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Duration and Multidimensionality in Poverty Measurement

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ABSTRACT

This paper unites two strands of the literature on subgroup decomposable poverty measurement originating from Foster, Greer and Thorbecke (1984) by incorporating information on both multiple dimensions and multiple periods. This generalises the Alkire and Foster (2011a) measure into a dynamic setting. In doing so, it introduces two variants of the ‘transfer’ axiom: one that gives increasing weight to individuals whose deprivations are concentrated as repeated dimensions in a specific period (what we term ‘breadth’) versus one that gives increasing weight to individuals whose deprivations are concentrated as repeated periods in a specific dimension (‘length’). The measure is able to differentiate between both aspects of poverty and consequently allows the assignment of different weights to each aspect. This makes it well suited to make comparison across subgroups when individual longitudinal data is available. We apply the proposed measure to longitudinal data from China where we compare differences in the estimate of poverty relative to existing measures.

Keywords: Multidimensional Poverty; Duration of Poverty; Transfer Axiom; Subgroup Decomposability.

JEL classification: I31, I32
1. Introduction

Traditional measures of poverty based on income and expenditure have been extended along two major directions: the broadening of the measures to incorporate a wider set of dimensions that together give a more accurate representation of welfare; and the deepening of the measures to incorporate information that spans over several periods of observations.

Extensions along the first direction, largely influenced by the writings of Sen (1985), move away from unidimensional measures and into a multidimensional approach based on the individual’s lack of access to a wide set of dimensions that include both market and nonmarket goods. Sen (1976)’s pioneering contribution introduced the axiomatic approach to the measurement of poverty and provided the basis for the recent axiomatic approach to the multidimensional measurement of poverty – examples include Tsui (2002), Bourguignon and Chakravarty (2003), and Alkire and Foster (2011a) [henceforth AF].

Extensions along the second direction move away from static measures into a dynamic approach where repeated observations (or ‘spells’) of poverty are treated differently to cases where poverty is temporary. In this strand of the literature, the issue of the duration of the spell of poverty endured by an individual or household is considered important in the measurement and analysis of poverty. There is now increasing realisation that long, uninterrupted spells of poverty may lead to social exclusion from which recovery may be very difficult; see, for example, Walker (1995). Examples of extensions based on such a view include Foster (2009), Calvo and Dercon (2009), Hojman and Kast (2009), Duclos et.al. (2010), Hoy and Zheng (2011), Bossert et al (2012), and Gradin et al (2012).

Despite the usefulness provided by extensions along both directions, the literature has largely considered both extensions independent of each other, retaining either the unidimensional or static property of traditional measures. This paper aims to provide a generalised framework for measuring poverty that jointly incorporates both aspects – multidimensionality and the duration of deprivation – with a particular emphasis on the information gained by considering the joint distribution of both aspects. The highlight of such a measure is its ability to take advantage of panel data when making poverty comparisons between different countries or subgroups of a population. With the increasing availability of panel data in both developing and developed countries, the proposed measure can be usefully applied in a variety of contexts as our illustrative application on panel data from China shows.

See, also, Chakravarty and D’Ambrosio (2006), Bossert et al (2007), and Jayaraj and Subramanian (2010) for closely related work on the measurement of multidimensional deprivation.
Recently, there have been attempts to unite both strands of the literature: Nicholas and Ray (2012) [NR], Bossert, Ceriani, Chakravarty and D’Ambrosio (2012) [BCCD] and Alkire, Apablaza, Chakravarty and Yalonetzky (2013) [AACY] consider a class of subgroup decomposable poverty measures based on the ‘count’ of an individual’s deprivations (Atkinson, 2003). The unique contribution of these papers is to define each deprivation as belonging to a particular period of time and a particular dimension – an individual’s poverty score is then simply a function of the double-sum of these deprivations over time and over dimensions. However, such a method leads to ‘duration-dimension path-independence’, meaning that the measure is invariant to whether the deprivations were summed first over periods of time or over dimensions. A consequence of this is that the measure is unable to make two types of distinctions.

Firstly, it cannot differentiate between individuals for whom deprivation is concentrated in particular dimensions or time periods versus individuals for whom deprivations are uncorrelated neither across time nor dimensions. Consider a simple example, where an individual can be deprived in up to three dimensions, and a maximum of three periods for each dimension. In terms of counts, this means an individual can experience a maximum of nine deprivations. Let us then compare an individual deprived for all three periods in the first dimension, versus an individual deprived for one period in the first dimension, one period in the second dimension and one period in the third dimension. While both individuals share the same count of deprivations (three), the distribution of the deprivations differ, and indeed, we argue that the first individual should count as more deprived. From the multidimensional perspective, this is consistent with the idea that at any given time, “the consequences for quality of life of having multiple disadvantages [across different domains] far exceed the sum of their individual effects” (Stiglitz et al, 2009).^{2} From the duration perspective, this is consistent with the implementation in Gradin et al (2012) and Hoy and Zheng (2011) where there is an underlying belief that recurring deprivations incur an increasing cost on the individual (for example, this is notably the case for unemployment – see Sengupta, 2009).

Secondly, duration-dimension path-independence also implies that the measure cannot differentiate individuals for whom we observe deprivations over multiple dimensions for specific periods of time (what we term the ‘breadth’ component) from individuals for whom we observe deprivations in repeated periods of time for specific dimensions (what we term the ‘length’ component). This distinction is important in allowing the policy-maker a choice over how important deprivation in

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^{2} The measures in Bourguignon and Chakravarty (2003), Chakravarty and D’ambrosio (2006), Jayaraj and Subramaniam (2010) allow for this possibility in the static multidimensional case. In their conclusion AF also discuss how their measure can be extended to include increased (or decreased) sensitivity for individuals with more deprivations. Datt (2013) explores this extension and discusses its merits.
repeated dimensions is relative to deprivation in repeated time periods. There may be a case to believe, for example, that an additional period of deprivation is more costly than an additional dimension of deprivation, or vice versa. This is closely linked with the issue of unbalanced panels; for example, due to data availability, one may have data with 12 dimensions of deprivation but with only 4 periods of observations for each dimension. Using the models adopted in NR, BCCD or AACY will result in implicitly assuming that being deprived in all 12 dimensions for 1 period is equivalent to being deprived for all 4 periods in 3 dimensions.

The principal motivation of this paper is to propose a poverty measure and, more generally, a framework that incorporates the importance of both the distinctions discussed above. We do so by introducing new variants of the transfer axiom, specifically as they relate to the breadth and length components of deprivation. In incorporating these additional properties, we retain the subgroup decomposability property from Foster, Greer and Thorbecke (1984) [FGT], the dimensional decomposability property from Alkire and Foster (2011a) and the dynamic decomposability property from Foster (2009). We also introduce decomposability according to breadth and length components, which gives us an indication of the sensitivity of the measure to different assumptions regarding the trade-off between additional periods versus additional dimensions of deprivation. Such decomposability also allows identification of the contribution of duration vis-à-vis multidimensionality to overall poverty.

We apply the proposed dynamic multidimensional poverty measure to China. The absence of information on a panel of households in developing countries for a sufficiently long time period containing information on a reasonably wide set of dimensions has made applications of dynamic multidimensional poverty measures to developing countries quite limited. Such panel data sets are rare even in the context of developed countries and, until recently, almost non-existent in the case of developing countries. For example, Mishra and Ray (2012) have recently compared multidimensional deprivation in the static framework between China and India, but their study was not on panel data and, consequently, was unable to incorporate any of the dynamic elements of the present study.

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3 The transfer axiom originated with Pigou (1912), Dalton (1920), with a modern characterisation in Sen (1976).
4 As BCCD report, the problem of missing information at the household level on material deprivation in several individual dimensions is present in the data sets of developed countries as well. This forces one to adopt either the unsatisfactory practice of treating missing information as the household having access to the dimensions concerned as done in BCCD, or simply not including such households as done in the present study.
5 There is now increasing availability of such panel data in developing countries. Besides the CHNS data set from China that has been used here, there is the IFLS data set from Indonesia used in AF, and household surveys conducted by the Research Centre on Rural Economy (RCRE) in Beijing used in Duclos, et al (2010).
The rest of the paper is organised as follows. The proposed dynamic multidimensional poverty measure is introduced in Section 2 along with a discussion of its principal properties. The data sets are described in Section 3 along with some summary features of the data that are relevant for this study. The empirical results are presented in Section 4 while Section 5 concludes the paper.

2. Analytical Framework

2.1. Notation

Assume we observe, for $N$ individuals in a population of interest, $J$ different dimensions of deprivation and $T$ equally-spaced periods of time. We say that an individual $n$ is deprived in dimension $j$ at time $t$ when $x_{njt} < F_j$, where $n \in \{1,2,\ldots,N\}, j \in \{1,2,\ldots,J\}, t \in \{1,2,\ldots,T\}$, $x_{njt}$ is individual $n$'s achievement in dimension $j$ at time $t$, and $F_j$ is a cut-off point that determines whether or not an individual is considered deprived in a particular dimension at a particular time. For example, in the dimension 'health', $x$ may be the individual’s Body Mass Index, in which case $F_{health}$ would be some threshold below which the individual would be considered underweight and therefore deprived in the health dimension.

Each individual $n$ can be said to have an individual deprivation profile, which is a matrix $D_n = \begin{pmatrix} d_{n11} & \cdots & d_{n1T} \\ \vdots & \ddots & \vdots \\ d_{nJ1} & \cdots & d_{nJT} \end{pmatrix}$ where $d_{njt}^\alpha = \begin{cases} (1 - \frac{x_{njt}}{F_j})^\alpha & \text{if } x_{njt} < F_j \ \forall j \in \{1,2,\ldots,J\} \ \& \ \forall t \in \{1,2,\ldots,T\} \\ 0 & \text{otherwise} \end{cases}$

$\alpha \geq 0$ is a sensitivity parameter along the lines of the poverty measure in FGT. Call $d_{njt}^\alpha$ deprivation inputs. When observed achievement levels are discrete or ordinal in at least one dimension, it is common to restrict $\alpha = 0$ such that $d_{njt}^\alpha \in \{0,1\}$.

