Developing Pre-service Teachers’ Pedagogical Content Knowledge Through Lesson Study

By

Meiliasari
B.Ed., M.Sc.

Submitted in fulfilment of the requirements for the degree of
Doctor of Philosophy

Deakin University
March 2018
I am the author of the thesis entitled
Developing of Pre-service Teachers’ Pedagogical Content Knowledge through Lesson Study
submitted for the degree of Doctor of Philosophy

This thesis may be made available for consultation, loan and limited copying in accordance
with the Copyright Act 1968.

'I certify that I am the student named below and that the information provided in the form is
correct'

Full Name       : Meiliasari

Signed          : [Signature Redacted by Library]
Date            : 3 August 2018
DEAKIN UNIVERSITY
CANDIDATE DECLARATION

Declaration

I certify the following about the thesis entitled: Developing of Pre-service Teachers’ Pedagogical Content Knowledge through Lesson Study submitted for the degree of Doctor of Philosophy

a. I am the creator of all or part of the whole work(s) (including content and layout) and that where reference is made to the work of others, due acknowledgment is given.
b. The work(s) are not in any way a violation or infringement of any copyright, trademark, patent, or other rights whatsoever of any person.
c. That if the work(s) have been commissioned, sponsored or supported by any organisation, I have fulfilled all of the obligations required by such contract or agreement.
d. That any material in the thesis which has been accepted for a degree or diploma by any university or institution is identified in the text.
e. All research integrity requirements have been complied with.

'I certify that I am the student named below and that the information provided in the form is correct'

Full Name : Meilasari

Signed : [Signature Redacted by Library]
Date : 31 March 2018
Acknowledgements

I would like to express my greatest appreciation to my three supervisors. My principal supervisor – Dr. Wanty Widjaja, and my associate supervisors – Associate Professor Susie Groves, and Professor Colleen Vale for all the advice and encouragement. I remain forever grateful for the learning experience, for making me do things that I thought I could not do. Thank you for making the impossible possible.

This study is supported by BLN DIKTI scholarship. I would like to send my gratitude to DIKTI – Ministry of Research and Higher Education, Republic of Indonesia for granting me the scholarship to pursue my doctoral degree at Deakin University. I would also like to thank the family of Glendon A. Lean for awarding me the Glendon Lean Research Scholarship towards the end of my study.

I would like to acknowledge the university, the university lecturers, the pre-service teachers, the schools, the mentor teachers and the students for participating in this study.

I would like to thank my colleagues at Department of Mathematics, State University of Jakarta for their support.

My special gratitude to my friends and PhD colleagues at Deakin University. Thank you for making this journey more enjoyable.

Lastly, I could never finish this journey without the support from my family. A big special thanks to my mother – Ibu Munirah, my father – Bapak Samikin, and my sister – Purnama Agustina for their endless love, support, and encouragement.
# Table of Contents

List of Tables ........................................................................................................................................ v  
List of Figures ...................................................................................................................................... vii  
Abbreviations and Acronyms ........................................................................................................... viii  
Abstract ............................................................................................................................................... xi  
1 Introduction ......................................................................................................................................... 1  
   1.1 Background of the Study ........................................................................................................... 1  
   1.2 Education System in Indonesia ............................................................................................... 2  
   1.3 Teacher Education System in Indonesia ............................................................................... 7  
   1.4 Organisation of the Thesis ..................................................................................................... 9  
2 Literature Review ............................................................................................................................. 11  
   2.1 Pedagogical Content Knowledge (PCK) ............................................................................... 11  
   2.2 Research on Teachers’ Pedagogical Content Knowledge (PCK) Development ............... 13  
   2.3 Frameworks of Pedagogical Content Knowledge ................................................................ 15  
      2.3.1 Mathematical Knowledge for Teaching (MKT) ............................................................... 16  
      2.3.2 Knowledge Quartet (KQ) .............................................................................................. 18  
   2.4 Lesson Study .......................................................................................................................... 22  
      2.4.1 Japanese Lesson Study ................................................................................................. 22  
      2.4.2 Lesson Study for Teacher Professional Development ............................................... 25  
      2.4.3 Adaptation of Lesson Study ........................................................................................ 26  
      2.4.4 Lesson Study in Indonesia .......................................................................................... 28  
      2.4.5 Lesson Study in Teacher Education .......................................................................... 32  
   2.5 Implications for this Study ...................................................................................................... 36  
   2.6 Research Questions ............................................................................................................... 37  
3 Methodology ................................................................................................................................... 38  
   3.1 Research Paradigms ................................................................................................................. 38  
   3.2 Educational Research .............................................................................................................. 39  
      3.2.1 Quantitative Research in Education ............................................................................. 40  
      3.2.2 Qualitative Research in Education ............................................................................... 40  
   3.3 Research Trustworthiness ....................................................................................................... 42  
      3.3.1 Credibility .................................................................................................................... 42  
      3.3.2 Transferability .............................................................................................................. 43
3.3.3 Dependability ......................................................................................................... 43
3.3.4 Confirmability ....................................................................................................... 43
3.4 Implications for This Study ....................................................................................... 44
3.5 The Researcher’s Role .............................................................................................. 44
3.6 Ethics ......................................................................................................................... 45
3.7 Conclusion ................................................................................................................. 46

4 Research Process ............................................................................................................... 47
4.1 The Teaching Practicum ........................................................................................... 47
4.2 The Schools ............................................................................................................... 47
4.3 Participants ................................................................................................................ 49
4.4 Research Design ........................................................................................................ 50
4.4.1 Lesson Study Workshop ........................................................................................ 50
4.4.2 Observation and Preparation ................................................................................. 51
4.4.3 Lesson Study Implementation ............................................................................... 51
4.5 Data Collection .......................................................................................................... 56
4.5.1 Written Test ........................................................................................................... 56
4.5.2 Video Recording .................................................................................................... 57
4.5.3 Field notes .............................................................................................................. 57
4.5.4 Lesson Study Artefacts .......................................................................................... 57
4.5.5 Interviews .............................................................................................................. 58
4.6 Data Analysis ............................................................................................................ 58
4.6.1 Data Organisation .................................................................................................. 59
4.6.2 Transcription .......................................................................................................... 60
4.6.3 Creation of Codes .................................................................................................. 60
4.6.4 Development of a KQ Rubric ................................................................................ 61
4.6.5 Coding ................................................................................................................... 65
4.6.6 Data Interpretation ................................................................................................. 69
4.7 Conclusion ................................................................................................................. 70

5 The Development of Pre-service Teachers’ Pedagogical Content Knowledge (PCK) ..... 71
5.1 Changes Evident in Pre-service Teachers’ PCK ....................................................... 71
5.1.1 Foundation Dimension .......................................................................................... 71
5.1.2 Transformation Dimension .................................................................................... 88
List of Tables

Table 1.1 Number of schools, student, teachers and student to teacher ratio by school level (2012/2013) (Source: BPS, 2014) .............................................................................................. 4
Table 1.2 Structure of the Secondary Mathematics Education program ......................................................... 8
Table 2.1 The Knowledge Quartet .............................................................................................................. 18
Table 3.1 The ontology, epistemology and methodology underpinning research paradigms 39
Table 4.1 Classroom arrangement in SMPN E Jakarta ........................................................................ 48
Table 4.2 Lesson study participants ...................................................................................................... 49
Table 4.3 Lecturers’ and mentor teachers’ experience ......................................................................... 49
Table 4.4 Mathematical topic for each lesson study cycle .................................................................... 52
Table 4.5 Participants’ attendance during lesson study Group D7 .......................................................... 53
Table 4.6 Participants’ attendance during lesson study Group D8 .......................................................... 54
Table 4.7 Participants’ attendance during lesson study Group E7 .......................................................... 55
Table 4.8 Participants’ attendance during lesson study Group E8 .......................................................... 55
Table 4.9 Types of data to answer the research questions ...................................................................... 56
Table 4.10 Written test items ................................................................................................................ 57
Table 4.11 Types of data collected before, during, and after lesson study implementation .................. 58
Table 4.12 Data organization of participants .......................................................................................... 59
Table 4.13 The Knowledge Quartet (KQ) Rubric ................................................................................. 63
Table 4.14 Illustration of coding .............................................................................................................. 65
Table 4.15 Frequency of Connection dimension in cycle 1 Group D7 .................................................. 67
Table 4.16 Examples of coding using the KQ Rubric ............................................................................. 68
Table 4.17 Frequency of Use of Terminology Levels in Lesson Study Cycle 1 Group D7 .......................... 70
Table 5.1 Highest Level of Overt Subject Knowledge (OSK) .................................................................. 72
Table 5.2 Examples of pre-service teachers’ responses to item 4.c ....................................................... 81
Table 5.3 Examples of pre-service teachers’ interview responses .......................................................... 82
Table 5.4 Highest Level of Theoretical Underpinning of Pedagogy (TUP) ............................................. 82
Table 5.5 Pre-service teachers’ responses to item 7 .............................................................................. 90
Table 5.6 Highest Level of Choice of Representation (COR) ................................................................. 91
Table 5.7 Pre-service teachers’ responses to Item 5 .............................................................................. 97
Table 5.8 Highest Level of Anticipation of Complexity (AOC) ............................................................. 98
Table 5.9 Highest Level of Responding to Students’ Ideas (RSI) ............................................................ 106
Table 5.10 The KQ components enacted in C2D8 ................................................................................. 112
Table 5.11 KQ components enacted in C2D8 ....................................................................................... 115
Table 5.12 Pre-service teachers’ learning from the research lessons ..................................................... 130
Table 5.13 The pre-service teachers’ opinion about the mentor teachers’ contribution ....................... 132
Table 5.14 The pre-service teachers’ opinion about the knowledgeable other’s contribution ............. 134
Table 6.1 Benefits of anticipating students’ responses according to the pre-service teachers ............... 138
Table 6.2 Difficulties of anticipating students’ responses according to the pre-service teachers ................................................................. 138
Table 6.3 Benefits from participating in lesson study according to the participants.............. 141
Table 6.4 Difficulties in participating lesson study according to the participants............... 144
Table 6.5 The pre-service teachers’ opinion about the lecturer’s role in the teaching practicum ................................................................................................................................................ 150
List of Figures

Figure 2.1 Domain of Mathematical Knowledge for Teaching (Ball, 2008, p. 403) ............... 17
Figure 2.2 Interconnection of the Knowledge Quartet dimensions (Weston, 2013, p. 287) ... 19
Figure 2.3 Incorporation of Shulman’s (1987) teachers’ knowledge in the MKT (Ball et al., 2008) and the KQ (Rowland et al., 2009) ................................................................. 21
Figure 2.4 Lesson study cycle (Lewis, 2009, p. 97) ................................................................ 23
Figure 4.1 Phases of the lesson study implementation .......................................................... 50
Figure 4.2 Data organisation and labelling in NVivo ............................................................. 60
Figure 4.3 How many candies are in one bag? ................................................................. 65
Figure 4.4 Balance problem .............................................................................................. 68
Figure 5.1 Fishing boat problem ...................................................................................... 75
Figure 5.2 Pipit’s strategy of finding square root of 1369 .................................................... 75
Figure 5.3 Number line strategy to estimate the square root of a number ......................... 76
Figure 5.4 Item 4 of the written test .................................................................................. 80
Figure 5.5 Examples of students’ strategies ...................................................................... 84
Figure 5.6 Zayn’s work- using the two-point formula ....................................................... 85
Figure 5.7 Sofie’s group work .......................................................................................... 87
Figure 5.8 Written test Item 7 .......................................................................................... 89
Figure 5.9 An example of student work of finding gradient ................................................ 93
Figure 5.10 Yanti explaining the direction of the lines ....................................................... 94
Figure 5.11 Written test item 5 ......................................................................................... 96
Figure 5.12 Raya’s responses to item 5 ............................................................................. 98
Figure 5.13 Balance model discussed in the planning ....................................................... 99
Figure 5.14 Balance problem used in the lesson ............................................................... 101
Figure 5.15 Raya explaining the balance problem .............................................................. 103
Figure 5.16 Students used guessing and thinking backward strategies ............................. 108
Figure 5.17 Balance problem with variables on both hands ............................................. 110
Figure 5.18 The interconnection between the Contingency dimension and the Foundation dimension ............................................................................................................... 114
Figure 5.19 The interconnection between the Connection dimension and the Transformation dimension ............................................................................................................... 118
Figure 5.20 Diana’s observation sheet of Raya’s research lesson (C1D7) ......................... 121
Figure 5.21 School D’s pre-service teachers’ observation ................................................ 123
Figure 5.22 School E’s pre-service teachers’ observation ................................................ 124
Figure 5.23 Ida’s observation sheet of Gina’s research lesson (C1D8) ............................... 127
Figure 7.1 The interconnections of the Knowledge Quartet dimensions in lesson study ...... 176
Abbreviations and Acronyms

**BPS**  
*Badan Pusat Statistik*  
Statistics Centre Bureau

**CoRe**  
Content Representation

**DIKTI**  
*Direktorat Jenderal Pendidikan Tinggi*  
Directorate General of Higher Education

**FMIPA**  
*Fakultas Matematika dan Ilmu Pengetahuan Alam*  
The Faculty of Mathematics and Natural Sciences

**IMSTEP-JICA**  
Indonesian Mathematics and Science Teacher Education Project - Japan International Cooperation Agency

**JICA**  
Japan International Cooperation Agency

**KQ**  
Knowledge Quartet

**LPTK**  
*Lembaga Pendidikan dan Tenaga Kependidikan*  
Teacher Education Institution

**MI**  
*Madrasah Ibtidaiyah*  
Islamic Primary School

**MTs**  
*Madrasah Tsanawiyah*  
Islamic Lower Secondary School

**MA**  
*Madrasah Aliyah*  
Islamic Higher Secondary School

**MoEC**  
*Kementerian Pendidikan dan Kebudayaan*  
Ministry of Education and Culture

**MoRA**  
*Kementerian Agama*  
Ministry of Religious Affairs

**MoRTHE**  
*Kementerian Riset, Teknologi dan Pendidikan Tinggi*  
Ministry of Research, Technology and Higher Education

**MKT**  
Mathematical Knowledge for Teaching

**SMK**  
Subject Matter Knowledge
CCK Common Content Knowledge
HCK Horizon Content Knowledge
KCS Knowledge Content and Students
KCT Knowledge Content and Teaching
KCC Knowledge Content and Curriculum
MGMP Musyawarah Guru Mate Pelajaran
OECD Organisation for Economic Co-operation and Development
PCK Pedagogical Content Knowledge
PGSD Pendidikan Guru Sekolah Dasar
PLPG Pendidikan dan Latihan Profesi Guru
PPG Program Profesi Guru
PPL Program Pengalaman Lapangan
PISA Programme for International Student Assessment
SISTTEMS Strengthening In-Service Teacher Training in Mathematics and Science at Secondary Level
SD Sekolah Dasar
SMP Sekolah Menengah Pertama
SMPN Sekolah Menengah Pertama Negeri
SMA Sekolah Menengah Atas
SD Sekolah Dasar
SMP Sekolah Menengah Pertama
SMPN Sekolah Menengah Pertama Negeri
SMA Sekolah Menengah Atas
<table>
<thead>
<tr>
<th>Institution</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM3T</td>
<td><em>Sarjana Mendidik di Daerah Terluar, Terdepan, dan Tertinggal</em>&lt;br&gt;Bachelors of Education teach at isolated, outermost, and underdeveloped areas</td>
</tr>
<tr>
<td>S1</td>
<td>Bachelor’s Degree</td>
</tr>
<tr>
<td>S2</td>
<td>Master’s Degree</td>
</tr>
<tr>
<td>S3</td>
<td>Doctoral Degree</td>
</tr>
<tr>
<td>TIMSS</td>
<td>Trends in International Mathematics and Science Study</td>
</tr>
<tr>
<td>UM</td>
<td><em>Universitas Negeri Malang</em>&lt;br&gt;State University of Malang</td>
</tr>
<tr>
<td>UNY</td>
<td><em>Universitas Negeri Yogyakarta</em>&lt;br&gt;State University of Yogyakarta</td>
</tr>
<tr>
<td>UPI</td>
<td><em>Universitas Pendidikan Indonesia</em>&lt;br&gt;Indonesia University of Education</td>
</tr>
</tbody>
</table>
Abstract

Pedagogical Content Knowledge (PCK) embodies teachers’ pedagogical knowledge in subject-specific knowledge. To teach effectively teachers must possess PCK and be able to enact it in their teaching. Lesson study, with its planning, teaching, and post-lesson discussion cycle, has been employed in teacher professional development and teacher education. The aim of this study is to investigate secondary mathematics pre-service teachers’ PCK using lesson study and the characteristics of lesson study that contribute to the pre-service teachers’ PCK development.

Lesson study was embedded in a teaching practicum unit at one university in Jakarta, Indonesia. It involved two lower secondary schools, ten pre-service teachers, five mentor teachers, and two university lecturers. The pre-service teachers, mentor teachers and university lecturers worked together in four lesson study groups. Each group conducted two cycles of planning, teaching research lessons, and post-lesson discussions. This study used case study methodology. Data were collected using a written test, video-taping of the lesson study meetings, field notes, and lesson study artefacts. The Knowledge Quartet (KQ) framework (Rowland, Turner, Thwaites, & Huckstep, 2009) was used to analyse the data.

The KQ consists of four dimensions – Foundation, Transformation, Connection, and Contingency. The Foundation dimension concerns the teachers’ mathematical knowledge and beliefs about teaching mathematics. The Transformation dimension is about how the teachers use their mathematical knowledge to support student learning. This includes translating their mathematical knowledge into representations, examples, and demonstrations that students can understand. The Connection dimension concerns the coherence of the lessons as shown in the planning and within the lesson. It includes making clear links between mathematical concepts and aligning them with students’ prior knowledge. Lastly, the Contingency dimension refers to the teachers’ action in the moment of teaching, particularly their responses to unanticipated situations in the lesson.

In order to identify the development of the pre-service teachers’ PCK, this study developed a KQ Rubric. The KQ Rubric has three levels to specify the quality of each KQ component so that any improvement of any KQ component can be identified. Those improvements indicate the development of the pre-service teachers’ PCK. By using the KQ Rubric to analyse different data over different phases of lesson study, this study found the development of the pre-service teachers’ PCK over time.
This study found empirical evidence of the development of the pre-service teachers’ PCK during lesson study in all four KQ dimensions. Lesson study enabled the pre-service teachers to enact their PCK in the planning, research lessons, and post-lesson discussions. In the planning, the pre-service teachers demonstrated a development in the Connection dimension that is when they anticipated students’ responses. Moreover, while anticipating the students’ responses, the pre-service teachers re-thought and gained new knowledge of the mathematical concept which resulted in the improvement of the Foundation dimension. They also demonstrated a development in the Transformation dimension when they selected appropriate mathematical representations for the lesson. By discussing students’ work in the post-lesson discussion, the pre-service teachers could anticipate students’ responses better for the next lesson, thus they improved their Contingency Knowledge.

Moreover, this study found that the development of the pre-service teachers’ PCK is a complex process. By examining the pre-service teachers’ PCK throughout lesson study phases, this study found the interconnection of the KQ dimensions. The development of one KQ dimension is built on other KQ dimensions. While the Foundation Knowledge underpins the Transformation, Connection and Contingency dimension (Rowland et al., 2009), this study found new interconnections. Firstly, the findings showed that the Connection dimension underpins the pre-service teachers’ Transformation Knowledge. Secondly, the Connection Knowledge also contributes to the pre-service teachers’ Foundation Knowledge. Lastly, the Contingency Knowledge contributes to the pre-service teachers’ Foundation Knowledge.

The pre-service teachers found that lesson study was highly rewarding. They appreciated the constructive feedback they received during lesson study. This study found that the role of the mentor teachers and the knowledgeable other was important in the lesson study for pre-service teachers. Mentor teachers provided some support related to students’ characteristics and school curriculum. The knowledgeable other has created contingency in the lesson study which revealed the pre-service teachers’ Foundation Knowledge. Moreover, the knowledgeable other also directed the lesson study discussion on focusing on students’ thinking.

Nevertheless, this study found some constraints in the implementation of lesson study in the teaching practicum. The majority of the participants agreed that time was a major challenge in the lesson study. The other constraints were the dominant role of the knowledgeable other has weakened the autonomy of the participants, and the dynamics of the groups, which highlighted the challenge of establishing productive collaboration in lesson study.
1 Introduction

This study focuses on the implementation of lesson study for pre-service teachers during their teaching practicum program to develop their Pedagogical Content Knowledge (PCK). This chapter provides the background of the study and an overview of the context in which this study took place.

1.1 Background of the Study

Starting out as a secondary mathematics teacher, and then as a lecturer in a mathematics education program and teacher trainer, my belief about teaching has evolved. I used to think teaching is individual work. In my first year of teaching at a school, I did all my teaching duties by myself. Then I moved to another school with a different work environment. In this school, I had a weekly meeting with a mentor teacher where I showed him my lesson plan and we discussed it. As a result, I made a refined lesson plan. For me, this meeting helped build my understanding and competences in teaching. I learned about designing a lesson that supports student learning. Sometimes, I sat in my colleague’s classroom and observed how she conducted her lesson. I started to appreciate the work environment at this school and viewed teaching as collaborative work. Having this experience as a lecturer and teacher trainer, I often put my students or teacher participants of in-service training in groups to discuss issues in the classroom - for example students’ common misconceptions. In group discussion, students and teachers engage in a deeper analysis of the topic and gain a deeper understanding of student learning. So now I believe that collaborative inquiry into teaching practice can enhance teachers’ understanding of student learning and result in a better lesson.

From 2009 to 2012, The Faculty of Mathematics and Natural Sciences – Fakultas Matematika dan Ilmu Pengetahuan Alam (FMIPA) – State University of Jakarta was awarded a Lesson Study Grant from the Directorate General of Higher Education (DIKTI). This grant aimed to improve the teaching and learning at FMIPA through lesson study. Each department – Mathematics, Biology, Physics, and Chemistry formed lesson study groups. Each group consisted of a number of lecturers who taught the same course, designed the lesson plan together, invited other lecturers for observation during their teaching, and participated in post-lesson discussions afterwards. In 2010, I participated as a member of one lesson study group. After participating in lesson study, I appreciated how collaborative inquiry helped me improve my teaching.
In 2011, I participated in the training program *Improvement of Quality of Education through Lesson Study* in Asia, organised by the Japan International Cooperation Agency (JICA) in Hiroshima, Japan. There, I learned about Japanese Lesson Study through workshops and school observation. Returning from Japan, I embedded the lesson study principles in my teaching. In a unit called *Pembelajaran Matematika SMP* or *Teaching and Learning Secondary School Mathematics*, I asked my students to work in a group to design a lesson and conduct a micro teaching lesson. During the micro teaching, one group acted as the teacher, one group as observers, and the rest as students. We then had a discussion about the micro teaching. By doing this, I found that my students developed skills of conducting a more student-centred lesson. But, I was surprised that, when they went to school for their teaching practicum, many of them said that it was difficult to conduct student-centred lessons; the situation of a real classroom is so complicated. It seems that all the pedagogical courses and practices they have engaged with prior to their teaching practicum are difficult to incorporate into their classroom practicum.

Having seen a pleasing result of lesson study implementation in a micro teaching environment, I wanted to know if lesson study could also help pre-service teachers in a teaching practicum setting. Therefore, in 2012, I carried out a research of using lesson study for pre-service teachers during their teaching practicum program (see Meiliasari, 2013). The study was conducted in a single school with six pre-service teachers, three mentor teachers and three lecturers. The result showed that pre-service teachers developed their mathematics knowledge as well as knowledge about teaching mathematics. They carefully chose a teaching method or learning tool to help students build their understanding on a specific mathematics topic. For example, after having a lesson plan meeting, pre-service teachers decided to use graphing software when teaching about the gradient of parallel and perpendicular lines. They found that this software helped the students to understand the gradient of parallel and perpendicular lines. The result of the study implies that lesson study can be used as a model for improving teacher education programs. This has motivated me to continue working on lesson study for pre-service teachers, particularly to help pre-service teachers develop their Pedagogical Content Knowledge (PCK).

### 1.2 Education System in Indonesia

Indonesia is an archipelago with more than 17,000 islands and a population of 240 million people (BPS, 2014). Indonesia’s national language is Bahasa Indonesia, but more than
700 local languages are spoken in different places across the nation. Indonesia is culturally very diverse, therefore the education system in Indonesia is one of the most complicated systems in the world (Chang et al., 2014). As the biggest Muslim populated country, traditional Islamic education has a long history and has become a strong part of Indonesian society. Therefore, the Indonesian education system is managed by the Ministry of Education and Culture (MoEC) and the Ministry of Religious Affairs (MoRA). MoEC is in charge of national education policy while MoRA follows the national policy and manages Islamic schools. Islamic schools are equivalent to regular schools: primary schools – Sekolah Dasar (SD) as Madrasah Ibtidaiyah (MI), lower secondary school – Sekolah Menengah Pertama (SMP) as Madrasah Tsanawiyah (MTs) and high secondary schools – Sekolah Menengah Atas (SMA) as Madrasah Aliyah (MA). The curriculum in Islamic schools follows the national curriculum set by the MoEC with adjustment in the Islamic studies.

Until 2014, MoEC held the responsibility for managing all aspects of education in Indonesia, which involved formal education from early childhood education to higher education and informal education. In 2014, under the newly elected cabinet, basic education and higher education were separated into two different ministries. Basic education which includes early childhood, primary and secondary education is under MoEC, while higher education is under the Ministry of Research, Technology and Higher Education (MoRTHE). It is expected that under MoRTHE, universities can boost their research and contribute more to the development of technology (Santoso, 2014). However, this change does not apply to Islamic education. Islamic schools and Islamic universities are still the responsibility of MoRA.

Indonesian children’s school lives start when they enter kindergarten by the age of five or six. Kindergarten in Indonesia is not compulsory, thus many parents, especially those who live in rural areas, do not send their children to kindergarten. Compulsory education is nine years long – from primary to lower secondary schools. At the age of seven, children enter primary schools for six years, and then continue on to lower secondary for three years. After graduating from lower secondary, they continue for another three years at either higher secondary schools or vocational schools. The curriculum of higher secondary schools is designed for those who would like to continue their study in universities. Meanwhile, in vocational schools, the curriculum is designed so that students can enter the workplace after they finish.
With huge population and area, Indonesia has a large number of schools and teachers. The Table 1.1 shows the total number of schools, students, and teachers for each level of education in Indonesia. Despite having a low student-teacher ratio, the quality of Indonesian education is lower than that of its neighbouring countries. Results from the Programme for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS) reveal that Indonesian students perform among the lowest in the world (Mullis, Martin, Foy, & Arora, 2012; OECD, 2014). Inefficient teacher distribution and poor teacher quality might cause this problem (Chang et al., 2014). For example, schools in rural and remote areas have a shortage of teachers. Moreover, they tend to have teachers with lower academic qualifications. Furthermore, Chang et al. (2014) reported that in 2006, 83% of primary school teachers, 38% of lower secondary school teachers, and 18% of higher secondary school teachers were under qualified, that is they did not have a Bachelor’s degree.

Table 1.1 Number of schools, student, teachers and student to teacher ratio by school level (2012/2013) (Source: BPS, 2014)

<table>
<thead>
<tr>
<th>School Level</th>
<th>Number of Schools</th>
<th>Number of Students</th>
<th>Number of Teachers</th>
<th>Student : Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kindergarten</td>
<td>71,356</td>
<td>3,993,929</td>
<td>213,823</td>
<td>19 : 1</td>
</tr>
<tr>
<td>Primary School (SD)</td>
<td>148,272</td>
<td>23,939</td>
<td>3,269,771</td>
<td>18 : 1</td>
</tr>
<tr>
<td>Madrasah Ibtidaiyah (MI)</td>
<td>23,939</td>
<td>3,269,771</td>
<td>336,843</td>
<td>10 : 1</td>
</tr>
<tr>
<td>Lower Secondary (SMP)</td>
<td>35,527</td>
<td>9,653,093</td>
<td>552,083</td>
<td>18 : 1</td>
</tr>
<tr>
<td>Madrasah Tsanawiyah (MTs)</td>
<td>15,594</td>
<td>2,781,647</td>
<td>324,351</td>
<td>9 : 1</td>
</tr>
<tr>
<td>Higher Secondary (SMA)</td>
<td>12,107</td>
<td>4,272,860</td>
<td>252,405</td>
<td>17 : 1</td>
</tr>
<tr>
<td>Vocational Schools (SMK)</td>
<td>10,673</td>
<td>4,189,519</td>
<td>176,856</td>
<td>24 : 1</td>
</tr>
<tr>
<td>Madrasah Aliyah (MA)</td>
<td>6,728</td>
<td>1,065,922</td>
<td>198,359</td>
<td>5 : 1</td>
</tr>
</tbody>
</table>

One of the key factors in improving the quality of education is the quality of the teachers (Darling-Hammond, 2000). Therefore, to improve education, the government must improve teacher quality. In 2005, the Indonesian government passed the Teachers and Lecturers Law (hereafter termed as the Teacher Law). This law was derived from the 2003 National Education System Law which intended to improve the management of education in Indonesia, stressing the improvement of teacher quality and budget allocation. The National Education System Law mandates 20% of the national budget for educational expenditure. The large sum of money indicates that the government is serious about improving education in Indonesia.
Teacher certification is one major reform in the Teacher Law that is intended to improve teacher quality and teacher welfare in Indonesia. To be qualified for certification, a teacher first must hold a Bachelor’s degree (S1) from a 4-year university education and follow professional development program. Qualified teachers will be certified and receive additional income – the certification allowance which is the same amount as their monthly salary. In other words, once teachers are certified, their income will be doubled. When the certification was launched in 2007, 180,000 teachers were certified, and 200,000 teachers were certified in 2008 (Jalal et al., 2009).

