did not understand what she was doing (P9.1 “In a test I think it’s better to know
just how to get the answer.” p. 65). Thus, during tests, Pippa reported she used instrumental understanding. When the purpose of the task was ‘normal learning’, Pippa reported wanting to focus on understanding rather than finding answers (P9.3 “But if it’s normal learning, then I would rather focus on understanding why that happens,” p. 65) and was found to demonstrate a more meaningful level of understanding than instrumental understanding (4.3.2.6 Summary of Pippa’s understanding, p. 75). This has been summarised in the last two rows of Table 8.

<table>
<thead>
<tr>
<th>Row no.</th>
<th>Influence</th>
<th>Researcher identified level of understanding</th>
<th>Help seeking behaviour</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Prior task experience (comparative to other students)</td>
<td>More task experience and able to keep up with her group</td>
<td>More meaningful understanding than instrumental</td>
</tr>
<tr>
<td>2</td>
<td>Less than or equal task experience and worried about lagging behind her group</td>
<td>Instrumental understanding</td>
<td>Executive help seeking</td>
</tr>
<tr>
<td>3</td>
<td>‘Normal learning’</td>
<td>More meaningful understanding than instrumental</td>
<td>No changes observed in this study</td>
</tr>
<tr>
<td>4</td>
<td>Assessment</td>
<td>Instrumental understanding</td>
<td></td>
</tr>
</tbody>
</table>

Table 8 Summarising influences on Pippa’s help seeking behaviours and researcher identified level of understanding

The purpose of the task, while it was found to influence Pippa’s level of understanding, was not found to influence help seeking behaviours. This is possibly because none of the tasks included in this study were used as assessments. That is, tasks during which Pippa had the opportunity to ask for help were all in the
context of ‘normal learning’. Therefore, no comparisons could be made between Pippa’s help seeking during ‘normal learning’ and an assessment task. This has been represented in as a grey shading for help seeking in the last two rows of the table.

These factors are represented in Figure 10 (p. 145) using different coloured shadings. The grey shading represents Pippa’s help seeking and self-reported level of understanding when the purpose of the task was an assessment or she felt she did not have more task experience than other students (See rows one and three in Table 8). The green shading represents Pippa’s help seeking and level of understanding when she felt she did have more task experience than other students and the purpose of the task was not an assessment (See rows two and four in Table 8). The total area of shading has significnatly more grey than green showing that Pippa approached her work using instrumental understanding and executive help seeking more often than a more meaningful level of understanding with adaptive help seeking.

These results suggest that for Pippa, executive help seeking (asking for answers or the required steps rather than explanations) is more likely to be accompanied by instrumental understanding while appropriate help seeking (asking for help when necessary and asking for explanations rather than answers) is more likely to be accompanied with a more meaningful level of understanding than instrumental. More cases would need to be studied to see if this association can be generalised to other activities Pippa might engage in or even to the activity of other students.

The next section conducts a similar analysis for Lilly. While Pippa’s activity related to help seeking and researcher identified level of understanding is represented in quadrants three and four of Figure 10 (p. 145), the next section will show that Lilly’s behaviour places her in the opposite quadrants: quadrants one and two.
5.2.2 Lilly’s help seeking and researcher identified level of understanding

Lilly was predominantly found demonstrating relational understanding (See Figure 8, p. 117), and in one instance from the four tasks Lilly demonstrated logical understanding (4.4.1.5 If I can explain it to others, then I understand, p. 107), that is, forming generalisations and connecting ideas together, as well as being able to explain these generalisation to others (Skemp, 1979). Thus relational understanding was considered the most prevalent form of understanding that Lilly demonstrated as it was observed during multiple tasks (See 4.4.1.4 Making connections is a helpful strategy, p. 104 and 4.4.1.3 Recognising patterns in similar questions is a helpful strategy, p. 102). Lilly was not observed approaching her work with instrumental understanding.

The type of help seeking that Lilly was observed using most often was appropriate help seeking (4.4.2.5 Summary of help seeking, p. 128). Lilly, in general, asked for help when necessary, and asked for explanations rather than direct answers. She was less frequently observed using executive help seeking (See Figure 9, p. 131).

The association between Lilly’s help seeking and researcher identified understanding is represented in Figure 11 which uses the same horizontal and vertical axes as was used for Pippa in Figure 10 (p. 145). The diagram was constructed by combining Lilly’s understanding (Figure 8, p. 117) and help seeking behaviours (Figure 9, p. 131). The shaded area represents the combination of understanding and help seeking observed for Lilly. The shading places Lilly predominantly in quadrant one, that is, she was found to use adaptive help seeking and relational understanding most often. The width of shading in quadrant one narrows towards logical understanding showing that while Lilly did demonstrate logical understanding, it was observed less frequently than relational understanding (4.4.1.9 Lilly’s researcher identified understanding, p. 116). There is also some shading in quadrant two representing that Lilly was
### Key

<table>
<thead>
<tr>
<th>Type of help seeking</th>
<th>Level of understanding (researcher observed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lilly’s help seeking and understanding when she felt she was spending an appropriate amount of time on a problem</td>
<td>Lilly’s help seeking and understanding when she felt she was spending too long on a problem relative to other students</td>
</tr>
</tbody>
</table>

Influences on Lilly’s help seeking behaviours. Includes **peer comparisons** (awareness she has spent longer than other students on a problem) and **negative emotions**. Both influences result in a horizontal shift from quadrant one to quadrant two.

---

**Figure 11 Relationship between Lilly’s help seeking and researcher identified understanding**
observed using relational understanding and non-adaptive help seeking together (4.4.2.5 *Summary of help seeking*, p. 128).

Factors that influenced Lilly’s choice of help seeking behaviours are also represented in Figure 11 but are discussed in the next section. No influences were identified in this study affecting Lilly’s level of understanding. In other words, Lilly was found to approach her work with relational understanding during all tasks used throughout this study, with the exception of the one instance where she used logical understanding.

5.2.2.1 Influences on Lilly’s researcher identified understanding and help seeking behaviours

Two influences were identified in this study: time spent working on a problem and emotions.

It was found that the amount of time Lilly spent on a problem, in comparison to other students, influenced her help seeking. When she felt she was taking longer than others to finish, she used executive help seeking (L10.1 “But how are they already done with that sheet? Help!” p. 111). When she worked at a pace that was equivalent or faster than other students, she used appropriate help seeking (4.4.1.6 *If I cannot finish a problem quickly, then I will not be able to finish it*, p. 110).

The second influence identified from this study is emotions where an emotional response occurred before a change in help seeking. When Lilly experienced positive emotions such as joy or excitement, she was found to use appropriate help seeking (4.4.1.7 *Understanding is related to an emotional response*, p. 112). When Lilly experienced negative emotions such as panic or frustration, she was instead observed changing from appropriate to executive help seeking (4.4.2.5 *Summary of help seeking*, p. 128). After moments of frustration, if Lilly was able to resume progress on her work, she was found to experience relief, that is, a reaction in response when a negative event has stopped (Lumby, 2011; Pekrun et al., 2002). The
corresponding help seeking behaviour then shifted from executive, back to appropriate. These findings are summarised in the last three rows of Table 9 and are in agreement with Pekrun et al. (2002) who found that positive emotions lead to more adaptive learning strategies while negative emotions lead to less adaptive strategies. These are also in agreement with Hannula (2006) who argues that emotions have the potential to control and regulate behaviours in the classroom.