The population deprivation profile is a vector $\rho = (D_1,\ldots,D_N)$.

Define the identification vector $v = (C_1,\ldots,C_N)$ where $C_n$ takes the value 1 if the individual is considered poor, and 0 otherwise. An individual is considered poor if he has at least $z$ count of deprivations; this can be based on a minimum number of periods, or dimensions, or a combination of both.

The poverty index is a function $g: (\rho, v) \rightarrow \mathcal{R}_+$. The union method of identification would set $z = 1$ while the intersection method would set $z = (J \ast T)$. Clearly the choice of who to consider poor will affect the final measure of poverty. However, the contribution of our proposed measure is the expansion of ways in which to think about
the depth of poverty among the poor, rather than whom to consider poor. We therefore restrict our attention to the union method of identification but define our axioms consistent with any choice of $z$.

### 2.2. The breadth versus length of deprivation

Unlike the simpler unidimensional case in AF, each deprivation input $d^z_{njt}$ in the dynamic multidimensional case contributes to both the ‘breadth’ of deprivation (number of dimensions deprived at a particular time $t$) as well as the ‘length’ of deprivation (number of periods deprived in a particular dimension $j$). In dynamic multidimensional applications such as NR, BCCD and AACY, an individual’s deprivation score is simply the sum of deprivation inputs regardless of which dimension or period they belong to. Therefore, the poverty index is insensitive to permutations of the individual’s deprivation profile. Consider the example below of three individuals with different deprivation profiles with $T = 3, J = 3$ and $\alpha = 0$ where the rows represent the dimensions and the columns, the periods.

$$D_A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad D_B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad D_C = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Assuming all three individuals are poor, current dynamic multidimensional measures would consider all three individuals as equally deprived. We first argue that in most applications, $D_B$ and $D_C$ are more deprived than $D_A$. To make the question grounded in application, let us say that the periods (columns) are 1999, 2001 and 2003. The dimensions of deprivation are access to clean water, access to toilets and access to clinics. Notice that none of the three individuals have access to clean water in 1999. Imagine the government had the option of providing clean water access in 1999 to one of these individuals – who should it have been? We can develop a simple rule based on the following question: who has had the least access to clean water over all three periods? Answer: Person B. But this question ignores the fact that in 1999 person B at least had access to toilets and clinics. So we can ask a second question: who has had access to the least dimensions in 1999? Person C. Person A clearly fall out of the scope of these two considerations and is therefore unambiguously the least likely to be considered for access to water. The answer of whether person B or C should gain access to water is ultimately something to be decided by the policy maker; for example in the case of a permanent solution such as the construction of a well or water-pump, the case for Person B may be stronger since it would remove his deprivation not only in 1999 but for all the years. Clearly there is

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6 Interested readers may refer to AACY where they consider the definition of the poverty cut-off in two stages: firstly, an individual is poor in a particular period if they are deprived in $v$ dimensions, and an individual is chronically poor if they are poor for $\tau$ periods (as introduced in Foster, 2009). Poverty is then calculated over the set of chronically poor. AACY also generalise the cut-offs to allow different weights across dimensions.
no hard and fast rule of whether $D_B$ or $D_C$ is to be considered more deprived; however, it is this context-dependence that highlights the need for a measure that offers the analyst flexibility in deciding which one to weight more heavily. In the next section we highlight in detail how our measure allows the analyst to vary the weight given to individual B versus C.

Question 1 can simply be stated as increased sensitivity towards additional periods of deprivation given any particular dimension. Question 2 can equally be stated as increased sensitivity towards additional dimensions of deprivation given any particular period. As mentioned in the introduction, both extensions have been considered independently in the dynamic unidimensional case (question 1) and static multidimensional case (question 2). These properties can be seen as broadly analogous to FGT’s transfer axiom which requires that individuals deeper in poverty be allotted increasing weights. The key difference is that while in the case of FGT, depth of poverty is defined purely in terms of the achievement gap (i.e. income gap), here depth of poverty is further supplemented with information on multidimensionality and duration. It would therefore seem inconsistent to require that the transfer axiom apply to only differences in the achievement gap but not to other aspects of the extent of individual poverty.

We now state formally the two properties associated with the two questions by firstly defining two types of permutations of the deprivation profile.

Define $D_{nj}$ as the row vector of individual $n$’s deprivation profile for dimension $j$. A **dynamic permutation** occurs if there is a rearrangement of any two deprivation inputs $d_{nj|t=t_{tr}}^\alpha$ and $d_{nj|t=t_{tr}}^\beta$ along $D_{nj}$. Furthermore, $d_{nj|t=t_{tr}}^\alpha > d_{nj|t=t_{tr}}^\beta$, prior to rearrangement and $\sum d_{nj|t=t_{tr}}^\alpha$ remains the same after rearrangement for all $j$.

Define $D_{nt}$ as the column vector of individual $n$’s deprivation profile for period $t$. A **dimensional permutation** occurs if there is a rearrangement of any two deprivation inputs $d_{nj|t=j_{tr}}^\alpha$ and $d_{nj|t=j_{tr}}^\beta$ along $D_{nt}$. Furthermore, $d_{nj|t=j_{tr}}^\alpha > d_{nj|t=j_{tr}}^\beta$, prior to rearrangement and $\sum d_{nj|t=j_{tr}}^\alpha$ remains the same after rearrangement for all $t$.

**{(Axiom 1): Dimensional Transfer}**

$g(\rho, \nu) > g(\rho', \nu)$ if $\rho'$ is obtained from $\rho$ by a dynamic permutation of individual $n$’s deprivation profile in dimension $j = j'$ and $\sum (j-1) d_{nj|t=t_{tr}}^\alpha < \sum (j-1) d_{nj|t=t_{tr}}^\beta$, where $k \neq j'$ and individual $n$ is poor.
The *Dimensional Transfer* axiom requires that the poverty measure register an increase if deprivation was transferred to a dimension where there are more periods of deprivation.

**(Axiom 2): Dynamic Transfer**

\[ g(\mathbf{\rho}, \mathbf{v}) > g(\mathbf{\rho}', \mathbf{v}) \] if \( \mathbf{\rho}' \) is obtained from \( \mathbf{\rho} \) by a dimensional permutation of individual \( n \)'s deprivation profile at period \( t = t' \) and \( \sum_{m}^{(T-1)} d_{njt|j=j,t}^{m} < \sum_{m}^{(T-1)} d_{njt|j=j,t}^{m} \) where \( m \neq t' \) and individual \( n \) is poor.

The *Dynamic Transfer* axiom requires that the measure register an increase if deprivation was transferred to a period where there are more dimensions of deprivation.\(^7\)

From our three individual example, deprivation profile \( \mathbf{D}_B \) can be constructed from \( \mathbf{D}_A \) by performing two dimensional permutations (on \( t = 2 \) and \( t = 3 \) respectively). Similarly, deprivation profile \( \mathbf{D}_B \) can be constructed from \( \mathbf{D}_A \) by performing two dynamic permutations (on \( j = 2 \) and \( j = 3 \) respectively). Therefore *Dimensional Transfer* implies that \( \mathbf{D}_B \) is more deprived than \( \mathbf{D}_A \) while *Dynamic Transfer* implies that \( \mathbf{D}_C \) is more deprived than \( \mathbf{D}_A \).

### 2.3. The Proposed Dynamic Measure of Multidimensional Poverty

Consider the following functional form for \( g(\mathbf{\rho}, \mathbf{v}) \):

\[
\Omega_{\alpha\beta} = \left[ \frac{\sum_{n}^{N} \left( \left( \frac{\sum_{i}^{T} \sum_{j}^{J} \alpha^{\alpha} d_{njt}^{\alpha}}{s_{njt}} \right) \cdot c_{n} \right)}{N} \right]
\]

(1a)

where \( \alpha, \beta \geq 0; c_{n} = \begin{cases} 1 & \text{if } \sum_{i}^{T} \sum_{j}^{J} d_{njt}^{\alpha=0} \geq z \\ 0 & \text{otherwise} \end{cases} \)

Like NR and BCCD, the measure continues to be a double-sum across time and dimensions, therefore preserving a form of duration-dimensional path-independence (and consequently, decomposition according to time and dimensions). However, each deprivation input is now weighted by \( s_{njt}^{\alpha\beta} \), which we now define.

\[
s_{njt}^{\alpha\beta} = \beta \left( \frac{s_{njt}^{\alpha}}{t} \right) + (1 - \beta) \left( \frac{s_{njt}^{\alpha}}{t} \right)
\]

(1b)

\[ 1 \geq \beta \geq 0 \]

\(^7\) Unlike AF, these two axioms are defined over deprivation inputs rather than achievements. This distinction allows the axioms to be satisfied even when \( \alpha = 0 \) (which is common in applications using counting-based measures). When \( \alpha = 0 \), changes in achievements have no effect on deprivation inputs unless they bring the individual above/below a deprivation threshold. Also, unlike AF, the transfer is occurring within, rather than across individuals. Therefore while the transfer axiom generally represents a social preference for an equal distribution of achievements across individuals, the variants defined here represents a social preference for an equal distribution of deprivations across dimensions and across time within each individual.
$S_{njt}^{\alpha\beta}$ endogenously weights each deprivation input according to deprivations in all the dimensions associated with that period [first term of (1b)] and according to deprivations in all the periods associated with that dimension [second term of (1b)]. This would therefore give more weight to individuals whose deprivations are located along the same period/dimension relative to those whose deprivations are unrelated by time or dimensions.