Teacher certification was conducted in two ways, first, for in-service teachers through an assessment procedure. Second, for pre-service teachers through Teacher Profession Education program. The assessment for in-service teachers’ certification was based on academic qualifications and portfolio assessment. Teachers with a Bachelor’s degree must undergo professional trainings, and then they must include the certificates of accomplishment in their portfolio. However, portfolio assessment did not support the improvement of teachers’ daily teaching practice. Teachers attended seminars and conferences to fulfil the professional development requirement but this did not necessarily translate into any improvements in their teaching practice. Therefore, in 2007, following the evaluation of the portfolio assessment mechanism, MoEC decided to stop the portfolio assessment and change it to the Teacher Profession Training Program or Pendidikan dan Latihan Profesi Guru (PLPG). It is a 90-hour training program, designed to improve teachers’ competencies in pedagogy, knowledge of subject matter, and skills in action research. A test is given at the end of the program. A teacher who passes the test will be certified, otherwise they will repeat the training program. All in-service teachers were expected to be certified under this scheme by 2015.

The second procedure is targeted to pre-service teachers. Therefore it is designed to be an integrated teacher education program. Teacher education is conducted by teacher education institutions or universities that offer education programs. All higher education institutions under MoRTHE and MoRA that produce teachers are called Lembaga Pendidikan dan Tenaga Kependidikan (LPTK). These institutions offer Bachelor’s (S1) degrees in education to fulfil the demand created by the Teacher Law. Some LPTKs offers Master’s (S2) and Doctoral (S3) degrees. LPTKs offer programs for primary education, secondary education, and vocational education. Primary education is conducted in a special program called Primary School Teacher Education, Pendidikan Guru Sekolah Dasar (PGSD). Secondary education and vocational
LPTK graduates with Bachelor of Education degree must continue their study in a Teacher Profession Education program - *Program Profesi Guru* (PPG). This is a one year program concerned with pedagogy and teaching methodology in a specific subject area. It also includes a field experience program or *Program Pengalaman Lapangan* (PPL) in which the participants undertake their teaching practicum in schools and are supervised by a mentor teacher and university lecturer (Jalal et al., 2009). More specifically, PPG participants learn about pedagogy and content specific in the first semester, then they have teaching practice in the second semester (Kusumah & Nurhasanah, 2017). Moreover, PPG also facilitates non-teaching degree graduates with relevant academic background who are interested in becoming a teacher.

Kusumah and Nurhasanah (2017) reported that there are two types of PPG – General PPG and PPG-SM3T (*Sarjana Mendidik di Daerah Terluar, Terdepan, dan Tertinggal*/ Bachelor of Education teach at isolated, outermost, and underdevelop areas). General PPG is offered for LPTK graduates or non-LPTK graduates who are interested of becoming a certified teacher. In this scheme, the participants self-fund their education. PPG-SM3T is specifically for LPTK graduates who have served in a one-year SM3T program. SM3T was initiated by MoEC in 2011. MoEC, now MoRTHE recruited best LPTK graduates to teach in remote and isolated area in Indonesia. After serving SM3T, MoEC/MoRTHE awarded these LPTK graduates with a government-fully funded PPG (Kusumah & Nurhasanah, 2017).

Furthermore, regarding teacher quality, the Teacher Law defines teacher competencies in four areas, that is, pedagogical, professional, personal and social competencies. Jalal et al. (2009) explained that the pedagogical competence is about having knowledge of and skill in teaching, including for example understanding psychological aspect of learners. Teachers must have adequate knowledge of the subject they teach. This is referred to in The Teacher Law as the professional competence whereas social competence refers to teachers’ skills in communicating, being able to work cooperatively in a team, and having empathy for others. The personality competence refers to good personal characteristics and being a role model for the students.

Whilst Jalal et al. (2009) highlighted these four teacher competencies as a mark of teacher quality, they did not make explicit links to Shulman’s (1986) seminal notion of
Pedagogical Content Knowledge (PCK). It seems that pedagogical and professional competences relate closely to teachers’ PCK because these two competencies encapsulate teachers’ knowledge of subject matter and of pedagogy. Over the last decades, PCK has attracted global interest that researchers from different countries have studied PCK to understand teacher knowledge (Ball, Thames, & Phelps, 2008; Ben-Peretz, 2011; Gess-Newsome, 1999; Rowland et al., 2009) and improve teaching (Hill, Rowan, & Ball, 2005; Tirosh, Tsamir, Levenson, & Tabach, 2011).

However, there are only few studies investigating teachers’ PCK in Indonesia. Some studies investigated the impact of teachers’ PCK to their ability to teach using integrated approach (Adi Putra, Widodo, & Sopandi, 2017) and students’ motivation (Maryani & Martaningsih, 2015). Recently, an integrated approach is used in Indonesian elementary schools. It is where one theme is used to teach different subjects. This requires teachers to have adequate knowledge of different subject matter. Adi Putra et al. (2017) found that teachers’ PCK impact how they deliver integrated teaching, however, they did not clearly elaborate how teachers’ PCK is enacted when they teach science in an integrated learning approach. Maryani and Martaningsih (2015) used a quantitative approach to investigate the relationship between teachers’ PCK and students’ motivation. They found that there is a positive correlation between teachers’ PCK and students’ motivation (Maryani & Martaningsih, 2015). However, these studies did not explain how the teachers’ PCK are enacted in the teaching. To better understand teacher’s PCK in the action of teaching, it implies the need for more studies on how teachers enact their PCK in their teaching especially in Indonesian context.

1.3 Teacher Education System in Indonesia

As mentioned above, teacher education in Indonesia is conducted by LPTKs. A pre-service teacher must pass at least 144 credits to graduate and obtain a Bachelor of Education degree. To be a certified teacher, an LPTK graduate must complete PPG (Kusumah & Nurhasanah, 2017). In that sense, PPG is an integrated part of teacher education. MoRTHE highlighted the need of coherence in LPTKs’ and PPG’s curriculum. Therefore, MoRTHE called for LPTKs to reform their curriculum.

The curriculum of Bachelor of Education at LPTKs is developed by each study program, aligned with the guidelines from DIKTI. The curriculum for secondary school education programs typically consists of general subject units, pedagogy units, subject-specific units, teaching practicum, and research. On the other hand, PPG is more teaching oriented,
with 60% of its curriculum allocated for developing instructional materials, and 40% for the teaching practicum (Direktorat Pembelajaran, 2017).

One major change resulting from this curriculum reform is cutting down of the teaching practicum credits. In the previous curriculum, teaching practicum was weighted four credits. This meant pre-service teachers spent five days a week at school for one semester. In contrast, the new curriculum only allocated two credits for the teaching practicum, in which pre-service teachers spend two to three days a week at school for one semester. The reason for this change was because the teaching practicum is heavily weighted in the PPG, so LPTKs did not want to add more to it. Table 1.2 shows the distribution of credits in the mathematics education program at one university in Jakarta.

Table 1.2 Structure of the Secondary Mathematics Education program

<table>
<thead>
<tr>
<th>Types of units</th>
<th>Total credits</th>
</tr>
</thead>
<tbody>
<tr>
<td>General units</td>
<td>11</td>
</tr>
<tr>
<td>Pedagogical units</td>
<td>30</td>
</tr>
<tr>
<td>Mathematical units</td>
<td>90</td>
</tr>
<tr>
<td>Teaching practicum</td>
<td>2</td>
</tr>
<tr>
<td>Research</td>
<td>8</td>
</tr>
<tr>
<td>Others</td>
<td>3</td>
</tr>
</tbody>
</table>

In 2013, MoEC launched a pilot PPG under PPG-SM3T involving 2,475 pre-service teachers (Sobri, 2012). MoEC/MoRTHE select a few number of LPTKs with criteria such as the LPTK’s accreditation status (Direktorat Pembelajaran, 2017) to conduct PPG-SM3T (Kusumah & Nurhasanah, 2017). In 2017, 23 LPTKs from all over Indonesia were designated to organise PPG-SM3T (Direktorat Pembelajaran, 2017). While PPG-SM3T is running fully on MoRTHE support, the general PPG remains under-exposed. By the time this research was conducted, there are no data about general PPG.

The fact that only a small number of LPTKs in Indonesia offer PPG implies that the majority of pre-service teachers get little opportunity to learn in their teaching practicum. Studies have shown the importance of the teaching practicum for pre-service teachers’ learning, particularly for providing opportunity to integrate theory into practice (Allen & Wright, 2014). Moreover, pre-service teachers highly value the opportunity to learn from practice (Smith & Lev-Ari, 2005). While LPTKs’ curriculum only gives little weight to the teaching practicum,
it still has the potential to support pre-service teachers’ learning. Therefore, an innovative approach is needed in the teaching practicum to support pre-service teachers to have a better teaching and learning experience.

My prior research in using lesson study in the teaching practicum (Meiliasari, 2013) suggests that lesson study offers a promising opportunity to optimise pre-service teachers’ learning. Other studies using lesson study in Indonesia were done through a school-university partnership focusing on teachers’ learning (Saito, Imansyah, Kubok, & Hendayana, 2007; Suratno & Cock, 2009). These studies showed that the collaboration of teachers and university lecturers in lesson study has improved the teaching (Saito et al., 2007). The focus of school-university partnership in lesson study can be extended to pre-service teachers’ learning. However, studies have not explored this in Indonesia. Therefore, this study fills the gap by implementing lesson study in the teaching practicum to develop pre-service teachers’ PCK.

1.4 Organisation of the Thesis

This chapter has addressed the background of the study and provided some context of Indonesian teacher education in which this study took place.

Chapter 2 is the literature review, including two major components – Pedagogical Content Knowledge (PCK) and lesson study. It elaborates on the original conception of Shulman’s (1986) Pedagogical Content Knowledge and the criticisms about it in the literature, the research on developing teachers’ PCK, and the PCK frameworks used by researchers. The section on lesson study discusses Japanese Lesson Study and the use of lesson study for teacher professional development. It also focuses on the adaptation of lesson study in different countries and different contexts, especially lesson study in Indonesia and in teacher education. Finally, this chapter identifies the research questions.

Chapter 3 is about the research methodology focusing on different research paradigms and their implications for research methodologies. After elaborating on a wide range of research methodologies, this chapter provides justification of the methodology employed by this study – case study methodology. Moreover, this chapter also addresses the research trustworthiness, researcher’s role, and ethics.

Chapter 4 describes the research process, including the school and participant recruitment process, the context of the schools, the teaching practicum, and the implementation
of the lesson study. It provides a detailed description of the data collection and the data analysis processes.

Chapter 5 and 6 present the findings of the study. More specially, Chapter 5 relates to Research Questions 1 and 2. The findings in Chapter 5 are organised based on the Knowledge Quartet (KQ) dimensions with evidence of the development of the pre-service teachers’ PCK. Chapter 6 reports the findings related to Research Questions 3 and 4. The participants’ views are defined as the pre-service teachers’ opinion on the benefits and challenges of anticipating students’ solutions, and the participants’ opinion on the benefits and challenges of participating in lesson study. Lastly, this chapter addresses the affordances and constraints relating to the implementation of lesson study in pre-service teachers’ mathematics teaching practicum

Chapter 7 provides the discussion of the findings comparing it with previous studies. It also elaborates on the significance and the limitations of the study. Lastly, it provides implications for teacher education and future research.
2 Literature Review

This chapter presents a review of the literature regarding Pedagogical Content Knowledge (PCK) and lesson study with a focus on lesson study for pre-service teachers. It is followed by the implications for this study, that is the need to expand research in developing pre-service teachers’ PCK using lesson study that addresses the interconnections of PCK components across phases of lesson study. Research questions of this study are stated at the end of this chapter.

2.1 Pedagogical Content Knowledge (PCK)

The knowledge that teachers need to possess in order to teach effectively has been a major discussion in the teacher learning community. It has been widely accepted that teacher knowledge is considered as a key factor in the student achievements, underpinning policy in teacher requirement in many countries (European Commission Directorate General for Education and Culture, Organisation for Economic Cooperation Development, & University of Twente, 2010; Jalal et al., 2009; Porter-Magee, 2004). However, teaching requires more than subject matter knowledge, knowledge of students’ characteristics and understanding of curriculum also play an important role in successful teaching.

One of the seminal works on teacher knowledge is Shulman’s notion of Pedagogical Content Knowledge (PCK) in 1986. Shulman was concerned with the absence of attention paid to teachers’ subject matter knowledge in the research community and among policy makers (Shulman, 1986). He defines PCK as the combination of content knowledge and pedagogical knowledge which “represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction” (Shulman, 1987, p. 8). Moreover, PCK is “most likely to distinguish the understanding of the content specialist from that of the pedagogue” (Shulman, 1987, p. 8).

He proposed categories of knowledge that are important for teachers to be able to conduct effective teaching (Shulman, 1987) which consist of:

- general pedagogical knowledge,
- knowledge of learners’ characteristics,
- knowledge of educational context,
- knowledge of educational purposes and values,
- content knowledge,
- curriculum knowledge, and
- pedagogical content knowledge.

The first four categories relate to generic knowledge of teaching and the last three categories are specifically about subject-matter knowledge. Shulman focused on content knowledge, curriculum knowledge, and PCK. To be able to teach a particular subject, a teacher must have a mastery of the subject they teach and understand its structure – the facts, concepts and process of the subject and the links between them. Moreover, the teacher also needs to understand how such concepts are established and what grounds validate it. The curricular knowledge addresses how a teacher is familiar with topics from different subjects and subject topics taught at different year levels.

While the notion of PCK is considered an important milestone in the teacher education community, there have been a number of criticisms on PCK conceptualisation. Recently, Shulman (2015) himself points out some limitations of his original notion of PCK. The first limitation is the omission of non-cognitive attributes such as “emotion, affect, feelings and motivation” (p. 9). Other scholars highlighted the need to take into account teachers’ beliefs as elements of teachers’ PCK (Friedrichsen, Driel, & Abell, 2011; Henze & Van Driel, 2015; Lannin et al., 2013; Park & Oliver, 2008; Rowland et al., 2009). The second limitation of PCK according to Shulman (2015) is pedagogical mind instead of pedagogical action. He emphasised that there was a lack of attention on the teaching in action. The third limitation is that PCK does not seem to take into consideration social and cultural context (Shulman, 2015). Furthermore, he argues that teaching and learning cannot be separated from the social and cultural context, therefore PCK should encompass the social and cultural context. Tirosh et al. (2011) acknowledged that the social norms and the national or international curriculum were influencing factors in studying and interpreting teachers’ PCK. Lastly, Shulman (2015) points out that in the early work of PCK, he did not attend the students’ learning outcomes. Shulman’s original notion of PCK missed out the relationship between the way teachers teach and the student learning.

Other scholars also criticised Shulman’s PCK. Ball et al. (2008) argue that even though PCK is an important element in teacher knowledge and teaching, there were limited theoretical background and empirical studies supporting PCK conception, and the distinction between
each knowledge category was vague. Similarly, Marks (1990) criticised the unclear boundary of each knowledge category proposed in Shulman’s PCK might lead to an overlap of interpretation of PCK. However, Gess-Newsome (1999) had a different view about this boundary issue. She showed that there are two models of teacher knowledge – an integrative model and a transformative model. In an integrative model, distinguishing teacher knowledge is not possible because in the teaching, knowledge across different domains and contexts are integrated. On the other hand, a transformative model takes into account different categories of knowledge which teachers enact and transform in order to make students understand (Gess-Newsome, 1999). Fennema and Franke (1992) highlight that while it is important to define teachers’ knowledge, it is more important to understand how different categories of knowledge interact in the teaching.

Other scholars argue that Shulman holds a static view on teachers’ PCK, that is the “knowledge about teaching – which can be acquired and applied independently from the classroom context” (Depaepe, Verschaffel, & Kelchtermans, 2013, p. 13). As opposed to this static view, scholars hold a dynamic view of PCK – knowledge of teaching that is embodied in the act of teaching in a particular context (Alonzo & Kim, 2016; Depaepe et al., 2013; Petrou & Goulding, 2011). Consequently, this view influences current research on PCK. Depaepe et al. (2013) showed that most studies that hold static view of PCK are large-scale, involving tests to measure teachers’ PCK (e.g. Kleickmann et al., 2013). On the other hand, researchers who hold a dynamic view of PCK frame their studies using situated perspective, where PCK is observed in the action of teaching (e.g. Lannin et al., 2013; Park & Oliver, 2008; Petrou & Goulding, 2011). These studies are typically small-scale with data collected through observation, interviews or mentoring meetings.

2.2 Research on Teachers’ Pedagogical Content Knowledge (PCK) Development

Studies that aim to develop teachers’ PCK have been well documented (e.g. Park & Oliver, 2008; Vale, McAndrew, & Krishnan, 2011). Some studies used an ascertaining approach – examining the way teachers’ PCK evolves throughout their career or a training program. For example, Park and Oliver (2008) examined how teachers’ PCK evolves in their teaching. They found that PCK development is a complex process where “knowledge acquisition and knowledge use are interwoven within the context of instructional practices” (Park & Oliver, 2008, p. 278). They also found that teachers’ PCK becomes salient when they encountered contingent moments in the teaching (Park & Oliver, 2008). When teachers
encounter contingent moments, they have to incorporate all components of PCK in that moment to respond appropriately to students.

Some studies used an intervention approach – investigating the impact of a particular intervention to teachers’ PCK. For example, Vale et al. (2011) conducted a professional learning program for mathematics secondary teachers. They found that positioning practicing teachers as learners of mathematics could support them in making connection between mathematical concepts, thus support their PCK development (Vale et al., 2011). Other researchers used lesson study to develop teachers’ PCK. For example, Lucenario, Yangco, Punzalan, and Espinosa (2016) conducted a quasi-experimental study comparing a lesson study group and a non-lesson study group of chemistry teachers in The Philippines. They found that the teachers in the lesson study group showed higher teaching competence than those in the non-study group (Lucenario et al., 2016).

Teachers’ PCK is influenced by many factors. Studies show that to some extent, content knowledge is an influencing factor of PCK (Hill, Ball, & Schilling, 2008; Hill et al., 2005; Krauss et al., 2008). In line with this, most studies on teachers’ PCK are topic-specific (Henze & Van Driel, 2015; Lee, 2010). Other influencing factors of teachers’ PCK are teachers’ professional backgrounds, education qualifications and length of teaching experience are significantly associated with their PCK (Kleickmann et al., 2013; Lee, 2010). More specifically, Lannin et al. (2013) found that teachers’ focus on a particular PCK component shapes their PCK development. For example, when a teacher is focusing on developing tasks then his/her PCK will develop accordingly. Park and Oliver (2008) argue that students’ responses and misconceptions shape teachers’ PCK.

More specifically looking at pre-service teachers’ PCK, a number of studies aimed to develop pre-service teachers’ PCK. In science education, Nilsson and Loughran (2012) used Content Representations (CoRe) (Loughran, Mulhall, & Berry, 2004) over a semester long science method course. Also using CoRe, Aydin, Demirdogen, Nur Akin, Uzuntiryaki-Kondakci, and Tarkin (2015) investigated the interaction of pre-service teachers’ PCK components in a practicum course. In mathematics education, some researchers used intervention during a mathematics method course (Karp, 2010; Kinach, 2002). These studies found that pre-service teachers’ PCK development is a complex process. To be able to teach for understanding, pre-service teachers need to have a relational understanding of the subject knowledge (Kinach, 2002). Pre-service teachers’ mathematical knowledge and understanding
of the conception of mathematics determine their ability to listen and respond to the students (Karp, 2010). Using a structured tool such as CoRe to guide the focus of the intervention supports the pre-service teachers’ development of PCK (Nilsson & Loughran, 2012). Moreover, Aydin et al. (2015) found that pre-service teachers’ PCK is idiosyncratic – it is person-specific, and mentoring is found to be important for pre-service teachers’ PCK development.

2.3 Frameworks of Pedagogical Content Knowledge

The enactment of teachers’ knowledge in classroom practice is a critical aspect of PCK. Researchers argue that teachers’ mathematical knowledge for teaching is most visible during the act of teaching (Baxter & Lederman, 1999; Rowland et al., 2009). Therefore, it is important to identify evidence of teachers’ PCK occurs in their teaching. This section discusses methods of identifying and analysing the PCK of mathematics teachers.

Examining teachers’ knowledge in the teaching is complicated. The tacit nature of teacher knowledge is complex and often cannot be observed directly (Chick, Baker, Thuy, & Hui, 2006; Kagan, 1990), especially when it involves a large number of teachers. To overcome this methodological challenge, Chick et al. (2006) developed a framework to examine teachers’ PCK by using a questionnaire and interview, while Prescott, Bausch, and Bruder (2013) developed the Teacher Education Lesson Plan Survey (TELPS) to analyse pre-service teachers’ mathematics lesson plans to gather insight into their PCK. Other studies (Hill, Blunk, et al., 2008; Rowland, Huckstep, & Thwaites, 2005; Rowland et al., 2009) aimed to examine pre-service teachers’ PCK during the lesson and developed frameworks for analysing teachers’ PCK during the act of teaching. Different approaches have been used in developing PCK frameworks. Chick et al. (2006); Hill, Blunk, et al. (2008); Prescott et al. (2013) used a theoretical approach while Rowland et al. (2005) used a grounded approach.

Chick et al. (2006) developed a framework for investigating teachers’ PCK in teaching about decimal numbers. They proposed the categories of PCK – “clearly PCK”, “content knowledge in a pedagogical context”, and “pedagogical knowledge in content context”. “Clearly PCK” is the intertwining between content and pedagogy. For example, knowledge of students’ misconceptions, knowledge of the variety of mathematical models and representation. “Content knowledge in pedagogical context” includes knowledge about mathematics structure and understanding its connection, and the ability to decompose a mathematical concept into its key components so that it is appropriate for student learning.
“Pedagogical knowledge in content context” involves knowledge about teaching a particular content area. They used this framework to examine primary teachers’ PCK of decimals involving fourteen Australian Year 5 and 6 teachers. The teachers were asked to complete a questionnaire and undertake a follow-up interview. Their findings showed that not all aspects of the framework were displayed by the teachers, and teachers’ knowledge of decimal connection with other topics was complicated. While they claimed that their framework provided a lens to examine teachers’ PCK, it only captured teachers’ PCK as a result of learning or teaching experiences. Looking at PCK from a dynamic view (Moline-Dershimer & Kent, 1999), Chick’s et al. (2006) framework cannot unpack the interactions between the different categories of knowledge involved in a teaching situation. Chick et al. (2006) called for a follow-up study to examine the use of this framework in a lesson setting.

Some researchers have developed frameworks for examining mathematics teachers’ knowledge in the teaching. Some substantial work in this area are the Mathematical Knowledge for Teaching (Ball et al., 2008) and the Knowledge Quartet (Rowland et al., 2009) which will be discussed in the following sections.

### 2.3.1 Mathematical Knowledge for Teaching (MKT)

Ball et al. (2008) aimed to better understand the content knowledge that teachers need to conduct effective teaching in mathematics. They tested Shulman’s (1987) conception of content knowledge and PCK in mathematics lessons by conducting a qualitative analysis of the mathematics teaching practicum in a Year 3 classroom from 1989 to 1990 and designing measures of mathematical knowledge for teaching. After investigating the teaching, they came up with the notion “Mathematical Knowledge for Teaching” (MKT) that is “the mathematical knowledge needed to carry out the work of teaching mathematics” (Ball et al., 2008, p. 295).

Ball et al. (2008) unpacked the fundamental subject matter knowledge by elaborating on its sub-domains: Common Content Knowledge (CCK), Specialised Content Knowledge (SCK), and Horizon Content Knowledge (HCK) (see Figure 2.1). CCK is “the mathematical knowledge and skill used in settings other than teaching” (Ball et al., 2008, p. 399), for instance, performing the addition algorithm. While anyone may be able to perform the addition algorithm, a teacher must know more than that, including for examples identifying errors and understanding students’ reasoning behind these errors. Ball et al. (2008) categorised the knowledge and skills uniquely needed for teaching purposes as SCK, whereas HCK is an awareness of the connection of different mathematical topics across the curriculum. For
example, mathematics teachers teaching Year 7 need to know what mathematical topics students in Year 8 or higher learn and how those topics relate to the mathematics they teach. This knowledge enables teachers to set up a good mathematical foundation.

MKT also includes a more detailed description of PCK through its subdomains: Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum (KCC). KCS involves being able to anticipate students’ response and predict students’ thinking, difficulties and misconceptions. KCT is knowledge about mathematics and its teaching. This includes understanding how instructional design and pedagogical issues affect student learning. Even though Ball et al. (2008) placed Shulman’s curricular knowledge in the PCK domain, they were still unsure whether it may serve across several categories or stand on its own.

![Domains of Mathematical Knowledge for Teaching](image)

**Figure 2.1 Domain of the Mathematical Knowledge for Teaching (Ball, 2008, p. 403)**

MKT is a refinement of Shulman’s categories in two ways. First, by putting PCK in a larger picture – the Mathematical Knowledge for Teaching, helps mathematics teacher educators to understand the broad context in which PCK takes place. Second, MKT addresses subdomains in the Subject Matter Knowledge and Pedagogical Content Knowledge, and incorporates the curricular knowledge within PCK.
2.3.2 **Knowledge Quartet (KQ)**

Rowland et al. (2005) argue that teachers’ knowledge is developed and applied in the classroom. They used Shulman’s theoretical framework and developed a framework for analysing and understanding how pre-service teachers ‘mathematical knowledge is transformed in a classroom – the Knowledge Quartet (KQ). The KQ was developed by using a grounded approach. Through analysing video-taped lessons of twelve primary pre-service teachers, the authors come up with KQ components which are grouped into four KQ dimensions – Foundation, Transformation, Connection, and Contingency (Table 2.1). These dimensions embody Shulman’s teachers’ generic knowledge, subject matter knowledge, as well as Pedagogical Content Knowledge.

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Description</th>
<th>Components</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Foundation</strong></td>
<td>Teachers’ knowledge and understanding of mathematics pedagogy and their beliefs about it.</td>
<td>Awareness of Purpose; Identifying Errors; Overt Subject Knowledge; Theoretical Underpinning of Pedagogy; Use of Terminology; Use of Textbook; Concentration of Procedures.</td>
</tr>
<tr>
<td><strong>Transformation</strong></td>
<td>Representations and examples used by teachers as well as teachers’ explanation and questions.</td>
<td>Choice of Representations; Teacher Demonstrations; Choice of Examples; Use of Instructional Materials*.</td>
</tr>
<tr>
<td><strong>Connection</strong></td>
<td>Links made between different lessons, different math ideas, and between different parts of the lesson. Sequences of activities, awareness of possible difficulties and obstacles that students might have with different math topics and tasks.</td>
<td>Making Connection Between Procedures; Making Connection Between Concepts; Anticipation of Complexity; Decision About Sequencing; Recognition of Conceptual Appropriateness.</td>
</tr>
<tr>
<td><strong>Contingency</strong></td>
<td>Teachers’ readiness to respond to students’ questions, students’ wrong answers and ability to deviate from their lesson plan. (teachers’ readiness to react to situations that are almost impossible to plan for).</td>
<td>Responding to Students’ Ideas; Use of Opportunities; Deviation from Agenda; Responding to the (Un)Availability of Tools and Resources*.</td>
</tr>
</tbody>
</table>

* New components added by (Thwaites et al., 2011)

While the KQ was initially used to analyse primary pre-service teachers’ teaching, it has been refined overtime. One noticeable refinement of the KQ was carried out by Thwaites,
Jared, and Rowland (2011) who refined the codes of the KQ based on their studies using the KQ in a secondary mathematics classroom. They intended to test the ‘fit’ of the KQ as an analytical framework. Three pre-service secondary mathematics teachers were voluntarily involved in this study. Data were taken from observed and video-taped lessons and stimulated-recall interviews. While they did not find difficulties in using the KQ to analyse the lessons, they found some teaching episodes that did not correspond to the KQ codes. Hence, they proposed two additional codes (see Table 2.1). The first one emerged from the lesson episode about completing the square and using graphs to solve quadratic equations. This new code was placed under the Transformation dimension, related to teachers’ use of technology – “extended (interactive) explanation of the application of completing the square to graph-sketching, emphasising why the graphs must be as they are, rather than how to arrive at them” (Thwaites et al., 2011, p. 228). The second new code came under the Contingency dimension, concerning how the teacher responds to the (un)availability of tools and resources.

Underpinned by Rowland et al. (2005, 2009), Weston (2013) used a diagram to illustrate the interconnection among the dimensions (see Figure 2.2.). Note that Weston (2013) did not include the two additional components in the diagram. The Foundation dimension is concerned with the mathematical knowledge and beliefs about the teaching and learning of mathematics. Furthermore, Rowland et al. (2009) argue that the Foundation dimension underpins the other three dimensions, therefore in the classroom, it is manifested in the other three dimensions (Lin & Rowland, 2016). Moreover, the Foundation, Transformation, and Connection dimensions inform teachers’ decision making in a moment of contingency.

![Image](image.png)

Figure 2.2 Interconnection of the Knowledge Quartet dimensions (Weston, 2013, p. 287)

The Foundation dimension refers to the knowledge “acquired at schools or in teacher education, sometimes before it is put to use in the classroom” (Rowland et al., 2009, p. 29). In
contrast, the other three dimensions refer to knowledge-in-action, which are acted out by the teachers during planning and teaching. The Transformation dimension embodies Shulman’s PCK (Rowland et al., 2009). It refers to how teachers transform their mathematical knowledge in a way that is understood by students. This includes teachers’ choice and use of examples, representations and demonstrations.

The Connection dimension concerns “the coherence of the planning or teaching across an episode, lesson, or series of lessons” (Rowland et al., 2009, p. 31). It embodies teachers’ understanding of how mathematical concepts or procedures are connected and how to make this connection visible and understood by their students. The coherence in teaching includes teachers’ decision making about sequencing the structure of the instruction. Even though Rowland et al. (2009) argue that the Connection dimension is not directly linked to any of Shulman’s (1987) categories of knowledge for teaching, understanding the connection between mathematical concepts and making a logical sequence of teaching relates to the teachers’ knowledge of curriculum and subject matter (Turner, 2012).