<table>
<thead>
<tr>
<th>Row no.</th>
<th>Influence</th>
<th>Researcher identified level of understanding</th>
<th>Help seeking behaviour</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Time spent on task</td>
<td>Perceived to have spent an appropriate amount of time on a task</td>
<td>Relational understanding</td>
</tr>
<tr>
<td>2</td>
<td>Perceived to have spent too long on a task</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Emotions</td>
<td>Positive emotions such as joy or excitement</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Negative emotions such as panic or frustration</td>
<td></td>
<td>Relational understanding</td>
</tr>
<tr>
<td>5</td>
<td>Relief (in response to the cessation of a negative experience)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 9 Summary of influences on Lilly’s researcher identified level of understanding, help seeking behaviours

Table 9 shows that instances where Lilly used either executive or appropriate help seeking were accompanied with relational understanding. Thus, a change in help seeking behaviour did not cause a change in Lilly’s researcher identified understanding. No influences were identified in this study affecting Lilly’s level of understanding.
Returning to Figure 11 (p. 150), influences on Lilly’s help seeking are represented in different coloured shadings. The blue shading represents Lilly’s help seeking and understanding when she felt she was spending too long on a problem and demonstrated non-adaptive help seeking with relational understanding (Row two of Table 9). The grey shading represents Lilly’s help seeking and understanding when she felt she was spending an appropriate amount of time on a problem and represents adaptive help seeking with relational understanding (Row one of Table 9). This combination of help seeking and relational understanding was observed more often than non-adaptive help seeking with relational understanding. Thus, a larger portion of the shaded area is in grey rather than blue.

Influences that resulted in a change in help seeking behaviour are represented in Figure 11 as a black arrow. Because peer comparisons were found to influence only help seeking rather than both help seeking and understanding, the arrow has been drawn pointing horizontally rather than on the diagonal, showing that it only influenced help seeking behaviours. It represents that either when Lilly compared her progress with others and felt she was taking too long, or if she experienced negative emotions, her help seeking changed from adaptive to non-adaptive and is shown as a shift from quadrant one to quadrant two.

The next section now compares researcher identified understanding and help seeking behaviours for the activity examined from all four students to see if any associations can be found across the four students.

### 5.2.3 Researcher identified understanding and help seeking for all four students

When help seeking behaviours are compared with researcher identified understanding for all four students, some associations were found. In particular, relational understanding was more likely to be accompanied with adaptive help seeking while instrumental understanding was more likely to be accompanied with non-adaptive help seeking. This is represented in Figure 12 which compares all four
students. It combines the diagrams representing the combination of help seeking and researcher identified understanding for both Pippa (Figure 10, p. 145) and Lilly (Figure 11, p. 150). A similar analysis that was conducted for Pippa and Lilly was also carried out for the two additional students, Aaron and Kipp (See 4.5 Additional students, p. 131). Their results have also been included in Figure 12.

Kipp is placed predominantly in quadrant one representing that he was most frequently observed using relational understanding and adaptive help seeking (4.5.2 Kipp, p. 133). This is represented with light grey shading. A darker shading has been used to represent Kipp’s behaviour when, like Lilly, he felt he was spending too long on a problem (4.5.2 Kipp, p. 133). This behaviour is represented in quadrant two showing that when Kipp felt he was taking too long on a problem, he continued to use relational understanding but his help seeking behaviour changed from adaptive to non-adaptive. Like Lilly, Kipp reported that emotions were significant (4.5.2 Kipp, p. 133). This is represented by the horizontal black arrow pointing from quadrant one to quadrant two. Emotions, like Lilly, influenced Kipp’s help seeking behaviour. When Kipp experienced negative emotions, his help seeking changed from adaptive to non-adaptive. Like Lilly, emotions were not found to influence his researcher identified level of understanding. Therefore, the black horizontal arrow in can be considered to represented in Figure 12 in influences for both Lilly and Kipp.

Aaron is placed exclusively in quadrant three showing that he was only observed using instrumental understanding and dependent help seeking (4.5.1 Aaron, p. 132). This is represented in a gridded shading. No influences were found to affect Aaron’s behaviour, thus only one tone of shading has been used.

The diagrammatic representation of students’ help seeking behaviours and corresponding level of understanding in Figure 12 shows that student activity was observed for all four quadrants. Furthermore, Figure 12 suggests a positive association between help seeking behaviours and researcher identified understanding. Results from the four students suggests that instrumental understanding was more likely to be associated with non-adaptive forms of help seeking. Relational understanding was more likely to be associated with adaptive forms of help seeking.
Figure 12 Comparison of all four students and their observed understanding with help seeking
Figure 12 shows that overlaps existed for Lilly and Kipp as both their help seeking and level of understanding corresponded closely. Because Kipp reworked problems on his own after receiving help (4.5.2 Kipp, p. 133), he was considered to display more aspects of autonomous help seeking than Lilly and is therefore represented as spreading further to the right in Figure 12 than Lilly. Because Pippa displayed predominantly instrumental understanding (Figure 4, p. 77), never quite reaching relational understanding, she does not overlap with Lilly and Kipp who did not display instrumental understanding.

Although Aaron and Pippa were found to sit predominantly in quadrant three, they are not represented as overlapping. Both displayed predominantly instrumental understanding, but Pippa was able to demonstrate meaningful understanding (horizontal mathemtaizing, Figure 4, p. 77) while Aaron’s understanding remained an activity in memorisation and there was no evidence of anything more (4.5.1 Aaron, p. 132). Additionally, Aaron was found to be a dependent help seeker who asked for help when unnecessary and was not observed using the help he received to improve subsequent problem solving. Pippa on the other hand did use the help she received to try subsequent problems on her own (4.3.3.3 Subsequent problem solving, p. 91). This places Pippa between quadrants three and four, while Aaron is exclusively represented in quadrant three.

Potential influences on help seeking behaviours and level of understanding are also represented in Figure 12 and are discussed in the next section.

### 5.2.3.1 Peer comparison and its influence on help seeking behaviours

For three of the four students, peer comparison emerged as significant with regards to help seeking. For Lilly and Kipp, the combination of relational understanding with non-adaptive help seeking (quadrant two) only occurred when they felt that they did not understand the problem but others around them did (Row one of
This relationship was causal in that a change in help seeking occurred after Lilly felt she was spending longer on a problem than others. This is represented as a shift from quadrant one to quadrant two. For Pippa, the combination of instrumental understanding with adaptive help seeking (quadrant four) only occurred when the task she worked on was not being assessed \textit{and} her prior task-experience was comparatively higher than other students (Rows one and two of , p. 148). It was during these moments that Pippa used a more adaptive form of help seeking by asking for explanations rather than answers. Evidence from the data could not confirm whether this association was causal.

Thus, for three students, comparison with other students was a significant factor with regards to help seeking behaviours, associated with a shift from one quadrant to another in Figure 12 (p. 155). No factors were found to influence Aaron.

### 5.2.3.2 Summary

The data suggests that an association exists between help seeking behaviours and researcher identified level of understanding. Instrumental understanding was likely to be accompanied with non-adaptive help seeking while relational understanding was likely to be accompanied with adaptive help seeking.

Peer comparisons were found to influence help seeking behaviours for three of the four students.

### 5.3 Student perceptions of understanding, help seeking, and goals

Currently, it has been shown that one particular perception of understanding was not associated directly with one particular help seeking behaviour. This section instead explores the nature of any associations
between student perceptions of understanding and help seeking behaviours. It will be argued that perceptions of understanding and help seeking behaviours are not directly related, but are instead mediated through learning goals. The model of self-regulated learning will be used as a lens to explain how perceptions of understanding and help seeking might relate. First, results for Pippa and Lilly will be discussed. The section then concludes by comparing the results for Pippa and Lilly where an overall model is proposed showing that perceptions of understanding can be seen to fit within the process of self-regulated learning.