The exogenously chosen parameter $\beta$ allows the analyst to assign more weight to either the breadth or length aspects. By setting $1 > \beta > 0$ both dimensional and dynamic transfer are satisfied. When $\beta = 0$ only dynamic transfer (sensitivity to length) is satisfied and when $\beta = 1$ only dimensional transfer (sensitivity to breadth) is satisfied (proof in Appendix A).

**Figure 1:** Generalisation-tree of the class of subgroup decomposable poverty measures

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A: Note that the strands of literature referred to in the figure do not necessarily represent the origin of the extensions; for example, multidimensional extensions had appeared as early as Bourguignon and Chakravarty (2003). We simply refer to measures that are direct special cases of $\Omega_{\alpha\beta}$. 

$$1 \geq S_{njt}^{\alpha\beta} \geq 0$$
Consistent with the class of subgroup decomposable measures, $\Omega_{a\beta}$ has a convenient interpretation as the average poverty score for individuals in the population of interest. NR can be seen as a special case of $\Omega_{a\beta}$ when $S_{njt}^{a\beta} = 1$ and $z = 1$. AF can be seen as a special case of NR when $T = 1$ and while Foster (2009) can be seen as a special case of NR when $J = 1$. Figure 1 presents a summary of this discussion with the arrow pointing to measures of greater generality.

2.4: Trading off Breadth versus Length

We now return to the original question in our three-individual example of whether $D_B$ is to be considered more or less deprived than $D_C$. This boils down to deciding whether being deprived in additional dimensions is to be weighted more heavily relative to being deprived in additional periods which, in our measure, translates into a choice of $\beta$. A priori, it seems reasonable to set $\beta = 0.5$, thereby giving equal weight to both breadth and length aspects of deprivation. However given that equal weighting is desirable, it turns out that setting $\beta = 0.5$ is only appropriate when $J = T$. When $J \neq T$, for any given deprivation input $d_{njt}^a$, an additional period of deprivation along the same dimension will affect the weighting of $d_{njt}^a$ differently to an additional period of deprivation along the same period. More specifically, an additional period of deprivation along the same dimension increases the weight to $d_{njt}^a$ by the factor $1/T$ while an additional dimension of deprivation along the same period increases the weight to $d_{njt}^a$ by the factor $1/J$. Therefore, when $J < T$, every additional dimension has a larger impact on the weights than additional periods, and vice versa. This is undesirable since the number of dimensions and periods are often dictated by data availability rather than any theoretical prior.

One may be tempted to perceive this is a limitation introduced in the use of weights that satisfy dimensional and dynamic transfer. While it does imply that the weights have to be adjusted according to $J$ and $T$, in the simpler case in NR and other studies where the weights are effectively set to 1, there is no option to increase the weight of one component relative to the other. Therefore, for the added cost of having to specify a different $\beta$ for each application, we gain both dimensional and dynamic transfer, as well as flexibility with regards to the weight of the breadth relative to the length components of poverty.

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8 To be precise, $\Omega_{a\beta}$ is only a generalisation of NR if we ignore NR’s $\delta$ parameter. $\delta$ is assumed to be equal to 1 in $\Omega_{a\beta}$ since the dimensional and dynamic transfer properties are more refined than the property gained with $\delta > 1$. We discuss this in detail in Appendix B.
The contribution of an additional dimension of deprivation to a particular weight $S_{njt}^{a\beta}$ at $t = t'$ and $j = j'$ can be represented by $\frac{\Delta S_{njt}^{a\beta}(t = t', j = j')}{\Delta d_{njt}^{a}(t = t', j = j')}$ and correspondingly by $\frac{\Delta S_{njt}^{a\beta}(t = t', j = j')}{\Delta d_{njt}^{a}(t = t', j = j')}$ for an additional period of deprivation. Define $x = \frac{\Delta S_{njt}^{a\beta}(t = t', j = j')}{\Delta d_{njt}^{a}(t = t', j = j')}$. $x$ can be chosen by the analyst and defines the ratio of the sensitivity of any weight $S_{njt}^{a\beta}$ to additional dimensions relative to additional periods. Choosing $x = 2$ for example means an additional dimension of deprivation increases $S_{njt}^{a\beta}$ by a factor twice that of the contribution from an additional period of deprivation and in the example would lead to $D_C$ being classified as being more deprived than $D_B$.

**Proposition 1:** Given $J$ and $T$, for any choice of $x$, $\beta$ must be set equal to $\frac{xJ}{T+xJ}$

**Proof:** Appendix C

From Proposition 1 we can see that when $x = 1$, $\beta$ should be set equal to 0.5 only when $T = J$.

Recall that the main point of dynamic multidimensional measures is to compare the deprivation of different populations, or the subgroups within a given population. Therefore even when the precise choice of $x$ (and hence, $\beta$) is unclear to the analyst, it may be useful to consider whether the group rankings from any of these comparisons change with the value of $\beta$. We can term groups whose rankings do not change with $\beta$ as $\beta$ invariant and avoid the need to worry about the choice of $\beta$ (at least, for ranking purposes).

While the insensitivity of the measure to $\beta$ means we can avoid the question of whether $D_B$ or $D_C$ is more deprived, $\Omega_{a\beta}$ remains useful relative to the simpler case of NR because it still ranks $D_B$ and $D_C$ as more deprived than $D_A$. We explore sensitivity to $\beta$ further in Section 2.6.

2.5. Subgroup decomposability and comparisons across groups

Because $\Omega_{a\beta}$ remains the average of the sum of individual poverty scores, it retains the subgroup decomposability property common to the class of measures originating in FGT.

Define $N_S$ as the number of individuals in a population subgroup $s \in \{1 \ldots S\}$ where $S$ is the total number of subgroups and $\sum_{s=1}^{S} N_S = N$ and define $g(N_S, \rho, \nu)$ as the deprivation index calculated over subgroup $s$.

**(Axiom 3): Subgroup Decomposability**

A poverty measure $g(N, \rho, \nu)$ is subgroup decomposable if $g(N, \rho, \nu) = \sum_{s=1}^{S} \frac{N_S}{N} g(N_S, \rho, \nu)$. 

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The axiom allows deprivation to be decomposed into a subgroup’s proportion of contribution towards aggregate level deprivation. For our measure, define \( g(N_S, \rho, \nu) \) as \( \Omega^s_{a\beta} \). The subgroup’s proportion of contribution can be calculated by:

\[
\Omega \left( \frac{N_S}{N} \right)_{a\beta}^s = \frac{N_S}{N} \cdot \frac{\Omega^s_{a\beta}}{\Omega_{a\beta}}
\]

This means that \( \Omega_{a\beta} \) can effectively produce two subgroup-specific measures of poverty: \( \Omega^s_{a\beta} \) allows the ranking of subgroups according to the highest average poverty score per person while \( \Omega \left( \frac{N_S}{N} \right)_{a\beta}^s \) gives the percentage contribution of each particular subgroup to the overall poverty level. The two measures need not give the same ranking; for example consider a 'subgroup' consisting of only one individual, who is deprived in every dimension for every period. While his \( \Omega^s_{a\beta} \) will be 1 (i.e. the highest possible poverty score), his contribution to overall poverty in the population may be extremely small assuming there are numerous others in the population who are deprived, even if their deprivations are nowhere near the extent of this one individual.

### 2.6. Identifying the contribution of Breadth versus Length

One way to get an idea of the sensitivity of the various subgroup-specific poverty measures, \( \Omega^s_{a\beta} \), to changes in \( \beta \) is to consider the differences in the relative importance of the breadth and length components of poverty across the subgroups. Consider the following formulation:

\[
Bp = \frac{\sum_{i}^{n} \left( \sum_{j}^{i} \sum_{i}^{j} \left( \sum_{i=1}^{j} \frac{d_{i,j}}{T_i} \right) d_{i,j}^a \right)}{\sum_{i}^{n} \left( \sum_{j}^{i} \sum_{i}^{j} \left[ \left( \frac{\sum_{i=1}^{j} d_{i,j}^a}{T_i} \right) + \left( \frac{\sum_{i=1}^{j} d_{i,j}^a}{T_i} \right) \right] \right)}
\]

where \( Bp \) is the proportion of overall poverty attributable to a concentration of multiple dimensions of deprivation in particular periods (i.e., breadth of deprivation) and \( Lp \) is the proportion of overall poverty attributable to a concentration of multiple periods of deprivation in particular dimensions (i.e., length of deprivation). Since \( Bp + Lp = 1 \), for convenience we focus solely on \( Bp \). \( Bp \) can be calculated for the population as a whole, or for each subgroup separately by replace \( N \) with \( N_S \).

When calculated for each subgroup, relative differences in \( Bp \) across subgroups give us an indicator of the relative sensitivity of the subgroup scores \( \Omega^s_{a\beta} \) to changes in \( \beta \). For example, consider two possible subgroups \( s \in \{1, 2\} \). Assume \( Bp_{s=1} > Bp_{s=2} \). This means subgroup 1 has more breadth of deprivation relative to subgroup 2. Increasing \( \beta \) would therefore increase subgroup 1’s poverty score relative to subgroup 2 while decreasing \( \beta \) does the opposite.
Note that a $Bp$ above 0.5 indicates that deprivation is more likely to be concentrated within specific periods of time in the form of multiple dimensions. A $Bp$ below 0.5 indicates that deprivation is more likely to be concentrated within specific dimensions in the form of multiple periods. The $Bp$ can therefore serve as an indicator of whether a policy-maker or researcher should be concentrating their attention on specific periods of time ($Bp > 0.5$) or concentrating their attention on specific dimensions ($Bp < 0.5$).\(^9\)

2.7. Other properties

One of the attractive features of duration-dimension path-independence is the ability to decompose the measure according to the individual contribution of each dimension, or the individual contribution of each time period. While NR and BCCD have dimensional and dynamic decomposability, decomposition across time in these measures is equivalent to calculating a static multidimensional measure (as per AF) for each time period separately. The decomposition therefore only uses information on deprivations associated with the particular period of time. Similarly, decomposition across dimensions is equivalent to calculating a dynamic unidimensional measure (as per Foster, 2009) for each dimension separately. The decomposition therefore only uses information on deprivations associated with the particular dimension. Our measure has the advantage of allowing the decompositions to be sensitive to information outside the period or dimension being decomposed since each deprivation input is transformed to be a function of the entire deprivation profile.