The Contingency dimension concerns teachers’ capacity to respond in the contingent moments. This includes teachers’ responses to unanticipated students’ questions and how they manage unplanned events in the classroom. Rowland, Thwaites, and Jared (2015) argue that contingent moments can be initiated by the students, the teachers themselves, and the availability or unavailability of tools and/or resources. Teachers’ decision making in contingent moments and responding to unanticipated situations require an interplay of mathematical knowledge, pedagogical knowledge, and Pedagogical Content Knowledge (Rowland et al., 2009; Rowland & Zazkis, 2013).

Moreover, the KQ provides tools for understanding how teachers’ mathematical knowledge and pedagogical knowledge interacts and comes into play in the classroom (Petrou & Goulding, 2011). Even though the distinction between one dimension and others may not always be clear, Rowland, Turner, and Thwaites (2014) argue that the distinction is less significant than the categories of situations in which that knowledge occurs during teaching.

These categories of situation are typically captured as a snapshot taken at one particular event in the teaching. Because teaching is a continuous process in which the teachers’ knowledge is constantly played out, it is important to look at the knowledge in a broader way – throughout the lesson, the semester, etc. An attempt to answer this challenge has been done by Weston (2013). She argues that teacher knowledge exists in a continuum and that a
particular situation does not indicate the relative quality of the teacher knowledge, thus she developed a coding protocol to quantify the teacher’s mathematical knowledge using the KQ framework. Weston (2013) developed levels of the KQ components, which indicate their quantity and quality.

To sum up, the MKT and the KQ provide ways to examine and understand teachers’ knowledge in the act of teaching. Both frameworks extended Shulman’s PCK. Figure 2.3 illustrates how Shulman’s (1987) seven categories of knowledge for teaching are incorporated in the MKT and the KQ. The MKT redefines and clarifies the notion of subject matter knowledge, and Pedagogical Content Knowledge (Ball et al., 2008). On the other hand, the KQ categorises the situations where this knowledge come into play in the teaching (Rowland et al., 2014). The Contingency dimension is one element in the KQ that is not explicitly defined in the MKT. The Contingency dimension offers a way to investigate the complex relationship between planning and teaching. More precisely, how the planned instruction and anticipation of students’ responses or the unplanned instruction and unanticipated responses impact the teaching. Moreover, it also offers a way to investigate what knowledge is acted out by the teachers in contingent moments.

![Figure 2.3 Incorporation of Shulman’s (1987) teachers’ knowledge in the MKT (Ball et al., 2008) and the KQ (Rowland et al., 2009)](image-url)
2.4 Lesson Study

Lesson study is a collaborative practice in teaching conducted by a group of teachers and sometimes supported by an external expert, the knowledgeable other (Fernandez, 2002; Groves & Doig, 2010b). Lesson study mainly consists of a cycle of formulating student learning goal and long-term development, collaboratively planning the lesson, conducting the research lesson, and reflecting the research lesson based on evidence of students’ learning (Lewis, 2009). In the planning sessions, the teachers design a lesson in which the learning goal will be brought to life, that is what they want their students to achieve through their instruction (Fernandez, 2002). The outcome of the planning sessions is a well-developed lesson plan that describes the lesson in detail (Fernandez, 2002). In line with the learning goal, in the planning sessions the teachers will also formulate the focus of observation of the lesson by observers (Lewis, Perry, & Murata, 2006). Next, in the teaching phase, which is often called the research lesson. One teacher will teach in the classroom while others will observe the students’ learning. Post-lesson discussion provides opportunities for a deeper analysis of the lesson in light of students’ learning process and teacher reflection, which results in a refinement of the lesson (Lewis, 2009).

2.4.1 Japanese Lesson Study

Lesson study is a model of teacher professional development established in Japan. The Japanese term of lesson study is “jugyo kenkyu” which means the study of lessons (Shimizu, 2014). Its origin can be traced down to early 1900s (Fernandez & Yoshida, 2004). To this day, lesson study has evolved to meet the educational needs. There are three different levels of lesson study – school level, district or regional level, and national level (Fujii, 2014).

In Japan, lesson study typically consists of four steps (Fernandez & Yoshida, 2004; Lewis, 2002, 2009) as shown in Figure 2.4. Lewis constructed this model based on her observation of Japanese Lesson Study (Lewis, 2002). The first step is goal setting. It includes identifying the learning problems by considering students’ characteristics and their long-term learning goals (Huang & Shimizu, 2016). This becomes the research theme for the lesson study (Fujii, 2016). This process is called “kyozaikenkyu” – it is the process of transforming planned curriculum into the classroom instruction (Baba, 2007). In this process, teachers study the curriculum, tasks or problems, and instructional tools before planning the lesson (Takahashi, 2015). However, Fujii (2016) suggested that kyozaikenkyu can also take place in the planning.
Watanabe, Takahashi, and Yoshida (2008) described kyozaikenkyu as an intensive and complex process, which focuses on studying instructional materials and encompasses two stages.

![Lesson Study Cycle](image)

Figure 2.4 Lesson study cycle (Lewis, 2009, p. 97)

The first stage involves teachers studying various instructional materials, asking themselves questions such as how the materials can be used to achieve the learning goals and are they suitable for teaching the concept? This process helps teachers deepen their understanding of the teaching materials as well as the subject knowledge (Doig, Groves, & Fujii, 2011; Takahashi, 2015; Watanabe et al., 2008). The second stage in kyozaikenkyu is concerned with how the instructional materials help the student learning. This includes anticipating students’ solutions and difficulties (Watanabe et al., 2008). Through this process, teachers become aware of students’ common mistakes and address them when planning the lesson (Doig et al., 2011).

The second step – planning – is where the teachers study teaching material to address the research theme (Fujii, 2016; Huang & Shimizu, 2016; Lewis, 2009). Moreover, Fujii (2016) demonstrated the way in which Japanese teachers engaged in a detailed discussion about the tasks including the selection of the numbers in the tasks, the context, connection to the curriculum and learning goals, and the anticipated student solutions. They could take the tasks from textbooks, modify them from the textbooks, or develop new tasks (Fujii, 2016). This process could take more than half a year to design a task and plan a lesson (Fujii, 2014), and results in a detailed lesson plan. A Japanese lesson plan typically consists of: (1) name of the
unit, (2) objective of the unit, (3) research theme, (4) students’ characteristics, (5) learning plan of the unit, (6) plan for the research lesson, and (7) background information and data collection form for observers (Fujii, 2016). Logistics and organisation of the research lesson are not discussed in the planning.

The third step is a research lesson where one member of the lesson study group teaches the lesson while other members observe and collect evidence of students’ learning (Fujii, 2016). Sometimes teachers from outside the lesson study group also observe the research lesson. In addition, staff members of the school, teachers from other schools, and the knowledgeable other observe the research lesson too. In the research lesson, the observers carefully pay attention to how students learn (Takahashi, 2015). Murata (2011, p. 3) argues that the “live research lesson creates a unique learning opportunity for teachers” where they share each other’s classroom experience and thus they can notice certain aspects of teaching and learning. Finally, the last step is reflection, commonly known as post-lesson discussion, where observers share evidence of students’ learning. It includes discussing how students learn from the task in the research lesson, which may result in revision of the task (Fujii, 2016). However, Fujii (2016, p. 421) emphasised that “this does not imply that re-teaching is necessarily part of Japanese Lesson Study”.

Typically, in mathematics, Japanese lessons are built on structured problem-solving. It is intended to make mathematics interesting for the students, and to trigger creativity in learning mathematics (Takahashi, 2006) – “it is not for students to solve a problem, but through solving the problem to learn mathematical ways of thinking” (Fujii, 2014, p. 9). The problem-solving activities provide rich opportunity for the students to re-invent the mathematical concepts themselves (Takahashi, 2006). Therefore, the instruction is structured as follow: (1) presenting a single problem that provokes students’ thinking; (2) while students are working on the problem, the teacher monitors students’ work, or in other words undertakes purposeful scanning; (3) students compare and discuss different strategies they use; and (4) the teacher sums up and guides the students to understand the reason for selecting the most sophisticated strategy (Fujii, 2016; Groves, Doig, Vale, & Widjaja, 2016).

The structure of the instruction emphasises the critical selection of the problems and activities. This requires teachers’ abilities to select or develop tasks that enable students to use their prior knowledge to construct an understanding of the intended concepts (Takahashi, 2006). Because of this, kyozaikenkyu is an important process in lesson study (Fujii, 2014;
Groves et al., 2016). Furthermore, Groves et al. (2016) argue that students’ comparing and discussing different strategies is the ‘heart’ of the instruction. It is where students learn the important mathematical ideas and concepts.

### 2.4.2 Lesson Study for Teacher Professional Development

Lesson study supports teachers’ professional development through focusing on student learning and teacher collaborative practice. Focusing on student learning supports teachers’ Pedagogical Content Knowledge (Lewis, 2009). This is embedded throughout the planning, research lessons, and post-lesson discussions (Huang & Shimizu, 2016). During planning teachers set learning goals (Fujii, 2014), design proper tasks to make students’ thinking visible (Lewis, Perry, & Hurd, 2009; Murata, 2011), and discuss possible students’ solutions (Fujii, 2016; Widjaja, Vale, Groves, & Doig, 2017). In the research lessons, the focus of the observation is the students’ learning not the teacher’s performance (Cajkler, Wood, Norton, & Pedder, 2014; Fernandez & Yoshida, 2004), thus it requires skilful observation of students’ learning (Lewis, Perry, Hurd, & O’Connell, 2006). Lewis, Perry, Friedkin, and Roth (2012) argue that observers in a research lesson can develop their knowledge of student thinking and use it to improve their own teaching. Lastly, in the post-lesson discussions, the teachers analyse the evidence of students’ learning to improve the tasks (Fujii, 2015; Widjaja et al., 2017).

Focusing on the students’ learning enhances teachers’ learning and teaching practice development through deeper exploration of the content of the subject, how to teach it (Dudley, 2013; Lewis et al., 2009; Stigler & Hiebert, 1999; Takahashi, 2015; Widjaja et al., 2017), and the ways in which the students learn the concepts (Lewis, 2009). Murata (2011) argues that lesson study brings different categories of knowledge together, enables them to interact with one another thus, it helps teachers transform their knowledge in the teaching context.

Next, teacher professional development is facilitated through the collaborative learning communities of the lesson study groups (Doig & Groves, 2011; Lewis et al., 2012; Takahashi, 2015; Warwick, Vrikki, Vermunt, Mercer, & van Halem, 2016). Lieberman (2009) argues that lesson study supports the openness and collaborative community norms, which enable teachers to develop their professional identity. The collaborative planning and reflecting of lessons serve as an effective means of developing teachers’ individual expertise (Cajkler et al., 2014; Lieberman, 2009). Lesson study provides a collaborative environment for teachers to discuss the content of the subject they teach, the ways of teaching it, and student thinking. Through this discussion, teachers have opportunities to learn about the content and the instruction
methods, which are aligned with the Pedagogical Content Knowledge (Fernandez, 2005; Lewis et al., 2009; Warwick et al., 2016).

2.4.3 Adaptation of Lesson Study

In the late 1990s lesson study started to gain international attention when Stigler and Hiebert (1999) wrote about how the Japanese teaching practice outshone that of other developed Western countries in terms of student achievement. Some challenges in launching lesson study are commonly found in different countries. Simply importing lesson study in a ‘copy-paste’ approach to a new country without acknowledging the country’s cultural differences to Japan can risk successful adaptation of lesson study (Ebaeguin & Stephens, 2014; Ebaeguin, 2018). Furthermore, Sato, as cited in Kusanagi (2014) pinpointed the importance of acknowledging the diversity of local cultures where lesson study is to be implemented.

Globally lesson study is understood as a collaborative teacher learning through planning, conducting and observing lessons and post-lesson discussions. However, the implementation of lesson study is influenced by local factors such as national regulations and teaching norms at the school. Huang and Shimizu (2016) classified these influencing factors into macro-level and micro-level factors. Macro-level factors are concerned with the educational system, and the micro-level factors are conditions related to the participant. Furthermore, they argue that these factors influence the effectiveness of adaptation of lesson study (Huang & Shimizu, 2016). Some researchers argue that inappropriate adaptation of lesson study makes lesson study ineffective (Fujii, 2014; Yoshida, 2012).

Studies highlight the complexity of adapting lesson study in different countries. Time arrangements and teachers’ discourse are found to be challenging in implementation of lesson study in the United States (Chokshi & Fernandez, 2004). American teachers associate teaching as an individual private work where their classroom is isolated from others, therefore they are reluctant to open their classroom for a research lesson. Similarly, Lewis (2015) reported some problems in the implementation of lesson study in the United States, such as the individualistic rather than collaborative teaching culture, and participants’ understanding their role in lessons study. For example, she noted in the United States, the knowledgeable other told the teachers what to teach instead of being a co-researcher. In other countries such as Indonesia and The Netherlands, constraint comes from the pressure of preparing students for high-stake examinations (Kusanagi, 2014; Verhoef, Tall, Coenders, & Smaalen, 2014).
Moreover, unlike the Japanese curriculum, structured problem-solving is not always included in other countries’ curriculums (Lewis, 2015), thus teachers may not be familiar with developing and working with such problems (Groves et al., 2016; Widjaja et al., 2017). Groves et al. (2016) showed that one of the difficulties of working with structured problem-solving for Australian teachers is anticipating students’ solutions. To resolve this problem, they tried out the problem in different classes at the same level before using it in research lessons. By doing this, they were able to identify a wide range of students’ possible solutions, predict students’ difficulties as well as develop prompts for class discussion (Groves et al., 2016). Fujii (2014) found that even though teachers in Malawi and Uganda used structured problem-solving, they treated it as another way of solving problems rather than using it to build students’ understanding.

Even though Japanese Lesson Study in mathematics is usually associated with structured problem-solving, research on mathematics lesson study outside Japan does not always focus on structured problem-solving. Some researchers have investigated the impact of lesson study to the quality of mathematics teaching and learning (e.g., Lim, Kor, & Chia, 2016; Verhoef et al., 2014), and mathematics teacher learning (Warwick et al., 2016).

Organisational challenges were also reported in the literature. For example, in Australia getting replacement teachers for the teachers who are observing research lessons is costly (Groves et al., 2016). Therefore, expanding lesson study outside Japan requires willingness from teachers and school management to use lesson study for the schools’ professional development and individual teachers’ expertise development (Cajkler et al., 2014; Groves & Doig, 2010a; Saito, Harun, Kuboki, & Tachibana, 2006).

Sustainability and scaling up have been reported as a challenge in some countries (Akiba & Wilkinson, 2016; Lim, Lee, Saito, & Syed Haron, 2011). Unlike Japan, where the teachers have internal motivation from the benefits of participating in lesson study, other countries typically initiate lesson study through a top-down approach. It is initiated by the government (Akiba & Wilkinson, 2016) hence it is often project-oriented. Therefore, continuity is one important aspect in Japanese Lesson Study that seems to be missing in the adaptation in other countries (Fujii, 2014). Lack of support from district or state government in securing funding is reported to be one of the major challenges for lesson study in the United States (Akiba & Wilkinson, 2016; Murata, 2011; Yoshida, 2012). Lim et al. (2011) conducted
a survey with school leaders and teachers in Singapore, they found that the following factors are important for lesson study sustainability:

(i) the school leader (principal or vice-principal) is critical in providing supporting conditions for LS to take root in a school and flourish;
(ii) school leaders feel that LS is able to impact on student learning and outcomes;
(iii) school leaders feel that LS is able to impact on teacher knowledge, particularly subject matter knowledge and pedagogical content knowledge;
(iv) protected time for LS meetings; and
(v) presence of an advocator among the teachers.

(Lim et al., 2011, p. 362)

The important role of the knowledgeable other for an effective lesson study has been shown in some studies (Chichibu, 2016; Lewis, Perry, Hurd, et al., 2006; Takahashi, 2014). In Japan, the knowledgeable other is an external content or educational expert invited by the school to observe the research lesson and provide ‘final comments’ in the post-lesson discussions (Takahashi, 2014). When possible, the knowledgeable other also joins the planning (Fuji, 2016). In the United States, beside observing lessons, collecting data of students’ work and commenting on the lesson, the knowledgeable other is also asked by the school to teach public lessons (Lewis, Perry, Hurd, et al., 2006). Some studies examined the final comments of the knowledgeable other in Japanese Lesson Study and showed that effective final comments focus on curriculum and its connection to the topic taught (Takahashi, 2014), including ideas, concretes examples and suggestions to help teachers improve the teaching (Chichibu, 2016; Takahashi, 2014). However, there is a missing area in the literature concerning how the knowledgeable other supports teachers in the planning and the role of the knowledgeable other in lesson study outside Japan.

2.4.4 Lesson Study in Indonesia

Unlike in Japan where lesson study was established organically within the teacher community, in Indonesia, lesson study was first initiated by the central government. In 1999 the Indonesian Government, that is the Ministry of Education, together with the Japanese International Cooperation Agency (JICA) started a joint project, the Indonesian Mathematics and Science Teacher Education Project – Japan International Cooperation Agency (IMSTEP – JICA). The project aimed to develop good practice in mathematics and science teaching and learning (Marsigit, 2007). Three universities participated in this project: Indonesia University of Education (UPI) in Bandung, State University of Yogyakarta (UNY), and State University of Malang (UM). These universities conducted ‘piloting activities’ (PA) of lesson study – that is a school-university partnership project in which a team of faculty members and school
teachers collaborated in lesson study. The lesson study was intended to develop teaching methods, materials, as well as teaching evaluation (Marsigit, 2015). Within this project, they developed lesson plans, conducted research lessons, and reflection on the lessons (Saito et al., 2006). The participating teachers perceived lesson study as an insightful professional development. They improved their teaching skills such as questioning and orchestrating discussion during lesson study (Marsigit, 2015).

JICA and Indonesian experts conducted an evaluation of IMSTEP and proposed a successor program, a follow up IMSTEP (Hendayana, 2014; Suratno, 2012). Some changes and improvements were made based on the evaluation of IMSTEP. For example, the reflection or post-lesson discussion was carried out as soon as possible after the lesson, and was intended to focus on students’ learning activities (Hendayana, 2014). Therefore, the team of faculty members and teachers developed observation and reflection guidelines (Suratno, 2012). However, there was a problem related with the teachers’ understanding of lesson study. This was especially prominent during the observation of the research lessons. The observers focused on the teacher instead of the student learning. As a result, teachers were discouraged by the fear of criticism and perceived lesson study as an evaluation of their teaching (Kusanagi, 2014; Marsigit, 2015).

To achieve sustainable improvement and broader impact by engaging more schools and teachers in lesson study, a successor program was launched in 2006. It was called, *Strengthening In-Service Teacher Training in Mathematics and Science at Secondary Level* (SISTTEMS). This program was intended to improve mathematics and science education in lower secondary schools at a district level by empowering the Subject Teacher Group Work or *Musyawarah Guru Mata Pelajaran* (MGMP) (Suratno, 2012). Three districts in Java Island participated in SISTTEMS – Sumedang in West Java Province, Bantul in Yogyakarta Province, and Pasuruan in East Java Province. To maintain the school-university partnership, the implementation of lesson study in each district was under the supervision of a teacher education institution. Sumedang district’s program was under the supervision of UPI, Bantul district’s was supervised by UNY, and Pasuruan district’s was supervised by UM. In the case of Sumedang district, SISTTEMS involved 94 schools, 556 teachers, and eight superintendents divided into eight working groups (Hendayana, 2014; Suratno, 2012). Each working group created a lesson study group, called MGMP-based lesson study. Group members were subject teachers from different schools located in a cluster, together with a superintendent. The groups were supported by faculty members from UPI. Suratno (2012) reported that SISTTEMS laid
the foundation of learning communities involving different stakeholders such as teachers, principals, superintendents, and university lecturers. Moreover, Suratno (2012) pointed out that even though SISTTEMS has a positive impact on student learning, Indonesian teachers’ teaching skills such as questioning and classroom management still need to be improved.

Studies showed some characteristics of Indonesian lesson study. In Indonesia, lesson study is widely understood by the teachers and teacher educators as cycles of planning the lessons together, conducting the lesson and reflecting on the lesson; or as “Plan – Do – See” (Kusanagi, 2014; Suratno & Cock, 2009). This is probably a result of interpreting and simplifying the four steps of Japanese Lesson Study cycle (Fernandez & Yoshida, 2004; Huang & Shimizu, 2016). Goal setting, which encompasses identifying the problems in achieving students’ long-terms goals is missing in Indonesian lesson study. Setting up long-term goals is not a common practice for Indonesian teachers. Due to the fact that most Indonesian teachers teach for national examination (Kusanagi, 2014), it can be assumed that their goal is to help students pass their exams. Without connecting lesson study to the current teaching problem, this indicates the Indonesian lesson study is less grounded in teacher professional practice. Furthermore, lesson study might be perceived by the teachers and teacher educators as a temporary professional development program without any relevance to long-term teaching and learning goals.

The next characteristic of Indonesian lesson study is the school-university partnership. Because the introduction of lesson study in Indonesia was done through the teacher education institutions, the lesson study initiatives were mostly done as a school-university partnership (Suratno, 2012). This partnership provided a great opportunity for the university lecturers to improve their understanding of classroom realities (Saito et al., 2007). However, some challenges were also identified such as the issues of time-consuming lesson study (Saito et al., 2007) and hierarchical differences between the lecturers and the teachers that weakens the teachers’ autonomy and sense of ownership of their teaching (Suratno, 2012). Moreover, Saito et al. (2006) found that during lesson study implementation in Indonesia, teachers and university lecturers participating in lesson study showed more interest in teaching methods rather than students’ learning processes and misconceptions. Unfortunately, how this partnership supports teachers to improve their teaching is missing in the literature.

As well as school-university partnership, some universities implemented lesson study in their internal teaching and learning improvements. Saito, Hawe, Hadiprawiroc, and
Empedhe (2008) reported an implementation of lesson study at the Faculty of Mathematics and Natural Sciences, State University of Yogyakarta (FMIPA UNY). They found that the practice of lesson study in teacher education institutions has promoted a reform in teacher education. Faculty members shifted from working individually to collaboratively in groups, while teaching practice became more student-centred. However, this study did not examine the pre-service teachers’ learning during the lesson study implementation.

Only a few studies showed the cultural obstacles in Indonesian lesson study. Kusanagi (2014) conducted an ethnography study of an Indonesian lesson study at one school in Java Island. She found some sociocultural factors that distinguished Indonesian lesson study to Japanese Lesson Study. The first factor is the teacher responsibilities. In Indonesia, there is a strong pressure of the National Examination – *Ujian National* (UN) for students to pass school level. This is a high-stake test, which often causes teachers to teach for the test and limit teaching for understanding. Kusanagi (2014) argues that this shaped how teachers perceived lesson study. The teachers interpreted lesson study as an evaluation of the lesson and that it had little relevance to their daily teaching practice.

Moreover, Kusanagi (2014) found that in the planning the teachers showed more concerned toward the administrative aspect of lesson study, such as preparing students’ seating chart to help the observers recognise the students. In the case of mathematics lesson study, even though the teachers claim that lesson study encouraged them to conduct student-centred lessons through hands-on and group activities, they did not embody structured problem-solving in their lesson. The teacher designed the research lesson to ‘activate the student’. In contrast to their regular teaching, where the students learned mainly through lecturing and rote memorisation, during research lessons students are engaged in group work and discussion. This resulted in new classroom norms, consequently challenging the routine classroom management. It was not surprising then that in the post-lesson discussions the observers commented on classroom management and students’ general activities with no interpretations of the process of the students’ learning (Kusanagi, 2014; Saito et al., 2008). Furthermore Kusanagi (2014) found that hierarchical collegiality among the teachers did not support openness to share and discuss their lessons.

There is a gap in recent literature on Indonesian lesson study in recent years. This might be because the JICA-supported projects ended in 2013. This suggests that even though the initiation of lesson study was done intensively, it lacks sustainability. Recent studies of lesson
study in Indonesia mostly lack empirical evidence (e.g. Bastiana, 2017; Triyanto, 2015). JICA continues to implement lesson study projects until present time, but it is limited to trainings for teacher educators. Lecturers from Indonesian teacher education institutes were sent to Japan for a short training of lesson study. However, the outcome of this project is not clear and whether these lecturers initiate or contribute to the lesson study program at their institution or its partner schools remains unknown.

2.4.5 Lesson Study in Teacher Education

The previous section has discussed that lesson study is an effective strategy in teacher professional development. Much evidence on how lesson study supports teachers to develop their teaching competencies has raised a growing interest among teacher educators to utilise lesson study in pre-service teacher education programs. A number of recent studies have been done in this area. Because of the different nature of in-service teachers and pre-service teachers (such as pre-service teachers’ limited or non-existent teaching experience) (Mostofo & Zambo, 2015), an adaptation to the lesson study implementation is needed. For in-service teachers, lesson study can be embedded in the school’s professional development program while for pre-service teachers this might seem impossible because they have not yet entered the profession. Moreover, pre-service teachers do not have as much opportunity teaching in school as in-service teachers.

Lesson study in teacher education takes place in different models. There are three models found in the literature: (1) on campus, (2) school-based practicum, and (3) a combination of both. An on campus model is where the lesson study was embedded in units in the teacher education curriculum. For example, microteaching lesson study (Fernández, 2010; Susetyarini & Miharja, 2017), in which pre-service teachers conducted cycles of lesson study, within a microteaching setting. Fernández (2010) argues that the simplified classroom setting allowed pre-service teachers to focus on the overarching learning goals – the students’ mathematical reasoning. By doing this, pre-service teachers learned to develop tasks that support students’ reasoning and at the same time, they also developed their mathematical understanding. Similarly, Mostofo and Zambo (2015) used lesson study in a method class to help pre-service teachers prepare their lessons before their teaching practicum. The pre-service teachers designed, taught, revised, and re-taught the lesson in the method class. This lesson was then taught in the teaching practicum classroom. The study found that lesson study provided a direct connection between the method class and the teaching practicum. Using lesson study to prepare
the lessons for the teaching practicum has helped the pre-service teachers gain confidence (Mostofo & Zambo, 2015).

The school-based practicum is when the lesson study is conducted at schools and embedded in the teaching practicum. The pre-service teachers and mentor teachers, sometimes with a knowledgeable other, form lesson study groups and conduct cycles of planning, research lessons and post-lesson discussions (Chassels & Melville, 2009). This model supports situated learning in which the pre-service teachers get to experience teaching in a classroom (Lee & Choy, 2017; McMahon & Hines, 2008; Meiliasari, 2013; Rock, 2003). These studies found that the collaboration between pre-service teachers and in-service teachers supports pre-service teachers’ skills and knowledge of teaching (McMahon & Hines, 2008; Meiliasari, 2013; Rock, 2003). However, pre-service teachers also reported that the mentor teachers did not always provide good teaching practice examples (Rock, 2003). Furthermore, Meiliasari (2013) found that time constraint associated with the mentor teachers’ teaching schedule hindered them from participating fully.

A combination model blends the content of the units with a little bit of school experience. Typically, this is done by completing the planning and post-lesson discussions on campus, and the research lesson component at a participating school (Corcoran & Pepperell, 2011; Myers, 2012, 2013; Ricks, 2011; Sims & Walsh, 2009). Some studies also reported that they involved pre-service teachers and in-service teachers in the collaborative lesson study group (Burroughs & Luebeck, 2010; Leavy & Hourigan, 2018; Myers, 2012, 2013; Parks, 2008; Post & Varoz, 2008). These studies show that pre-service teachers appreciate the collaboration in a group and the constructive feedback from colleagues (Post & Varoz, 2008; Sims & Walsh, 2009). Pre-service teachers were able to develop their content knowledge through engaging in lesson study (Corcoran & Pepperell, 2011). The structure of lesson study allowed pre-service teachers to experience reflective processes as they collaboratively prepared, tested, refined, and re-tested their lessons (Ricks, 2011). However, these studies also showed some challenges while implementing this model. For example, the pre-service teachers did not show a sense of ownership of the lesson (Sims & Walsh, 2009). Post and Varoz (2008) showed that both pre-service teachers and in-service teachers noted the difficulties in expressing balanced critique without intimidating the feelings of others.

Recently, researchers have implemented lesson study in teacher education to develop pre-service teachers’ PCK (Corcoran & Pepperell, 2011; Leavy, 2015; Leavy & Hourigan,
Using Mathematical Knowledge for Teaching (MKT) (Ball et al., 2008), the studies by Leavy (2015) and Leavy and Hourigan (2016) revealed the interrelation between the knowledge subdomains in the MKT framework (Leavy, 2015; Leavy & Hourigan, 2016). Leavy (2015) found that knowledge of content and teaching (KCT) and knowledge of content and students (KCS) lead to the development of specialised content knowledge (SKC). Lesson study was instrumental in supporting the pre-service teachers’ mathematical knowledge development through understanding relevant concepts for teaching a topic and awareness of the need to identify the source of mathematical errors (Leavy & Hourigan, 2018). Due to the structured lesson analysis provided by lesson study (i.e. discussing teaching problems that occurred in the research lessons), pre-service teachers were able to revisit and improve their mathematical understanding.

Similarly, Shuilleabhain (2016) found that the pre-service teachers involved in his study shifted their focus from the organisation of the teaching to students’ thinking. More specifically, he found evidence of pre-service teachers’ PCK development in planning when anticipating students’ responses, identifying and incorporating students’ prior knowledge in the lesson plan, determining the sequence of instructions, and using appropriate language to support students’ understanding. Moreover, the post-lesson discussions provided opportunities for the pre-service teachers to notice and interpret students’ thinking in the research lessons and to evaluate the mathematical representations or models used in the research lessons (Shuilleabhain, 2016).