5.3.1 Self-regulated learning

The results from this study, as will be argued through this section, suggested that students’ help seeking formed part of regulatory behaviours geared towards achieving set learning goals. This has similarities with self-regulated learning which will be used as a lens through which to analyse the relationship between help seeking and perceptions of understanding.

The process of self-regulation involves planning and setting goals for learning prior to engaging with the task, monitoring one’s activities and progress during the task, and checking the outcomes against one’s set goals (Pintrich, 1999). If the goals have not been met, then the learner may engage in strategies with the intention of repairing any breakdown in understanding. Help seeking can be included as one of these strategies (Ibid.).

Pintrich (2004) argues that the model of self-regulated learning is a useful model to use when students hold multiple learning goals or multiple approaches to learning. Because students in this study were found to hold multiple goals, self-regulated learning was seen as an appropriate choice of model.

Muis (2007) found epistemological beliefs (beliefs about knowledge and how it is learnt) influence the process of self-regulated learning whereby one’s personal beliefs on knowledge and how it is learnt
influence how one chooses to approach a problem and which strategies will be used, including help seeking. Op’t Eynde et al. (2002), studying mathematical beliefs, did not draw attention to the way in which mathematical beliefs influence self-regulated learning but did find that mathematical beliefs form a subset of epistemological beliefs and also influence the way in which students approach problem solving.

Schoenfeld (1988) showed that some of the perceptions of understanding already identified (If I can memorise all the rules then I have understood, and If I can finish a problem quickly, then I have understood) formed part of students’ mathematical beliefs. Given the interpretation of perceptions of understanding for this study, some mathematical beliefs identified by Schoenfeld, some perceptions of understanding can therefore be seen as a subset of mathematical beliefs. Because epistemological beliefs have been shown to influence self-regulated learning (Hofer & Pintrich, 1997; Muis, 2007), then perceptions of understanding, as a subset of epistemological beliefs, might also influence self-regulated learning.

From this, the model of self-regulated learning provides a potential lens through which to analyse the relationship between help seeking behaviours and perceptions of understanding. The next two sections analyses the data for both Pippa and Lilly through the lens of self-regulated learning.

5.3.2 Pippa’s perceptions of understanding, help seeking, and goals

This section explores the way in which Pippa’s help seeking and reported views on understanding might interact. Some of Pippa’s learning goals and the means to achieve them emerged from her views on understanding (Section Pippa’s perceptions of understanding, p. 78). These goals will be used to draw possible associations between different styles of help seeking and different perceptions of understanding.
<table>
<thead>
<tr>
<th>Row no.</th>
<th>Perception of understanding</th>
<th>Learning goals and way in which to achieve them</th>
<th>Help seeking behaviour</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Getting the answer does not imply understanding has occurred</td>
<td>Goal: Getting high marks.</td>
<td>Executive help seeking</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Means: Knowing how to get the answers</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Recognising patterns implies understanding has occurred</td>
<td>Goal: Comprehension</td>
<td>Appropriate help seeking</td>
</tr>
<tr>
<td></td>
<td>Making connections implies understanding has occurred</td>
<td>Means: Being an active participant in the solution process</td>
<td></td>
</tr>
</tbody>
</table>

Table 10 Summary of Pippa’s perceptions of understanding, help seeking behaviours, and corresponding learning goal

One of Pippa’s goal was achieving high marks (P9.2 “Because you want to know how to get good marks.” p. 65). The way in which she could achieve this was by getting the answers. She perceived that ‘getting the answers’ did not count as understanding (4.3.2.3 Relationship between the purpose of the task and Pippa’s reported understanding, p. 65). When a goal of high marks was in effect, Pippa did not think that understanding was important (P10.1 “I don’t think it’s [understanding] necessarily completely important.” p. 65). With regards to help seeking, Pippa was frequently observed using executive help seeking where one of the intention of help seeking was to find out from others the steps needed to complete the problem (4.3.3.2 Types of questions Pippa asks, p. 85). This is not unexpected as other studies have found that students who do not emphasise or value understanding tend to use executive help seeking (Webb et al., 2002). Because knowing how to get the answers was necessary in order to achieve high marks (P9.1 “In a test I think it’s better to know just how to get the answer.” P. 65), and executive help seeking is one way in which Pippa can find the answers from other students, executive help seeking can be seen as a way for Pippa to achieve her goal of high marks. This suggests that for Pippa, executive help
seeking can be seen as a behaviour that is directed towards achieving one’s goal (Hannula, 2006; Pintrich, 2004). This information is summarised in the first row of Table 10.

Pippa held a second learning goal occurred during ‘normal learning’: comprehension (P9.3 “But if it’s normal learning, then I would rather focus on understanding why that happens.” p. 65). The way in which to achieve this was to be an active participant in the solution process. This related to her views on what counted as understanding. Pippa reported that understanding involved recognising patterns (P11.8 “So over time you just kind of get the pattern.” p. 67) as well as drawing connections between ideas (P15.1-P15.3 “Suddenly everything just links together.” p. 72). What was important was that recognising patterns were her own patterns she could identify, rather than pointed out by the teacher. Additionally, making connections related to understanding when she was able to identify her own connections between various ideas (4.3.2.6 Summary of Pippa’s understanding, p. 75). Similarly, Pippa’s help seeking while problem solving included an underlying goal of wanting to be an active participant in the solution process (P19.2 “But I made no contribution towards this.” p. 83). This suggests that adaptive help seeking can be seen as a way to achieve Pippa’s goal of comprehension. By using adaptive help seeking, Pippa becomes an active member in the solution process from which she is then able to generate comprehension. This information is summarised in the second row of Table 10.

Table 10 summarises the results from this section showing two learning goals evident from the way in which Pippa described understanding. For Pippa, it is likely that her help seeking was directed towards her goal of either comprehension or achieving high marks.

Muis (2007) found that epistemological beliefs influence the choice of learning goals during the process of self-regulated learning. This seems to be similar to the relationship between Pippa’s reported perceptions of understanding and her help seeking behaviours. Like self-regulated learning, Pippa’s help seeking seemed directed towards achieving her learning goals. Similar to Muis’ model where beliefs influence the choice of which goal to pursue, results here suggest that Pippa’s perceptions of understanding were also evident when deciding which goal to pursue for the particular task she was
working on. While this study is not looking specifically at beliefs, perhaps there is a connection between student perceptions of understanding and beliefs that warrants further investigation.

Influences were also identified from this study with regards to Pippa’s choice of learning goal. This is discussed in the next section.

5.3.2.1 Influences on Pippa’s perceptions of understanding, help seeking, and learning goals

When the purpose of the task was an assessment, Pippa reported wanting to aim for high marks instead of understanding (P9.1-P9.2, p. 65). When the task was ‘normal learning’, she pursued comprehension (P9.3 “But if it’s normal learning, then I would rather focus on understanding why that happens.” p. 65). This is summarised in . Similar findings were reported by Reid et al. (2003) who found that some students who have the intention of understanding their work will use memorisation and approach their work with instrumental understanding when the demands of the task are low, such as a traditional skills based test (See also Callejo & Vila, 2009; Crawford et al., 1994).