(Axiom 4): Dimensional Decomposability

$$g(d_{njt}^a, \rho, \nu) = \omega_1 g(d_{njt1}^a, \rho, \nu) + \cdots + \omega_j g(d_{njtj}^a, \rho, \nu),$$

where $\omega_j = \frac{1}{j}$ are exogenously imposed dimensional weights, and $g(d_{njtj}^a, \rho, \nu)$ is the deprivation index calculated using only the row vector $D_{njj}$ of each individual’s deprivation profile.

Dimensional decomposability is useful since it allows the policy maker to calculate the percentage contribution of each dimension towards the overall deprivation index.

\(^9\) Note for any individual, the breadth and length aspects of deprivation can only differ when the count of deprivations, $\sum_{j}^{i} \sum_{t}^{T} d_{njt}^a$, is greater than zero and less than $(j * T)$ – i.e., the theoretical maximum number of deprivations. When the count of deprivations is $(j * T)$, the deprivation profile is ‘full’ and therefore both breadth and length aspects are at their respective maxima. For values of $\sum_{j}^{i} \sum_{t}^{T} d_{njt}^a$ that lie between zero and $(j * T)$, the differentiation of breadth and length becomes the most meaningful and the $Bp$ can take on different values depending on the deprivation profile. In applications, the average deprivation count $\sum_{j}^{i} \sum_{t}^{T} d_{njt}^a / n$ can be used as an indicator of the usefulness of distinguishing length from breadth.
(Axiom 5): Dynamic Decomposability

\[ g\left(d^a_{n|t}, \rho, v\right) = \omega_1 g\left(d^a_{n|t|t=1}, \rho, v\right) + \cdots + \omega_T g\left(d^a_{n|t|t=T}, \rho, v\right), \]

where \( \omega_T = \frac{1}{T} \) are exogenously imposed dynamic weights, and \( g\left(d^a_{n|t|t}, \rho, v\right) \) is the deprivation index calculated using only the column vector \( D_{n|t} \) of each individual’s deprivation profile.

Dynamic decomposability allows the policy maker to calculate the percentage contribution of each period towards the overall deprivation index.

\( \Omega_{\alpha\beta} \) also satisfies the following properties, which are based on AF but extended to the dynamic framework. The difference with the following over the axioms in NR is the compatibility with the poverty focus axiom (i.e. allowing the possibility of non-union identification).

(Axiom 6): Replication Invariance

Define a single replication of \( N \) as \( A = (N,N) \); of \( \rho \) as \( B = (\rho, \rho) \); of \( v \) as \( E = (v, v) \).

Replication invariance implies \( g(N, \rho, v) = g(A, B, E) \) for any number of replications of \( N, \rho \) and \( v \) so long as the number of replications are the same for \( N, \rho \) and \( v \).

This ensures that larger populations do not automatically count as more deprived.

(Axiom 7): Symmetry (anonymity)

\[ g(\rho, v) = g(\rho', v) \] where \( \rho' \) is any permutation of the vector \( \rho \).

Therefore, all individuals are identical except for their deprivation profiles.

(Axiom 8): Normalisation and Nontriviality

\( g(\rho, v) \) achieves at least two distinct values: a minimum value of 0 and a maximum value of 1.

(Axiom 9): Poverty Focus

\[ g(\rho, v) = g(\rho', v) \] if \( \rho' \) is obtained from \( \rho \) by having a non-poor individual experience an achievement increase in any deprivation.

Therefore, if an individual is not considered poor, then improvements in that individual’s achievements are irrelevant to the measure, similar to a Rawlsian social welfare function.

(Axiom 10): Deprivation Focus

\[ g(\rho, v) = g(\rho', v) \] if \( \rho' \) is obtained from \( \rho \) by an increase in the achievement in a deprivation where an individual is not considered deprived.
Therefore, if an individual is not deprived in a particular period and dimension, then improvements in that particular deprivation input are irrelevant to the measure.

**Proposition 2:** $\Omega_{a\beta}$ satisfies the following **core properties:** Subgroup Decomposability, Replication Invariance, Symmetry, Nontriviality, Normalisation, Poverty Focus, Deprivation Focus.

**Proof:** Since individual scores are simply summed and averaged over individuals, $\Omega_{a\beta}$ always satisfies Subgroup Decomposability, Replication Invariance, Symmetry, Normalisation and Nontriviality. It also satisfies Poverty Focus based on the cutoff $z$ and Deprivation Focus based on the cutoff $F_j$.

Define a *deprivation decrement* as a decrease in any one of an individual’s deprivation inputs.

**(Axiom 11): Deprivation Monotonicity**

$g(\rho, \nu) > g(\rho', \nu)$ if $\rho'$ is obtained from $\rho$ by a deprivation decrement among the poor.

**Proposition 3:** The poverty index $\Omega_{a\beta}$ satisfies the **core properties** (axioms 3,6,7,8,9 and 10), and in addition, Dimensional Transfer, Dynamic Transfer, Dimensional Decomposability, Dynamic Decomposability, Deprivation Monotonicity when $\alpha \geq 0; 1 > \beta > 0$.

**Proof:** Dynamic Transfer, Dimensional Transfer and Deprivation Monotonicity are proved in Appendix A. Dynamic Decomposability and Dimensional Decomposability are straightforward implications of duration-dimension path-independence.

Further extensions to the measure, including the assignment of different weights to different dimensions, can be found in Appendix D.

3. Data set and summary features

We apply the proposed dynamic measure of multidimensional poverty to a panel data set from China from 1993-2009. The empirical evidence is of particular interest since they cover periods that include the Asian Financial Crisis and the Global Financial Crisis.

The Chinese data came from the China Health and Nutrition Survey (CHNS). This is an ongoing international project between the Carolina Population Center at the University of North Carolina at Chapel Hill and the National Institute of Nutrition and Food Safety at the Chinese Centre for Disease Control and Prevention. This project was designed to examine the effects of health, nutrition and family planning policies and programs implemented by the national and local governments and to see how the social and economic transformation is affecting the health and nutritional status of the
population. A detailed description of the CHNS data base has been presented in Popkin, Du, Zhai and Zhang (2010). The surveys took place over a three day period using a multi-stage, random cluster process to draw a sample of over 4000 households in nine provinces that vary substantially in geography, economic development, public resources and health indicators. We converted household level information to the individual level by assuming that the household’s access to a facility such as drinking water or electricity is the same for all individuals in that household. In a departure from previous applications on CHNS data, we supplemented the individual and household level information with the community level information available in the CHNS community identifiers. The community questionnaire (filled out for each of the primary sampling units) collected information from a knowledgeable respondent on community infrastructure (water, transport, electricity, communications, and so on), services (family planning, health facilities, retail outlets), population, prevailing wages, and related variables. Only individuals aged 18 years and above in the first year of the panel were included in construction of the balanced panel.

The dimensions at the household, individual and community levels considered in this study are described in Appendix E1. Appendix E2 describes the two different samples used for China. Appendix E2 also provides the deprivation cut offs used in the quantitative dimensions (years of education, BMI, BP). While the CHNS data set is longitudinal, there is a trade-off between the length of the time interval and the number of dimensions on which information is available for the panel of individuals. The first sample considers different combinations of time and dimensions through the use of exclusively ‘qualitative’ dimensions where an individual is either deprived or not deprived in a dimension – there is no information on gradations of deprivation. In contrast, sample two contains only dimensions that are quantitative, meaning there is information on the size of the achievement gap. This means that setting $\alpha > 0$ in $d_{ij}^\alpha$ will have an effect on the measure.

For the analysis, we focus initially on Sample 1 given the breadth of dimensions it covers: Appendix E3 presents the summary statistics, year and dimension-wise, of the deprivation rates in China for the panel of individuals in sample one. While some dimensions such as electricity, drink water, radio/TV recorded large improvements over the period, the opposite is true for other dimensions; for example, the proportion of individuals with abnormal blood pressure and the proportion of individuals without access to a vehicle increased over the period.

\[\text{Note that though in case of some of these dimensions, for example, BMI, blood pressure and years of schooling, the quantitative information is available, we converted them into qualitative dimensions for consistency with the others, such as access to toilet, fuel, etc.}\]
4. Results

This section provides an empirical illustration of the use and interpretation of the proposed measure, \( \Omega_{\alpha\beta} \), and its associated properties. As the main use of this class of measures is the comparison of poverty across different population subgroups, in Section 4.1 we present the results according to this decomposition, along with a comparison to NR. In Section 4.2 we consider how sensitive the results are to changes in \( \beta \). We also present results based on dimensional and dynamic decomposition in Section 4.3. Such decompositions allow direct comparisons with special cases of Foster (2009) and AF. In all our calculations we assume \( z = 1 \) (the union approach).

The results presented in Tables 1-4 contain three models each. Model 1 is the base case, where \( S_{njt}^{\alpha\beta} = 1 \). This leads to the measure used in NR, and in the case of dimensional and dynamic decomposition, also leads to particular specifications of Foster (2009) and AF respectively.\(^{11} \) Model 2 allows \( S_{njt}^{\alpha\beta} \) to endogenously vary as defined in equation (1b), therefore satisfying dimensional and dynamic transfer properties. However it also sets \( \beta = 0.5 \). Since \( J > T \) in Sample 1, this implicitly amounts to allotting more weight to repeated periods of deprivation relative to repeated dimensions of deprivation. In Sample 2, \( J < T \) and the reverse is true. To account for this, in Model 3 we set \( \beta \) according to Proposition 1, assuming that \( x = 1 \) (that is, the weights are equally sensitive to additional periods and dimensions).