Even though many studies reported success in the implementation of lesson study in teacher education, the process was not always smooth. Incorporating lesson study for pre-service teachers can be challenging. Some studies have showed that when the research questions and the focus of observation are missing from lesson study, it was very difficult for the pre-service teachers to achieve the desired learning goals (Bjuland & Mosvold, 2015; Parks, 2008). More specifically Bjuland and Mosvold (2015) found that the pre-service teachers and mentor teachers in their study focused on what the students should learn but paid no attention to what the pre-service teachers should learn. While it is important to focus on student learning, Bjuland and Mosvold (2015) suggested that the pre-service teachers should also establish their own learning goals “which includes posing a research question targeting the student teachers’ own learning” (p. 89).
Researchers recommended the need of guidance in lesson study for pre-service teachers (Parks, 2008; Sims & Walsh, 2009). This implies the central role of the mentor teachers or lecturers in lesson study for pre-service teachers. However, there are only few studies focused on the role of the mentor teachers and the knowledgeable others in lesson study for pre-service teachers. Some studies focused on investigating the learning of pre-service teachers, mentor teachers and knowledgeable others during lesson study (Amador & Weiland, 2015; Cajkler & Wood, 2016a, 2016b, 2016c). Cajkler and Wood (2016c) found that the collaborative practice in lesson study allows not only pre-service teachers, but also mentor teachers, to develop pedagogical skills. Amador and Weiland (2015) showed that lesson study supports professional noticing, focusing on students’ mathematical thinking not only for the pre-service teachers, but also for the mentor teachers and university lecturers. Moreover, Amador and Carter (2018) pinpointed the essential role of the lesson study facilitator in supporting professional noticing through focusing on students’ thinking.

Only recently Bjuland and Helgevold (2018) investigated the mentoring conversation in lesson study. They found that when the mentoring was supported by scaffolding tools and the mentor teachers served as the knowledgeable other, lesson study creates dialogic space in which the pre-service teachers together with the mentor teachers interact and interlink. More specifically, they found that this dialogic space encouraged the pre-service teachers to focus on the predicting and observing of student learning, as well as reflecting on their own teaching.

Despite differences of the lesson study type and models, these studies share some commonalities regarding the benefits of incorporating lesson study in teacher education. Lesson study supports the pre-service teachers’ understanding of student learning, curriculum, and teaching strategies (Chassels & Melville, 2009; Sims & Walsh, 2009), and pedagogical and mathematical knowledge (Corcoran & Pepperell, 2011; Lamb, 2015; Leavy, 2015; Leavy & Hourigan, 2016, 2018). Studies show that lesson study reinforces pre-service teachers learning in many ways. The structure of lesson study allows pre-service teachers to experience reflective processes as they collaboratively prepare, and refine their lessons (Cajkler & Wood, 2015; Myers, 2012, 2013; Ricks, 2011) and engage in noticing to students’ thinking (Amador & Weiland, 2015; Lee & Choy, 2017).

The literature also reveals the challenges of incorporating lesson study in teacher education. Many studies agree that time is a major constraint; it deals with the participants’ external commitment and the school timetable causes difficulties in setting up time for the
lesson study meeting (Chassels & Melville, 2009; Post & Varoz, 2008; Rock, 2003). Other aspects regarding pre-service teachers’ knowledge and experience were also highlighted. With no or little teaching experience, it is difficult for pre-service teachers to predict students’ responses and address misconceptions (Burroughs & Luebeck, 2010). Because of pre-service teachers’ lack of experience, they tend to focus on the logistics of the lesson rather than the process of students’ learning (Bjuland & Mosvold, 2015; Lee & Choy, 2017; Meiliasari, 2013; Post & Varoz, 2008; Sims & Walsh, 2009). Moreover, pre-service teachers’ lesson analysis and reflection often only described the lesson (Galani & Kostas, 2014), and was very unlikely to make a critical evaluation of the lesson (Myers, 2013; Ricks, 2011). This implies that constant guidance might be needed for pre-service teachers to direct their focus on student learning throughout the lesson study implementation (Sims & Walsh, 2009).

### 2.5 Implications for this Study

Taking a dynamic view of PCK, developing PCK is best in situated learning. Lesson study is teacher-directed and grounded in their teaching, therefore it can serve as a vehicle to develop pre-service teachers’ PCK. In the curriculum of secondary pre-service teacher education in Indonesia, pre-service teachers have field experience in a teaching practicum unit. Therefore, this study implements lesson study in a teaching practicum to develop pre-service teachers’ PCK.

In adapting lesson study into different countries, studies have suggested the incorporation of local culture in the implementation lesson study and some modifications in the organisation of lesson study. Therefore this study employed some adjustments in the implementation of lesson study for pre-service teachers in the teaching practicum. Detailed information about the lesson study implementation will be presented in Chapter 4.

Most research on lesson study in teacher education focuses on pre-service teachers learning. This is done by engaging the pre-service teachers in lesson study. Some researchers maintained a so called ‘traditional lesson study’ in which a group of pre-service teachers, sometimes involving an expert as the knowledgeable other, plan a lesson together, then one pre-service teacher teaches the lesson while others observe. Lastly, they discuss the lesson that was taught and make improvements for future lessons, then another member of the group teaches the next lesson in a different classroom (Fernandez, 2002). Even though researchers incorporate this traditional lesson study, because of logistical difficulties their data collection and data analysis can only partially capture the lesson study. For example, some studies do not
include research lessons in their analysis and only use data from planning and post-lesson discussions (Cajkler & Wood, 2016c; Lee & Choy, 2017; Shuilleabhain, 2016). Consequently, the detailed learning process in each lesson study phase and the interconnection between the learning in one phase and the other are still under examined. This study attempts to fill this gap by investigating the interconnection of the learning process in each lesson study phase. Furthermore, this study is particularly interested in including an investigation of the pre-service teachers’ PCK at contingent moments, therefore, the KQ framework is chosen to guide the analysis of the data.

2.6 Research Questions

This study poses the following research questions:

RQ 1 What changes are evident in pre-service teachers’ PCK during lesson study in their teaching practicum?

RQ 2 What characteristics of lesson study contribute to the development of pre-service teachers’ PCK?

RQ 3 What are pre-service teachers’, mentor teachers’, and university lecturers’ views about the incorporation of lesson study into pre-service teachers’ mathematics teaching practicum?

RQ 4 What are some of the affordances and constraints relating to the implementation of lesson study in pre-service teachers’ mathematics teaching practicum?
3 Methodology

This chapter addresses the research paradigm underpinning the study, the case study methodology, the rationale for the selection of case study as the research methodology, the researcher’s role and ethics.

3.1 Research Paradigms

Guba and Lincoln (1994) define a paradigm as a set of basic beliefs or metaphysics about fundamental principles. These paradigms are related to ontology, epistemology and methodology. Ontology is a study about being (Crotty, 1998). It concerns the ways in which the researcher views reality and builds knowledge about how things really are and how things really work (Guba & Lincoln, 1994; Ling & Ling, 2017; Scotland, 2012; Willis, 2007). Epistemology is the philosophy of knowledge – it is about how one acquires knowledge (Willis, 2007) and “the nature of the relationship between the knower or would-be knower and what can be known” (Guba & Lincoln, 1994, p. 108). Methodology is about the strategy or plan of action in order to know something. It underpins the selection and use of particular methods (Scotland, 2012). Researchers’ views regarding ontology, epistemology, and methodology often determine their choice of research paradigm. Borg, Gall, and Gall (2005) stated that there are two main paradigms – positivism and interpretivism.

Positivism assumes that there is a single reality and it exists and is driven by natural laws and mechanisms (Guba, 1981; Guba & Lincoln, 1994). Objectivity is the main concern for the positivists (Ling & Ling, 2017) that the epistemological position held by positivists is that the researcher is independent from the object being researched (Guba & Lincoln, 1994). This implies that researchers should be able to conduct their investigations without influencing the object or being influenced by it (Guba & Lincoln, 1994). Thus, the subject of interest is defined “in terms of observable behaviour” (Borg et al., 2005, p. 14) so that it can be measured. Many quantitative research are underpinned by a positivist paradigm. Their data collections are carried out in quantitative ways, using measurement instruments. Data analysis is intended to find cause and effect relationships which can be generalised (Atkins & Wallace, 2012).

In interpretivism, reality is regarded as subjective – different people might perceive it differently (Atkins & Wallace, 2012; Scotland, 2012), hence there are multiple realities (Guba, 1981). Using an interpretivist lens, knowledge emerges through one’s consciousness about the world. Interpretivism aims to understand a phenomenon from an individual’s perspective.
Thus, knowledge does not exist independently without someone who gives it meaning (Borg et al., 2005). Data collection methods are intended to capture many aspects of the individual as well as their environment – for example by using open-ended interviews and observations. Instead of providing generalisation, interpretivism provides a detailed description of the social phenomena (Wahyuni, 2012). Even though it is subjective, a conclusion is drawn from “a systematic evidenced investigation supported by a coherent argument” (Ling & Ling, 2017, p. 8). Qualitative research is typically built on interpretivism paradigms.

To sum up, Table 3.1 shows the research paradigms relate to their ontological, epistemological and methodological background.

<table>
<thead>
<tr>
<th></th>
<th>Positivism</th>
<th>Interpretivism</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ontology</strong></td>
<td>There is only single reality. Knowledge are driven by existing laws and</td>
<td>There are multiple realities. Knowledge</td>
</tr>
<tr>
<td></td>
<td>theories</td>
<td>are socially constructed</td>
</tr>
<tr>
<td><strong>Epistemology</strong></td>
<td>Objective. The researchers maintain distance from the object under research</td>
<td>Subjective. The researchers and the</td>
</tr>
<tr>
<td></td>
<td></td>
<td>phenomena under research are interrelated</td>
</tr>
<tr>
<td><strong>Methodology</strong></td>
<td>Quantitative. To provide generalisation.</td>
<td>Qualitative. To provide detailed description</td>
</tr>
</tbody>
</table>

This study investigates the development of pre-service teachers’ PCK through an implementation of lesson study in their teaching practicum program. The aim is to understand how pre-service teachers develop their PCK through the lesson study process. Thus, the focus is on the pre-service teachers’ enactment of their PCK as well as the aspects of the environment in which the lesson study is being implemented, such as the interaction among the lesson study participants, contributions of the mentor teachers and university lecturers. For these reasons, this study employs an interpretivism paradigm.

### 3.2 Educational Research

Research in education covers a wide range of topics and is often multidisciplinary. For example, researchers in education and neuroscience might collaboratively investigate how the brain works when an individual is learning a new thing. Borg et al. (2005, p. 3) define educational research as “the systematic collection and analysis of data in order to develop valid, generalizable descriptions, predictions, interventions, and explanations relating to various aspects of education”. Furthermore, they explain that descriptive research is intended to
describe an educational phenomenon through detailed observations, while predictive research is used to predict future behaviour or events based on current data (Borg et al., 2005). Researchers often use intervention to improve educational practice whereas other researchers aim to explain individual or group behaviour, or cause-effect relationships (Borg et al., 2005).

3.2.1 Quantitative Research in Education

Quantitative research in the educational field seeks to find out the relationship between variables especially the cause and effect relationship (Kervin, Vialle, Herrington, & Okely, 2006). To conduct quantitative research, researchers need to pay attention to clearly determine the variables of interest and control that no other variables influence the outcome. Then they need to select standardised measurement or establish the measurement device, finally use statistical means to get the result (Kervin et al., 2006).

Quantitative research has some strengths and limitations. Its strengths include the high degree of confidence in the findings resulted from the precision and control in the design. It also provides information about the causes behind the observed effects. Nevertheless, educational is a complex field, where many factors come into play and influence the educational outcomes. Therefore, the mechanistic characteristic of quantitative research has been criticised because it does not allow researchers to capture the complexity of human behaviour (Kervin et al., 2006).

3.2.2 Qualitative Research in Education

Qualitative research aims to understand social phenomena in their natural settings (Kervin et al., 2006). It is more interested in understanding the process instead of the outcome by making sense of the whole context. In contrast to quantitative research, qualitative research addresses the complexity of an educational setting through exploring the factors influencing the educational outcomes.

However, because qualitative research uses small numbers of participants it has been criticised for the validity, reliability and generalisability (Kervin et al., 2006). The relationship between the researcher and the participants raises the problem of objectivity in qualitative research. The following section addresses some qualitative research methodologies used in educational field.
**Ethnography**

Historically ethnography was used by anthropologists and sociologies because it aims to describe and analyse the practice and beliefs of a community (Freebody, 2003). In the educational field, ethnography offers ways to describe and interpret an educational phenomenon through immersion in its formal or informal settings using interview and observation (Atkins & Wallace, 2012; Freebody, 2003). More specifically, Pole and Morrison (2003, p. 16) define ethnography as:

An approach to social research based on the first-hand experience of social action within a discrete location, in which the objective is to collect data which will convey the subjective reality of the lived experience of those who inhabit that location.

In that point of view, ethnography captures the reality in its natural setting. This means ethnography aims at understanding the practice, not to change the practice (Hammersley & Atkinson, 2007).

**Action research**

The focus of action research is to improve practice (Atkins & Wallace, 2012). Therefore it is typically deliberate – it is designed and focused to change practice, rather than explanatory (Freebody, 2003). Moreover, because education is an ongoing process, to improve practice, it is important to respond, reflect and make adjustment in the practice. Therefore, action research is iterative (Atkins & Wallace, 2012). In the educational field, action research is usually conducted by education practitioners in their own settings to improve their practice and their students’ learning (Efron & Ravid, 2013). From this point of view, action research provides ways to improve education from the inside – the practitioners, instead of outside researchers.

**Case Study**

Yin (2014, p. 16) defines case study as “an empirical inquiry that investigates a contemporary phenomenon in depth and within its real-world context especially when the boundaries between phenomenon and context may not be clearly evident”. Stake (1998, p. 86) claims that case study “is not a methodological choice but a choice of object to be studied”. The focus of case study is to understand a certain social behaviour or activity in its natural settings (Opie, 2004). Data collection methods in case study can be done in various ways (Yin, 1981) including methods that are closely associated with quantitative research (Baxter & Jack,
However, in educational research, understanding aspects of learning from an individual or group perspective within a certain context attracts many researchers to use qualitative case study.

Case study offers a flexibility to explore a wide range of contexts and situation, as the case can be a person, a group of people, an organisation, etc. (Yin, 2014). It can be a means to investigate “connection, patterns and context, and of reflecting on the bigger picture as well as on detail” (Atkins & Wallace, 2012, p. 108). However, Atkins and Wallace (2012) point out that case study findings cannot be used to generalise cause and effect relationship beyond the evidence presented.

3.3 Research Trustworthiness

Establishing rigor is a fundamental aspect in doing a qualitative research. In quantitative research, researchers use validity and reliability to evaluate their research findings. Validity is concerned with the accuracy of the instruments used to measure the phenomenon under study and the generalisation of the research findings that is how the findings can be applied to a larger population (Krefting, 1991). Reliability is concerned with the stability and consistency of the study (Krefting, 1991). However, the purpose of qualitative research is not to generalise but to explore holistic phenomenon in multidimensional contexts (Thomas & Magilvy, 2011). Thus, some scholars argue that due to the differences between the nature and purpose of quantitative and qualitative research, the terms validity and reliability do not fit when evaluating qualitative research. Therefore, different terms such as trustworthiness or credibility are used to describe the rigor of qualitative research (Guest, MacQueen, & Namey, 2012; Krefting, 1991; Thomas & Magilvy, 2011). To avoid confusion, this study uses trustworthiness to indicate rigor.

According to Carnine (1995) trustworthiness is concerned with how the research findings are based on a well-designed study with clear specification of its context. Krefting (1991) described Guba’s (1981) four aspects of trustworthiness for qualitative studies as: (1) credibility; (2) transferability; (3) dependability; and (4) confirmability.

3.3.1 Credibility

Credibility deals with the question of how researchers establish confidence in the truth of their findings (Krefting, 1991). This concerns how the data is drawn from credible sources. Data in this study were collected using multiple sources, such as observations, interviews, and
a written test. To maintain the credibility, data instruments such as interviews and written test were developed using the KQ framework. A detailed description of the data collection will be addressed in Chapter 4. Moreover, interpretations during data analysis also determines credibility (Creswell, 2014; Wiersma & Jurs, 2009). To strengthen the credibility of the analysis in this study, data were organised in such a way that enabled easy access for the researcher to revisit them. To establish a credible interpretation, data were analysed systematically and the evidence from different data source were triangulated. A detailed description of the data organisation and analysis will be presented in Chapter 4.

3.3.2 Transferability

Transferability is concerned with the degree to which the findings are applicable to other contexts and settings (Krefting, 1991). Transferability is often associated with generalisation of the research. Some scholars argue that generalisation refers to how the findings apply to other people’s situations (Merriam, 1998; Stake, 1978; Yin, 1981). In qualitative research, the researcher needs to provide a detail explanation of their research design and the context in which the research was conducted so that the readers can make judgement whether it is applicable in different contexts (Mertens, 2010). The detailed description of research design and the context of this study will be addressed in Chapter 4.

3.3.3 Dependability

Dependability is related to the question of whether the findings would be consistent if the study was replicated in a similar context (Krefting, 1991). In qualitative research, replicability might be problematic since there are many interpretations. For this reason, replication of a qualitative study might not produce the same result (Merriam, 1998). However, there are strategies for strengthening dependability in qualitative research. Thomas and Magilvy (2011) suggested that in order to establish dependability, researchers must provide explanations for the reasons for the data collection sources and methods; triangulate the data collection and data analysis methods; and explain how the data were reduced and transformed for analysis. The detailed description of these processes is presented in Chapter 4.

3.3.4 Confirmability

According to Thomas and Magilvy (2011), confirmability is established when credibility, transferability and dependability are fulfilled. It concerns with the objectivity of the
researcher (King, 2017). Krefting (1991) suggests that confirmability can be strengthened through reflexive analysis where the researcher shows their awareness of their influence on the data. The reflexive analysis of my dual role in this study will be addressed in Section 3.5 and Section 6.2.1. Furthermore, to distinguish my role as the knowledgeable other in lesson study with my role as the researcher, when reporting the findings throughout Chapter 5 and 6, I clearly described the role of the knowledgeable other in the lesson study.

3.4 Implications for This Study

This study seeks to understand the development of pre-service teachers’ PCK through the implementation of lesson study in their teaching practicum program in an Indonesian university. Therefore the context is inseparable from the phenomenon being studied. Lesson study is an intervention, consequently the phenomena under study is not in its natural setting. Because of that, ethnography is not a suitable methodology for this study. One might associate cyclic process of lesson study to the iterative characteristic of action research. Furthermore, lesson study may be a promising means to improve practice; however, it is not the intention of this study. Instead, it focuses on understanding how pre-service teachers enact and develop their PCK during their participation in lesson study. Because of that, action research was not suitable for this study. Case study is the appropriate methodology for this research because it allows the researcher to examine the development of the pre-service teachers’ PCK within the lesson study context. The single case in this study is defined as “one lesson study cycle of one lesson study group”.

3.5 The Researcher’s Role

Many studies have highlighted the important role of a knowledgeable other in lesson study (see, for example, Corcoran & Pepperell, 2011; Lewis, Perry, Hurd, et al., 2006; Takahashi, 2011, 2014; Takahashi & McDougal, 2016). In order to carry out effective lesson study and support pre-service teachers’ PCK development, this study takes into account the role of a knowledgeable other. A knowledgeable other is required to be present at each research lesson and post-lesson discussion, thus time availability is an important factor. Due to their busy schedule, the university lecturers had limited time to participate in the study. They were not able to attend all research lessons. Because of that, I took the role of the knowledgeable other. I observed and interacted with the participants of this study.
Domenico, Laura, and Phillips (2009) claim that when a researcher observes and interacts with the subject of interest while actively participating in the setting, as well as gaining knowledge of practice through intense immersion in the field of the study, the researcher acts as a participant observer. As a participant observer, the researcher can learn about the activities under study through observing and participating in those activities (Kawulich, 2005). Especially in educational research, participant observers immerse themselves in the learning community to understand the participants’ experiences and perspectives as well as to support their learning (Fowler-Amato, 2017). My role as a participant observer included organising and facilitating the lesson study meetings. More precisely, as a researcher, I set up schedules of the lesson study meetings with the participants, and as the knowledgeable other, I chaired the meetings and directed the focus of the discussions.

3.6 Ethics

This research involved interacting with human beings in the implementation of lesson study. According to National Health and Medical Research Council, Australian Research Council, and Australian Vice-Chancellors’ Committee (2007) such research raises ethical issues. Therefore, this study required ethical clearance. Data for this study were collected in Indonesia, however, as a Deakin University student, I followed Deakin University Research Ethics standards. There were no requirements to get ethics clearance from Indonesia.

Pre-service teachers, mentor teachers, and university lecturers were informed about the consequences of participating in this study. For example, lesson study implementation requires their extra work and time commitment. Once they were aware of the rights and responsibilities of participants in this study, they were asked to voluntarily consent to participating in the research. Parents of students whose classroom was used for the research lesson were asked for their consent, together with students themselves.

Deakin University Human Research Ethics Committee approved the Plain Language Statement and Consent Form before they were distributed to each of the participants. All participants – university lecturers, mentor teachers, pre-service teachers as well as students and their parents/guardians were provided with consent forms (Appendix 2) to indicate their willingness to participate in the research before the fieldwork commenced. Those who consented to participate in the study gave their permission to have their photos or videos taken
as data. Furthermore, participants’ identities were confidential. Participants’ faces in the video or photos were blurred and pseudonyms were used in the data analysis and research report.

There is no power relation issue regarding the researcher’s role as the knowledgeable other. The researcher does not evaluate the pre-service teachers’ individual performance during the teaching practicum. The research does not influence the pre-service teachers’ evaluation or assessment of their teaching practicum. This is mentioned in the Plain Language Statement. Further discussion of the dual role of the researcher/knowledgeable other will be presented in Chapter 6.

To maintain the security of data storage, data were stored in two external hard disks, as well as in Deakin University’s online data storage system. The hard disks are kept in a locked drawer, while the online data storage is protected by account identification and password.

3.7 Conclusion

The aim of this study is to investigate the development of pre-service teachers’ PCK during the implementation of lesson study in the teaching practicum. Because the context of teaching practicum cannot be separated from the focus of the study, this study employs case study methodology. Detailed descriptions of the research process, data collection and data analysis will be presented in Chapter 4.
4 Research Process

This study investigated the development of pre-service teachers’ Pedagogical Content Knowledge (PCK) during the implementation of lesson study in their teaching practicum program at a university in Jakarta, Indonesia. The pre-service teachers’ PCK investigated in this study is embedded in lesson study within a teaching practicum program. Therefore, the data collection and data analysis process cannot be separated from the context. This chapter describes the research process and the context in which the research takes places.

4.1 The Teaching Practicum

Teaching practicum is one of the compulsory units offered for third year pre-service teachers. To be enrolled in the teaching practicum, the pre-service teachers are required to have passed some prerequisite units on pedagogy and mathematics. By the time the fieldwork was conducted, the university employed the new curriculum in which the teaching practicum is allocated two credit points. Unlike other units where two credits are equivalent to two 50 minutes face-to-face sessions per week, the teaching practicum requires pre-service teachers to be at school at most three days a week during the whole semester, or roughly about four months. At the end of the teaching practicum, the lecturers and the pre-service teachers set up the time for the exam.

The university collaborates with the partner schools where a group of maximum six pre-service teachers conduct their teaching practicum. One lecturer is assigned as the supervisor for those pre-service teachers at each school. The school assigns the mathematics teachers as the mentor teachers for the pre-service teachers. There is no specific qualification for mentor teachers but the schools usually assign senior teachers as the mentor teachers.

4.2 The Schools

The lesson study was carried out in two lower secondary schools in Jakarta where pre-service teachers undertook their teaching practicum. The schools were selected using purposive sampling. In case study research, purposive sampling allows researchers to select participants or sites that provide rich information to help answer the research questions (Borg et al., 2005; Creswell, 2014). The selection of the schools was based on the following criteria. First, the selected schools have been collaborating with the university for several years; thus, they are
open to research and innovation. Second, due to practical reasons, the locations of the schools were close to the university to allow the researcher a convenience in travelling to the schools during the implementation of the lesson study program.

The recruitment process of schools and participants started by getting permission from the head of Mathematics Department of the University to conduct the research within the teaching practicum unit. Next, two lower secondary schools were selected from 14 schools, namely SMP D and SMPN E Jakarta. These two schools were in close proximity to the university and have been the partner schools of the university in previous research projects.

The lecturers of both schools were approached personally. After having a brief introduction to the research, they voluntarily agreed to participate. The pre-service teachers who had the practicum at both schools were invited for an introductory session of the research. After hearing the explanation of the research, they agreed to participate. The next step was getting approval from the selected schools to participate. I asked the principals of both schools for permission to conduct the research. After permission was granted, I approached the mentor teachers and explained the research to them. They all agreed to participate in the research.

SMP D is a private school located within walking distance from the university. The school complex is home for lower secondary school, higher secondary school and vocational school. The lower secondary has three parallel classrooms for each year level. SMPN E is a public school located in East Jakarta. It has eight parallel classrooms for each year level. The school building was under renovation, thus some of the classrooms were unavailable. To accommodate the classes, besides using its own school building, the school used the building of a primary school nearby – SDN D Jakarta. With this arrangement, classes were conducted in the morning and in the afternoon. Table 4.1 shows the classes arrangement for each year level. This arrangement influenced the lesson study schedule and will be discussed in 4.4.6.

<table>
<thead>
<tr>
<th>Year level</th>
<th>Location</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>SMPN E Jakarta</td>
<td>Morning (07.00 AM – 12.00 AM)</td>
</tr>
<tr>
<td>8</td>
<td>SMPN E Jakarta</td>
<td>Afternoon (12.30 PM – 17.30 PM)</td>
</tr>
<tr>
<td>7</td>
<td>SDN D Jakarta</td>
<td>Afternoon (12.30 PM – 17.30 PM)</td>
</tr>
</tbody>
</table>
4.3 Participants

Two lower secondary schools and five mentor teachers, ten pre-service teachers and two university lecturers participated in this study (see Table 4.2). Names are pseudonyms. The practicum was conducted in Year 7 and Year 8 classes at both schools. Each year level forms a lesson study group. The researcher was the knowledgeable other at both schools.

Table 4.2 Lesson study participants

<table>
<thead>
<tr>
<th>Schools</th>
<th>Lecturers</th>
<th>Lesson Study Group</th>
<th>Year</th>
<th>Mentor teachers</th>
<th>Pre-service teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMP D</td>
<td>Siti (f)</td>
<td>D7, D8</td>
<td>7, 8</td>
<td>Nur (f), Ida (f), Raya (f)</td>
<td>Diana (f), Gina (f), Pipit (f)</td>
</tr>
<tr>
<td>SMPN E</td>
<td>Farida (f)</td>
<td>E7, E8</td>
<td>7, 8</td>
<td>Rusdi (m), Irwan (m)</td>
<td>Jeni (m), Jamal (m), Umar (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Name</th>
<th>Years of teaching</th>
<th>Years of mentoring</th>
<th>Lesson Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Siti (L)</td>
<td>31</td>
<td>No data</td>
<td>Participant in a lesson study program as observer</td>
</tr>
<tr>
<td>Farida (L)</td>
<td>28</td>
<td>20</td>
<td>Participant in a lesson study training Participant in a lesson study program as observer and lead lecturer</td>
</tr>
<tr>
<td>Nur (MT)</td>
<td>5</td>
<td>0</td>
<td>None</td>
</tr>
<tr>
<td>Hani (MT)</td>
<td>7</td>
<td>0</td>
<td>None</td>
</tr>
<tr>
<td>Rusdi (MT)</td>
<td>6</td>
<td>0</td>
<td>None</td>
</tr>
<tr>
<td>Irwan (MT)</td>
<td>9</td>
<td>0</td>
<td>None</td>
</tr>
<tr>
<td>Nani (MT)</td>
<td>37</td>
<td>No data</td>
<td>None</td>
</tr>
</tbody>
</table>
4.4 Research Design

Since most of the participants have no knowledge and experience of lesson study, it is important that they have adequate knowledge about lesson study before they carry it out. Therefore, lesson study workshops were carried out at both schools. Moreover, because the lesson study was embedded in the teaching practicum, the lesson study implementation was designed to fit together with the school schedule. Because of those reasons, a lesson study workshop was conducted in each school. Following the practicum structure, the pre-service teachers observe the mentor teachers’ classroom before teaching the class themselves. Finally, the participants start the lesson study in which they work in a group doing cycles of planning, research lessons and post-lesson discussions. Figure 4.1 shows the design of the lesson study implementation.

4.4.1 Lesson Study Workshop

Because the initial design of this study was to focus on pre-service teachers’ Pedagogical Content Knowledge (PCK) in algebraic reasoning, the lesson study workshop aimed to inform the participants about lesson study and teaching algebraic reasoning. The workshop was conducted in each school separately. Due to room availability, the workshop in SMP D was conducted over two days. Both workshops were attended by the mentor teachers, the pre-service teachers, and the lecturers. SMPN E had the workshop in one day which took about three hours. The lecturer and one pre-service teacher did not attend this workshop because they had another appointment.

In each school, the workshop had two sessions. The first session was about lesson study. Participants were presented with information about what lesson study is and how to conduct...
lesson study. Participants were made aware of the importance of student thinking in planning and teaching including predicting student’s responses and discussing students’ work. Pre-service teachers were expected to present their predictions of students’ responses in their lesson plans. Examples of lessons plans from Japanese and Australian schools were distributed to participants during the workshop.

Because this study initially intended to develop pre-service teachers’ PCK on algebraic reasoning, the second session of the workshop was on algebraic reasoning, meaning of variable in school algebra (Kaput, 1999). The reason for this is because the concept of variable is new for students in lower secondary school and often the concept of variable is simplified which leads to some misconceptions (Lucariello, Tine, & Ganley, 2014; Rosnick, 1981; Sahin & Soylu, 2011). For example, a common misconception is “fruit seller algebra” that is when students or teachers use letters to represent physical object rather than numbers (e.g., a is for apples, rather than the number of apples) (Arcavi, Drijvers, & Stacey, 2017). Patterning activity as a means to build students’ understanding of function was used as one example of a rich algebraic lesson. Finally, in line with lesson study focus, participants were asked to predict students’ responses for a particular algebraic problem and discussed examples of student works. At the end of the workshop, the pre-service teachers and mentor teachers formed lesson study groups based on the year level taught.