What remains unclear is at what stage during this process perceptions of understanding enter. Does the learning goal set the way Pippa perceives understanding, or does her preconceived perception of understanding influence the type of goal she pursues? Muis (2007) found that within the process of self-regulation, epistemological beliefs influence the choice of goal to pursue suggesting that perhaps perceptions of understanding might function in the same way in that they too influence learning goals. However, this could not be confirmed by the data collected in this study. Classroom observations did not provide sufficient data to allow any inferences to be made. Information from interviews suggest tentatively that the purpose of the task influenced Pippa’s perception of understanding. For example, Pippa reports that when the task is an assessment, she does not value understanding (P9.4 “But if it’s for an exam, then I don’t really care.” p. 65). From Pippa’s comments, it appears that whether she values
represents Pippa’s different perceptions of understanding and corresponding help seeking which synthesises the information summarised in Table 10 (p. 160) and (p. 163). The first row represents whether the purpose of the task is an assessment or ‘normal learning’. The second row shows Pippa’s different goals for learning she pursued as well as the different ways she was found to perceive understanding. This has been placed in the second row as it was found that Pippa set her goals and perceptions of understanding in response to the purpose of the task (P9.1, p. 65; P9.3, p. 65). Because it cannot be determined whether perceptions of understanding or learning goals occurred first, goals and perceptions of understanding are represented as occurring in the same row. This is not to imply that they occur at the same time, but that it is unclear as to the precise order in which they occur.

The third row represents the way in which Pippa chose to pursue her goal. These include finding the answers and participating in the solution process. The fourth row represents the corresponding help seeking Pippa used given the relevant classroom influence, perception of understanding, goal, and means. This includes executive and appropriate help seeking. Help seeking is represented as occurring last because it is assumed that help seeking forms part of self-regulatory behaviour that is geared towards
Figure 13 Relationship between Pippa’s help seeking, perceptions of understanding, learning goals, and way in which to achieve the learning goal.

- **CLASSROOM INFLUENCES**
  - Task is an assessment
  - Task is normal learning

- **PERCEPTIONS OF UNDERSTANDING AND GOALS**
  - **PERCEPTION OF UNDERSTANDING**
    - Finding answers (forms part of Pippa’s perceptions of what does not count as understanding)
    - Understanding is not important
  - **GOAL**
    - High marks
  - **PERCEPTION OF UNDERSTANDING**
    - Perceptions of understanding related to generating own knowledge and insights (making connections and recognising patterns)
  - **GOAL**
    - Comprehension

- **WAY IN WHICH TO ACHIEVE THE LEARNING GOAL**
  - Getting the right answers
  - Participation in the solution process

- **HELP SEEKING**
  - Executive help seeking
  - Asks for explanations
achieving one’s goal (Hannula, 2006). Thus, help seeking is seen as a response by the student after goals have been set.

From , one type of help seeking behaviour and one perception of understanding occur together. The left side represents one combination of understanding and help seeking. It shows that executive help seeking occurred when Pippa’s perceptions of understanding included finding the answer does not imply understanding has occurred. The right-hand side represents the second pair. It shows that appropriate help seeking occurred when Pippa viewed understanding as making connections or recognising patterns. Because perceptions of understanding and help seeking can be seen to change as a pair, it implies that a relationship exists between the two. However, also demonstrates that perceptions of understanding and help seeking are not directly related. Instead, they are mediated via goals which are influenced overall by the purpose of the task.

In the next section, a similar analysis is conducted for Lilly. Similar findings are drawn out, showing that, like Pippa, Lilly’s perceptions of understanding and help seeking behaviours are also indirectly associated via goals for learning.

5.3.3 Lilly’s Perceptions of understanding, help seeking, and goals

This section explores the ways in which Lilly’s help seeking and perceptions of understanding might interact. The data from this study revealed one learning goal for Lilly: comprehending her work. The way in which Lilly went about achieving this goal was by being able to solve problems quickly and fluently (4.4.1.9, Lilly’s researcher identified understanding, p. 116). Fluency, throughout this section, refers to Lilly either being able to solve a problem in an appropriate amount of time, which was dependent on how
long other students were spending on the problem, or knowing what the next step to the problem is without needing to stop and think.

It will be argued that Lilly’s help seeking changed depending on whether or not she was able to work through a problem fluently. It will also be argued that Lilly’s need for fluency was evident in the way in which she described understanding, showing that Lilly’s perceptions of understanding and help seeking behaviours are not directly related but instead mediated by whether or not she is able to finish a problem in an appropriate amount of time.

### 5.3.3.1 Lilly’s perceptions of understanding and corresponding learning goal and help seeking behaviour

One of Lilly’s perceptions of understanding was *If I cannot finish a problem quickly, then I don’t understand* (Section 4.4.1.6 p. 110). Evident from this perception is Lilly’s need for fluency. Lilly is concerned with being able to finish problems in an appropriate amount of time. Lilly also experienced feelings of embarrassment if she felt she was the only one who did not understand a question but others around her did (L12.3 “And it’s kind of embarrassing.” p. 113) reporting that how well other students were doing played a “big part” in how she feels (L11.5 “So definitely everyone else’s reactions play a big part in how I feel.” p. 112). Both these themes on understanding relate to Lilly comparing herself with others in the class. Additionally, when Lilly was unable to finish a problem quickly, but others around her were able to, she experienced negative emotions, such as panic or frustration (4.4.1.7 *Understanding is related to an emotional response*, p. 112). This also formed part of her perceptions of understanding in that a negative emotion, for Lilly, was an indication that she had not understood the problem.

At such times these emotions were accompanied by Lilly displaying non-adaptive help seeking. When Lilly was aware she was spending too long on a problem, she then experienced negative emotions. Her
help seeking then changed from adaptive to non-adaptive (4.4.2.5 Summary of help seeking, p. 128). This is summarised in the first row of Table 12.

<table>
<thead>
<tr>
<th>Row no.</th>
<th>Perception of understanding</th>
<th>Ways in which to achieve learning goal (Fluency)</th>
<th>Emotion</th>
<th>Help seeking behaviour</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Not being able to finish a problem quickly implies one does not understand the problem</td>
<td>Unable to finish a problem in an appropriate amount of time</td>
<td>Negative emotions such as panic or frustration</td>
<td>Executive help seeking</td>
</tr>
<tr>
<td></td>
<td>Not understanding the problem, but other students do results in feelings of embarrassment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>A negative emotion means understanding has not occurred</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Not part of Lilly’s perceptions of understanding</td>
<td>Able to finish a problem in an appropriate amount of time</td>
<td>Positive emotion such as joy or excitement</td>
<td>Appropriate help seeking</td>
</tr>
<tr>
<td>3</td>
<td>Fluency resumes after a period of frustration</td>
<td>Relief (as cessation of frustration, Lumby, 2011)</td>
<td></td>
<td>Help seeking changes from executive to appropriate</td>
</tr>
</tbody>
</table>

Table 12 Summary of Lilly’s perceptions of understanding, help seeking behaviours, and ways in which to achieve her learning goal

During moments when fluency resumed (i.e., she was stuck and then experienced a ‘breakthrough’) Lilly experienced feelings of relief (Barnes, 2000; see also 4.4.1.7 Understanding is related to an emotional response, p. 112) and her help seeking returned to appropriate (4.4.2.5 Summary of help seeking, p. 128). This
however, did not relate in any way to Lilly’s perceptions of understanding. This is summarised in the third row of Table 12.