4.1. Subgroup comparisons: NR and \( \Omega_{\alpha\beta} \)

Table 1 presents the subgroup comparisons for Sample 1.

We define subgroups according to three different groupings: 1) male/female; 2) province; 3) rural/urban. The reported calculations present four numbers: the first is the proportion of overall poverty contributed by each population subgroup (equation 4); the second number is the rank of each subgroup according to this proportion (from the most to least poor); the third is the poverty index calculated for each subgroup, \( \Omega_{\alpha\beta}^{z} \); and lastly, the rank of each subgroup (from the most to least poor) according to \( \Omega_{\alpha\beta}^{z} \). Note that while the proportion calculation is sensitive to the size of the subgroup, the actual score \( \Omega_{\alpha\beta}^{z} \) is not, by virtue of it being an average across individuals (as per the replication invariance axiom). However, the actual score \( \Omega_{\alpha\beta}^{z} \) suffers from not being comparable across models because of changes in the choice parameters. For example in Model 1 we see that the Henan province contributes the least proportion to overall poverty, even though it is has the 3rd highest deprivation score, which is attributable to Henan’s relatively smaller population.

\(^{11} \)In the case of Foster (2009), this is equivalent to a specification where \( \tau = 1/T \); in the case of AF, this is equivalent to a specification where \( \alpha = 0 \) for Sample 1 and \( \alpha = 1 \) for Sample 2.
A comparison of proportion contributions between Model 1 and 2 of Table 1 reveal relatively minor changes in terms of the rural/urban and male/female comparison. However, while the rankings of the provinces according to \( \Omega_{\alpha \beta}^{\text{s}} \) do not change across the two models, the rankings according to proportion contributions do, notably, for the Henan, Hunan and Guangxi provinces. The increased relative poverty level associated with Henan suggests that deprivation in Henan is more likely to be concentrated, whether in particular periods or particular dimensions. These changes highlight that \( \Omega_{\alpha \beta} \) can lead to rankings that are different from NR due to the introduction of the *dimensional* and *dynamic transfer* properties.

Table 2 presents the results from Sample 2, which contains quantitative information on deprivations, and in which we set \( \alpha = 1 \). As in Table 1, we see several provinces continue to switch ranks in terms of proportion contribution as we move from Model 1 to Model 2. Furthermore the Jiangsu and Hubei provinces switch ranks in terms of \( \Omega_{\alpha \beta}^{\text{s}} \) as well. Table 2 also suggests that the use of quantitative dimensions increases the differences in proportion contributions between males and females considerably: the proportion of deprivation attributable to females increases by 8% (to 75%) in Model 2.

### 4.2. Subgroup comparisons: \( \Omega_{\alpha \beta} \) and \( \beta \)

In all the tables, Models 2 and 3 differ by the choice of \( \beta \). Where the former sets \( \beta = 0.5 \), the latter sets it by assuming that \( x = 1 \) and through the use of Proposition 1. For Sample 1 (Table 1), this results in \( \beta \) increasing to 0.722. As highlighted in Section 2.6, the sensitivity of the poverty score to changes in \( \beta \) can be partially gauged by looking at the proportion of deprivation due to the breadth aspect, \( Bp \). Looking at \( Bp \) for the sample as a whole, the first thing we notice is that it is less than 0.5. This value of \( Bp \) can be interpreted thus: 36% of overall poverty in the sample can be attributed to multiple dimensions within specific periods and the remaining 64% can be attributed to repeated periods within specific dimensions. The overall poverty score, \( \Omega_{\alpha \beta} \), should therefore decrease as \( \beta \) increases.

Along the same lines, the sensitivity of the subgroup measure, \( \Omega_{\alpha \beta}^{\text{s}} \), to changes in \( \beta \) can be ascertained by looking at the differences between the \( Bp \) calculated for each subgroup. Looking at the provinces in Table 1, \( Bp \) lies within the range of 34%-38%, therefore suggesting that \( \Omega_{\alpha \beta}^{\text{s}} \) is unlikely to vary wildly with \( \beta \). However, for subgroups whose poverty scores are close, sensitivity to \( \beta \), regardless of how low, may have an impact on poverty rankings. Consider, for example, the Shandong and Henan provinces, who, in Model 2, have scores of 0.2541 and 0.2514 respectively. Because Henan’s \( Bp \) score is higher than Shandong, we can expect Henan’s poverty to increase...
relative to Shandong as $\beta$ is increased. In Model 3 the corresponding scores are 0.2215 and 0.2202. While their ranks have not switched, it is clear that the choice of $\beta$ may have an influence on the final ranking. Figure 2 depicts the poverty score of each province as a ratio of the sum of the scores of all the provinces according to all possible choices of $\beta$ – we see that at values of $\beta$ close to 1, the Henan province is considered to be in deeper poverty than Shandong.

**Figure 2:** Contribution to poverty across different values of $\beta$

The quantitative sample in Table 2 sets $\beta = 0.375$ in Model 3. Therefore while in Table 1, the movement from Model 2 to 3 involved an increase in $\beta$, in Table 2 it instead involves a decrease to $\beta = 0.375$. This is because in the quantitative sample we have three dimensions but six periods. Similar to Table 1, we do not see any change in rankings, though the closeness of the poverty scores of Jiangsu and Shandong suggest that further decreases in $\beta$ may switch their rankings. One interesting thing to note is that in both Tables 1 and 2, the $Bp$ score of Guizhou – consistently the poorest district in both measures and in all models across both tables – is relatively higher than the other provinces. This suggests that a wider breadth of deprivation, i.e. deprivation in multiple

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12 Decreases because the $Bp$ for Shandong is higher than Jiangsu’s, therefore decreasing $\beta$ decreases Shandong’s poverty relative to Jiangsu.
dimensions for certain periods, may be a key driver of Guizhou’s consistent relatively higher poverty score.

4.3: Dynamic and Dimensional Decomposition

Like NR before it, $\Omega_{a\beta}$ is decomposable according to each period and dimension. As mentioned in Section 2.7, decomposition according to each period using NR is equivalent to a specific case of AF calculated separately for each period while decomposition according to each dimension using NR is equivalent to a specific case of Foster (2009) calculated for each dimension separately.\(^{13}\) Model 1 in Tables 3 and 4 therefore represents the estimate based on NR, which is also equal to AF for the dynamic decomposition calculations and Foster (2009) for the dimensional decomposition calculations. Since Models 2 and 3 use $\Omega_{a\beta\delta}$ they allow the decomposition to be made with the additional properties of dimensional and dynamic transfer.

Table 3 depicts the dynamic and dimensional decompositions for Sample 1 in terms of the proportion contribution to overall poverty. In terms of dynamic decomposition, we see that 1993 remains the most deprived period in all three models and that deprivation has been falling over time. The qualitative conclusions are the same for all 3 models, with very minor changes in the percentage contributions. In terms of dimensional decomposition, we see that Blood Pressure and No Access to Toilets switch ranks as we move from Model 1 to Model 2. The increase in $\beta$ reflected in moving from Model 2 to Model 3 has little effect on the results. No Access to Fuel consistently remains the dimension that contributes the most to poverty, while No Access to Electricity, the least.

Table 4 depicts the dynamic and dimensional decompositions for Sample 2. While the rankings of the decompositions across time and dimensions do not change across the models, two features are worth noting. First, while the year 2006 remains fourth ranked in terms of poverty in Table 3, it is second ranked in Table 4. This highlights that the choice of dimensions clearly has an effect on our conclusions; the unrepresentative dimensions for the quantitative sample should not be taken as indicative of overall poverty in the sample. Secondly, the proportion contribution of the blood pressure dimension falls by 19% as we move from Model 1 to Model 2. This is primarily attributable to the increase in the proportion contribution of the education dimension. We had chosen the education variable because of its correlation over time; i.e. an adult individual who does not have a primary education today is not likely to obtain one over time. Because Models 2 and 3 are sensitive to the repeated deprivation in this dimension, the proportion contribution has increased dramatically.

\(^{13}\) Since Foster’s application is unidimensional and focussed on the traditional income measure, such a comparison is not immediately obvious.
Clearly this leads to the question of whether education is an appropriate dimension to include in measures that are interested in the duration of deprivation, or whether it should be, at least, discounted in the overall calculations (Appendix D details how this can be done). We have included it here to highlight one aspect that can be captured by our measure: sensitivity to repeated deprivations over time within the same dimension. The flexibility of our measure allows the policy maker to set $\beta = 1$ when sensitivity to repeated periods (dynamic transfer) may be deemed unnecessary.\footnote{Indeed, it is also possible to think of other ways to decrease the weight allotted to repeated time periods; for example, the term $\left( \sum_{n=1}^{N} \frac{d_i}{T} \right)$ in the weighting function $S_n^\beta$ can be raised to a power of less than one but greater than zero. This will clearly violate dynamic transfer, but may be desirable in some cases such as when a dimension like education is deemed a necessary component of the measure.}

We should also highlight as this point that even if there were no differences between the results in Model 1 and Model 2, it may be useful to still calculate $Bp$, which tells us whether attention should be focussed on particular periods, or particular dimensions. In Sample 1, for example, we know that $Bp = 0.36$. We therefore know that deprivation is more likely to be concentrated in particular dimensions, which in turn suggests that attention should be paid to dimensional, rather than dynamic decompositions.