4.4.2 Observation and Preparation

In the first two weeks of the teaching practicum, pre-service teachers usually undergo a school orientation where they are introduced to the school’s staff, facilities, and environment. Aligned with this research, they observed their mentor teacher’s lessons, and identified students’ prior knowledge and learning needs. Through this task, they had an opportunity to practise their observation skills. They were also asked to study some literature on teaching algebra in secondary schools (e.g. Jupri, Drijvers, & van den Heuvel-Panhuizen, 2014).

4.4.3 Lesson Study Implementation

The allocated time of the teaching practicum allowed each lesson study group to conduct two cycles of lesson study. Each cycle consisted of one planning meeting, a number of research lessons and post-lesson discussions, depending on the number of pre-service teachers in the group and the lesson schedules. The groups decided the topic taught, how to teach it and set the schedule of the research lesson and post-lesson discussion during the
planning. In the research lessons, each pre-service teacher in the group taught the same lesson in a different classroom. The other pre-service teachers outside the lesson study group, mentor teachers and lecturer were invited to be the observers in the research lessons. To help the observers record the important events during the research lesson, they were given an observation sheet (Appendix 3). The post-lesson discussions were held as soon as possible after each research lesson. A protocol (Appendix 4) was established to guide the post-lesson discussion. For example, the lead teacher started the discussion by sharing their reflection of their own lesson. Next, the lecturer, mentor teachers, members of the planning team, and observers reported their observations. The knowledgeable other then gave a final remark focusing on the evidence of students’ learning. In some cases, due to their teaching obligations, the mentor teachers could only attend half of the post-lesson discussion. Hence, they were given the time to share their observation right after the lead teacher. Following the post-lesson discussion, the next research lesson was taught in different classroom by different pre-service teacher. This process was repeated multiple times depending on the number of pre-service teachers in the group.

This study is aware of the limited time in the teaching practicum which only allowed the pre-service teachers to carry out two lesson study cycles. Within this short time, it may seem unrealistic for the pre-service teachers to develop their PCK. However, the lesson study cycles, especially the post-lesson discussions would allow the pre-service teachers to reflect on their teaching and make immediate changes in the next lesson. Therefore, even though the pre-service teachers in this study only carried out two lesson study cycles, evidence of the development of the pre-service teachers’ PCK was expected to be seen within each lesson study cycle or over the two lesson study cycles.

Initially the lesson study was intended to focus on algebraic topics but due to the school schedule and curriculum, the topics for the second cycle were ratio and proportion, Pythagorean Theorem, and financial mathematics. Table 4.4 presents the mathematics topic taught by each lesson study group.

Table 4.4 Mathematical topic for each lesson study cycle

<table>
<thead>
<tr>
<th>Lesson study group</th>
<th>Topics taught in lesson study cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cycle 1</td>
</tr>
<tr>
<td>D7</td>
<td>Linear equation</td>
</tr>
<tr>
<td>D8</td>
<td>Linear equation</td>
</tr>
<tr>
<td>E7</td>
<td>Linear equation</td>
</tr>
<tr>
<td>E8</td>
<td>Gradient of straight line</td>
</tr>
</tbody>
</table>
To provide context of the analysis, a detailed description of the lesson study organisation of each group will be presented below.

**Group D7**

Table 4.5 shows the participants’ attendance during lesson study cycle 1 and cycle 2 Group D7. The pre-service teachers from Group D8 also participated in the research lessons and post-lesson discussions.

The schedules for the research lessons in the second cycle of Group D7 were changed several times due to the school schedule. Moreover, the lecturer availability was also another factor taken into account when scheduling the research lessons and the post-lesson discussion. Even though the schedule was approved by the lecturer – meaning that she would be able to attend the research lessons and post-lesson discussion, she cancelled her participation at the very last minute because of her other duties. Ida did not attend the first research lesson and post-lesson discussion because of health problems.

Table 4.5 Participants’ attendance during lesson study Group D7

<table>
<thead>
<tr>
<th>Lesson study cycle</th>
<th>Lesson study phase</th>
<th>KO Meili</th>
<th>UL Siti</th>
<th>MT Nur</th>
<th>Diana</th>
<th>Ida</th>
<th>PST Raya</th>
<th>Pipit</th>
<th>Gina</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Planning</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Research lesson 1</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td></td>
<td>Lead teacher</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 1</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Research lesson 2</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td></td>
<td></td>
<td>Lead teacher</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 2</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Research lesson 3</td>
<td>√</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td>Lead teacher</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 3</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>2</td>
<td>Planning</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Research lesson 1</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td></td>
<td></td>
<td>Lead teacher</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 1</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Research lesson 2</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td></td>
<td></td>
<td>Lead teacher</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 2</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Research lesson 3</td>
<td>√</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td>Lead teacher</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 3</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>√</td>
</tr>
</tbody>
</table>
Group D8

Table 4.6 presents the participants’ attendance during Group D8’s lesson study. In the second cycle, the lecturer only attended half of the research lesson due to her busy schedule.

Table 4.6 Participants’ attendance during lesson study Group D8

<table>
<thead>
<tr>
<th>Lesson study cycle</th>
<th>Lesson study phase</th>
<th>KO</th>
<th>UL</th>
<th>MT</th>
<th>Pipit</th>
<th>Gina</th>
<th>PST</th>
<th>Diana</th>
<th>Ida</th>
<th>Raya</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Planning</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Research lesson 1</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Research lesson 2</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 2</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Planning</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Research lesson 1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Research lesson 2</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 2</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>

Group E7

Table 4.7 shows the participants’ attendance during Cycle 1 and 2 Group E7. The lecturer could not attend all the lesson study because the lesson study schedules did not suit hers. In the second cycle, the mentor teachers did not attend the post-lesson discussion fully because they had a class to teach. They left the post-lesson discussion after giving comments and feedback on the lesson.

Because of the schedule of the school the lesson study schedules in Group E7 had to be modified. In the first cycle, Research Lesson 3 was scheduled right after Research Lesson 2. Consequently, there was no time to conduct the post-lesson discussion immediately after the Research Lesson 2. The post-lesson discussion of Research Lesson 2 and 3 were conducted together. In cycle 2, the first research lesson was on the last period. Because the school building had to be closed after school hours, the group could not conduct the post-lesson discussion immediately after the lesson. Therefore the post-lesson discussion was delayed and combined with the third one.
Table 4.7 Participants’ attendance during lesson study Group E7

<table>
<thead>
<tr>
<th>Lesson study cycle</th>
<th>Lesson study phase</th>
<th>KO</th>
<th>UL</th>
<th>MT</th>
<th>PST</th>
<th>Participants</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Me li</td>
<td>Farida</td>
<td>Irwan</td>
<td>Rusdi</td>
<td>Deni</td>
</tr>
<tr>
<td>1</td>
<td>Planning</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Research lesson 1</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 1</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Research lesson 2</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Research lesson 3</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 2</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
</tbody>
</table>

Group E8

Table 4.8 presents the participants’ attendance during Group E8’s lesson study. The mentor teacher could not attend Yanti’s research lesson because of her health condition.

Table 4.8 Participants’ attendance during lesson study Group E8

<table>
<thead>
<tr>
<th>Lesson study cycle</th>
<th>Lesson study phase</th>
<th>KO</th>
<th>UL</th>
<th>MT</th>
<th>PST</th>
<th>Participants</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Me li</td>
<td>Farida</td>
<td>Nani</td>
<td>Irwan</td>
<td>Vina</td>
</tr>
<tr>
<td>1</td>
<td>Planning</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Research lesson 1</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 1</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Research lesson 2</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 2</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
</tbody>
</table>
4.5 Data Collection

Even though the fieldwork included lesson study workshop and observation and preparation before the lesson study implementation, the data in this research were only taken around the lesson study implementation. The reason for this is to maintain the focus of the research on the lesson study implementation. In order to capture the complexity of pre-service teachers’ PCK, multiple instruments (Kagan, 1990) were required. To answer the research questions, data were collected using a number of different research instruments (Table 4.9). Detailed description of each instrument is presented in the sections below.

Table 4.9 Types of data to answer the research questions

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Written test</th>
<th>Video recording</th>
<th>Interviews</th>
<th>Field notes</th>
<th>Lesson study Artefacts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>2</td>
<td>√</td>
<td></td>
<td>√</td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>√</td>
<td></td>
<td>√</td>
<td></td>
<td>√</td>
</tr>
</tbody>
</table>

4.5.1 Written Test

A written test (Appendix 5) was designed to assess pre-service teachers’ PCK and given prior to the implementation of lesson study. The test items were developed based on the Knowledge Quartet (KQ) framework and were intended to identify the current state of pre-service teachers’ PCK in algebra (Table 4.10). There are seven items covering Mathematical Content Knowledge and PCK, especially for algebra topics. The Mathematical Content Knowledge part of the test assessed pre-service teachers’ understanding of algebra. Items were modified from TEDS-M 2009 test (IEA, 2009) and mathematics teachers’ content knowledge framework (Kahan, Cooper, & Bethea, 2003). The PCK part of the test assessed pre-service teachers’ understanding of teaching and learning algebra in lower secondary school. Items were constructed in line with the Indonesian lower secondary school mathematics curriculum and school textbooks. Consequently, algebraic operation and functions were selected. In line with the lesson study focus, the items asked pre-service teachers to anticipate students’ responses and analyse students’ mistakes or misconceptions.
### Table 4.10 Written test items

<table>
<thead>
<tr>
<th>Test Item</th>
<th>Knowledge Quartet</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Foundation</td>
</tr>
<tr>
<td>1</td>
<td>√</td>
</tr>
<tr>
<td>2</td>
<td>√</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>√</td>
</tr>
<tr>
<td>5</td>
<td>√</td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

#### 4.5.2 Video Recording

Video recording was used to capture the lesson study implementation. This includes the processes of lesson study, the interactions among the participants in the lesson study meetings and the pre-service teachers’ teaching during the research lessons. The planning and post-lesson discussions were recorded using one static camera. The research lessons were recorded using two cameras – one camera followed the lead teacher and the other one captured student’s group discussions.

#### 4.5.3 Field notes

Field notes were used to record information about the lesson study implementation. These include for example records of the attendance list of lesson study groups in each meeting, and schedule of each meeting. Because of the researcher’s dual role as the knowledgeable other, field notes were also used to record important moments in the research lessons which then would be discussed in the post-lesson discussions. Moreover, the notes of important moments were used to help the researcher locate important data segments in the data analysis process.

#### 4.5.4 Lesson Study Artefacts

Lesson study artefacts such as observation sheets, students’ work, pre-service teachers’ teaching materials (worksheets, handouts, and presentation slides), and lesson plans were collected as data. Worksheets (Appendix 6), and PowerPoint presentations (Appendix 7) offered insights into how the pre-service teachers represent and transform their mathematical knowledge for their students. Students’ work provided context and insight of the pre-service teachers’ PCK. Observation sheets provided evidence of the pre-service teachers’ noticing during the research lessons.
4.5.5 Interviews

Pre-service teachers, mentor teachers, and university lecturers participated in an individual interview at the end of the lesson study implementation. The interviews took about 30 minutes for each participant and were audio-recorded. A semi-structured interview (Appendix 8) was used to gather information on how they view lesson study in the teaching practicum program. Items were about the teaching practicum and lesson study, more specifically about the differences between the regular teaching practicum and the lesson-study embedded teaching practicum, and the benefits and difficulties in conducting lesson study. In addition, to gather information whether lesson study affected pre-service teachers’ PCK, they were asked to justify their written test responses.

Qualitative research may sometimes carry out data collection, analysis and interpretation simultaneously thus “one activity can alter the direction of the others” (Iacono, Brown, & Holtham, 2009). In this research, new interview questions were added based on the actual lesson study process. For example, Pipit taught Pythagorean Theorem on her second research lesson and students were having problems working with square roots. At the interview she was asked “In the research lesson, there were problem in finding the roots, what happened in the next lessons?” This kind of questions aimed to gather information if pre-service teachers took the advices from the post-lesson discussion and used it in their next lesson.

To sum up, data were taken before, during, and after the lesson study implementation. Table 4.11 presents the types of data collected before, during and after the lesson study.

Table 4.11 Types of data collected before, during, and after lesson study implementation

<table>
<thead>
<tr>
<th>Types of data</th>
<th>Before lesson study implementation</th>
<th>During lesson study implementation</th>
<th>After lesson study implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Written test</td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Videos</td>
<td></td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>Field notes</td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>Lesson study artefacts</td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>Interviews</td>
<td></td>
<td></td>
<td>√</td>
</tr>
</tbody>
</table>

4.6 Data Analysis

Data analysis in this research includes data organisation, coding, and interpretation.
4.6.1 Data Organisation

Organising data is critical at the early stage of a qualitative data analysis (Creswell, 2011). The huge amount of data gathered in this study needed special care regarding practicability and data accessibility. First, segments from the video data, interviews, field notes and lesson study artefacts were carefully selected, with only segments that showed evidence of the pre-service teachers’ PCK being selected for further analysis. Since the research lessons were videotaped by using two cameras, some videos were duplicated, in which case the videos with best quality of visual and audio were selected. Moreover, irrelevant video segments (i.e. ones that did not show evidence of the pre-service teachers’ PCK) from the planning, research lessons and post-lesson discussions were removed from the data. Similarly, irrelevant interviews, written test responses, pre-service teachers’ reflection notes, field notes and lesson study artefacts were discarded from the data.

Next, a qualitative data analysis software – NVivo was used to organise and analyse the data. Data were organised in two ways – the data type and the case. First, data were stored according to their type, for example videos, interviews, reflection notes, etc. Second, to easily identify the data and the lesson study group pre-service teachers belong to, a systematic labelling was used. For example, participants were labelled using initials that represent their role in lesson study, school, year level, and name. Table 4.12 shows how the label was used.

Table 4.12 Data organization of participants

<table>
<thead>
<tr>
<th>Role in lesson study</th>
<th>School</th>
<th>Year level</th>
<th>Participant’s name</th>
<th>Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lecturers</td>
<td>D</td>
<td>7 and 8</td>
<td>Siti</td>
<td>LDS</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>7 and 8</td>
<td>Farida</td>
<td>LEF</td>
</tr>
<tr>
<td>Mentor teachers</td>
<td>D</td>
<td>7</td>
<td>Nur</td>
<td>MDSN</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>Hani</td>
<td>MDEH</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>7</td>
<td>Rusdi</td>
<td>MESR</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>Irwan</td>
<td>MESI</td>
</tr>
<tr>
<td>Pre-service teachers</td>
<td>D</td>
<td>7</td>
<td>Diana</td>
<td>PDSD</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ida</td>
<td>PDSI</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>Raya</td>
<td>PDSR</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Gina</td>
<td>PDEG</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Pipit</td>
<td>PDEP</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>7</td>
<td>Deni</td>
<td>PESD</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Jamal</td>
<td>PESJ</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>David</td>
<td>PESU</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Vina</td>
<td>PEEV</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Yuma</td>
<td>PEEY</td>
</tr>
</tbody>
</table>
It is important to identify the participants and their role in the lesson study. Thus, cases function on NVivo was used to code the participants. All data belonging to a participant were coded with their corresponding case.

To illustrate how the data are stored and organised, the video data is presented as an example. Video data were labelled based on lesson study cycle and group. Folders of Planning, Research Lesson (RL) and Post-Lesson Discussion (PLD) were created to store video data from corresponding stages of lesson study cycle. Subfolders were used to store videos from lesson study Cycle 1 and Cycle 2. For example, in Figure 4.2, folder C1 RL D7 indicates it contains videos of research lessons from lesson study Cycle 1 Group D7. Subfolder C1 RL D7 PDSD contains videos of research lesson of the first cycle of Group D7 whom Diana taught. Each of the videos in the folder used the same label with an extension which indicates the event in the lesson. For example, a video C1 RL D7 PDSD OPENING indicates that it is from Cycle 1 Group D7 in which Diana – the lead teacher opened the lesson.

![Figure 4.2 Data organisation and labelling in NVivo](image)

### 4.6.2 Transcription

After all data were selected and organised according to the relevance to the research questions and the quality of the data, the next process was transcribing the video and interview data. This was done by using a technique called “gisting”. Gisted transcription summarises the essence in the data and maintains its context (Dempster & Woods, 2011). All transcriptions were done in NVivo so that the data were attached to their transcript.

### 4.6.3 Creation of Codes

In qualitative research, coding is about data retention (Richards, 2009), that is to allow the researchers to learn from and revisit the data to understand the phenomena. There are three types of coding creation (Miles & Huberman, 1994). The first one comes from the conceptual
framework, research questions and key elements that the researchers bring to the study, the second one is inductive coding where the codes are emerged from the data, the last one is creating general codes which then can be developed inductively (Miles & Huberman, 1994).

This study employs the first and the second coding creation. The research questions underpinned the creation of the codes. Research Question 1 and 2 are about pre-service teachers’ PCK development during lesson study and the lesson study characteristics that contribute to the development. To answer Research Question 1, it is important to examine the content of the lesson study meetings. This is done by using the Knowledge Quartet (KQ) framework that is the four dimensions of the KQ (FOU – Foundation, TRA – Transformation, CON – Connection, and CNT – Contingency) and their components (see Table 2.1). Next, to answer Research Question 2, 3 and 4 codes were constructed from the data. From analysing the connections between the codes, patterns were emerged.

Furthermore, to answer Research Question 1, it is necessary to identify the changes in pre-service teachers’ PCK during lesson study. The changes referred to any shift in the quality of the KQ components demonstrated by the pre-service teacher. Coding the data with the KQ components did not indicate any of these changes. Therefore, later on in this study, a KQ Rubric was developed to indicate the pre-service teachers’ PCK quality. This rubric enabled the researcher to identify the changes. The development of the KQ Rubric is presented below.

4.6.4 Development of a KQ Rubric

In Chapter 2, the KQ and its components were discussed. This study investigates changes in pre-service teachers’ PCK after participation in lesson study. Therefore, it is important to qualify their PCK at any particular stage of lesson study. Comparing pre-service teachers’ PCK across the stages of lesson study provides evidence whether any changes have occurred.

A KQ Rubric was developed to help the researcher identify the changes in the pre-service teachers’ PCK. The KQ components are classified into three levels. Level one is for when there is no evidence or low level of the KQ. Level two is for evidence of middle comprehension and level three is for high comprehension. The descriptions of each of the KQ components at a particular level in the rubric are underpinned by Rowland’s et al. (2009) framework. They were carefully stated to avoid overlapping interpretation over different components.
To ensure the descriptions fit the data, the rubric was tested by applying it to the data. This resulted in refined descriptions. Note that the rubric was made for the data for this study, hence some of the KQ components were redefined to fit the data. For example, Rowland et al. (2009) described Theoretical Underpinning of Pedagogy as the pre-service teachers’ beliefs about teaching mathematics and them being well-informed to recent research in mathematics education. However, since the pre-service teachers in this study did not study recent research, Theoretical Underpinning of Pedagogy in this study is redefined as the pre-service teachers’ beliefs about teaching mathematics. This is shown in the descriptions of the KQ components in the rubric. The final rubric is presented in Table 4.13. The example of the use of the KQ Rubric will be presented later in this section.
Table 4.13 The Knowledge Quartet (KQ) Rubric

<table>
<thead>
<tr>
<th>KQ Dimensions</th>
<th>KQ Components</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOU</td>
<td>Adheres to Textbook (ATB)</td>
<td>PSTs strictly follow the school textbook</td>
<td>PSTs critically use and modify the problems from the school textbook to meet students’ needs</td>
<td>PSTs critically use wide range of appropriate resources.</td>
</tr>
<tr>
<td></td>
<td>Awareness of Purpose (AOP)</td>
<td>PSTs do not address the lesson goals in the lesson plan</td>
<td>PSTs address the lesson goals in the lesson plan but not in the lesson</td>
<td>PSTs address the lesson goals in the tasks, learning materials and assessment throughout the lesson.</td>
</tr>
<tr>
<td></td>
<td>Concentration on Procedure (COP)</td>
<td>PSTs rely on memorisation of procedures and algorithms</td>
<td>PSTs discuss the reasoning behind an algorithm with the students</td>
<td>PSTs build students’ understanding of concept as well as procedures and algorithm.</td>
</tr>
<tr>
<td></td>
<td>Identifying Errors (IER)</td>
<td>PSTs are unable to identify students’ errors.</td>
<td>PSTs are able to identify students’ errors.</td>
<td>PSTs identify the misconceptions underpinning students’ errors</td>
</tr>
<tr>
<td></td>
<td>Overt Subject Knowledge (OSK)</td>
<td>PSTs have inadequate knowledge of the topic taught.</td>
<td>PSTs have adequate knowledge of the topic taught</td>
<td>PSTs have in-depth knowledge of the topic taught</td>
</tr>
<tr>
<td></td>
<td>Theoretical Underpinning of Pedagogy (TUP)</td>
<td>PSTs show beliefs towards traditional teaching</td>
<td></td>
<td>PSTs show beliefs towards student-centred teaching</td>
</tr>
<tr>
<td></td>
<td>Use of Terminology (UTL)</td>
<td>PSTs use incorrect language, symbols and terminology</td>
<td>PSTs use correct terminology and symbols but are unable to link them with everyday language.</td>
<td>PSTs use correct terminology and symbols and are able to link them with the everyday language.</td>
</tr>
<tr>
<td>TRA</td>
<td>Choice of Examples (COE)</td>
<td>PSTs choose incorrect examples</td>
<td>PSTs choose correct examples but do not use them properly</td>
<td>PSTs choose correct examples and use them properly</td>
</tr>
<tr>
<td></td>
<td>Choice of Representation (COR)</td>
<td>PSTs choose incorrect representations</td>
<td>PSTs choose correct representations but do not use them properly</td>
<td>PSTs choose correct representations and use them properly</td>
</tr>
<tr>
<td></td>
<td>Demonstration (COD)</td>
<td>PSTs choose incorrect demonstrations</td>
<td>PSTs choose correct demonstrations but do not use them properly</td>
<td>PSTs choose correct demonstrations and use them properly</td>
</tr>
<tr>
<td></td>
<td>PSTs use incorrect instructional materials</td>
<td>PSTs choose correct instructional materials but do not use them properly</td>
<td>PSTs choose correct instructional materials and use them properly</td>
<td></td>
</tr>
<tr>
<td>-----------------------------</td>
<td>--------------------------------------------</td>
<td>-------------------------------------------------------------------------</td>
<td>------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td><strong>Use of Instructional Materials (UIM)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>CON</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anticipation of Complexity (AOC)</td>
<td>PSTs do not anticipate the complexity of the lesson</td>
<td>PSTs anticipate the complexity of lesson vaguely</td>
<td>PSTs anticipate the complexity of the lesson clearly</td>
<td></td>
</tr>
<tr>
<td>Decision About Sequencing (DAS)</td>
<td>PSTs do not arrange the sequence of lessons or activities in a logical order</td>
<td>PSTs arrange the sequences of activities in the lesson plan</td>
<td>PSTs arrange the sequences of activities in the lesson plan and maintain it throughout the lesson.</td>
<td></td>
</tr>
<tr>
<td>Making Connection Between Procedure (CBP)</td>
<td>PSTs only use one procedure and ignore different strategies used by the students</td>
<td>PSTs do not discuss the connection between different procedures.</td>
<td>PSTs discuss the connection between different procedures.</td>
<td></td>
</tr>
<tr>
<td>Making Connection Between Concepts (CBC)</td>
<td>PSTs focus on one concept</td>
<td>PSTs do not discuss the connection between different mathematical concepts</td>
<td>PSTs discuss the connection between different mathematical concepts</td>
<td></td>
</tr>
<tr>
<td>Recognition of Conceptual Appropriateness (RCA)</td>
<td>PSTs have inadequate knowledge of students’ current knowledge</td>
<td>PSTs have an adequate knowledge of students’ current knowledge underpinning the lesson plan</td>
<td>PSTs have a good knowledge of students’ current knowledge underpinning the lesson plan and the lesson</td>
<td></td>
</tr>
<tr>
<td><strong>CONT</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deviation From Agenda (DFA)</td>
<td>PSTs deviate from the lesson plan and lesson goal because they spend too much time on students’ behaviour and classroom management</td>
<td>PSTs strictly adhere to the lesson plan without any flexibility to improvise to meet students’ need</td>
<td>PSTs deviate from lesson plan because they take time to discuss students’ ideas or difficulties.</td>
<td></td>
</tr>
<tr>
<td>Responding to Children’s Ideas (RSI)</td>
<td>PSTs ignore students’ ideas</td>
<td>PSTs do not respond to students’ ideas properly</td>
<td>PSTs respond to students’ ideas properly</td>
<td></td>
</tr>
<tr>
<td>Use of Opportunities (UOP)</td>
<td>PSTs are not aware of the opportunities to reinforce students’ learning</td>
<td>PSTs do not use the opportunities properly to reinforce the learning</td>
<td>PSTs use the opportunities properly to reinforce the learning</td>
<td></td>
</tr>
</tbody>
</table>
4.6.5 Coding

The video data were the main source of data for answering Research Question 1. The coding of the video data were conducted in two stages. The first stage was identifying the pre-service teachers’ PCK during the planning, research lessons, and post-lesson discussions. This was done by coding the data with KQ components (see Table 2.1). The second stage was by applying the KQ Rubric.

To illustrate the process of the first stage, a video episode is presented as an example of the coding. The lesson was about linear equation, especially on the concept of variable and equivalence in a linear equation using balance model. The video was labelled “C1 RL D7 PDSI - NR1” which indicates that it is from a research lesson, Cycle 1 of Group D7 where Ida - the lead teacher was interacting with a student-NR. Ida was helping NR on a problem – to find the number of candies in each bag (Figure 4.3). The bold text in the table indicate the coding references. NR could understand that the balance represents equal values of both sides. However, she misunderstood the use of a variable in the equation. Table 4.14 illustrates how the coding was applied.

Table 4.14 Illustration of coding

<table>
<thead>
<tr>
<th>Data</th>
<th>Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A student was having a difficulty working on a problem (Figure 4.4). She was trying to find the number of candies in one bag.</td>
<td>TRA Choice of Representation</td>
</tr>
<tr>
<td>PDSI was helping the student.</td>
<td></td>
</tr>
<tr>
<td>Ida : How many candies?</td>
<td></td>
</tr>
<tr>
<td>NR : [counting one by one] 20</td>
<td></td>
</tr>
<tr>
<td>Ida : How’s the position of the balance scale?</td>
<td>CON Anticipation of Complexity</td>
</tr>
<tr>
<td>NR : The same. equal both side</td>
<td></td>
</tr>
<tr>
<td>Ida : OK, you said this is equal, can you make the equation?</td>
<td></td>
</tr>
<tr>
<td>Ida : What is the unknown?</td>
<td></td>
</tr>
<tr>
<td>NR : This [pointing the bag]</td>
<td></td>
</tr>
<tr>
<td>Ida : That means you can express it using what?</td>
<td>CON</td>
</tr>
</tbody>
</table>
NR : $x$
Ida : Yes, you may use $x$. $x$ equals to what?
NR : $x$ equals two bags of candies
Ida : $x$ is two bags of candies, and then what is the equation? Can you write it down?
NR : [writing]
Ida : You said it is balance. discuss this with your group. What does it mean?
Ida left the student
NR looked confused

The pre-service teachers chose the balance model to introduce the concept of equations. Therefore it was coded as TRA – the Transformation dimension, especially on the Choice of Representation component. Ida prompted the students to understand the equivalence of the equation by using the balance. This indicates she has knowledge of the benefit of balance model to illustrate the equivalence in an equation and she used that to guide NR to understand the equation. Ida might have anticipated students’ confusion of using the balance to represent the equation, hence this was coded as CON – the Connection dimension especially on the Anticipation on Complexity component. Next, NR used $x$ to represent the two bags which indicated a misconception about the use of $x$ as a variable. Ida might have not predicted the complexity of using the balance model to introduce the concept of variables. The researcher interpreted it as an indication of Anticipation of Complexity component in the CON – the Connection dimension. Lastly, as Ida was unable respond to this student’s misconception, this provides an evidence of her Contingency dimension (CNT) – particularly on the Responding to Students’ Ideas component.