What about instances when Lilly was able to work through a problem without getting stuck? When Lilly was able to work fluently, she experienced positive emotions such as joy or excitement (4.4.1.7 Understanding is related to an emotional response, p. 112). When Lilly was able to work fluently through a problem, or she felt she was stuck but others around her were too, she used adaptive help seeking. Lilly did not report being able to finish a problem with understanding. Thus, achieving fluency and positive emotions did not relate to Lilly’s perceptions of understanding. This is summarised in the second row of Table 12.

When Lilly was able to work through a problem fluently, rather than relate it to understanding, Lilly instead spoke of learning strategies she used to help her achieve fluency. Lilly reported using strategies such as recognising patterns (4.4.1.3 Recognising patterns in similar questions is a helpful strategy, p. 102) and making connections (4.4.1.4 Making connections is a helpful strategy, p. 104). While these strategies related to perceptions of understanding for Pippa, they did not relate to perceptions of understanding for Lilly. Instead, Lilly saw recognising patterns and making connections as helpful in achieving fluency (Lilly’s perceptions of understanding, p. 118) in that they were learning strategies that would allow her to solve a problem more efficiently and smoothly, reducing the need to stop and think. She reported such strategies as helpful (L4.3 “I think it helps.” p. 103. See also L5.2, L5.3, L5.13, L6.4, p. 104) because they provided her a set of steps she can directly apply from one problem to another (L4.5 “I could apply some of the steps to this one.” p. 103). This would thereby reduce the amount of ‘thinking time’ needed when first starting a problem. This shows that during moments when Lilly demonstrated relational understanding, and might be expected to report it as understanding, she does not necessary describe it as understanding.

The next section explores the nature of the association between Lilly’s perceptions of understanding and help seeking that has been outlined here through the model of self-regulated learning.
5.3.3.2 Relationship between Lilly’s perceptions of understanding, help seeking, and learning goals

From Table 12 it can be seen that perceptions of understanding were not evident when Lilly was able to work through a problem fluently. The corresponding appropriate help seeking behaviour can be seen as geared towards achieving her goal of comprehension, in that she tended to want justifications and explanations when she received help (L18.1 “The method that they [Aaron and Xander] were using, well it didn’t have much foundation to it.” p. 126).

also shows that perceptions of understanding were only relevant for Lilly when she was unable to finish a problem fluently. The corresponding help seeking behaviour was executive. Executive help seeking can be seen as a useful way to achieve Lilly’s need to finish a problem quickly. If Lilly felt she is taking too long, then executive help seeking would help speed up the solution process by allowing her to find out what steps are needed to solve the problem.

Like Pippa, there are similarities between the findings here and that of self-regulated learning. As mentioned earlier (5.3.2 Pippa’s perceptions of understanding, help seeking, and goals, p. 159), Muis (2007) found that epistemological beliefs influence the choice of learning goals during self-regulation. This seems to be similar to the association between Lilly’s perceptions of understanding and help seeking. As with the process of self-regulated learning, Lilly’s help seeking seemed directed towards achieving her learning goals. Similar to Muis’ model where beliefs influence the choice of which learning goal to pursue, as well as the way in which to achieve one’s goal, results here suggest that Lilly’s perceptions of understanding were evident when her goal was at risk of not being achieved.

Furthermore, for Lilly, this process appeared to be regulated through emotions. Lilly continued to use appropriate help seeking when she experienced positive emotions. The experience of negative emotions instead resulted in a change in help seeking behaviour from appropriate to executive. This is in agreement with findings from Hannula (2002, 2006) who argues that emotions are significant in changing and
influencing behaviour and sees emotions as part of the process of self-regulation. In other words, a change in emotion should generate a change in behaviours such as help seeking behaviours.

<table>
<thead>
<tr>
<th>GOAL</th>
<th>WAY IN WHICH TO ACHIEVE GOAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mastering task and Competence</td>
<td>Fluency – being able to solve a problem either by taking a short amount of time, by immediately knowing what step to use, or by not taking longer than other students around her</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LEARNING STRATEGIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Making connections, recognising patterns, and memorisation are helpful strategies that make solving problems easier</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>IS FLUENCY ACHIEVED?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Completes problem in perceived reasonable amount of time</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PERCEPTIONS OF UNDERSTANDING</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not related to understanding</td>
</tr>
</tbody>
</table>
This is summarised in Figure 14 and represents Lilly’s different perceptions of understanding and corresponding help seeking behaviour. The first row shows Lilly’s main learning goal of comprehension. This then leads to the second row which shows Lilly’s means of achieving her goal is one of fluency. The third row includes the learning strategies of recognising patterns and making connections. They are shown in the diagram as following fluency as they are strategies that Lilly uses in response to wanting to be able to finish a problem smoothly and efficiently.
The subsequent rows in Figure 14 represent changes in Lilly’s behaviour when she was or was not able to solve a problem fluently. The diagram shows two possible paths. The right represents Lilly’s behaviour when she felt she was taking too long to solve a problem. The left pathway represents Lilly’s behaviour when she was able to solve a problem fluently. The fifth row shows Lilly’s perceptions of understanding which only became relevant when Lilly felt she spent too long on a problem. The next row represents Lilly’s subsequent emotional reactions which then impacts on the choice of help seeking behaviour shown in the seventh row.

The right-hand side includes three more rows than the left. These represent Lilly’s behaviour once she resumed fluency and replicate rows five through seven in a cyclic fashion.

What Figure 14 highlights is how Lilly’s choice of help seeking behaviour are influenced which is summarised more succinctly in Figure 15. When Lilly compared her progress to others, this produced an emotional response which then influenced her choice of help seeking behaviour. Perceptions of understanding were only relevant for Lily when fluency was at risk. Thus perceptions of understanding only had a limited role to play in influencing Lilly’s help seeking behaviours. This association between what Lilly saw as lack of understanding and executive help seeking (Row one of , p. 168) was not directly related. Instead the two are mediated via whether fluency is or is not achieved, as well as Lilly’s emotional responses.

![Figure 15](image)

*Figure 15* How Lilly’s choice of help seeking behaviours are influenced
The next section will compare findings from both Pippa and Lilly. Similarities will be drawn out between the two students where it will be argued that help seeking and perceptions of understanding were indirectly related for both students.

5.3.4 Comparing Pippa’s and Lilly’s perceptions of understanding, help seeking, and goals

This section discusses how perceptions of understanding and help seeking behaviours are not directly related, but are instead mediated through learning goals. The model of self-regulated learning will be used to explain how perceptions of understanding and help seeking might be associated. Perceptions of understanding and self-regulated learning are discussed where an overall model is proposed showing that perceptions of understanding might be at play during the process of self-regulated learning.

5.3.4.1 Comparing perceptions of understanding, help seeking, and goals for Pippa and Lilly

For both Pippa and Lilly, when they spoke about understanding, their underlying goals for learning, as well as the way they went about achieving their learning goals sometimes became evident. Pippa’s perceptions of understanding related to both her learning goals and her way in which to achieve them (Table 10, p. 160). Lilly’s perceptions of understanding did not relate to her learning goals. Instead it related only to her need fluency, and only when this was at risk (
Emotions was also relevant for Lilly with regards to her need for fluency. It was then argued that for both students, help seeking was directed toward achieving their personal goals for learning.

In short, the results from Pippa and Lilly suggest that help seeking behaviours are influenced by the choice of learning goals (5.3.2.1 Influences on Pippa’s perceptions of understanding, help seeking, and learning goals and 5.3.3.1 Lilly’s perceptions of understanding and corresponding learning goal and help seeking behaviour). Moreover, help seeking behaviours for both students appear to be geared towards achieving their learning goal suggesting that help seeking is a regulatory behaviour (Pintrich, 1999). Perceptions of understanding, if relevant, surfaced at the stage of setting the learning goal.