5. Discussion and Conclusion

There has been a large and rapidly proliferating literature on the multidimensional measurement of poverty and its application to household survey data. The increasing availability of micro datasets containing a wealth of household and individual information has helped in the development of increasingly sophisticated measures that take into account the richness of the available information.

We have proposed a dynamic multidimensional measure of poverty that unites two strands of a literature that finds its origins in Foster, Greer and Thorbecke (1984). Our measure allows population subgroups to be compared when information regarding both multiple dimensions of deprivation and multiple periods of time are available. Following the axiomatic framework in Alkire and Foster (2011a) we lay out two unique properties of our measure that are only identifiable when such panel data is available. We argue that the separate identification and weighting of the breadth and length aspects of poverty are useful to both policy-makers and researchers for two reasons: 1) due to the general idea that a concentration of deprivations (whether within specific dimensions or within specific periods of time) should be given more attention than the simple count of deprivations; 2)
there may be cases where a concentration of deprivation within specific periods is more important than deprivation within specific dimensions and vice versa, in which case the ability to change the relative weights of the two components becomes important.

The empirical application has highlighted that our measure allocates subgroup poverty scores, as well as subgroup, dimensional, and dynamic shares that may differ from other models in the literature that exist as special cases of ours. Another point illustrated by our application has been the importance of the length relative to the breadth aspect of poverty in China, suggesting that for several dimensions, deprivation remains chronic. Despite this, the province of Guizhou, which consistently appears as the province with the deepest extent of poverty, has a relatively higher breadth aspect. Overall therefore, while chronic deprivation within specific dimensions best explains overall poverty, deprivation over multiple dimensions within specific periods explains Guizhou’s relatively higher poverty score.

The generality and flexibility of our measure comes at a cost since any researcher or policy-maker will have to make a choice with regards to the additional parameter $\beta$. This additional complication may then encourage interested users who have panel data in defaulting to use Alkire and Foster (2011a) when comparing poverty of a fixed population across time, and using Foster (2009) when comparing poverty of a fixed population across dimensions. That is, our extension may be deemed unnecessary if comparison across subgroups is not required. However, we have also highlighted that even if one was only interested in comparing across dimensions, or comparing across periods, our measure, when decomposed, may still lead to different conclusions than that generated by existing models given its sensitivity to the overall distribution of deprivations within individual deprivation profiles.

For multidimensional poverty measures in general, the choice of deprivation dimensions, cut-offs, as well as the appropriate weights to attach to them remains a fertile research agenda beyond the scope of this paper [for a brief discussion, see Alkire and Foster (2011b)]. When faced with a large degree of undecidedness with regards to parameter choices, we would advocate calculating the results for various combinations of the parameters. Indeed, understanding how and why poverty measurement changes with the various parameters is arguably more informative than poverty conveyed by a single ‘definitive’ number.
REFERENCES


Table 1: Multidimensional Deprivation and its Subgroup Decomposition for CHNS Sample One

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Table 2: Multidimensional Deprivation and its Subgroup Decomposition for CHNS Sample Two

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<td>( 0.1130, 7 )</td>
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<td>Guangxi</td>
<td>285</td>
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<td>( 0.1179, 6 )</td>
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<td>Urban</td>
<td>760</td>
<td>0.2738</td>
<td>2</td>
<td>( 0.1236, 2 )</td>
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Table 3: Dynamic and Dimensional Decomposition (CHNS Sample One)

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<tr>
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<tr>
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<td>$\Omega_{\alpha\beta}$</td>
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<td>2004</td>
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<td>No access to toilet</td>
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<td>3</td>
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<td>0.1325</td>
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<td>No access to fuel</td>
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<td>0.0016</td>
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<td>No access to drink water</td>
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<td>0.1274</td>
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<tr>
<td>No access to at least one vehicle type</td>
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<td>0.0525</td>
<td>9</td>
<td>0.0533</td>
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<td>No access to radio/TV</td>
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<td>BMI not in normal range</td>
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<td>Blood pressure not normal</td>
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<td>Individual below primary education</td>
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<td>No access to road in community</td>
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<td>No access to bus station in community</td>
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<td>No access to school in community</td>
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Table 4: Dynamic and Dimensional Decomposition (CHNS Sample Two)

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<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$S_{nj}^{a}\beta = 1$</td>
<td>0.5</td>
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<td>$\Omega_{a\beta}$</td>
<td>0.1396</td>
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### Dynamic Decomposition

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<td>0.2030</td>
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<td>0.1785</td>
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### Dimensional Decomposition

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<td>0.1351</td>
<td>2</td>
<td>0.1262</td>
<td>2</td>
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<tr>
<td>Individual below primary education</td>
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<td>0.8625</td>
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APPENDIX A: Proofs

(1) **Proposition:** $\Omega_{\alpha\beta}$ satisfies *Dynamic Transfer* when $\delta > 0$, $\beta < 1$.

**Proof:**

We restrict our attention to individual $n$’s deprivation profile since that is the only thing that changes in the definition. A dimensional permutation ensures that $\sum_j d_{njt}^\alpha$ remains the same after the permutation, for all $t$. It also ensures that $\sum_j \sum_t d_{njt}^\alpha$ remains the same after the permutation. Therefore a measure that is path-independent in terms of $d_{njt}^\alpha$ (as in NR) will not change with a dimensional permutation. This means that when $\beta = 0$, *dynamic transfer* is clearly not satisfied.

First, define $d_{njt}' = d_{njt}(t=tr,j=jr)$ and $d_{njt}''' = d_{njt}(t=tr,j=jr')$.

Dimensional permutation results in $d_{njt}' + \varepsilon$ and $d_{njt}''' - \varepsilon$ where $\varepsilon$ is any non-negative real number. We need to show individual $n$’s deprivation score $\Omega_{\alpha\beta n}$ has increased with the permutation when $\sum_{m}^{(T-1)} d_{njt|j=m}^\alpha < \sum_{m}^{(T-1)} d_{njt|j=m}^\alpha$, where $m \neq t'$, which is equivalent to showing $\frac{\partial \Omega_{\alpha\beta n}}{\partial d_{njt}'''} > 0$ under those conditions.

Define $I_{njt}^{\alpha\beta} = S_{njt}^{\alpha\beta} d_{njt}^\alpha$

$$\frac{\partial \Omega_{\alpha\beta n}}{\partial d_{njt}'''} = \frac{\partial \Omega_{\alpha\beta n}}{\partial (\sum_j l_{njt}^{\alpha\beta})} \frac{\partial (\sum_j l_{njt}^{\alpha\beta})}{\partial d_{njt}'''}$$

(4a)

Consider the first term on the right.

$$\frac{\partial \Omega_{\alpha\beta n}}{\partial (\sum_j l_{njt}^{\alpha\beta})} = \left[ \delta \left( \frac{\sum_j l_{njt}^{\alpha\beta}}{j^T} \right)^{\delta-1} \right]$$

(4b)

Equation (4b) is positive when $\delta > 0$. This is a necessary condition for the proof since it ensures that increases in $\sum_j l_{njt}^{\alpha\beta}$ translate into increases in the individual’s deprivation score.

Now consider the second term on the right of equation (4a). Note, $\frac{\partial (\sum_j l_{njt}^{\alpha\beta})}{\partial d_{njt}'''} = \sum_j \frac{\partial l_{njt}^{\alpha\beta}}{\partial d_{njt}'''}$

Recall that $l_{njt}^{\alpha\beta} = \beta \left( \frac{\sum_j d_{njt}^\alpha}{j} \right) d_{njt}^\alpha + (1 - \beta) \left( \frac{\sum_t d_{njt}^\alpha}{T} \right) d_{njt}^\alpha$

Since $\sum_j d_{njt}^\alpha$ does not change with dimensional permutation, we can ignore its influence on $l_{njt}^{\alpha\beta}$ and instead define $l_{njt}^{\alpha\beta} = (1 - \beta) \left( \frac{\sum_t d_{njt}^\alpha}{T} \right) d_{njt}^\alpha$. 

We now need to show that \( \frac{\partial I_{njt}^{\alpha \beta \ast}}{\partial d_{njt}^{\alpha \beta \ast}} \) is a positive function of \( d_{njt}^{\alpha \beta \ast} \) where \( d_{njt}^{\alpha \beta \ast} \) is any \( d_{njt}^{\alpha \beta} \) where \( t \neq t' \) but \( j = j' \). If so this automatically implies \( \frac{\partial I_{njt}^{\alpha \beta \ast}}{\partial d_{njt}^{\alpha \beta \ast}} > \frac{\partial I_{njt}^{\alpha \beta \ast}}{\partial d_{njt}^{\alpha \beta \ast}} \) and \( \frac{\partial I_{njt}^{\alpha \beta \ast}}{\partial d_{njt}^{\alpha \beta \ast}} > \frac{\partial I_{njt}^{\alpha \beta \ast}}{\partial d_{njt}^{\alpha \beta \ast}} \) given \( \sum_{m}^{T} d_{njt|j' \neq j}^{\alpha \beta} < \sum_{m}^{T-1} d_{njt|j = j'}^{\alpha \beta} \). From equation (4c) it can be seen that \( \frac{\partial I_{njt}^{\alpha \beta \ast}}{\partial d_{njt}^{\alpha \beta \ast}} > 0 \) for any \( \beta < 1 \).

(2) Proposition: \( \Omega_{ab} \) satisfies Dimensional Transfer when \( \delta > 0, \beta > 0 \).

Following the proof for dynamic transfer, define \( I_{njt}^{\alpha \beta \ast} = \beta \left( \frac{\sum_{j} d_{njt}^{\alpha \beta}}{j} \right) d_{njt}^{\alpha \beta} \).

\[
\frac{\partial I_{njt}^{\alpha \beta \ast}}{\partial d_{njt}^{\alpha \beta}} = \beta \left( \frac{\sum_{j} d_{njt}^{\alpha \beta} + d_{njt}^{\alpha \beta \ast}}{j} \right) \quad (4d)
\]

From equation (4d) it can be seen that \( \frac{\partial I_{njt}^{\alpha \beta \ast}}{\partial d_{njt}^{\alpha \beta}} > 0 \) for any \( \beta > 0 \) where \( d_{njt}^{\alpha \beta \ast} \) is any \( d_{njt}^{\alpha \beta} \) where \( j \neq j' \) but \( t = t' \).

