Topic study group no. 27: learning and cognition in mathematics

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Topic Study Group No. 27: Learning and Cognition in Mathematics

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The Programme

Learning and cognition is a classical and very vital area in research in mathematics education. Different to many other special and related TSGs, such as teaching and learning of algebra, geometry, measurement, statistics, calculus, to mention a few, this TSG has a more general focus.

Originally, research was focused mainly on the cognitive processes taking place in the individual. The past twenty-five to thirty years, however, the research has expanded. Research on learning as well as mathematical cognition are now frequently framed with socio-cultural theories, and closer connections are being made between social and cognitive theories. In addition, influences of materials, classroom contexts, and affective factors such as emotions, beliefs, and attitudes on learning and cognition are foci of interest.

In what follows, we will briefly report on the presentations that took place in the four regular sessions of TSG27. Each session started with an invited plenary speaker, after which a number of papers were presented.

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Session 1

The main focus for Session 1 was on the development of mathematical cognition in individual learners. The focus was strongly, but not exclusively, on cognitive aspects of learning and cognition in mathematics.

Erno Lehtinen (University of Turku, Finland) was the invited keynote speaker. He focused on the development and extensions of number concept before formal schooling and during primary school. He showed how individual level cognitive analysis helps in understanding learning trajectories, crucial changes, and different processes leading to deep and flexible understanding of number systems in some individuals, and superficial and inflexible number knowledge in others. Drawing on research conducted in his group, he described inter-individual differences in children's spontaneous quantitative focusing tendencies, using them to partially explain differences in how children learn to understand natural numbers that prepare students for conceptual changes needed in learning rational numbers. Educational consequences of these findings were also discussed.

The two subsequent papers focused on a sub-aspect of this talk: students' rational number understanding and difficulties experienced in the development of this understanding.

Jo Van Hoof (University of Leuven, Belgium) focused on natural number bias, a tendency in learners to apply natural number properties to rational numbers even when not applicable. She drew attention to three natural number properties that were inappropriately applied in rational number tasks: density, size, and operations. A test constructed to characterize the development of 4th to 12th graders' natural number bias was administered to 1343 elementary and secondary school students. Results showed an overall natural number bias weakest in size tasks, and strongest by far in density tasks. An overall decrease of the natural number bias with grade was found. Educational implications were provided.

David Maximiliano Gomez (Universidad de Chile, Chile) examined learners' understanding of fraction magnitudes. His research extended beyond learners' understanding of fraction magnitudes interpreted through natural number bias to the idea that several qualitative differences may exist among learners regarding their fraction magnitude understanding. The biases possessed and strategies employed by a large group of middle school children in a computerized fraction comparison task were studied. Overall findings suggested the presence of a strong bias for mistaking component magnitude for fraction magnitude. A clustering analysis revealed the coexistence of at least five distinct manners of reasoning. The findings hold promise not only for research purposes, but also as a contribution to teaching practices because of their potential to expose common mistake patterns.

Session 2

In this session, the scope of mathematical cognition was broadened to a situated process distributed over individuals and objects.

Dor Abrahamson (University of Berkeley, USA), the invited keynote speaker, focused on the conceptualization of the mind as embodied, extended, and enactive activity in natural and sociocultural ecologies. He showed how students' immersive hands-on dynamical experiences become formulated within semiotic registers typical of mathematical discourse. He presented analyses of integrated videography, action logging, and eye-tracking data from tutor–student clinical interviews using a technologically enabled embodied-interaction learning environment, the Mathematical Imagery Trainer for Proportion, to describe the emergence of mathematical concepts from the guided discovery of sensorimotor schemes. A central notion was the theoretical construct of attentional anchor. Abrahamson reconciled constructivist and sociocultural models by underscoring the role of artifacts and facilitation in the micro-events of mathematical ontogenesis.

Anke Lindmeier's (University of Kiel, Germany) presentation focused on structured representations for whole numbers. Such representations which have a long tradition in mathematics education are used to foster the formation of mental models and according specific strategies, so that fast, accurate, and flexible solutions for whole numbers tasks can be retrieved. However, evidence of how children actually use these strategies is rare. An eye-tracking experiment explored possibilities to assess strategies when working with structured representations. By comparing the strategies of first-graders and mathematics proficient adults in a basic whole number problem, and the strategies in different representations, she showed characteristic affordances of structurally equivalent representations. The study informs the further use of structured whole number representations and the potential of eye-tracking to infer students' cognitive processes when working with these representations.