This coding scheme allowed the researcher to identify the dominant components that is components that are apparent across the planning, research lessons and post-lesson discussions. For example, Table 4.15 was derived from an NVivo matrix query of the components under the Connection dimension in the first cycle of Group D7. The cells show the frequency of each component coded in each video data. This information provides insight of the consistency of the KQ components during one cycle and across two cycles. For example, Decision About Sequencing occurred in all phases within the cycle, thus it indicates a dominant code.
Table 4.15 Frequency of Connection dimension in cycle 1 Group D7

<table>
<thead>
<tr>
<th>Components of Foundation</th>
<th>Planning</th>
<th>Research lesson 1</th>
<th>Post-lesson discussion 1</th>
<th>Research lesson 2</th>
<th>Post-lesson discussion 2</th>
<th>Research lesson 3</th>
<th>Post-lesson discussion 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anticipation of Complexity</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Making Connection Between Concepts</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Making Connection Between Procedures</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Decision About Sequencing</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Recognition of Conceptual Appropriateness</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

However, having the dominant codes did not help the researcher to identify changes in the pre-service teachers’ PCK. Therefore, the second stage of coding is using the KQ Rubric to indicate the quality of the pre-service teachers’ PCK. The following examples are used to illustrate the use of the KQ Rubric (Table 4.16). Both examples show the Use of Terminology component in the Foundation dimension acted out by two pre-service teachers. In example 1, Ida discussed a problem - *3 kg of eggs contain 48 eggs. How many eggs are in 1 kg egg?* She asked students to define $x$, that is the missing item that $x$ represents. She drew on students’ answers and defined $x$ as the kilograms of eggs instead of the number of eggs in one kilogram. This suggests her lack of understanding the connection of mathematical terminology and languages used in a concrete problem, therefore it was coded as Level 1. In example 2, Diana helped the students to write the equation from a balance problem (Figure 4.4) and to find the number of candies in one bag. First, she asked the students to determine what the variable represents. When students stated that it was the candies and the bag, Diana told them the correct items were the number of candies in one bag. This suggests her ability to connect the mathematical terminology with the problem, thus it was coded as Level 3.
Table 4.16 Examples of coding using the KQ Rubric

<table>
<thead>
<tr>
<th>No</th>
<th>Data source: C1 RL D7 PDSI OPENING</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ida : 3 kg of eggs contain 48 eggs. How many eggs are in 1 kg egg? Look at your note, what did you learn yesterday? How do we define (x)? To get one kg of eggs?</td>
</tr>
<tr>
<td></td>
<td>Student : Kg of egg</td>
</tr>
<tr>
<td></td>
<td>Ida : Kg of egg, that means, we assume (x = ) one kg of egg. Now we have 3 kg of egg, what is that?</td>
</tr>
<tr>
<td></td>
<td>Student : [Quiet]</td>
</tr>
<tr>
<td></td>
<td>Ida : One kg of eggs is symbolized with an (x). That means 3 kg of eggs is ...?</td>
</tr>
<tr>
<td></td>
<td>Student : (3x = 48)</td>
</tr>
<tr>
<td></td>
<td>Ida : From here, can we find the (x)?</td>
</tr>
<tr>
<td></td>
<td>Student : (x = 48 \div 3. x = 16).</td>
</tr>
<tr>
<td></td>
<td>Ida : So, in one kg of egg, we have how many eggs?</td>
</tr>
<tr>
<td></td>
<td>Student : 16</td>
</tr>
</tbody>
</table>

| 2  | Data source: C1 RL D7 PDSD - GW1 |

![Figure 4.4 Balance problem](image)

Diana : What is the problem?  
Student 1 : The candies in one bag  
Diana : Then what does \(x\) represent?  
Student 1 : Candies  
Student 2 : The bag  
Diana : You assume \(x = \) the number of candies in one bag.  

The written test was analysed using the KQ framework. This gives an indication of the pre-service teachers’ PCK before the implementation of the lesson study. To see if their PCK changes after the lesson study, the KQ framework was also used to code some of the interview responses after the implementation of lesson study. In some cases, the written test and interview responses closely related to the video data and indicated the quality of the pre-service teachers’ PCK. In these cases, they were coded with the KQ Rubric as well. The teaching materials such as lesson plans, worksheets, and PowerPoint presentation and the students’ work were used to
provide context to the pre-service teachers’ PCK. Therefore, they were not coded, instead the coding was applied to the pre-service teachers’ performance while using the teaching materials or while responding to the students’ work. Similarly, field notes provided context and additional information of the data. The field notes were not coded, but in order to make sense of the data, they were used in interpreting the data. Some of the interview questions were used to answer Research Question 2, 3, and 4. Therefore the codes were constructed from the data. The list of the codes can be found in Appendix 11.

4.6.6 Data Interpretation

This section describes the interpretation process to answer the research questions. Because the lesson study groups taught different topics in Cycle 1 and Cycle 2, it was difficult to trace the pre-service teachers’ PCK development over two cycles. Consequently, the PCK development in this study is observed within each lesson study cycle. It is also difficult to trace an individual pre-service teachers’ PCK development because one pre-service teacher only taught once in every cycle. Therefore, the pre-service teachers’ PCK development was observed for the lesson study group, not for the individual pre-service teacher.

To make sense of the data and to understand the development of the pre-service teachers’ PCK during lesson study, this research only focuses on the instances where the KQ components were observed in the planning, research lessons and post-lesson discussions. For example, Table 4.17 is derived from an NVivo query of the levels of Use of Terminology component during the first lesson study cycle of Group D7. The table shows the frequency of each level coded at the planning, research lessons and post-lesson discussions.

Since the aim of the study is to examine the pre-service teachers’ PCK development, rather than focusing on the frequency, this study focuses on the situations or contexts where the components were observed. The frequency helped the researcher locate the lesson study phases in which the high and low levels were observed. After that, to understand the improvement of a particular KQ component, and what factors of the lesson study contribute to the improvement, the researcher looked closely at the situations where the component was apparent. This included tracing the KQ component over every stage in one lesson study cycle. In many cases one KQ component observed was overlapped with other KQ components, hence it suggests the interconnections of the KQ components.
Table 4.17 Frequency of *Use of Terminology* Levels in Lesson Study Cycle 1 Group D7

<table>
<thead>
<tr>
<th>Lesson study cycle</th>
<th>Use of Terminology</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level 1</td>
</tr>
<tr>
<td>Planning</td>
<td>0</td>
</tr>
<tr>
<td>Research lesson 1</td>
<td>4</td>
</tr>
<tr>
<td>Post-lesson discussion 1</td>
<td>1</td>
</tr>
<tr>
<td>Research lesson 2</td>
<td>8</td>
</tr>
<tr>
<td>Post-lesson discussion 2</td>
<td>0</td>
</tr>
<tr>
<td>Research lesson 3</td>
<td>3</td>
</tr>
<tr>
<td>Post-lesson discussion 3</td>
<td>1</td>
</tr>
</tbody>
</table>

For Research Question 3 and 4, the researcher looked for similarities and differences in participants’ interview responses. Next, the researcher grouped the same responses together to gather the patterns. Then to make sense of the data, the researcher looked closely at each group by connecting it to other data sources such as field notes and video. This triangulation process allows for better interpretation of the data.

4.7 Conclusion

This chapter has described the research process including the context in which the lesson study took place, the recruitment of the schools and the participants, the data collections and data analysis process. To identify the development of the pre-service teachers’ PCK, different data sets from different phases of lesson study were analysed by using the KQ Rubric. The findings from this analysis will be reported in the next two chapters. Chapter 5 will report the findings related to Research Questions 1 and 2, while Chapter 6 will report the findings related to Research Questions 3 and 4.
5 The Development of Pre-service Teachers’ Pedagogical Content Knowledge (PCK)

This chapter addresses Research Question 1 – What changes are evident in pre-service teachers’ PCK during lesson study in their teaching practicum? Since the pre-service teachers’ PCK development occurred in lesson study, this chapter also addresses Research Question 2 – What characteristics of lesson study contribute to the development of pre-service teachers’ PCK? This chapter is organised into two sections, each of which addresses Research Questions 1 and 2.

5.1 Changes Evident in Pre-service Teachers’ PCK

This section presents the findings from analysis of the written test, videos, interviews, and lesson study artefacts to answer Research Question 1. The analysis was framed using the Knowledge Quartet (KQ) framework (Rowland et al., 2009). The KQ consists of four dimensions: Foundation, Transformation, Connection, and Contingency. Each dimension contains some components which encompass the situations in which the pre-service teachers enact their PCK. This section presents the findings by organising it according to the KQ dimensions.

5.1.1 Foundation Dimension

The Foundation dimension is about pre-service teachers’ mathematical knowledge and beliefs about teaching mathematics (Rowland et al., 2009). It includes a number of components – Adherence to Textbook, Awareness of Purposes, Concentration of Procedures, Identifying Errors, Overt Subject Knowledge, Theoretical Underpinning of Pedagogy, and Use of Terminology. This study found evidence of the development of the pre-service teachers’ Overt Subject Knowledge (OSK), which is about the pre-service teachers’ mathematical knowledge and Theoretical Underpinning of Pedagogy (TUP) which refers to their beliefs about teaching mathematics.

5.1.1.1 Overt Subject Knowledge (OSK)

The Overt Subject Knowledge (OSK) component is concerned with pre-service teachers’ mathematical knowledge (Rowland et al., 2009). This section presents the findings from video data and interviews. As discussed in Chapter 4, the written test was designed to capture the pre-service teachers’ mathematical knowledge of functions. Unfortunately, all
groups did not teach functions during the lesson study as anticipated during the design phase of the study. Therefore, the written test data was not used in this analysis.

Table 5.1 presents the highest level of OSK observed during the two lesson study cycles of each group. These OSK levels were taken from the videos of planning, research lessons, and post-lesson discussions. It is important to note that the levels in one group’s cycle or across its two cycles’ might be observed for different topics. For example in Cycle 1 Group D7 (C1D7), Level 2 in the planning was about open and closed number sentences, and Level 3 in the Post-lesson Discussion 3 was about variable. Therefore increasing levels in one cycle or two cycles does not necessary mean an improvement.

Chapter 4 has shown that there were only two pre-service teachers in Group D8 and in Group E8, therefore they only had two research lessons and post-lesson discussions. For Group E7, there were some special circumstances that did not enable the post-lesson discussion to be conducted immediately after the research lessons. Therefore, some post-lesson discussions of Group E7 were combined.

Table 5.1 Highest Level of Overt Subject Knowledge (OSK)

<table>
<thead>
<tr>
<th>Lesson study cycle</th>
<th>Lesson study phase</th>
<th>D7</th>
<th>D8</th>
<th>E7</th>
<th>E8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Planning</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Research lesson 1</td>
<td>2</td>
<td>NE</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 1</td>
<td>2</td>
<td>NE</td>
<td>NE</td>
<td>NE</td>
</tr>
<tr>
<td></td>
<td>Research lesson 2</td>
<td>NE</td>
<td>NE</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 2</td>
<td>NE</td>
<td>NE</td>
<td>NE</td>
<td>NE</td>
</tr>
<tr>
<td></td>
<td>Research lesson 3</td>
<td>NE</td>
<td>NA</td>
<td>3</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 3</td>
<td>3</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>2</td>
<td>Planning</td>
<td>NE</td>
<td>3</td>
<td>NE</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Research lesson 1</td>
<td>NE</td>
<td>NE</td>
<td>NE</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 1</td>
<td>NE</td>
<td>3</td>
<td>NE</td>
<td>NE</td>
</tr>
<tr>
<td></td>
<td>Research lesson 2</td>
<td>NE</td>
<td>3</td>
<td>NE</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 2</td>
<td>NE</td>
<td>NE</td>
<td>NE</td>
<td>NE</td>
</tr>
<tr>
<td></td>
<td>Research lesson 3</td>
<td>NE</td>
<td>NA</td>
<td>NE</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 3</td>
<td>NE</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

NE : not evident  
NA : not applicable

Table 5.1 shows that OSK was apparent in all cases except Cycle 2 Group D7 (C2D7) and Cycle 2 Group E7 (C2E7). The reason for this is because the OSK observed in the lesson study is bound to the nature of the discussions in the planning session and post-lesson discussions. The focus of the post-lesson discussions in these two cycles did not reveal any of
the pre-service teacher’s OSK. Therefore, the pre-service teachers’ OSK was not evident in these two cases. Moreover, in this study, the presence or absence of OSK was not always clear. In some cases, it became visible when triggered by students’ difficulty. For example in Cycle 2 Group D8 (C2D8), the pre-service teachers’ OSK of square root was only visible during the first post-lesson discussion when the group discussed ways to help students with difficulties in finding square roots.

C2D8 shows evidence of improvement of the pre-service teachers’ OSK of square root. The video and interview data showed evidence of pre-service teachers’ progression on their mathematical knowledge, especially of finding square roots, and it resulted in some changes in their teaching of this topic. Even though some groups also showed OSK improvement, its impact to the pre-service teachers’ teaching was not as obvious as in C2D8. For that reason, C2D8 is presented as an illustration of the impact of subject knowledge in the pre-service teachers’ teaching.

Group D8 taught the Pythagorean Theorem in their second lesson study cycle. In the planning, the pre-service teachers, the mentor teacher, and the knowledgeable other focused on choosing the most appropriate Pythagorean Theorem proof and problems about Pythagorean Theorem. They discussed different ways of proving and verifying Pythagorean Theorem and the materials needed to help students working with the proof. Even though Group D8 demonstrated a Level 3 OSK in the planning (see Table 5.1), the OSK was not about square roots. For example, when the pre-service teachers discussed the types of triangles used to verify the Pythagorean Theorem, they came to an understanding that some of them only work for Pythagorean triples (Level 3 OSK). The concept of square roots was mentioned in the planning but was not discussed deeply. Gina – one of the pre-service teachers in Group D8 asked about how students would find the square roots. The mentor teacher informed that the Year 7 curriculum does not include square roots, suggesting that the students only learned it at primary school and they might have difficulties with square roots. Instead of discussing ways to help the students, they decided to use simple perfect squares to reduce the complexity of the problems.

Gina : How will the students find the roots?
Hani : There are no root lessons in Year 7. The last time they did roots was probably in primary. They might forget it.
Pipit : They could find the simple and small square numbers.
Gina : For tomorrow’s lesson, we’re only using Pythagorean triples because we don’t want students to get in trouble only for finding the square root.
Meili: Can they use a calculator?
Hani: We don’t encourage them to use a calculator. They’re not used to using calculators.
Meili: Okay, we use small numbers, but you can try giving them one or some hundreds, I wonder how students would find the roots.

Gina’s question indicated that she could identify a potential problem in the teaching. The mentor teacher’s comment on the square roots in the curriculum magnified the problem. However, it did not trigger a discussion about strategies of finding square roots. Instead, they decided to use perfect squares and Pythagorean triples to reduce the complexity of the problem and avoid difficulties. As the knowledgeable other, I could foresee that students would have difficulties with square roots, but I assumed the pre-service teachers have had a good mathematical knowledge about square numbers and square roots. I did not oppose the pre-service teachers’ idea to use perfect squares and Pythagorean triples, but I challenged them to give the students some problems with perfect squares involving larger numbers. The reason for this was that I wanted to test my assumption and ‘create’ a contingency in the lesson. Because of time limitations, the group only discussed the key ideas of the problems used in students’ exercise. They did not discuss other details such as the numbers used in the problems.

The first research lesson was taught by Pipit. In the first half of the lesson, the students verified Pythagorean Theorem using a puzzle-like activity. In the second half of the lesson, students worked on an exercise applying Pythagorean Theorem. When working on a problem about a fishing boat (Figure 5.1), the students showed difficulties in finding the square root of 1,369. Apparently, the pre-service teachers took my challenge of using larger perfect squares but unfortunately, they did not predict students’ responses and did not prepare prompts to help the students either. Without a calculator, the students took much time to solve the problem. They used a guessing strategy by taking a number randomly and then calculating the square. Pipit only encouraged the students to keep on trying to find the square root without giving them any specific clues.
The post-lesson discussion revealed that Pipit had limited knowledge about finding square roots. The transcript below shows Pipit’s understanding of an algorithm of finding a square root. She explained an algorithm of finding the square root of 1,369 (Figure 5.2) without discussing why the algorithm works. She only mentioned that she found the algorithm on the internet. Based on her explanation, it seems that she only understood the procedure but not the reasoning behind it. Gina did not know other strategies of finding square roots (Level 1 OSK).

Pipit : First, we group them

\[
\begin{array}{c|c}
13 & 69 \\
\end{array}
\]

Because this is a thousand, the roots will be a two digit number. When it’s a hundred, it also has a two digit number root. This is 13. What square number is close to this? I asked a student, she said 4 × 4 is 16. But it’s larger than 13. Then it must be 3 × 3. So we write 3 in here (the underlined 3 at the bottom). Now 3 × 3 is 9. The difference of 13 and 9 is 4. So we write 4 in here (the circled 4 on the left) [Moving to the right side, 69]. Look, this is 9. What squared numbers could make a 9? She said 3 × 3 and 7 × 7 (Pipit wrote 3 and 7 next to 9) Then I asked her, whether 4 is larger or smaller than 3. Because it’s larger than 3, then the answer is 7. [Pipit circled the 7]. So the answer is 37.
Knowing only how to use one strategy to find square roots procedurally without knowing the reasoning behind indicates Pipit’s and Gina’s lack of subject knowledge about finding square roots (Level 1 OSK). Moreover, because they did not know other strategies, it hindered their critique of the procedure. These pre-service teachers did not ask why the procedure works and how students would understand it. As the knowledgeable other, I discussed how to find square roots using a number line strategy (Figure 5.3). For example, in finding the square root of 1,369, students would see the number line and locate 1,369. Since it is in between 1,000 and 1,600, then the square root should be between 30 and 40. Using this estimation, students could minimise the number of possible answers. Since the last digit of 1,369 is 9, that gives two possible answers 33 and 37. The number lines are not the proper mathematical representation for square numbers. Because it is a quadratic function, the proper representation is a parabola. However, the number line is a good analogy and it could help the students understand the idea of pairing the whole numbers and the square numbers.

![Figure 5.3 Number line strategy to estimate the square root of a number](image)

While the video data of Pipit’s lesson showed her lack of subject knowledge, the interview provided some information regarding Pipit’s subject knowledge development. When asked for her opinion about lesson study, she specifically mentioned that she gained new knowledge about finding square roots and was able to use that new knowledge in her classroom (Level 2 OSK).

Meili: In general, what do you think of lesson study?

Pipit: It’s very helpful. Especially in making the lesson plans. By doing it in [a] group we get more ideas and get advice from the mentor teacher and the lecturer. For example, when you told us about finding the square root, I never thought of that technique before. I used that in my classroom, and some students understood that.

The interview signified that Pipit started the lesson study cycle with little knowledge of finding square roots. She learned a new strategy of finding square roots in the post-lesson
discussion. She used it in her next lesson and found that some students understood. Moreover, the interview asked what the pre-service teachers did in the lesson following their research lesson of the second cycle, and also what they would change if they could repeat the lesson.

Meili: In the research lesson, there are problems in finding the square roots. What happened in the next lessons?
Pipit: I told them to find the roots using two ways. Some understood, some still used trial and errors.
Meili: So, they did not use estimation?
Pipit: Only [a] few of them. They did not understand it when I explained it on the board, but when I showed them individually on their notebook, then they understood.
Meili: If you could repeat the Research Lesson 2, what would you change?
Pipit: I would review how to find the square roots at the beginning of the lesson, so that students would not have difficulties when working with Pythagorean Theorem. I would also [put] emphasis on the notation. I found many students made mistakes in writing the notation. For example, \(c^2 = 49\) then \(c^2 = 7\). They always wrote the square even though it’s actually the root.

Pipit’s responses suggested that she learned a new strategy for finding square roots and developed an understanding about the need to review the square roots before commencing the Pythagorean Theorem lesson. She has become more aware of the connection of square roots and Pythagorean Theorem in deciding the sequence of activity of the lesson. This indicates that Pipit’s Foundation Knowledge contributed to the development of her Connection Knowledge.

Gina, the lead teacher of the second research lesson did not change her lesson plan following the post-lesson discussion of Pipit’s lesson but she changed the teaching approach. The students in her class also struggled with finding square roots. Learning from the previous lesson, Gina included another example of finding square roots of 529 to help students work on the fishing boat problem. Gina showed the students how to find the square root of 529 by pairing the whole numbers with the corresponding square numbers (Level 3 OSK). The students in Gina’s class could find the square roots more easily with this strategy.

Gina: How to find the square root of a big number. 529?
Students: ...
Gina: I’ll show you how. What is \(10^2\)?
[showing the PowerPoint Presentation]
When noticing the students were struggling with square roots, Gina paused the lesson and used the opportunity to explain the pairing of the whole numbers and the square numbers. Gina did not use the number line strategy but she used the idea of pairing the whole numbers and the square numbers. This is an example of how her Overt Subject Knowledge influenced her Contingency Knowledge. Moreover, the use of pairing the whole numbers to help students find square roots showed evidence of Gina’s Transformation Knowledge, especially in the Choice of Representation.

In the interview with Gina, when asked what she had learned from the planning sessions, Gina specifically mentioned Cycle 2 as an example of the need to know students’ prior knowledge when designing lessons and selecting appropriate teaching methods.

Meili: What did you learn from planning?

Gina: In designing the lessons and selecting the teaching methods we need to know our students and their understanding of the topic. For example, in Cycle 2, we thought students have understood about square numbers and square roots when they learn about Pythagorean Theorem, but we were wrong.
Gina was the one who raised a question regarding students’ prior knowledge about square roots. At that time, she knew that square roots might be a problem in the lesson. But since the other pre-service teachers in her group did not raise and discuss this matter any further, she might have assumed that other members in the group did not see it as a potential problem. After observing Pipit’s lesson, she found that it was important to acknowledge students’ prior knowledge in the lesson planning for a successful lesson.

To sum up, the data from C2D8 showed that lesson study has enabled the pre-service teachers’ OSK development of finding square roots. Since the topic taught was Pythagorean Theorem, the pre-service teachers did not focus on square roots in the planning. Consequently, the first lead teacher – Pipit encountered a contingent moment when she found the students had difficulties finding the square root. Through discussion of students’ difficulties working with square roots in the post-lesson discussion, Pipit and Gina’s OSK on square roots were exposed. It was identified that they lacked knowledge of strategies for finding square roots. With help from the knowledgeable other, Gina and Pipit were able to gain new knowledge of finding square roots using the number line strategy. The new strategy was implemented in the second research lesson taught by Gina. Gina was able to help students working with square root using the idea of pairing the whole numbers and the square numbers in the number lines.

5.1.1.2 Theoretical Underpinning of Pedagogy (TUP)

The Theoretical Underpinning of Pedagogy (TUP) component is concerned with pre-service teachers’ awareness of current research in mathematics education and their beliefs about teaching mathematics (Rowland et al., 2009). In this study, the pre-service teachers did not study current research in mathematics education to inform their lesson plans, therefore it is not the focus of this study. However, the written test, lesson study observation, and interview indicated the pre-service teachers’ beliefs about teaching mathematics. Because of that, the TUP in this study only focuses on the pre-service teachers’ beliefs about teaching mathematics. It was observed in the pre-service teachers’ written test responses, and videos of the lesson study meetings. The pre-service teachers’ beliefs, particularly about traditional teaching and student-centred teaching, were indicated by the way they responded to the written test, and the way they designed and taught the lessons.

The written test and interviews were compared in order to investigate the pre-service teachers’ beliefs before and after the lesson study implementation. Video data provided information about the pre-service teachers’ beliefs during the lesson study. The results from
the written tests showed that the pre-service teachers entered the teaching practicum with beliefs about traditional teaching. During lesson study, the pre-service teachers’ beliefs were slightly challenged and moved towards beliefs about a more reformed mathematics teaching. This was seen especially during the planning sessions across the two cycles where the pre-service teachers designed lessons and tasks that were more student-centred. The data from the research lessons suggested that the changes were not consistent. In the teaching, the pre-service teachers carried out the student-centred lesson that they had designed. However, whenever they encountered unanticipated situations, they went back to the traditional way of teaching. The interview also revealed that the pre-service teachers held on to their beliefs about traditional teaching when helping students to correct their mistakes.

Item 4 of the written test (Figure 5.4) assesses the pre-service teachers’ knowledge of students’ mistakes and misconceptions. The pre-service teachers’ responses on Question 4.c revealed their beliefs about mathematics teaching.

![Figure 5.4 Item 4 of the written test](http://mathmistakes.org/wp-content/uploads/2013/05/Dist-2.png)

4. The picture below shows a student’s answer.

4a. Simplify \((x + 3)(x + 5)\)

\[2x + 18\]

4b. How sure are you about your answer? (1 // 2 // 3 // 4 // 5)

4c. Simplify \((x + 7)^2\)

\[2x + 49\]

4d. How sure are you about your answer? (1 // 2 // 3 // 4 // 5)

The pre-service teachers’ responses showed three different ways of helping students correct their mistakes. The first way is by explaining the procedure using the distributive property step-by-step, and using arrows to show the multiplication when necessary. The second way is by giving the students more exercises. Lastly, by using numbers before letters – these pre-service teachers believed using numbers would make the concept more concrete rather than using variables and therefore it would be easier for students to understand. Table 5.2 presents some examples of the pre-service teachers’ responses. Earlier studies (Handal, 2003; Stipek, Givvin, Salmon, & MacGyvers, 2001) associate teachers who focus on the procedure using step-by-step instruction and rely on drilling with traditional approaches of teaching.
mathematics. Therefore, the written test suggested that the pre-service teachers held traditional beliefs about teaching mathematics.

Table 5.2 Examples of pre-service teachers’ responses to item 4.c

<table>
<thead>
<tr>
<th>PSTs</th>
<th>Responses to item 4.c</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD8P</td>
<td>I would remind and explain students about algebraic operation.</td>
</tr>
<tr>
<td>(Pipit)</td>
<td>[I would use arrows to show the multiplication of each algebraic item]</td>
</tr>
<tr>
<td>PE7U</td>
<td>I would give students more exercises and help them when it is needed.</td>
</tr>
<tr>
<td>(Umar)</td>
<td>[Expanding the power], [Keeping them engaged]</td>
</tr>
<tr>
<td>PD7I</td>
<td>Students could benefit from practising distributive properties with numbers before algebra.</td>
</tr>
<tr>
<td>(Ida)</td>
<td>[An example with whole numbers], [Using expansion]</td>
</tr>
</tbody>
</table>

Similarly, the pre-service teachers’ responses to the interview question also revealed their beliefs about traditional teaching approaches. The interview question was modified from the written test item 4.c. It asked “Did you find any similar mistakes that students made during your teaching?” If their response was “yes” the next question was “How did you help them overcome the mistakes?” If the response was “no” the next question was “What kind of mistakes did you find?” Table 5.3 presents some examples of the pre-service teachers’ interview responses when they found their students made similar mistakes. In general, the pre-service teachers used a procedural approach to help students working with algebraic manipulation.
Table 5.3 Examples of pre-service teachers’ interview responses

<table>
<thead>
<tr>
<th>PSTs</th>
<th>Interview responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD8P (Pipit)</td>
<td>I gave them an example and showed the procedure of solving it.</td>
</tr>
<tr>
<td>PE7U (Umar)</td>
<td>I believe students need to practice more. I think they don’t practice enough at home. And I would explain the concept in a simpler way.</td>
</tr>
<tr>
<td>PD7I (Ida)</td>
<td>Many students did not understand this. I reviewed the concept individually to the students who needed reinforcement. I prompted them with some guiding questions such as: How to do the operation when variables are involved? What do you call the items without variables? I told them to group the like items, and when they move sides the value change.</td>
</tr>
</tbody>
</table>

While the written test and the interviews suggested pre-service teachers’ beliefs of traditional teaching, data from the lesson study implementation across all lesson study groups suggested that the pre-service teachers’ beliefs shifted into a more student-centred teaching approach. Table 5.4 shows the highest level of TUP observed in each lesson study cycle. Level 1 was assigned when the pre-service teachers showed beliefs about traditional teaching, and Level 3 was assigned when they showed beliefs toward a student-centred teaching. There was no Level 2 because it was difficult to define the middle ground between traditional teaching and student-centred teaching. Note that since beliefs in the lesson study were explicit and observed during the planning, research lessons, and post-lesson discussions, the references coded at TUP might also be coded as other KQ components.

Table 5.4 Highest Level of Theoretical Underpinning of Pedagogy (TUP)

<table>
<thead>
<tr>
<th>Lesson study cycle</th>
<th>Lesson study phase</th>
<th>Lesson study group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>D7</td>
</tr>
<tr>
<td>1</td>
<td>Planning</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Research lesson 1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 1</td>
<td>NE</td>
</tr>
<tr>
<td></td>
<td>Research lesson 2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 2</td>
<td>NE</td>
</tr>
<tr>
<td></td>
<td>Research lesson 3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 3</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Planning</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Research lesson 1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Research lesson 2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Research lesson 3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 3</td>
<td>3</td>
</tr>
</tbody>
</table>

NE : not evident
NA : not applicable
All cases showed a high level of TUP during the planning except C2E7. The discussions in C2E7 did not reveal the pre-service teachers’ TUP. This might have been caused by the dynamic of the group which will be discussed in Chapter 6. A high level of TUP in the planning was evident when the pre-service teachers focused on students’ thinking while designing their lesson. The data from C1D8 is presented to illustrate the way in which the pre-service teachers enacted their beliefs during lesson study.

The first lesson study cycle of Group D8 was about linear equations. The pre-service teachers presented their ideas of teaching the lesson in the planning meeting. The excerpt below shows Gina explaining the group’s plan of the lesson. She posed a linear function problem: “If your father gives you Rp. 200,000 monthly allowance, and you spend Rp. 6,000 each day, how much money you have left after any particular days?” Gina expected that the problem would lead to the use of a table of values. The column in the table represented the number of days and the row represented the amount of money left. This would then be replaced by using variables $x$ and $y$. Finally, students would be able to sketch the graph and determine the equation. This episode was coded as Level 3 TUP because it showed that the pre-service teachers considered the level of abstraction in line with expected students’ outcomes. They would start with a concrete problem, then move to using representations such as a table of values, a graph, and finally the equation. The pre-service teachers would use these representations to help students move forward from a concrete problem to the algebraic equation.

Moreover, during the planning, the pre-service teachers also predicted students’ responses. They considered students’ previous learning when predicting students’ strategies in determining the equation. They also predicted students’ possible mistakes (Level 3 TUP). These predictions were also included in their lesson plan (Appendix 9).

Gina : We are going to use problem based learning. This lesson is about the application of linear equation. We are going to pose a problem. If your father gives you Rp. 200,000 monthly allowance, and you spend Rp. 6,000 each day. At first we’re going to ask students “how much you have left after two days?” “After 5 days?” Then we will ask them to make a table. The columns represent the number of days, 2 days, 5 days, etc. The rows represent the amount of money left in any given day. Next, the table will be transformed in to the $x$-$y$ table. They will represent the number of days as $x$ and the amount of money left as $y$. They are able to draw the graph, and then determine the equation.

Gina : Our first prediction: students have learned to find the equation of a line that passes through two points. Then students can make the equation from the table. Second prediction [pause]. They will write the equation directly. They know they have Rp.
188,000 after two days, 176,000 after four days. Then they make an equation $188,000 = 200,000 - 2 \times 6,000$. From there, they will see a pattern that $y = 200,000 - 6,000x$.

Pipit: It’s possible students will get confused with the numbers, the numbers that should be the value of $x$ are written as the coefficient of $x$. They will think 6,000 is the $x$.