(3) Proposition: \( \Omega_{ab} \) satisfies Deprivation Monotonicity when \( 0 \leq \beta \leq 1 \).

Define \( I_{njt}^{\alpha \beta} = S_{njt}^{\alpha \beta} d_{njt}^{\alpha \beta} \). For the following, we restrict ourselves to individual \( n \)'s deprivation score \( \Omega_{ab \delta n} = \left( \frac{\sum_{j} I_{njt}^{\alpha \beta}}{j=1} \right) \delta \) since \( \Omega_{ab \delta} \) is simply the average across all individuals. Define \( d_{njt}^{\alpha \beta \ast} = d_{njt|t=t, j=j'}^{\alpha \beta} \).

Deprivation monotonicity requires that \( \frac{\partial \Omega_{ab \delta n}}{\partial d_{njt}^{\alpha \beta \ast}} = \frac{\partial \Omega_{ab \delta n}}{\partial \sum_{j} I_{njt}^{\alpha \beta}} \frac{\partial \sum_{j} I_{njt}^{\alpha \beta}}{\partial d_{njt}^{\alpha \beta \ast}} > 0 \).

\[
\frac{\partial \Omega_{ab \delta n}}{\partial \sum_{j} I_{njt}^{\alpha \beta}} \left[ \delta \left( \frac{\sum_{j} I_{njt}^{\alpha \beta}}{j=1} \right) \right] ^{-1} \frac{1}{j=1} > 0 \quad \text{when} \ \delta > 0
\]

\[
\frac{\partial \Omega_{ab \delta n}}{\partial d_{njt}^{\alpha \beta \ast}} = \sum_{t} \sum_{j} \frac{\partial I_{njt}^{\alpha \beta}}{\partial d_{njt}^{\alpha \beta \ast}}
\]

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\[
\frac{\partial \Omega_{\alpha\beta}^a}{\partial \alpha_{njt}^a} = \beta \left( \frac{\sum_j^n d_{njt}^a}{J} \right) \left( \alpha_{njt}^a + 1 \right) + (1 - \beta) \left( \frac{\sum_T^T d_{njt}^a}{T} \right) \left( \alpha_{njt}^a + 1 \right) > 0 \text{ for } 0 \leq \beta \leq 1
\]

**APPENDIX B: \( \delta \) and NR**

For ease of exposition, \( \delta \) is dropped from \( \Omega_{\alpha\beta} \) despite being present in NR. A formulation with \( \delta \) is straightforward:

\[
\Omega_{\alpha\beta\delta} = \left\{ \begin{array}{ll}
\frac{\sum_n \left( \frac{\sum_T^T \sum_j^n d_{njt}^a \delta_{njt}^a}{J \ast T} \right) \ast C_n}{N} & \text{if } \delta > 0 \\
\frac{\sum_n C_n}{N} & \text{otherwise}
\end{array} \right.
\]

\( \delta \)'s main use is twofold. At \( \delta = 0 \), \( \Omega_{\alpha\beta\delta} \) gives us the poverty headcount ratio. \( \delta > 1 \) was introduced in NR to allow for increasing sensitivity of the overall deprivation count. This property can be stated formally as follows.

Define an *averaging of deprivations* as a transfer of deprivations between A and B such that \( D_A' = [D_A \ast \kappa + D_B \ast (1 - \kappa)] \) and \( D_B' = [D_B \ast \kappa + D_A \ast (1 - \kappa)] \) where \( 1 > \kappa > 0 \). More generally, let \( Z \) be a \( N \times N \) bistochastic matrix whose elements are unity for every non poor person in \( N \). Then an averaging of achievements can be represented as: \( \rho' = Z \rho \).

(Axiom 12): *Deprivation Transfer*

\( g(\rho, \nu) \geq g(\rho', \nu) \) where \( \rho' \) is obtained from \( \rho \) by an averaging of deprivations among any two poor individuals A and B where \( D_A \neq D_B \); and \( D_B \) is derived by a deprivation decrement of \( D_A \).

A reduction in deprivation inequality between poor individuals must decrease poverty.

While this property is arguably desirable, it is problematic in that in assigns increasing weights to additional deprivations regardless of which dimension or period they belong to. *Dimensional* and *dynamic transfer* can be seen as more refined version of *deprivation transfer* since they are specific to two different aspects of poverty – multidimensionality and duration. A measure satisfying *dimensional* and *dynamic transfer* and not *deprivation transfer* would therefore not assign increasing weight to additional deprivations that do not share the same period or dimension as other deprivations.

The clear advantage of not needing \( \delta > 1 \) is the preservation of *dimensional* and *dynamic decomposition* properties.
For completeness we present the proof for the proposition that $\Omega_{\alpha\beta\delta}$ as defined above satisfies deprivation transfer for $\delta > 1$.

Deprivation transfer requires that 

\[
\frac{\partial^2 \Omega_{\alpha\beta\delta n}}{\partial (d_{\alpha n, j t}^*)^2} > 0 \quad \text{where } d_{\alpha n, j t}^* \text{ is any } d_{\alpha n, j t} \text{ not equal to } d_{\alpha n, j t}'.
\]

\[
\frac{\partial^2 \Omega_{\alpha\beta\delta n}}{\partial (d_{\alpha n, j t})^2} = \frac{\partial \Omega_{\alpha\beta\delta n}}{\partial (\Sigma_l \Sigma_j l_{n, j t}^\alpha)^2} \frac{\partial^2 (\Sigma_l \Sigma_j l_{n, j t}^\alpha)}{\partial (d_{\alpha n, j t})^2} + \frac{\partial \Omega_{\alpha\beta\delta n}}{\partial d_{\alpha n, j t}'} \frac{\partial d_{\alpha n, j t}'}{\partial (d_{\alpha n, j t}^*)} \frac{\partial^2 \Omega_{\alpha\beta\delta n}}{\partial (d_{\alpha n, j t})^2}.
\]

Assume that Deprivation monotonicity holds. Then, \(\frac{\partial \Omega_{\alpha\beta\delta n}}{\partial (\Sigma_l \Sigma_j l_{n, j t}^\alpha)}\) and \(\frac{\partial \Omega_{\alpha\beta\delta n}}{\partial d_{\alpha n, j t}'} > 0\).

**TERM (1)**

\[
\frac{\partial^2 (\Sigma_l \Sigma_j l_{n, j t}^\alpha)}{\partial (d_{\alpha n, j t}^*)^2} \left( \frac{\partial \Sigma_l \Sigma_j l_{n, j t}^\alpha}{\partial (d_{\alpha n, j t})} \right)^2 > 0 \quad \text{for } 0 < \beta < 1 \text{ but only for } d_{\alpha n, j t}^* \text{ that share the same dimension or same period as } d_{\alpha n, j t}'. \quad \text{Otherwise, } \frac{\partial^2 l_{n, j t}^\alpha}{\partial (d_{\alpha n, j t})^2} = 0.
\]

**TERM (2)**

\[
\frac{\partial^2 \Omega_{\alpha\beta\delta n}}{\partial (\Sigma_l \Sigma_j l_{n, j t})^2} = \left( \delta - 1 \right) \left( \frac{\Sigma_l \Sigma_j l_{n, j t}^\alpha}{J + T} \right)^{\delta - 2} \cdot \frac{1}{2T} > 0 \quad \text{for } \delta > 1.
\]

Since the Term (2) result holds regardless of the data, setting $\delta > 1$ ensures the property holds.

**APPENDIX C: Choosing $\beta$**

**Proposition:** Given $J$ and $T$, for any choice of $x$, $\beta$ must be set equal to $\frac{x J}{T + x J}$

**Proof:**

We illustrate the proof in the continuous case; that is, for marginal rather than discrete changes in the deprivation input to allow for $\alpha > 0$.

Define \(x = \frac{\partial S_{n, j t}^\alpha}{\partial d_{n, j t}^*} \bigg|_{t = \tau, j = \tau} / \frac{\partial S_{n, j t}^\alpha}{\partial d_{n, j t}^*} \bigg|_{t = \tau, j = \tau} \) where \(S_{n, j t}^\beta = \left[ \beta \left( \frac{\Sigma_j d_{\alpha n, j t}}{J} \right) + (1 - \beta) \left( \frac{\Sigma_T d_{\alpha n, j t}}{T} \right) \right] \)

Therefore, \(\frac{\partial S_{n, j t}^\alpha}{\partial d_{n, j t}^*} \bigg|_{t = \tau, j = \tau} = \beta \left[ \frac{1}{J} d_{\alpha n, j t} \right] \) and \(\frac{\partial S_{n, j t}^\alpha}{\partial d_{n, j t}^*} \bigg|_{t = \tau, j = \tau} = (1 - \beta) \left[ \frac{1}{T} d_{\alpha n, j t} \right] \)
Because our measure is a generalisation of the class of measures in NR and BC, it can incorporate the extensions used there, namely in terms of ‘persistence’ (Gradin et al, 2012; Bossert et al 2012) and ‘loss-aversion’ (Hojman and Kast, 2009). Given our focus on the interaction between ‘length’ and ‘breadth’ of deprivations, we avoided the use of such extensions in our empirical applications as they would make it harder to identify the effect of length versus breadth given the additional trade-offs introduced with such extensions. However, we briefly describe how these can be incorporated into our measure – the interested reader is advised to refer to the aforementioned papers directly for more details regarding their properties. Define \( I_{njt}^{ab} = S_{njt}^{ab}d_{njt}^{a} \)

\[
\Omega_{\alpha\beta}^{extend} = \left[ \frac{\sum_{n} \left( \left( \sum_{j} \frac{1}{T} I_{njt}^{ab} w_{njt} \right) * C_{n} \right) / N}{\delta > 0} \right]
\]

\( w_{njt} \) is a generic weight associated with each \( I_{njt}^{ab} \) (the transformed deprivation input). These weights can be defined in a variety of ways, though they clearly change the interpretation of the measure. ‘Persistence’, which gives increasing weight to deprivations that occur in consecutive periods, for example, can be incorporated by defining \( w_{njt} = \left( p_{njt} / T \right) \) where \( p_{njt} \) is the length of the deprivation spell associated with a particular \( I_{njt}^{ab} \). Another way of defining persistence (BCCD) could be \( w_{nt} = (p_{nt} / T) \) where the weights are increasing in the number of consecutive periods that an individual experiences breath of deprivation above a predetermined cutoff.