In terms of validity, students’ perceptions of understanding could not be directly observed (Pajares, 1992), but were rather inferred from what students said and did and it is the researcher’s interpretation of what the student said that has been taken as the perception of understanding. Depaepe, De Corte, and Verschaffel (2016) argue that such inferences assume a shared understanding between the participant and interviewer. To address the issue, they suggest that future research should include a member check, in which the participant is given the opportunity to view and confirm the interpretations that the researcher has made.

5.3.4.2 Perceptions of understanding and self-regulated learning

The data for Pippa and Lilly suggest that help seeking can be seen as a regulatory behaviour. When their goals have not been met, they engage in different help seeking strategies with the intention of repairing any risk of failure in achieving their goals (Pintrich, 1999). When perceptions of understanding and emotion are also taken into account as part of self-regulated learning, the results for Pippa and Lilly have similarities with models of self-regulated learning suggested by both Muis (2007) and Hannula (2006).
The following section will therefore use the model of self-regulated learning as a lens to analyse the nature of any existing associations between perceptions of understanding and help seeking.

Hannula (2006) acknowledges the importance emotions and mathematical beliefs have to play in the role of self-regulation within mathematics. This has relevance for Lilly as her need for fluency was notable when experiencing both positive and negative emotions. It also has relevance for this study in that the definition for perceptions of understanding used in this study presumed that perceptions of understanding formed a subset of mathematical beliefs (Op’t Eynde et al., 2002).

Hannula’s (2006) model is structured through students’ needs, goals, and means to achieve one’s goals. Needs includes psychological needs for autonomy, competence, and social belonging. Because students’ needs did not emerge significantly from the data in this study, they are not included here. On the other hand, students’ goals did emerge from the data when students discussed understanding and are therefore included as part of lens of self-regulated learning. According to Hannula, the accessibility of achieving ones’ goals can influence the process of self-regulation.

The second model of self-regulated learning considered for this study is Muis’ (2007). Muis proposed the following four phases form part of the process of self-regulated learning:

1. Task definition: before beginning a task, one develops perceptions about the task.
2. Planning and goal setting: one begins to develop a plan to approach the task using a set of strategies. The standards or means are set to achieve a particular goal.
3. Enactment: one begins to work on the task by applying selected strategies.
4. Evaluation: reflections and reactions occur to evaluate the success or failure of each phase.

The phases are cyclic in nature, where at any stage, one might return to phase one. Muis argued that epistemological beliefs facilitate self-regulate learning by influencing the goals that students set for
learning. Like Hannula, goals and the means to achieve them are significant in the role of self-regulation (phases 1 and 2). Muis hypothesised that beliefs influence the standards and means one sets in order to achieve their learning goal. That is, epistemological beliefs enter the process of self-regulated learning during phases 1 and 2.

Together, the two models have relevance for the findings in this study related to perceptions of understanding. The stages that perceptions of understanding entered for Pippa was during the task definition phase. For example, when the task was an assessment, Pippa did not perceive understanding to be valuable (p. 165). Similarly, Pippa reported that getting the answers related to her views on what did not count as understanding which related to wanting to achieve high marks (Table 10, p. 160). Therefore, perceptions of understanding for Pippa were evident during both phases 1 and 2. For Lilly, perceptions of understanding were relevant during phase 2 where goal setting and the means to achieve one's goals occur. For example, when Lilly was unable to complete a problem fluently, she reported a lack of understanding and described understanding using negative emotions (Figure 14, p. 171).

Thus, for both students, perceptions of understanding emerged during the same phases that Muis found epistemological beliefs to be relevant. While this study is not looking specifically at beliefs, perhaps there is a connection between student perceptions of understanding and beliefs that warrants further investigation.

With regards to Lilly, emotions were also relevant during self-regulation and results support findings from Hannula (2006) that regulatory behaviour can be directed through mechanisms that control emotions structured around goals. When Lilly was unable to achieve fluency, she experienced negative emotions when then caused a change in help seeking behaviour (Figure 16, p. 178).

These findings are summarised in Figure 16 where perceptions of understanding and emotions are shown to be relevant during the process of self-regulated learning. Figure 16 was created by combining
similarities between the cases of Pippa (p. 165) and Lilly (Figure 14, p. 171). It shows ‘Goals for learning’ (Phase 1) which then leads to ‘Ways in which to achieve one’s goal’ (Phase 2). Because both students were found to alter their help seeking based on either their choice of learning goal (Pippa) or whether their goal was at risk of not being achieved (Lilly), Figure 16 represents row one as influencing row two which in turns influences help seeking behaviours. Because perceptions of understanding were found to be active for Pippa during either phase 1 or 2, and Lilly during phase 2, they are represented on the diagram as associated with both goals and ways in which to achieve them. Emotions was found to be significant for Lilly (and Kipp) during phase 2 only and is presented on the diagram as associated this phase.

Studies have shown that the way in which students view what mathematics is (Hofer & Pintrich, 1997; Muis, 2004), or what it means to learn (Marton et al., 1993; Säljö, 1979), can influence the way in which they go about learning, which includes their choice of help seeking style. It is unclear from the data in this study whether perceptions of understanding can be said to influence students’ choice of goals or help seeking behaviours. This lack of clarity is supported by literature where there is a general lack of agreement regarding the way in which personal points of view and learning behaviours interact (Muis & Franco, 2009). Some studies have demonstrated a unidirectional relationship where a student’s perception might influence their choice of learning behaviour such as help seeking (Bromme, Pieschl, & Stahl, 2010; Hofer & Pintrich, 1997; Muis, 2007). Other studies have instead shown that it is the choice of learning behaviours that influence perceptions (Cobb, Yackel, & Wood, 1989; Richardson, 2013). Others instead argue that the relationship is reciprocal, where one influences the other in a cyclic nature (Crawford et al., 1994; Spray et al., 2013). It has also been suggested that it is not even possible to demonstrate a causal relationship between perceptions and learning behaviours (Callejo & Vila, 2009). The issue of how student perceptions and choice of learning behaviours interact remains open and further studies are needed in order to explore whether perceptions of understanding do or do not influence learning behaviours.
The results show that perceptions of understanding were found to be evident during the same phases of self-regulated learning as those of epistemological beliefs. In addition, Figure 16 shows that perceptions of understanding do not influence help seeking behaviours directly. Rather, perceptions of understanding were found to emerge during the task definition phase, and the goal setting phase. It is then these goals that influence the choice of help seeking behaviours. Therefore, the relationship between student
perceptions of understanding and help seeking behaviours can be said to be mediated via student goals. This is summarised with the following diagram in Figure 17.

Figure 17 Summary of how perceptions of understanding are associated with help seeking behaviours
Chapter 6  Conclusions

The intention of this study was to explore whether a relationship exists between students’ help seeking behaviours and the way in which they described and interpreted what it means to understand mathematics. This chapter provides the overall conclusions for this study. The first section summarises the key findings identified from the data. Next, limitations of this study are discussed. Implications for teachers and researchers follow where this chapter then concludes with suggested future research questions.

6.1 Overall conclusion

From the two case-studies, it was found that one particular perception of understanding was not related to one particular type of help seeking within the data collected for the case-study students. Because one particular perception of understanding was not found to be accompanied with one particular help seeking behaviour, perceptions of understanding were not found to relate directly to help seeking behaviours.