The evidence showed that in the planning, the pre-service teachers designed and discussed the tasks and the sequence of the activities in the lesson: a contextual problem, table of values, graph, and determining the equation. The pre-service teachers did not plan a traditional teaching lesson where the teacher worked through a problem as an example before asking the students to do similar problems. Instead they planned to give the students one problem and then while working on it students were expected to use mathematical models such as a table of values and a graph. The pre-service teachers spent a substantial amount of time discussing how the students would use the table of values or the graph to find the equation. They also demonstrated a focus on students’ thinking while predicting students’ responses.

Research Lesson 1 was taught by Pipit, she began by telling the students about the topic of the lesson and asking students what they learned over the previous lessons. Students mentioned that they learned about finding a linear equation when two points are given, when a point and the gradient are given, and when a point and another line parallel or perpendicular are given. Pipit did not review any further, she moved forward to the intended lesson. She distributed the worksheet and asked students to work in groups. She walked around the classroom and helped students during the group work. To complete the table of values students used either multiplying 6,000 by the number of days, and then taking away the result from 200,000 or repeated subtraction (Figure 5.5)

![Figure 5.5 Examples of students’ strategies](image)
Next, when asked to write the equation, most of the students could not use the table of values or the graph to help them find the equation. Even though they understood that the amount of money left after a certain day can be found by taking away the result of multiplying the number of days by 6,000 from 200,000 they could not see this as $200,000 - 6,000x$. However, one group could find the equation by using the two-point formula. Zayn, a student from the group was asked to present their work in front of the class (Figure 5.6).

![Figure 5.6 Zayn’s work- using the two-point formula](image)

Pipit used Zayn’s work to show the students how to find the equation using the two-point formula. Pipit did not elaborate on the reason for each step of the algorithm. She only mentioned that Zayn took two points, without emphasising how the selection of the two points was made and that students could take any two random points on the graph. She told students to put in the coordinate in the formula without asking them to identify the $x_1, x_2, y_1,$ and $y_2$. This episode was coded Level 1 TUP. Pipit might not know that it is a student’s common mistake to get the coordinates swapped. When working with an equation involving fractions, she only told students to do cross multiplication. She might assume that students at this level had learned about fractions and cross multiplication. Even though students had many encounters with fractions before, cross multiplication remains a common mistake for students.

<table>
<thead>
<tr>
<th>Pipit</th>
<th>Zayn took two points (2, 188,000) and (4, 176,000), then she used the two-point formula. She put the coordinates of the points into the formula, calculate[d] and [applied] cross multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>STUDENTS WERE NOISY</td>
<td>Pipit: The equation is $y = -6,000x + 200,000$. What does it mean? What is the $y$?</td>
</tr>
<tr>
<td>NO RESPONSE</td>
<td>Pipit: $y$ is the money left. 200,000 is the money given at the first place, and what is this $-6,000x$?</td>
</tr>
</tbody>
</table>
Student: \(-6,000\) multiplied by the number of days
Pipit: So if you're asked to find the money left after 4 days?
Student: \(-6,000 \times 4\)
Pipit: \(-6,000 \times 4 + 200,000\). Now do you understand?

The pre-service teachers did not predict that the students would have difficulty linking the table of values to the equation, thus they did not prepare any prompts to help students. Therefore, when her students could not find the equation, Pipit considered the previous lesson and chose the procedural approach of the two-point formula. During the post-lesson discussion, Pipit mentioned her struggle with the time. Pipit’s quick and mechanical explanation might have been caused by her concern of keeping up with the time rather than students’ understanding.

Even though most students could not see the connection between the table of values and the equation, one student – Aries, mentioned that finding the amount of money left after a certain day is achieved by multiplying the number of days by 6,000 and subtracting the result from 200,000. During the post-lesson discussion, as the knowledgeable other, I advised Pipit that she could have used the opportunity to discuss Aries’s answer and compare it with Zayn’s so that all students could see the connection between the two strategies.

Meili: Aries said “200,000 minus 6,000 times the number of days” but the group did not realise that it could be written as an equation. It indicates he understands the context, but not the generalised form. Too bad you erased Aries’s answer on the board. It could have been compared and connected to Zayn’s answer.

In the next research lesson, Gina who acted as the lead teacher took this advice. She found one group used multiplicative strategy to find the amount of money left after any given day. She helped the group to connect their answer with the equation. This was coded as Level 3 TUP.

Gina: It’s the money spent each day. It will increase as the number of days increase.

[Inaudible]
Do you know why it should be negative? Because 6,000 is taken out from the 200,000 every day. So the equation is?

\[y = \ldots?\]

Sofie: \(y = m\ldots\)
Gina: What’s the \(m\)?
Sofie: \(\ldots -6\)
\[y = -6 \text{ times } x + 200,000\]
Later on in the lesson, Gina asked this group to present and discuss their work in front of the class (Figure 5.7). Sofie started by calculating the amount of money left after 5 days, 10 days, 15 days, and 30 days as seen in point (a) to (f). They came up with these calculations from their discussion and help from Gina.

![Figure 5.7 Sofie’s group work](image)

Sofie: \[ y = -6 \times 5 + 200 \]
Gina: Where is it from?
Sofie: Because the numbers are 200 and 6. And 6 is taken out from 200.

...\[
y = -6x + 200
\]
Mahmud: It’s – 6 because 6,000 each day, and it’s taken away from 200,000. Therefore it’s negative.
Gina: Do you understand?

This evidence suggests that the post-lesson discussion has affected Gina’s decision in her teaching. She took the knowledgeable other’s advice to use students’ strategy to build on their understanding. Gina and Pipit used different strategies to help students find the equation. Gina’s strategy was more grounded to students’ reasoning that is “200,000 minus 6,000 times the number of days” whereas Pipit’s approach was using the two point-formula. However, Gina did not elaborate on this strategy any further in her lesson. Like Pipit, time was also her main constraint as she explained in the post-lesson discussion: “I spent too much time for the group discussion … Because of that, I skipped the reflection at the ending of the lesson”.

Overall, the findings suggest that the pre-service teachers believed in teaching procedurally before the lesson study implementation. Even though the pre-service teachers demonstrated slightly different beliefs during the planning, it was difficult for them to maintain their beliefs during the classroom teaching. The data showed that when the pre-service teachers encountered unpredicted students’ questions or difficulties, they tended to go back to the procedural approach to help their students. Time keeping also hindered the pre-service teachers from using a reformed approach in their teaching. However, the pre-service teachers learned
from participating in the post-lesson discussions and implemented some feedback from the knowledgeable other. They gained insight into how lessons could be better and this contributed to their beliefs about teaching.

5.1.2 Transformation Dimension

The Transformation dimension of the Knowledge Quartet resonates with Shulman’s (1987) idea of PCK that is, how the teacher transforms and unpacks their subject knowledge in the lesson using examples, analogies or demonstration to make it understandable by the students. The components under the Transformation dimension are Choice of Examples, Choice of Representations, Teachers’ Demonstration, and Teachers’ Use of Instructional Materials. This section presents the evidence of the pre-service Teachers’ Choice of Representations.

5.1.2.1 Choice of Representations (COR)

The Choice of Representations (COR) component is about how the pre-service teachers choose and use the representations in the lessons (Rowland et al., 2009). The findings in this section were taken from analysis of the written test and video data.

The written test item 7 (Figure 5.8) asked the pre-service teachers to evaluate the strengths and weaknesses of each teaching approach, select the best approach and justify their reasons. The first approach used geometrical representation, that is area model to represent the distributive property of multiplication over addition, whereas the second approach used a real world problem with symbolic representation.
The pre-service teachers’ responses showed that in evaluating the strengths and weaknesses of the approach, the pre-service teachers considered potential difficulties students might have when working with each approach. Table 5.5 presents the pre-service teachers’ responses to Item 7. For example, the pre-service teachers thought the geometric representation using an area model was simple but difficult for students to understand because it was not as realistic as using money context. They believed money is a part of students’ everyday life and therefore it is more familiar and easier for the students.
Table 5.5 Pre-service teachers’ responses to item 7

<table>
<thead>
<tr>
<th>Approach</th>
<th>7A</th>
<th></th>
<th>7B</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Strengths</td>
<td>PSTs</td>
<td>Weakness</td>
<td>PSTs</td>
</tr>
<tr>
<td>Simple (n = 6)</td>
<td>PD7D</td>
<td>Not realistic</td>
<td>PD7D</td>
<td>PD71</td>
</tr>
<tr>
<td></td>
<td>PD7R</td>
<td>(n = 3)</td>
<td>PE7U</td>
<td>PD7R</td>
</tr>
<tr>
<td></td>
<td>PD8P</td>
<td></td>
<td>PE7J</td>
<td>PE7J</td>
</tr>
<tr>
<td></td>
<td>PE7D</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>PE7U</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>PE8Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Connecting algebra and geometry (n = 4)</td>
<td>PD7I</td>
<td>Too abstract</td>
<td>PD7I</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PD8G</td>
<td>(n = 2)</td>
<td>PD8P</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PE7J</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>PE8V</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>Difficult if students do not have prior knowledge of area (n = 5)</td>
<td>PD7R</td>
<td>PD8G</td>
</tr>
<tr>
<td>Realistic (n = 10)</td>
<td>PD7D</td>
<td>Using large number (n = 2)</td>
<td>PD7I</td>
<td>PD7D</td>
</tr>
<tr>
<td></td>
<td>PD7I</td>
<td></td>
<td>PD8P</td>
<td>PD8P</td>
</tr>
<tr>
<td></td>
<td>PD7R</td>
<td></td>
<td>PE7U</td>
<td>PD8G</td>
</tr>
<tr>
<td></td>
<td>PD8G</td>
<td></td>
<td>PE7G</td>
<td>PE7U</td>
</tr>
<tr>
<td></td>
<td>PE7D</td>
<td></td>
<td>PE8V</td>
<td>PE8V</td>
</tr>
<tr>
<td></td>
<td>PE7U</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>PE7J</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>PE8V</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>PE8Y</td>
<td>Difficult to be used in different contexts (n = 3)</td>
<td>PE7D</td>
<td>PE7J</td>
</tr>
</tbody>
</table>

PE7D chose both approaches

In choosing the best approach (Item 7B), six pre-service teachers chose the second approach because it was realistic and considered easy for the students (n = 3) and it did not require extra work to review the geometry concepts (n = 3). Three pre-service teachers chose the first approach because it was simple (n = 1) and helped students to understand the connection between algebra and geometry (n = 2). One pre-service teacher chose both approaches because he believed that the two approaches complemented each other.

The pre-service teachers’ responses to the written test gave insights into how they chose the representation for teaching distributive properties. The pre-service teachers considered students’ prior knowledge and avoided approaches with potential difficulties. The practicality factor also underpinned the pre-service teachers’ choice. They preferred teaching approaches that did not require extra work. However, this selection process only reflected what the pre-
service teachers would choose from two given options. Whereas in everyday teaching, teachers must study the curriculum, understand the Choice of Representations available, and choose the appropriate representation and know how it is be used (Rowland et al., 2009). The data from the lesson study shed some insight into this matter.

During the lesson study, COR was most obvious in the designing the lesson which took place in the planning sessions. During the research lessons, the representation was used by the students. The students’ work provided insight into whether the pre-service teachers’ Choice of Representation helped students build their understanding or not. This triggered conversation to refine the representation during the post-lesson discussions. Table 5.6 shows the highest level of COR during the lesson study. COR appeared high in all cycles of the group except C2E7. Group E7 taught Financial Mathematics in their second cycles. The planning focused on developing tasks and problems which were mostly word problems with money context without using any mathematical representation. Therefore COR was not evident in this group.

Table 5.6 Highest Level of Choice of Representation (COR)

<table>
<thead>
<tr>
<th>Lesson study cycle</th>
<th>Lesson study phase</th>
<th>Lesson study phase</th>
<th>Lesson study group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Planning</td>
<td>D7</td>
<td>D8</td>
</tr>
<tr>
<td></td>
<td>Research lesson 1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 1</td>
<td>NE</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Research lesson 2</td>
<td>3</td>
<td>NE</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 2</td>
<td>NE</td>
<td>NE</td>
</tr>
<tr>
<td></td>
<td>Research lesson 3</td>
<td>2</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 3</td>
<td>NE</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>Planning</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Research lesson 1</td>
<td>NE</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 1</td>
<td>NE</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Research lesson 2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 2</td>
<td>NE</td>
<td>NE</td>
</tr>
<tr>
<td></td>
<td>Research lesson 3</td>
<td>NE</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 3</td>
<td>NE</td>
<td>NA</td>
</tr>
</tbody>
</table>

NE : not evident
NA : not applicable

Cycle 1 of Group E8 (C1E8) is chosen as an illustration for understanding how the pre-service teachers chose their mathematical representations. In the first cycle, Group E8 taught about gradient. In the planning, the group discussed ways to introduce the concept of gradient and the everyday language such as *tilt*, *slope*, *lean* that might help students make sense of it. They talked about using the leaning tower of Pisa to start introducing gradient to the students.

Vina: I looked for ideas of teaching gradient and I found something. Using the Pisa Tower to introduce the gradient. First, we’re going to ask students what is “slope/tilt/slant”
Yanti: It’s realistic.
Meili: From the example, what do you think it is?
Vina: The ratio of y and x. I hope students will get that, but I’m not sure.
Meili: How many degrees does the tower slanted?
Vina: 55
Meili: What if students think that the slope is the angle? How will you bring them to the concept of ratio of y and x?
Vina: Or using pictures. I will show them some pictures and ask them which one is the steepest then they will choose one. I’ll ask them why. They might say it’s the steepest because it almost falls down. Then I’ll ask them to define “slope”

Yanti argued that the Pisa tower was a realistic context therefore it was appropriate to be used in teaching about gradient. Both Yanti and Gina were not aware of how the gradient of the Pisa tower is represented in the story – that is by the angle. They did not connect the gradient as the angle of inclination to their lesson goal. For this reason, this episode was coded as Level 1 COR. When the knowledgeable other asked them about the potential misleading information in the Pisa tower problem, the pre-service teachers immediately looked for different ways to teach the gradient concept. Instead of working to refine the Pisa tower, they decided to use a Cartesian plane. The pre-service teachers expected that the coordinate points and the grids in the Cartesian plane would help them guide the students to the concept of \( \frac{\text{rise}}{\text{run}} \), and determine the gradient of a line in a Cartesian plane. This is an evidence of Level 2 COR.

Vina: They already know to find the gradient is \( \frac{y_2 - y_1}{x_2 - x_1} \), the y is \( y_2 - y_1 \), and the x is \( x_2 - x_1 \)
Meili: It requires them to draw the graph.
Vina: We can use the previous worksheet that is when they were given an equation and asked to draw the line. Then we’ll ask them to find the gradient.

The excerpt above shows Vina’s plan of the lesson. She used her knowledge of students’ previous learning and argued that students were able to draw linear graphs in a Cartesian plane. Starting from that point, she would continue by asking students to find the \( \text{rise} \) and the \( \text{run} \) in the Cartesian plane. The students could count the grids one by one or subtract the coordinate points, which then would lead to \( m = \frac{y_2 - y_1}{x_2 - x_1} \). In this episode, the pre-service teachers did not look for other ways to teach about \( m = \frac{y_2 - y_1}{x_2 - x_1} \).

Vina was the lead teacher in the first research lesson from Group E8 (C1E8). She gave students a task to help them understand the idea of \( \frac{\text{rise}}{\text{run}} \). The students were asked to find the gradient of lines by counting the grids to find the length of the \( \text{rise} \) and the length of the \( \text{run} \). Figure 5.9 is an example of students work when finding a gradient. Because the students only counted the length of the \( \text{rise} \) and the \( \text{run} \), this task did not raise the concept of negative and
positive gradient. Vina continued by showing another way to find the rise, that is by measuring the distance between the y coordinates of the two points and the run is by measuring the distance between the x coordinates of the two points. This resulted in $m = \frac{y_2 - y_1}{x_2 - x_1}$.

The next task was to determine the gradient of the lines in a Cartesian plane. Here, the students were given two coordinate points and asked to draw and find the gradient of the line. Because the students counted the grids in the previous task, it was not surprising that they still used this strategy in this task. This resulted in a common mistake where the students were not aware of the negative gradient.

This finding was discussed in the post-lesson discussion.

Vina: I thought students would have understood the Cartesian plane, the positive and the negative area of the plane. But they did not understand that, thus they only counted the grids and ignored the value.

Meili: We did not predict they would misunderstand that gradient is always positive. But that is understandable because the examples you gave in the introduction lead them to think that way. The goal of the second worksheet is to clarify the positive and negative gradient, then maybe you need to re-think the coordinates. For example mirroring. $(2, 4)$ and $(2, -4)$ then can be seen in the graph the lines are different, but if they only count the grids, they would end up with the same result – positive gradient. You might want to reconsider this, will this be easier for students?

Vina was surprised that the students counted the grids on a Cartesian plane instead of using $m = \frac{y_2 - y_1}{x_2 - x_1}$. The knowledgeable other showed the link of the students’ responses to the sequence of activities in the lesson. It was not surprising that the students counted the grids,
because the previous task asked them to do so. Both tasks misled the students to think that gradient is always positive. In order to address this misconception, the knowledgeable other suggested the pre-service teachers to mirror the lines. By mirroring the lines, students would see the lines are different, thus they have different gradient.

The second research lesson was taught by Yanti. She made some changes in her lesson based on the feedback during the post-lesson discussion. Figure 5.10 is a video screenshot when Yanti was showing the direction of the lines. She modified the worksheet by using mirroring as advised by the knowledgeable other.

Yanti: To measure a gradient, by determining the ratio of “height difference and horizontal difference”.

There is another thing you must consider when determining the gradient. There’s a positive gradient and a negative gradient. Look at this picture again line AB and CD. We can see that this line can be seen two ways, from A to B it goes up, from B to A it goes down. Now we need to make a convention. We start from the left. So we go from A to B, or the run then the rise. Look at the run. From A it goes to the right and then goes up to B, thus it’s positive.

Now look at CD. What was our agreement?

Students: Start from the left.

Yanti: The left side is C. We go from C to D. First we go down. What’s the value then?

Students: Negative.

Yanti: After that we go to the right. What does it mean?

Students: Positive.

However, Yanti did not use the idea of mirroring to raise the direction of the line and the value of the gradient. Instead, she started highlighting the value of the gradient early at the beginning of the lesson by making a convention with the students about the direction of the line. This episode was coded as Level 2 COR, because even though Yanti chose a better representation, she did not utilise it to build students’ understanding.
In summary, the written test revealed the pre-service teachers’ Choice of Representations on teaching distributive properties. Of two choices of representations, namely, the area model and the money context, most of them chose the money context because they believed it was more realistic for the students. The pre-service teachers argued that the area model might be difficult because students might not have a prior knowledge of area and consequently it required more work to review the concept of area. This suggests that the pre-service teachers considered students’ prior knowledge and practicality of the teaching when selecting the appropriate representation.

Similar findings were also shown in the planning. The pre-service teachers would choose a realistic context for introducing gradient. They understood realistic context as something that is real, not a made-up story. Students’ prior knowledge was another primary factor in deciding what representation they used in the lesson. These findings suggest that the Transformation Knowledge – the Choice of Representation is highly influenced by the Connection Knowledge – pre-service teachers’ knowledge of students’ prior knowledge. However, because of their lack of teaching experience, the pre-service teachers’ knowledge of students’ prior knowledge was not always accurate. Therefore, the expected students’ learning might not always occur in the research lessons. In the post-lesson discussion, the group discussed a better way to represent gradient by using mirroring. This resulted in the refinement of the representation. Unfortunately, Yanti did not use it to build students’ understanding about positive and negative gradients. It seemed that Yanti’s beliefs and knowledge about teaching students’ mathematical understanding led her to go back to the procedural teaching approach.

5.1.3 Connection Dimension

Mathematics is a connected and coherent body of knowledge. Learning mathematics meaningfully requires students to make the connection between their existing knowledge and the new knowledge they are learning to grasp. Teaching mathematics should support the students to build those connections. Therefore the Connection dimension concerns the pre-service teachers’ knowledge of students’ prior knowledge, mathematical concepts and procedure, anticipation of the complexity of student learning (Rowland et al., 2009). This section presents the evidence of the pre-service teachers’ Anticipation of Complexity.
5.1.3.1 Anticipation of Complexity (AOC)

Anticipation of Complexity (AOC) is about how the pre-service teachers predicted the students’ learning including possible student solutions and difficulties (Rowland et al., 2009). In addition, these predictions can help pre-service teachers prepare prompts to help the students overcome their difficulties. This section presents results from analysis of the written test, video of lesson study meetings, lesson study artefacts, and interviews.

The written test, Item 5 (Figure 5.11) asked the pre-service teachers to write their predictions of students’ answers and mistakes when working on a problem about a number pattern.

![Figure 5.11 Written test item 5](http://www.eworkshop.on.ca/edu/resources/guides/Guide_Patterning_and_Algebra_4_56.pdf)

Table 5.7 summaries the pre-service teachers’ responses to Item 5. For item 5A, five pre-service teachers solved the problem using arithmetic sequence formula, and three used addition. Only one pre-service teacher used both ways. In addition, two pre-service teachers provided an incorrect answer. One of the incorrect answers used a wrong formula, and the other one wrote the process to derive the formula.
Table 5.7 Pre-service teachers’ responses to Item 5

<table>
<thead>
<tr>
<th>Item</th>
<th>Responses</th>
<th>Pre-service teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PD7D</td>
<td>PD7I</td>
</tr>
<tr>
<td>5a</td>
<td>Formula arithmetic sequence</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Addition</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Incorrect answer</td>
<td>✓</td>
</tr>
<tr>
<td>5b</td>
<td>Addition</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Drawing the cans to the 10th level</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Formula arithmetic sequence</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Mistaken the sum with the 10th item</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Doubling : 15 × 2</td>
<td>✓</td>
</tr>
<tr>
<td>5c</td>
<td>Miscalculation</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Mistaken the sum with the 10th item</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Mistaken the formula</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Doubling : 15 × 2</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Misconception</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Misinterpretation of the problem</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Guessing</td>
<td>✓</td>
</tr>
</tbody>
</table>

The pre-service teachers’ responses to item 5B ranged from using pictorial representation, addition, to the arithmetic sequence formula. Two pre-service teachers – Ida (PD7I) and Gina (PD8G) answered all these three strategies. They were probably aware of the cognitive progression of students’ thinking when solving this problem. Moreover, for item 5C, the pre-service teachers who predicted students would draw the cans (n = 5) said that they would miscalculate. These pre-service teachers also predicted students would solve the problem using addition. Drawing the cans to the 10th level allows the students to count the cans one by one, or perform addition. Both strategies are very prone to miscalculation. However, the pre-service teachers did not provide detailed examples of the kind of miscalculation students might make. This is possibly because the pre-service teachers have written the strategies as a response to item 5B, thus they just wrote miscalculation referring to answers of item 5B. For example, Figure 5.12 shows PD7R’s (Raya) responses to item 5. Her predictions
of student’s solutions (Item 5B) were students would use addition, and draw the cans then count them one by one. Consequently, her predictions of students’ mistakes were miscalculation.

Table 5.8 presents the highest level of AOC during the lesson study. AOC was most evident in the planning where the group predicted how students would work on the task they designed. The C1D7 is presented as an illustration of the shift in the pre-service teachers’ AOC. Their lesson plan is also presented as an example of their prediction.

Table 5.8 Highest Level of Anticipation of Complexity (AOC)

<table>
<thead>
<tr>
<th>Lesson study cycle</th>
<th>Lesson study phase</th>
<th>D7</th>
<th>D8</th>
<th>E7</th>
<th>E8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Planning</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Research lesson 1</td>
<td>2</td>
<td>NE</td>
<td>NE</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Research lesson 2</td>
<td>2</td>
<td>1</td>
<td>NE</td>
<td>NE</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 2</td>
<td>2</td>
<td>1</td>
<td>NE</td>
<td>NE</td>
</tr>
<tr>
<td></td>
<td>Research lesson 3</td>
<td>2</td>
<td>NA</td>
<td>NE</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 3</td>
<td>NE</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>2</td>
<td>Planning</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Research lesson 1</td>
<td>NE</td>
<td>NE</td>
<td>NE</td>
<td>NE</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 1</td>
<td>NE</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Research lesson 2</td>
<td>NE</td>
<td>3</td>
<td>NE</td>
<td>NE</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 2</td>
<td>NE</td>
<td>NE</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Research lesson 3</td>
<td>1</td>
<td>NA</td>
<td>NE</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 3</td>
<td>NE</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

NE : not evident
NA : not applicable

The topic taught in the first cycle of Group D7 was solving linear equations. The group used the balance model to introduce the concept of equivalence in an equation. In the planning,
the focus of the discussion was about how students would make sense of the balance model and transform it into an equation. The pre-service teachers would start the lesson by giving the students a problem (Figure 5.13), students would be asked to find the number of candies in each bag.

![Figure 5.13 Balance model discussed in the planning](image)

As the knowledgeable other, I initiated the discussion about students’ thinking. I asked the pre-service teachers to predict students’ responses to the balance problem. The pre-service teachers’ predictions were vague thus, they were coded as Level 2 AOC.

- **Raya**: Prediction 1: student don’t do anything
- **Raya**: Prediction 2: students take away four from each sides.
- **Ida**: First we ask them why they take away four from both sides, and how is the balance position after they took away four from both sides?
- **Ida**: Prediction 3: they move the candies to different side but the balance stays equal.

The pre-service teachers’ predictions did not include students’ reasoning processes when solving the problem. In particular, they did not assume that each bag should contain the same number of candies. Therefore, I asked them to think about how students would make sense of the problem and translate the problem from a concrete object to a mathematical equation. This question provoked pre-service teachers to predict students’ understanding of variable.

- **Meili**: How do you help students to understand $10 = 2x + 4$?
- **Raya**: We’ll discuss ways to simplify the problem that is by symbolising the unknown with an $x$. But then it doesn’t come from their own construction?
- **Diana**: Maybe students will do it without using a variable $x$, like $10 = 2$ bags $+ 4p$. $p$ is for the candies.
- **Meili**: Have they learned about variable before?
- **Raya**: Yes.

This excerpt suggests that prompts from the knowledgeable other stimulated the pre-service teachers to predict students’ responses especially about the notion of variable. When asked how students would understand $10 = 2x + 4$, the pre-service teachers’ idea was to use a
symbol, in this case $x$ to represent the unknown. Presumably, the pre-service teachers referred the unknown to the number of candies in the bag. The pre-service teachers also thought about students’ possible mistakes such as students would not use any variable and students would use variables incorrectly. For example, in $10 = 2 \text{ bags } + 4p$, the use of $p$ to replace the “permen” – Indonesian word for candies is a common mistake for students who are just beginning to learn algebra. The pre-service teachers were able to make a more detailed prediction. Thus, this episode is evidence of Level 3 AOC. The pre-service teachers seemed to notice this common mistake. However, it is unclear if they fully understood what constitutes the mistake and what consequences might result from it.

The group continued the discussion with developing problems for students’ exercises. They wanted to move forward from the balance model to equations. They planned to give the students a word problem – “A number is multiplied by two then added to one, the result is 13. Find the number!”

Meili : Think of the possibilities of students’ answers!
Raya : Possibility 1: students might not understand at all.
Ida : Students might ask what number is that? Even number, odd number, or?
Nur : Tell them that it’s their job to figure it out.
Ida : Possibility 3: students guess the number.
Raya : Tell them to check their answer, put the number in the equation and see if the answer is correct.
Nur : It’s very unlikely they do the changing side, they’re likely to do guess and check.
Raya : They write the equation but wrong, $3x = 13$.

“reread the problem there’s a key word there “then” that means another mathematical operation”.

In this excerpt, the pre-service teachers also showed obscure predictions (Level 2 AOC). Their predictions suggested their assumption of the level of students’ mathematical competences. For example, possibility 1: students might not understand at all, likely refers to the lower achievers who do not show any effort at all to understand the problem. Whereas possibility 3: “guess and check” might be based on the pre-service teachers’ assumption of the average students. Possibility 2: “students might ask if it is an odd or even number” suggested that the pre-service teachers took into account the students’ behaviour into their prediction. While odd or even numbers are not relevant to the problem, the pre-service teachers used a behaviour lens for this prediction. The pre-service teachers often noticed that students mentioned random ideas before thinking of the problem.

Even though their predictions were still very vague, in this excerpt the pre-service teachers provided prompts for the students. This could have been triggered by the feedback
from the mentor teacher – Nur. In responding to the odd or even number, Nur suggested that the pre-service teachers encourage the students to find it out for themselves. When proposing the next prediction the pre-service teachers included the prompts corresponding to the prediction. The prediction of students’ responses and prompts to help students were also included in the lesson plan (Appendix 10). Even though the predictions and the prompts lacked depth and detail, it was a new practice that the pre-service teachers carried out during the lesson study.

The first research lesson was taught by Ida. When working with the balance problem (Figure 5.14), some students took away four from both sides. Even though students could solve the problem easily with this strategy, many of them could not write the equation corresponding to the problem. The students were having problem with determining the \( x \). For example, students mistook the \( x \) as two bags of candies or one bag of candies instead of the number of candies in one bag. Some students did not use any variable or algebraic representation because they could easily solve the problem using simple arithmetic.

![Figure 5.14 Balance problem used in the lesson](image)

How students would transform the balance model into an equation was not discussed deeply in the planning. The pre-service teachers did not predict that students would have difficulties in this area, thus it is a Level 1 AOC. The pre-service teachers were not aware that the balance problem was easy for the students to solve by only using simple arithmetic; it did not require the students to use algebraic representation. When students do not have a strong need to use algebra, forcing them to use it makes it artificial and difficult.

Even though as the knowledgeable other I had brought up this issue in the planning, I did not provide any suggestions for the pre-service teachers to deal with this. I followed up this matter in the post-lesson discussion. I brought out the evidence of students’ misconceptions. My objective was to trigger a discussion about helping students overcome these
misconceptions. Discussing students’ difficulties from the research lesson is a form of predicting students’ responses in the next lesson, therefore it is coded as Level 2 AOC.