Tine Degrande's (University of Leuven, Belgium) studied the extent to which children spontaneously focus on quantitative relations (SFOR), and the nature of this quantitative focus (types of quantitative relations that make up SFOR). Three different variants of a SFOR-task (multiplicative, additive or open task) were offered to second, fourth and sixth graders. Although most children focused on quantitative relations in the task, they focused not only on multiplicative relations but also on additive relations. SFOR. Multiplicative SFOR was found to increase with age whereas the evolution of additive SFOR depended on the task variant. The open SFOR-task was found to be best suited to capture SFOR. These results suggest further research on SFOR requires a broader conceptualization of quantitative relations than only multiplicative conceptualization.

Session 3

This session extends the focus to personal and environmental factors that influence students' learning.

Judy Anderson (University of Sydney, Australia), the invited keynote, and her colleagues captured the essence of various aspects of their multifaceted project. This longitudinal study of over 4000 students (and their teachers) from classrooms in 47 schools employed both quantitative and qualitative research methods including student interviews, teacher interviews, and observations of classrooms with high levels of student engagement. Constructs studied included student motivation and engagement, attitudes, the classroom environment, student achievement, and their 'switching on' and 'switching off' behavior. It was found that: (a) compared with Grade 6 students, those in Grades 7 and 8 significantly declined in mathematics engagement; (b) with regard to future intent, mathematics self-efficacy, valuing, enjoyment, perceived classroom enjoyment, and parent interest were significant predictors; and (c) additional predictors associated with disengagement were mathematics anxiety, perceived classroom disengagement, school ethnic composition, and school socioeconomic status.

The other two papers in this session examined ways in which students process learning.

Miguel Figueirado (Universidad de Lisboa, Portugal) utilised a questionnaire with Year 10 students to study components that build learning styles for mathematics. Her findings confirmed two learning styles previously identified in the literature: meaning-oriented and reproduction-oriented learning. It was found that meaning-oriented learners tended to be more successful than reproductive-oriented learners and more aware of their learning results.

Bishnu Khanal (Tribhuvan University, Nepal) found that students studying mathematics in secondary school in Nepal had difficulties in understanding investigating, and generalizing mathematical situations. Both quantitative and qualitative analyses were employed. The study raises questions about whether these difficulties are due to the ineffectiveness of the learning strategies students employed.

Session 4

The final session focuses on social and contextual influences on learning.

Minoru Ohtani (Kanazawa University, Japan), invited keynote speaker, employed a task designed to engage adolescents which conceived numerical tables, algebraic expressions, and graphs as traces or shadows of a function, and anthropomorphized this function as a Japanese "Ninja" with these representations as shadows of the stealthy and invisible "Ninja" who gives glimpses of its existence. They found students grasped properties of particular functions from bits and pieces

of the shadows thus enabling the reification of function as a mathematical object. This activity, designed collaboratively by the researchers and the teacher provided opportunity for dynamic and interactive representations to direct students' attention to features of changes of variables and succeeded in promoting discussions whose main topics were those features of function. Implications for teaching and learning are significant.

P Janelle McFeetors (University of Alberta, Canada) used a constructivist grounded theory study of Grade 12 students to study how they actively shaped their learning processes through the way they approached homework and study for tests. In particular she examined whether students can, through the process of learning to learn mathematics, bring into view how they learn mathematics. She found that students were authoring processes for learning, authoring mathematical ideas, and self-authoring as they began to see themselves as capable mathematical learners (where 'authoring' was conceptualised as a generative activity of making meaning of experiences).

Gaye Williams (Deakin University, Australia) used lesson video and video-stimulated interviews to examine the activity of a group of three Year 5/6 students undertaking an unfamiliar mathematical problem solving task in class. She found that neither a peer-tutoring model nor a collaborative development of new knowledge model fitted the learning that occurred. Instead, 'non-expert others' who were processing new ideas more slowly 'opened out' a new Zone of Proximal Development for a student more expert in relation to the mathematics emerging from the task. This study raises questions about influences of different paces of thinking on learning during problem solving.

Summing Up

These TSG27 Learning and Cognition sessions were well attended as were the other presentations associated with this TSG. The questions asked by participants, and the rich discussions that followed paper presentations enriched these sessions further.