Instead what was found from the two case-studies was that students’ perceptions of understanding and help seeking behaviours were indirectly related via learning goals as well as the way in which to achieve such learning goals. Student perceptions of understanding were associated with their choice of learning goals and ways in to achieve their goal. Help seeking behaviours were then influenced by whether or not students were able to achieve their learning goals, rather than directly to students’ perceptions of understanding.

The relationship between help seeking and perceptions of understanding was explained using the construct of self-regulated learning. Help seeking was seen as a behaviour geared towards the attainment
Perceptions of understanding were then found to appear during the phases of self-regulated learning related to goal setting.

Additionally, this study confirmed Hannula’s (2006) findings that emotions influence the process of self-regulated learning whereby emotions were found to direct help seeking behaviour. Negative emotions resulted in a change in help seeking from adaptive to non-adaptive. The feeling of relief resulted in a change from non-adaptive to adaptive help seeking. Positive emotions were generally found to be accompanied by adaptive help seeking. Because some perceptions of understanding related to emotions, it is possible that perceptions of understanding and help seeking behaviours can be seen as mediated via emotions though further studies would be necessary to explore whether such a relationship exists.

From this study, teachers are provided with a way in which to start asking students questions in order to tease out what students might or might not understand. The findings show that students speak about understanding in a number of ways by using a variety of language and descriptions. This is important for teachers who need to be able to tell whether students in their classroom do or do not understand. The data provides teachers with examples of ways in which students might articulate understanding giving teachers some insight into what students might be thinking.

Furthermore, it is important for teachers to get a sense of when students feel they understand. Results provide some indications of what student behaviour looks like when they feel they do not understand through either help seeking behaviours, emotional responses, or the language they use around understanding.
6.2 Limitations of study

In order to allow for further comparisons to be made between researcher identified level of understanding and help seeking, two additional students were included and, where appropriate, the reader has been referred to other case-studies identified in other research that showed similarities with Pippa and Lilly. The corroboration between cases strengthens findings within this study for the existence of an association between help seeking and researcher identified level of understanding. Findings regarding existence of an association between student perceptions of understanding and help seeking were strengthened through triangulation of data sources because multiple sources gave similar conclusions.

However, generalising from findings related to student perceptions of understanding and help seeking is more tentative. Only two cases were analysed in detail and the findings from the two cases did not corroborate each other fully. For example, perceptions of understanding were found to be relevant both for Pippa’s goals and the way in which she went about achieving her goals. For Lilly perceptions of understanding were only found to be relevant for the way in which she went about achieving her learning goals through fluency. Additionally, emotions were significant for Lilly while not significant for Pippa. Further exploration would be required to see if the findings from this study can be generalised more broadly with regards to the role perceptions of understanding has to play in students’ learning. Therefore, findings here form only a starting point into understanding the complex relationship between students’ perceptions of understanding and their corresponding behaviour in the class.

Because students in this study were part of an accelerated class, that is, Year 10 students were selected and invited to study Year 11 Mathematics, the participants in this study were not representative of a ‘typical’ class in Victoria, Australia. Further studies are needed in classrooms of varying abilities. Regardless, the results do provide a rich description of how two high-achieving secondary school students in an accelerated class both perceive understanding and seek out help which is lacking in current literature.
Some data was unable to be collected during classroom observations either because students were unwell, or because of equipment malfunction. This impacted on what findings could be drawn. For the case of Pippa, she was unwell during one lesson and she participated little during the triangle task. This did not impact the findings and in fact allowed for what might have been potentially unexpected insights into Pippa’s behaviours that occurred when she had more or less previous experience working on a problem than other students around her that might have been missed otherwise. For the case of Lilly, audio data failed to record during the velocity-graphs task. This did impact on results. During her follow-up interview, she reported a level of understanding representative of relational understanding. However, this level of reported understanding was unable to be confirmed through classroom observations. This reduced the strength for which claims about Lilly’s level of understanding during this one task could be made.

Furthermore, in relation to the reporting of emotions as understanding, it is possible that the interview question ‘How were you feeling during this moment?’ may have influenced the likelihood of students reporting understanding as emotions. In other studies, the interview question ‘What exactly do you mean by learning?’ resulted in many students reporting that understanding related to learning (Marton et al., 1993). This is not to say that students reporting understanding as either emotions or learning did not hold such perceptions, but that such questions used during interviews might have given such a perception more significance than necessary. It is encouraged that for future studies, interviewers use a variety of questions rather than rely often on ‘How were you feeling during this moment?’ as was used for this study.

### 6.3 Implications for teaching and research

Teachers need to be aware of the influence that tasks and assessment methods have on how students interact with the task. Tasks that do not require meaningful understanding might be approached by
students with low-level understanding, regardless of ability level, learning goals, or how one views understanding (Crawford et al., 1994; Reid et al., 2003). This was evident for one student whose choice of learning goals were found to change depending on whether the task was an assessment or not.

Additionally, the use of the word ‘understand’ is problematic. Many different interpretations were identified within this study and the term became problematic when taking into account questionnaire responses and careful consideration needs to be given if including problematic terms such as ‘understand’ within questionnaires.

Similarly, teachers need to be mindful when using the word understand in the classroom. If teachers tell their students that it is important they understand their work, this may be interpreted in a variety of ways by students. For example, one may think ‘It’s important that I memorise this’ when they hear that they need to focus on ‘understanding’.

This study has also shown one potential way to uncover what learning goals students hold for themselves. Teachers could ask students to write for them a sentence or two on what they feel it means to understand mathematics. Student responses may provide teachers with information on their students’ learning goals which provides teachers with a more rounded picture of the students they are working with.

6.4 Future research questions

The subsequent research questions are provided as possible directions for what additional studies might look like. Based on the results of this study, recommendations are made for future research.

a) Similar studies are needed in a wider variety of classrooms to see if the conclusions from this study apply more generally. For example, does a similar combination of help seeking behaviours and perceptions of understanding exist in low-ability classrooms, mixed-ability classrooms, and
middle-school and primary school classrooms? Furthermore, can the results from this study be replicated in other high-ability classrooms and therefore generalisable for high-ability students?

b) Further investigations would be needed in order to explore why perceptions of understanding related to students’ goals and the means with which to achieve their goals.

c) Further insight is also needed to help explain why some students identified understanding as emotions. It has already been identified that certain emotions encourage different learning behaviours (Pekrun et al., 2002). Potential avenues of research might include exploring whether emotions form part of students’ wider mathematical beliefs. One way to achieve this is to ask students to display emoticons, face icons expressing common feelings, while they are engaged in problem solving. For an explanation on this data-gathering technique, see Agostin (2014).

d) Lastly, there is the potential that the way students describe understanding may be related to how they perceive the classroom goals. Others have shown that students’ beliefs are greatly influenced by the classroom expectations and culture (Lampert, 1990; Yackel & Cobb, 1996). Students can find it difficult to differentiate between the social context and the discipline of mathematics itself (Kloosterman, 2002; Op’t Eynde et al., 2002). Therefore, asking students directly what they think it means to understand mathematics might have produced some problematic results, in that they may have reported understanding as what it means to do mathematics in the school classroom. Conducting similar studies in different classrooms with a range of cultures may be necessary to see whether the way in which students identify mathematical understanding relates in any way to the classroom context. In order to investigate whether students’ perceptions of understanding are related to what they believe learning mathematics in the classroom looks like, it would be worth investigating the students’ perceived classroom goals as well as their perceived teacher goals.
References


Clarke, D. J. (1997). Chapter 7: Studying the classroom negotiation of meaning: Complementary


Furinghetti, F., & Morselli, F. (2009). Every unsuccessful problem solver is unsuccessful in his or her