Persistence may not always be relevant to every dimension of deprivation. One can, for example, imagine that being unemployed for three consecutive periods and then being employed for three consecutive periods is superior to alternating in and out of employment for six periods since one incurs an ‘adjustment cost’ when changing states. This can be captured with the concept of loss-aversion (Kahneman and Tversky, 1979; applied to poverty indices in Hojman and Kast, 2009). Consider, for example:
This weighting scheme has two features: 1) it allows a particular period to be weighted heavier if it is proceeded by more dimensions of deprivation and to be weighted by less if it is proceeded by less dimensions of deprivation. 2) it captures the concept of loss-aversion by weighting increases in the breadth of deprivation in the subsequent period more heavily than an equivalent decrease in the breadth of deprivation in the subsequent period (in absolute terms).

Another potential use of the general weights is to assign different importance to different dimensions. As suggested by Atkinson (2003), it is not unreasonable in most applications to start with the case where $w_j = \frac{1}{J}$; that is, where each dimension is weighted equally. With additional information, however, a researcher or policy-maker may assign different weights (so long as $\sum_j w_j = 1$). Reasons behind changing the weights include ‘double counting’ in the sense that some dimensions may essentially be capturing the same aspect of deprivation, which justifies the discounting of the importance of the associated dimensions; another reason is that individuals may actually put very low importance on certain dimensions; e.g. in Bossert et al (2013) dimensions are weighted based on the views of society regarding the importance of those dimensions -- “consensus weighting”.

\[
w_{njt} = \begin{cases} 
\frac{2}{4} + \left( \frac{\sum_j I_{n(t+1)}^a}{j} - \frac{\sum_j I_{nj(t)}^a}{j} \right) \left( \frac{2}{4} \right) & \text{when } \sum_j I_{n(t+1)}^a > \sum_j I_{nj(t)}^a \\
\frac{2}{4} + \left( \frac{\sum_j I_{nj(t+1)}^a}{j} - \frac{\sum_j I_{nj(t)}^a}{j} \right) \left( \frac{1}{4} \right) & \text{otherwise}
\end{cases}
\] (5b)
## APPENDIX E1: Description of Dimensions used for analysis

<table>
<thead>
<tr>
<th>Type</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Household</strong></td>
<td>Access to Toilet (D1)</td>
<td>Individual’s household has access to improved toilet facility as per UN norms. UN defines improved facility as having own flush toilet, own pit toilet, traditional pit toilet, ventilated improved pit latrine, pit-latrine with slab, flush toilet, and composting toilet</td>
</tr>
<tr>
<td></td>
<td>Access to fuel (D2)</td>
<td>Fuel used for cooking by individual’s household is kerosene, electricity, LPG or biogas</td>
</tr>
<tr>
<td></td>
<td>Access to electricity (D3)</td>
<td>Individual’s household has access to electricity</td>
</tr>
<tr>
<td></td>
<td>Access to drink water (D4)</td>
<td>Individual’s household has access to improved drinking water source. UN defines improved drinking water source as piped water into dwelling, plot or yard, public tab/standpipe, tube well, borehole, protected dug well, protected spring and rainwater.</td>
</tr>
<tr>
<td></td>
<td>Access to atleast one vehicle type (D5)</td>
<td>Individual’s household owns at least one of the following mode of transport: a bicycle, motorcycle or car.</td>
</tr>
<tr>
<td></td>
<td>Access to radio/TV (D6)</td>
<td>Individual’s household owns at least a radio, b/w Television or a colour Television</td>
</tr>
<tr>
<td><strong>Individual</strong></td>
<td>BMI&lt;18.5 or BMI&gt;30 (D7)</td>
<td>If BMI of the individual is not normal (i.e., either greater than 30 or less than 18.5.)</td>
</tr>
<tr>
<td></td>
<td>Illness in the last four weeks (D8)</td>
<td>Whether individual suffered illness in the last four weeks.</td>
</tr>
<tr>
<td></td>
<td>Blood pressure not normal (D9)</td>
<td>If the blood pressure is normal (i.e., 90&lt;= systolic&lt; 120 and 60&lt;= diastolic&lt;80)</td>
</tr>
<tr>
<td></td>
<td>Individual atleast primary educated (D10)</td>
<td>Individual is educated up to primary (year 6).</td>
</tr>
<tr>
<td><strong>Community</strong></td>
<td>Access to road (D11)</td>
<td>If the village/neighbourhood has a stone/gravel or paved roads.</td>
</tr>
<tr>
<td></td>
<td>Access to bus/train station (D12)</td>
<td>If there is a bus stop in the village/neighbourhood.</td>
</tr>
<tr>
<td></td>
<td>Access to school (D13)</td>
<td>If there is a primary/middle/upper middle school in the village/neighbourhood.</td>
</tr>
</tbody>
</table>
## APPENDIX E2: Description of balanced samples used for analysis

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions</td>
<td>D1-D2-D3-D4-D5-D6-D7-D8-D9-D10-D11-D12-D13</td>
<td></td>
</tr>
<tr>
<td>Description</td>
<td>This sample comprises of household, individual and community dimensions. All dimensions are qualitative.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions</td>
<td>1. Years of education of individual (cut off is primary education i.e., 6 years)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. Individual's Body Mass Index (cut off is 18.5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. Individual's Blood Pressure (systolic/diastolic).i.e., more than 120/80 then high BP.</td>
<td></td>
</tr>
<tr>
<td>Description</td>
<td>All dimensions in this sample are quantitative</td>
<td></td>
</tr>
</tbody>
</table>
Appendix E3: Dimension wide poverty rates for China (Sample 1, Qualitative – balanced panel of 6 years and 13 dimensions)

<table>
<thead>
<tr>
<th>CHNS wave</th>
<th>No Access to Toilet</th>
<th>No Access to fuel</th>
<th>No access to electricity</th>
<th>No access to drink water</th>
<th>No access to atleast one vehicle type</th>
<th>No access to radio/TV</th>
<th>BMI not in normal range</th>
<th>Illness in the last four weeks</th>
<th>Blood pressure not normal</th>
<th>Individual below primary educated</th>
<th>No access to road in community</th>
<th>No access to bus/train station in community</th>
<th>No access to school in community</th>
<th>Income less than 0.5*Median *</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>0.6665</td>
<td>0.9189</td>
<td>0.0523</td>
<td>0.6526</td>
<td>0.1827</td>
<td>0.4823</td>
<td>0.1185</td>
<td>0.2391</td>
<td>0.4726</td>
<td>0.4454</td>
<td>0.8538</td>
<td>0.4885</td>
<td>0.2560</td>
<td>0.2344</td>
</tr>
<tr>
<td>1993</td>
<td>0.6573</td>
<td>0.8769</td>
<td>0.0164</td>
<td>0.6660</td>
<td>0.1637</td>
<td>0.4330</td>
<td>0.0944</td>
<td>0.1596</td>
<td>0.5239</td>
<td>0.4228</td>
<td>0.8964</td>
<td>0.4495</td>
<td>0.1837</td>
<td>0.2344</td>
</tr>
<tr>
<td>1997</td>
<td>0.6147</td>
<td>0.7414</td>
<td>0.0036</td>
<td>0.5557</td>
<td>0.2376</td>
<td>0.3740</td>
<td>0.0867</td>
<td>0.1585</td>
<td>0.6039</td>
<td>0.3982</td>
<td>0.7542</td>
<td>0.3217</td>
<td>0.1611</td>
<td>0.2488</td>
</tr>
<tr>
<td>2000</td>
<td>0.5952</td>
<td>0.6952</td>
<td>0.0092</td>
<td>0.5485</td>
<td>0.2550</td>
<td>0.2950</td>
<td>0.0841</td>
<td>0.1827</td>
<td>0.6485</td>
<td>0.3833</td>
<td>0.8481</td>
<td>0.3638</td>
<td>0.1432</td>
<td>0.2904</td>
</tr>
<tr>
<td>2004</td>
<td>0.5428</td>
<td>0.7086</td>
<td>0.0056</td>
<td>0.5141</td>
<td>0.2996</td>
<td>0.1652</td>
<td>0.1067</td>
<td>0.3479</td>
<td>0.6963</td>
<td>0.3366</td>
<td>0.9389</td>
<td>0.3628</td>
<td>0.2365</td>
<td>0.2776</td>
</tr>
<tr>
<td>2006</td>
<td>0.4772</td>
<td>0.6321</td>
<td>0.0056</td>
<td>0.4962</td>
<td>0.3258</td>
<td>0.1113</td>
<td>0.1042</td>
<td>0.2986</td>
<td>0.7045</td>
<td>0.4310</td>
<td>0.9656</td>
<td>0.4228</td>
<td>0.2817</td>
<td>0.3074</td>
</tr>
</tbody>
</table>

*income used as a benchmark and not an actual dimension