Appendices

Appendix 1  Questionnaire results
<table>
<thead>
<tr>
<th>Question</th>
<th>Pippa</th>
<th>Lilly</th>
<th>Aaron</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maths problems that take a long time to complete don’t bother me.</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I feel I can do maths problems that take a long time to complete.</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>If I can’t do a maths problem in a few minutes, I probably can’t do it at all.</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>If I can’t solve a maths problem quickly, I quit trying.</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>I’m not very good at solving maths problems that take a while to figure out.</td>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>No matter how many worked examples I read, and practice questions I try, there are some worded problems that just can’t be solved.</td>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Worded problems can be solved without having to remember formulas.</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Memorising steps is not that useful for learning how to solve worded problems.</td>
<td>3</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Learning how to do worded problems is mostly a matter of memorising the right steps to follow.</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Time used to work out why a solution to a maths problem works is time well spent.</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>A person who doesn’t understand why an answer to a maths problem is correct hasn’t really solved the problem.</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>It’s not important to understand why a set of steps works as long as it gives the correct answer.</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Getting a right answer in maths is more important than understanding why the answer works.</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>It doesn’t really matter if you understand a maths problem if you get the right answer.</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>The most satisfying thing for me in maths is trying to understand maths concepts as thoroughly as possible.</td>
<td>4</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Question</td>
<td>Pippa</td>
<td>Lilly</td>
<td>Aaron</td>
</tr>
<tr>
<td>-------------------------------------------------------------------------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>In my maths class, I prefer questions that really challenge me so I can</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>learn new things.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I like work that I’ll learn from even if I make lots of mistakes.</td>
<td>2</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>I like class work best when it really makes me think.</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>An important reason why I do my work is because I like to learn new</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>things.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>It’s important to me that I thoroughly understand my work.</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>I want to do better than other students in my class.</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Doing better than other students in class is important to me.</td>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Getting a good mark in maths is the most satisfying thing for me right</td>
<td>5</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>now.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>If I can, I want to get a better mark for maths than most of the other</td>
<td>5</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>students in my class.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>It’s important to me that I don’t look stupid in class.</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>I would avoid participating in class if it meant that other students</td>
<td>3</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>would think I know a lot.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>To avoid looking smart in front of others, I avoid answering the</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>teacher’s question in front of the class.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I try to avoid looking like I am having trouble doing my work in class.</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>I would prefer to do problems that are familiar to me, rather than</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>problems I would have to learn how to do.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I don’t like to learn a lot of new concepts in class.</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>I prefer to answer questions using familiar working out, rather than</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>trying something new.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I like maths concepts that are familiar to me, rather than those I</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>haven’t thought about before.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I would choose questions I knew I could do, rather than questions I</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>haven’t done before.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question</td>
<td>Pippa</td>
<td>Lilly</td>
<td>Aaron</td>
</tr>
<tr>
<td>-------------------------------------------------------------------------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>When I study for maths, I practice the same types of questions over and</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>over.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>When studying for maths, I memorise equations and steps.</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>I try to relate ideas in maths to those in other subjects whenever</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>possible.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>While in class, I try to relate the ideas I am learning to what I already</td>
<td>2</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>know.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>When studying for maths, I often try to explain ideas to another friend.</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>I try to work with other students from my class to complete maths</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>problems.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Even if I have trouble learning in this class, I try to do the work on</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>my own, without help from anyone.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I ask my teacher to help clarify concepts I don’t understand well.</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>When I can’t understand something in maths, I ask another student in</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>my class for help.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I try to identify students in my class whom I can ask for help if</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>necessary.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix 2  Ferris wheel activity
Below is the graph of \( y = \sin(x) \)

By drawing a unit circle, explain why

a) \( \sin(0) \), \( \sin(\pi) \), and \( \sin(2\pi) \) all equal 0. 

b) \( \sin\left(\frac{\pi}{2}\right) \) equals 1.

c) \( \sin\left(\frac{3\pi}{2}\right) \) equals -1.

Using your knowledge of transformations, sketch the graph of \( y = 1.5 \sin(2x) \).

Below is the graph of \( y = \cos(x) \)
By drawing a unit circle, explain why

a) \( \cos(0) \) and \( \cos(2\pi) \) equals 1.

b) \( \cos(\pi) \) equals -1.

c) \( \cos\left(\frac{\pi}{2}\right) \) and \( \cos\left(\frac{3\pi}{2}\right) \) equals 0.

Using your knowledge of transformations, sketch the graph of \( y = \cos\left(\frac{1}{2}(x + \frac{\pi}{2})\right) \).
Appendix 3 Velocity–graphs task
FERRIS WHEELS

**Diameter:** 40 m

**Axle Height:** 30 m

**Platform:** Bottom

**One Lap:** 4 minutes

---

**Diameter:** 40 m

**Axle Height:** 60 m

**Platform:** Bottom

**One Lap:** 4 minutes

---

**Diameter:** 30 m

**Axle Height:** 40 m

**Platform:** Middle

**One Lap:** 3 minutes

---

**Diameter:** 40 m

**Axle Height:** 30 m

**Platform:** Middle

**One Lap:** 4 minutes
**Diameter**
- 40 m

**Axle height**
- 30 m

**Platform**
- Top

**One lap**
- 4 minutes

**Diameter**
- 30 m

**Axle height**
- 40 m

**Platform**
- Bottom

**One lap**
- 2 minutes

**Diameter**
- 60 m

**Axle height**
- 40 m

**Platform**
- Top

**One lap**
- 4 minutes

**Platform**
- Middle

**One lap**
- 3 minutes
### Ferris Wheel Equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h(t) = 30 - 20 \sin\left(\frac{\pi}{2}(t - 1)\right)$</td>
<td>$h(t) = 30 + 20 \sin\left(\frac{\pi}{2}t\right)$</td>
</tr>
<tr>
<td>$h(t) = 30 + 20 \cos\left(\frac{\pi}{2}t\right)$</td>
<td>$h(t) = 60 + 20 \sin\left(\frac{\pi}{2}(t - 1)\right)$</td>
</tr>
<tr>
<td>$h(t) = 30 - 20 \sin\left(\frac{2\pi}{3}t\right)$</td>
<td>$h(t) = 40 + 15 \sin\left(\frac{2\pi}{3}t\right)$</td>
</tr>
<tr>
<td>$h(t) = 40 + 30 \sin\left(\frac{\pi}{2}(t + 1)\right)$</td>
<td>$h(t) = 40 - 30 \sin\left(\frac{\pi}{2}t - \frac{\pi}{2}\right)$</td>
</tr>
<tr>
<td>$h(t) = 40 - 15 \cos(\pi t)$</td>
<td>$h(t) = 30 + 20 \sin\left(\frac{\pi}{2}(t - 1)\right)$</td>
</tr>
<tr>
<td>$h(t) = 30 - 20 \cos\left(\frac{\pi}{2}t\right)$</td>
<td></td>
</tr>
</tbody>
</table>
FERRIS WHEEL GRAPHS

- Ferris wheel graphs showing height over time for different scenarios.
Appendix 4  Velocity–graphs activity
For the following pairs of graphs, the position-velocity graph is given. Sketch the velocity-time graph.
For the following pairs of graphs, the velocity-time graph is given. Sketch the position-time graph.
Appendix 5   Largest Triangle Problem
### Maximum Area of a Triangle

<table>
<thead>
<tr>
<th>Base (cm)</th>
<th>Height (cm)</th>
<th>Area (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Dimensions of A4 paper (in cm)