Meili: I noticed Nuri stated $x = 2$ bags of candies. Rani wrote $x = 1$ bag, Rafi wrote $x =$ the number of candies in one bag. But then he wrote on the board, $x =$ bag. Many students had misconception of the meaning of $x$. You need to correct this in the next meeting. Some students did not use $x$. They wrote $(20 - 4) ÷ 2 = 8$. What do you think happens here? Why they can solve it using basic calculation, but they cannot write the equation?

Raya: I think, looking at the syllabus, first they learn about numbers, then sets, and then linear equation with one variable. They have not learned basic algebra for linear equation with one variable in algebra.

Ida: In the previous meeting on Thursday, I introduced them to equation, variable, constant, and coefficient. Only the terminologies. We have not covered the operations at all.

Nur: In the 2013 Curriculum, algebra topic is allocated for Year 8. While in the previous curriculum it was for Year 7. We teachers find it problematic with 2013 Curriculum because the topic is not in order; we need to allocate time for introducing the pre-knowledge.

Raya argued that the reason for students’ misconceptions was because they have not learned basic algebraic concepts in this case the components of an equation such as variable, coefficient, and constant. She blamed it on the topic order in the curriculum. The mentor teacher agreed with this. Ida mentioned that she had taught the students the terminologies but not the algebraic operation. This suggests that the pre-service teachers relied on the Connection Knowledge that is, the order of topics in the curriculum that indicates students’ prior knowledge in addressing the problem. The discussion did not concern ways to help the students move from the balance model to the equation. This indicates the absence of mathematical knowledge about students’ reasoning in abstraction process of transforming the balance model in to an algebraic equation.

Because Raya was the next lead teacher, in the post-lesson discussion she was asked about her ideas for her lesson.

Meili: Who’s next? Raya? What do you think? Are you going to make some changes?

Raya: I want to revise the PowerPoint, but I think there’s not enough time to revise the worksheet. I’ll be in 7C. I know some of them are very quiet, some of them are low-motivated. They don’t show great effort to understand the lesson. Even after I repeat the explanation few times, they keep saying they don’t understand. On the other side, students who excel want to move on with the next lesson.

Raya’s lesson was on the same day as Ida’s with only a few hours difference. Because of this, Raya did not have enough time to revise the worksheet. She planned only to revise the PowerPoint presentation. However, she did not talk in detail about the revision she would make and the predictions of how students would learn from it. She expressed her concerns about the students’ low motivation instead.
Learning from Ida’s lesson, Raya provided some hints to the students before the balance activity. She started by discussing the homework and then showing the balance in the whiteboard (Figure 5.15).

Raya: Look at the whiteboard. What is that?
Students: Candies and bags.
Raya: What else?
Students: Balance weighs.
Raya: Look at the position of the balance. Are they equal or any side is heavier than the other?
Students: The same.
Raya: Look at the right side of the balance. What’s in there?
Students: 4 candies and 2 bags
Raya: What's in the left?
Students: ...
Raya: So far do you understand? You will be asked to write the equation. Just like the previous example, $10 + 2x = 50$. This is called the equation. Now from the picture here, you are asked to change it into an equation.

This excerpt shows Raya emphasises the equivalence and the intended students’ responses – transforming the balance into an equation. This indicates that she had anticipated students’ unawareness of the instruction might have caused students’ difficulties in writing the equation, therefore it was coded as Level 2 AOC. However, the prompts did not help the students’ transition from the concrete problems to equations as reported by the observers during the post-lesson discussion.

The observers reported students’ ways of solving the problem. Raya’s students also had the same difficulties as Ida’s. They could not translate the balance problem into an equation.
Nur – the mentor teacher observed some students still have not understood the concept of equivalence.

Nur : I think many students haven’t grasped the meaning of equation, left side is equal to right side. Dira, she’s one of the high achievers in the classroom. She wrote:
left side: \( 2x + 3 = \ldots \)
She should have continued the right side with \( x + 8 \). But instead she wrote it underneath.
\[
2x + 3 = \\
x + 8 = 
\]
Then she was confused, she didn’t know what to do because she has two equal signs. This indicates that she hasn’t understood the concept of right side = left side.
Similarly, Keyra wrote: \( k \) is for the basket. There are 3 baskets. Instead of left = right, she did it basket = balls. She wrote:
\[
3k = 11
\]
Rafli used logical thinking, he could find the answer correctly but was unable to explain and justify his answer. Some students are like Rafi, they are efficient in the sense that they don’t want to write long answers on the paper but they can’t explain their thinking.

The mentor teacher – Nur provided a very detailed description of students’ thinking. She noticed that students did not understand the idea of equivalence and the use of equal sign. She also observed that one student solved the problem using logical thinking but could not explain his reasoning. Nur commented that students were often reluctant to use long calculations when the problem could be solved easily. This suggests that the problem was not challenging enough to stimulate the students to use algebra. The knowledgeable other echoed the mentor teacher’s observation by sharing more evidence of students’ mistakes. However, the discussion did not proceed to finding ways of helping the students.

Diana taught the third research lesson. Even though Diana picked up a common observation in both Ida’s and Raya’s lesson – that the students could not translate the balance problem into an equation, she used the same approach as Raya that is giving the students some hints before working on the balance problem (Level 2 AOC).

Diana : Look at the picture, what is this?
Students : Bags and candies.
Diana : This is a picture of a balance weigh. What is on the left side?
Students : Candies.
Diana : How many candies?
Student : 4.
Diana : How’s the position of the balance? Is it balanced or not?
Students : Balance.
Diana : What’s on the right side?
Students : 20.
Diana : 20 candies.
This is the first problem, you will be asked to write the mathematical model and find how many candies are in one bag.
Similar to Raya, Diana also provided hints before the students worked on the balance problem. She stated clearly that the students were asked to write the mathematical model and to find the number of candies in one bag. However, this did not help the students to write the equation either.

To sum up, in the planning, the pre-service teachers anticipated students’ learning by making predictions and prompts to help the students. However, the predictions and the prompts both in the planning and lesson plans were very vague. The pre-service teachers’ anticipation of students’ thinking was underpinned by their knowledge of students’ prior learning and students’ behaviour. The research lessons showed that the students’ main difficulty was transforming the balance model into an equation. The group discussed the students’ difficulties in the post-lesson discussions. This is a form of AOC because the students’ responses in the first research lesson provided insight into how the students in the subsequent research lessons would respond. The data also showed that predicting students’ responses without preparing prompts did not help the pre-service teachers’ much. The mathematical reasoning involved when transforming balance models into equations was missing in the planning and post-lesson discussions. Therefore, the pre-service teachers were not prepared with support for the students.

5.1.4 Contingency Dimension

The Contingency dimension concerns the pre-service teachers’ reaction to unanticipated situations such as unpredicted students’ solutions and questions in the lesson (Rowland et al., 2009). It is concerned with how the pre-service teachers deviate from agenda, respond to students’ ideas, use opportunities, and respond to unavailability of teaching resources. The previous section addressed the pre-service teachers’ anticipation in the planning. This section focuses on the pre-service teachers’ responses to the unanticipated events in the lessons.

5.1.4.1 Responding to Students’ Ideas (RSI)

Even though the pre-service teachers had anticipated students’ responses, some of them did not occur in the lessons. On the contrary, unanticipated events occurred. This section presents the findings from video data that showed evidence of how the pre-service teachers reacted in contingent moments. Because the Contingency dimension is about the pre-service teachers’ responses to unanticipated moments, written test data were not applicable for this section.
Responding to Students’ Ideas is about the pre-service teachers’ reaction to unanticipated students’ responses (Abdulhamid & Venkat, 2014; Rowland et al., 2009). Moreover, it is important to see the alignment of the pre-service teachers’ anticipated students’ responses in the planning and the students’ actual responses during the teaching. To get an understanding of the connection between the AOC and the RSI, this section continues discussing C1D7.

Table 5.9 shows the highest level of the pre-service teachers’ RSI during the two cycles of lesson study. RSI was not observed in the planning because according to (Rowland et al., 2009) the Contingency dimension is for unanticipated events in the lesson.

Table 5.9 Highest Level of Responding to Students’ Ideas (RSI)

<table>
<thead>
<tr>
<th>Lesson study cycle</th>
<th>Lesson study phase</th>
<th>Lesson study group</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Planning</td>
<td>NE  NE  NE  NE</td>
</tr>
<tr>
<td></td>
<td>Research lesson 1</td>
<td>2  2  2  2</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 1</td>
<td>NE  1  NE  2</td>
</tr>
<tr>
<td></td>
<td>Research lesson 2</td>
<td>2  2  2  NE</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 2</td>
<td>2  NE  2  3</td>
</tr>
<tr>
<td></td>
<td>Research lesson 3</td>
<td>2  NA  2  NA</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 3</td>
<td>2  NA  NA  NA</td>
</tr>
<tr>
<td>2</td>
<td>Planning</td>
<td>NE  NE  NE  NE</td>
</tr>
<tr>
<td></td>
<td>Research lesson 1</td>
<td>2  2  3  2</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 1</td>
<td>3  2  1  1</td>
</tr>
<tr>
<td></td>
<td>Research lesson 2</td>
<td>2  3  NE  2</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 2</td>
<td>NE  NE  NE  3</td>
</tr>
<tr>
<td></td>
<td>Research lesson 3</td>
<td>NE  NA  NE  NA</td>
</tr>
<tr>
<td></td>
<td>Post-lesson discussion 3</td>
<td>NE  NA  NA  NA</td>
</tr>
</tbody>
</table>

NE : not evident  
NA : not applicable

The planning of this group has been presented in the previous section. This section presents the episodes from the research lessons where both anticipated and unanticipated moments occurred. More especially, it focuses on the pre-service teachers’ responses to these moments. Ida, the first lead teacher had an unanticipated moment when a student used $x$ for representing two bags of candies. Below is Ida’s reaction to the student.

Ida : How many candies?  
Nuri : [counting one by one] 20.  
Ida : How’s the position of the balance?  
Nuri : The same. Equal both side.  
Ida : Equals to what?  
Nuri : Two bags of candies and four candies.  
Ida : Okay, you said this is equal, can you make the equation? What is the unknown?  
Nuri : This [pointing the bag].  
Ida : That means you can express it using what?  
Nuri : $x$. 

106
Ida: Yes, you may use $x$. What equals to what?
Nuri: $x$ equals two bags of candies.
Ida: $x$ is two bags of candies, and then how is the equation? Can you write it down?
Nuri: [writing]
Ida: You said it is balanced. Discuss this with your group. What does it mean?
Nuri: Why do you use $x$ to represent 2 bags of candies?
Ida: I don’t know.
Nuri: Look at the question. What does it ask? The candies in one bag.
Ida: One bag.
Nuri: That means, what do you use $x$ for?
Nuri: [looked puzzled]

Nuri used $x$ to represent two bags of candies, not the number of candies in the bags. Ida seemed puzzled by Nuri’s misunderstanding. Ida did not predict this in the planning and she was not prepared with appropriate prompts. She asked Nuri to look at the problem “find the number of candies in each bag”. This sentence contains key words that lead to identifying the unknown. Ida’s hint to look for a key word was perfect for students who have understood the use of variable. However, Nuri has not understood this. She seemed to have a misunderstanding that variable is used to represent the unknown. Nuri thought the unknown was two bags, which was possibly because Ida kept referring $x$ for the unknown. Because Ida did not give Nuri a proper support, this instance was coded as Level 2 RSI. Moreover, Ida was not aware of Nuri’s main problem – understanding the variable. Ida did not seem to have knowledge about variable and how students construct an understanding of variable. It suggests that Ida needed to build on that knowledge to be able to help students with proper guidance hence, the Foundation Knowledge influences the Contingency Knowledge.

In the post-lesson discussion, the group discussed students’ difficulties but did not address Nuri’s misunderstanding specifically. Since many students could not apply the variable properly, as the knowledgeable other, I linked this to the example that Ida used in the opening of the lesson – “3 kgs of egg contain 48 eggs. How many eggs are in 1 kg egg?”.

Meili: You gave a problem about the eggs. What did you do there? $x$ equals what?
Ida: $x$ equals 1 kg of eggs
Meili: Why did you use $x$ for 1 kg of eggs?
Ida: I followed students’ responses when I asked them what is being assumed. They said the kilo grams of the eggs.
Meili: Okay, you wrote students’ responses. Is that correct? $x$ = 1 kg of eggs?
Ida: …[paused]
Meili: Do you think it’s correct?
Ida: Yes.
Meili: What do the others think?
Raya: $x$ is the number of eggs in one kg.
Meili: Yes. Be careful here. This is one example of the common mistake in algebra. You can read in Van De Walle’s book. I understand you want to shorten the sentence, but keep in mind that $x$ represents the quantity not the items.

My intention was for the pre-service teachers to become aware of this common mistake of the use of variables. Students or even teachers tend to use the variable as a shorthand for the object of the unknown not the quantity of the unknown (Arcavi, et al., 2017). Moreover, I wanted Ida and the other pre-service teachers to understand that when the teacher gave an incorrect example, the students would be more likely to make the same mistake.

Raya – the second lead teacher did not make any mistakes in referring the variable in the problem. It seemed that she had learned from the previous research lesson. The Contingency Knowledge, especially RSI, was observed when students worked on the second problem in the worksheet – “A number is multiplied by two and then added by one. The result is 13. What is the number?” The pre-service teachers’ predictions of students’ responses to this problem were presented in the previous section (5.1.3.1). The students’ actual responses were not predicted in the planning.

Many students used guessing and working backwards strategies (Figure 5.16). Student 1 stated the $x$ for the number and was able to write the equation correctly. However, he did not continue with algebraic operations. He might have guessed and replaced $x$ with 6 to make the equation true. Student 2 used the working backward strategy suggesting that he understood the word problem and was able to reverse the order of operations. He did not need any algebraic equations to solve the problem.

Figure 5.16 Students used guessing and thinking backward strategies

For this problem, students could not write the equation corresponding to the problem. Therefore, Raya discussed the solution with the equation.

Raya: [Reciting the problem]
What is the question? You must find that number. Do you know what number it is?

Students: No.
Raya: Because we do not know the number, we assume it with a variable. Let’s call it $x$.

Next, read the problem again. A number is multiplied by 2 [then writing $x \cdot 2$]

Then what? Added to one. [writing $x \cdot 2 + 1$]

What is the result? 13. [writing 13]

$$2x + 1 = 13.$$ 

Look this has $x$, this has no $x$. We put together the ones without the $x$.

1 is moved.

$$2x = 13 - 1.$$ 

$$2x = 12$$

Students: Six.
Raya: $x = 12 \div 2$

$$x = 6$$

Do you understand this?

Students: [choir] Yes.

Raya used written or verbal representation to link the words with the symbolic algebraic representations. For example, transforming “A number is multiplied by two” into the symbolic expression was done by first replacing the unknown – a number with an $x$. Next “a number is multiplied by two” becomes “$x \times 2$”. This was coded as RSI Level 2.

The next step after writing the equation was solving it. Raya used a procedural way of solving the equation that is, by isolating the variable and grouping the constants. (Tirosh, Even, & Robinson, 1998) named this approach as “collecting the like terms”. Raya emphasised the change of the sign when an item moves to the other side of the equation. This approach is not appropriate for students who have just started learning algebra because it is difficult to understand. Students need to understand that changing sign is a consequence of the equivalence. Unfortunately, Raya did not elaborate this. Tirosh et al. (1998) claim that this approach is a ritual procedure that does not motivate students to understand the goal of learning. While this type of problem can also be solved by using the working backward method, Raya did not use or make links to it. Therefore, this episode was coded as Level 2 RSI.

In the last research lesson, Diana was the lead teacher. Her students had some difficulties working with an equation involving variables on both sides of the equation (Figure 5.17). Assuming the baskets contain the same number of balls, students were asked to find the number of balls in one basket. A group of students could not make sense of the problem and they could not write the equation of the balance problem.
Similar to Raya, Diana used written or verbal representation in helping the students connect the problem to the equation as shown in the excerpt below. Diana also guided the students to solve the equation using “collecting the like terms”. Even though by observing two research lessons and participating in the post-lesson discussions Diana was able to identify students’ difficulties, she could not come up with a better way of helping the students. This was probably because the post-lesson discussions did not focus on that.

Diana : What is the variable?
Student 1 : $x$
Diana : How many $x$ do you have?
Student 1 : 2
Diana : $2x$ and …
Student : 3 balls.
Diana : They are together, what does it mean?
Student 1 : [no response]
Student 2 : They’re grouped.
Diana : Yes, what does it mean?
Student 2 : Addition.
Diana : Yes. $2x$ + …
Student 1 : 3 balls.
Diana : Write it down!
Student 1 : [writing $2x + 3$]
Diana : Are they balanced?
Student 1 : Yes.
Diana : What does it mean?
Student 1 : =
Diana : Yes. What do you have in this side?
Student 1 : 8 [writing 8]
Diana : [pointing the basket]
Student 2 : Plus $x$
Student 1 : 8 plus $x$?
Diana : Yes. Then you group all the $x$ together, and these together.
Student 1 : I really don't understand.
Diana : Now you move this to the other side. This is positive, when moved, it becomes negative. You put all the $x$ together on the right side. The one without the $x$ are together in the left side.
Diana used “collecting the like terms” for balance problem. She did not make use of the balance model to help students make sense of the problem, understand the concept of equivalence and solve the problem. Balance method – by crossing out one bucket and three balls from each side of the balance (Siemon, 2011; Van de Walle, Karp, & Bay-Williams, 2017) would have been easier for the students. Taking out \(x\) and 3 from each side, thus \(2x + 3 = x + 8\) becomes \(x = 5\). It seemed that Diana was not aware of this strategy, therefore it was coded as Level 2 RSI.

In summary, the pre-service teachers responded to unanticipated students’ responses similarly. They asked the students to look back at the problem so that they really understood it. Then they encouraged the students to translate the written and verbal expression into an algebraic expression. In solving the equation, the pre-service teachers used “collecting the like terms” approach without building students’ understanding of the equivalence by using the balance model. The data showed that even though the pre-service teachers could identify student’ difficulties from observing research lessons and participating in post-lesson discussions, they could not come up with effective ways of helping students to overcome those difficulties. As a result, when responding to unanticipated students’ responses, the pre-service teachers went back to the procedural teaching rather than a more appropriate method for solving equations. Given that the post-lesson discussions did not prepare them with some handy prompts, indicated that the pre-service teachers might not know any other ways beside the transformation procedure. This suggests the absence of the pre-service teachers’ Foundation Knowledge hindered them in responding properly in contingent moments.

5.2 The Characteristics of Lesson Study that Contribute to the Development of the Pre-service Teachers PCK

The previous section addressed the pre-service teachers’ PCK development during lesson study. This section addresses the characteristics of lesson study that contributed to the pre-service teachers’ PCK development namely the planning, research lessons and post-lesson discussion phase, observing research lessons, and support from the mentor teachers and the knowledgeable other.

5.2.1 Planning, Research Lessons, and Post-lesson Discussions

Section 5.1 has addressed the development of the pre-service teachers’ PCK in a particular KQ element of one particular lesson study group’s cycle. It addressed the pre-service
teachers’ PCK enactment in the planning, research lessons, and post-lesson discussions from Cycle 2 Group D8 (C2D8) and Cycle 1 Group E8 (C1E8). Furthermore, by tracking down the PCK in the planning, research lessons, and post-lesson discussion, it identified the development of the PCK and its complexity that revealed the interconnections of the Knowledge Quartet components.

From the data presented in Section 5.1, Cycle 2 of Group D8 (C2D8) showed the development of the pre-service teachers’ Foundation knowledge, especially the Overt Subject Knowledge component on finding square roots. In light with that development, Table 5.10 shows the KQ components enacted by the pre-service teachers during the planning, research lessons (RL) and post-lesson discussions (PLD).

Table 5.10 The KQ components enacted in C2D8

<table>
<thead>
<tr>
<th>KQ dimension</th>
<th>KQ components</th>
<th>Lesson Study Phases</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Planning RL 1 PLD 1 RL 2 PLD 2</td>
</tr>
<tr>
<td>Foundation</td>
<td>Overt Subject Knowledge</td>
<td>√ √ √</td>
</tr>
<tr>
<td>Connection</td>
<td>Anticipation of Complexity</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Making Connection Between Concepts</td>
<td>√ √ √</td>
</tr>
<tr>
<td>Transformation</td>
<td>Choice of Representation</td>
<td>√</td>
</tr>
<tr>
<td>Contingency</td>
<td>Responding to Students’ Ideas Using opportunities</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Deviation from agenda</td>
<td>√</td>
</tr>
</tbody>
</table>

Table 5.10 shows that, while designing the lessons the pre-service teachers enacted their Foundation, Transformation, and Connection Knowledge. More specifically on the Foundation Knowledge, when deciding which Pythagorean Theorem proof to be used in the lesson, they demonstrated the Overt Subject Knowledge on Pythagorean Theorem and Pythagorean Triple. However, the Overt Subject Knowledge on square roots was not observed here yet. The Connection Knowledge observed in the planning was the Anticipating of Complexity component, for example, while anticipating students’ responses and making connection between concepts, the pre-service teachers acquired an understanding of the connection between Pythagorean Theorem and Pythagorean Triples. The Transformation Knowledge particularly the Choice of Representation component was demonstrated when the pre-service teachers discussing which Pythagorean Theorem proof was best to use in the lesson. The
Contingency Knowledge was not noticeable in the planning. This is because Rowland et al. (1999) define it as how the teachers respond to unplanned situations in the lesson.

The Contingency Knowledge was observed in the first research lesson when Pipit – the lead teacher did not respond to the students’ struggle to find square roots. In the first post-lesson discussion, the reason why Pipit did not help students to work with square root was revealed. This unpacked her lack of Overt Subject Knowledge on square roots. Consequently, the discussion focused on ways to help students find square roots, which brought the Transformation Knowledge, especially Choice of Representation to life. This was evident when the knowledgeable other showed how to use the number line to represent the corresponding whole numbers and square numbers. This also highlighted the Connection Knowledge – Making Connection Between Concepts. Using the number line, the pre-service teachers learned about the connection between Pythagorean Theorem, numbers, and estimations.

In the second research lesson, Gina, the lead teacher used the pairing strategy to help students find the square roots (see Section 5.1.1.1). This indicates her improvement in the Foundation Knowledge, especially Overt Subject Knowledge on square roots, the Transformation Knowledge, especially on the Choice of Representation, the Connection Knowledge, especially on Making Connection between Pythagorean Theorem and Numbers. The Contingency Knowledge was observed in the second research lesson, but not on Responding to Students’ Ideas component. Gina enacted the Contingency Knowledge in making Use of Opportunities when she noticed students were struggling to find the square roots, she asked them to stop working so she could show them the strategy of finding square root. She also demonstrated Deviation from Agenda – when she realised she did not have enough time, she asked the students to do the first half of the worksheet as homework.

This suggests the interconnection of different KQ components in lesson study. The interconnection found in C2D8 is between the Contingency dimension and the Foundation dimension. More specifically, C2D8 showed that the Contingency dimension contributes to the Foundation dimension (Section 5.1.1.1). Figure 5.18 summarises the interconnection of the KQ dimensions in C2D8 focusing on the development of the Foundation dimension (FOU – OSK), in this case the pre-service teachers’ mathematical knowledge about finding square roots. The boxes indicate the KQ dimensions or components observed, the lines indicate that the components are connected, and the arrows show the contribution of the KQ component to another component. Dashed boxes represent the absence of a component. The dashed arrows
represent the absence of contribution caused by the absence of the dimension. The KQ Rubric was applied only to the component under focus. The focus was on OSK, the OSK levels were indicated by the numbers in the brackets.

![Diagram of interconnection between Contingency and Foundation dimensions](image)

Figure 5.18 The interconnection between the Contingency dimension and the Foundation dimension in Cycle 2 Group D8

Even though the concept of square roots is closely connected to the Pythagorean Theorem, the pre-service teachers’ subject knowledge of finding square roots was not noticeable in the planning. There are two reasons for this: (1) the focus of the planning was on verifying Pythagorean Theorem not on finding square roots; and (2) the knowledgeable other assumed that the pre-service teachers had a good understanding of school mathematics.

\[\text{FOU – OSK (1)}\]

Ways of finding square roots

\[\text{FOU – OSK (3)}\]

Ways of finding square roots

\[\text{FOU – IER}\]

Students could not find square roots

\[\text{CNT – RSI}\]

Pipit did not help the students

\[\text{TR A – COR}\]

Pythagorean Theorem proofs

\[\text{CNT – RSI}\]

Gina helped the students

\[\text{TR A – COR}\]

Using pairing

1 Note that the post-lesson discussion 2 was not included the diagram because the mentor teachers and the pre-service teachers mostly talk about classroom management as they did not discuss students’ difficulties.
including square roots. Consequently, the planning made no contribution to the pre-service teacher’s OSK in square roots, hence no arrows.

Even though there were some opportunities to discuss further about the square roots, the group decided to avoid potential problems by using small perfect squares. The knowledgeable other challenged the pre-service teachers to use larger numbers to see how students would work on it. This created a contingent moment in the first research lesson. When students had problems working with square roots, Pipit did not help them. This indicated her reaction to a contingent moment, Responding to Students’ Ideas (CNT – RSI). The post-lesson discussion revealed that Pipit and Gina had a limited knowledge of finding square roots. When discussing ways to help students with square roots, the pre-service teachers learned a new strategy and improved their subject knowledge (FOU – OSK). This resulted in some changes in the second research lesson. Gina responded quickly when she noticed students had difficulties with finding square roots. This indicates that Gina eliminated the contingent moment (CNT – RSI), which is represented by a dashed box. The use of pairing the whole numbers and the square numbers also indicates her Transformation Knowledge has improved, in this case specifically on the Choice of Representation (TRA – COR).

The development of the Transformation Knowledge, particularly on Choice of Representation component was shown in Cycle 1 Group E8 (C1E8). Table 5.11 shows the KQ components enacted by the pre-service teachers in C1E8.

Table 5.11 KQ components enacted in C2D8

<table>
<thead>
<tr>
<th>KQ</th>
<th>KQ dimensions</th>
<th>KQ components</th>
<th>Planning</th>
<th>RL 1</th>
<th>PLD 1</th>
<th>RL 2</th>
<th>PLD 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foundation</td>
<td>Overt Subject</td>
<td>Knowledge</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Awareness of Purpose</td>
<td>Concentration on Procedure</td>
<td>√</td>
<td></td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>Use of Terminology</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Connection</td>
<td>Anticipation of Complexity</td>
<td>Making Connection Between Concepts</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transformation</td>
<td>Choice of Representation</td>
<td></td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Contingency</td>
<td>Responding to Students’ Ideas</td>
<td>Using Opportunities</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In the planning, the pre-service teachers demonstrated the Foundation, Connection, and Transformation Knowledge. The Foundation Knowledge enacted by the pre-service teachers includes Overt Subject Knowledge, Awareness of Purpose, and Use of Terminology. While planning the lesson, the pre-service teachers discussed about the everyday languages used to represent the concept of gradient the pre-service teachers enacted their Use of Terminology component. The Overt Subject Knowledge and awareness of purpose were demonstrated when the pre-service teachers discussed whether the leaning tower of Pisa was appropriate for representing the concept of gradient. The pre-service teachers found that the concepts of gradient represented by the leaning tower of Pisa did not support the intended lesson goal that is gradient as the ratio \( \frac{\text{rise}}{\text{run}} \). When the pre-service teachers anticipated students’ responses, they demonstrated their Connection Knowledge on the Anticipation of Complexity. Moreover when they decided to use Cartesian plane because they claimed the students have learned about it in the previous lesson, they demonstrated the Connection Knowledge on the Making Connection Between Concepts and the Transformation Knowledge on the Choice of Representation.

The first research lesson showed the Transformation Knowledge when the student learning using a Cartesian plane did not run as expected. It was expected that the Cartesian plane would help students understand the concept of gradient. However, in the actual lesson, the Cartesian plane did not support students’ understanding of negative gradient. This triggered Vina – the lead teacher to enact her Contingency Knowledge in Responding to Students’ Ideas. Because this was not predicted, in responding to students’ misunderstanding of negative gradient, Vina showed the procedure of using \( \frac{y_2-y_1}{x_2-x_1} \) to the students. This suggested her Foundation Knowledge, particularly the Concentration on Procedure.

In the first post-lesson discussion, the pre-service teachers demonstrated evidence of the Transformation Knowledge – Choice of Representation and the Connection Knowledge – Anticipation of Complexity. Given that the students misunderstood the negative gradient, the pre-service teachers expected the same thing would happen in the next classroom. This suggested their Anticipation of Complexity component. To help the students with this problem, they decided to use mirroring, which indicated their Transformation Knowledge – Choice of Representation knowledge.
The second research lessons revealed the lead teacher – Yanti’s Transformation Knowledge – Choice of Representation when she used mirroring idea and included it on her PowerPoint presentation. However, she did not use it to support students’ own construction, instead she used it only to introduce her procedural explanation on negative gradient. This suggested her Foundation Knowledge – Concentration on Procedure. The second post-lesson discussion focused on students’ difficulties working with a Cartesian plane. The pre-service teachers demonstrated their Transformation Knowledge – Choice of Representation when discussing on students’ work with the Cartesian plane.

Data from C1E8 indicated that the Connection dimension influences the Transformation dimension. Figure 5.19 summarises the interconnection of the KQ components, especially the Choice of Representation (COR) development in the lesson study. Initially the pre-service teachers only considered students’ prior knowledge and realistic context when selecting the leaning tower of Pisa to introduce the concept of gradient.

The pre-service teachers did not consider the concept of gradient represented by the leaning tower of Pisa. This led to a discussion of how students would understand the concept of gradient represented by the leaning tower of Pisa. The leaning tower of Pisa could mislead the students to understand gradient as the size of the inclination angle. Whereas the syllabus states the gradient is \( \frac{\text{rise}}{\text{run}} \). When they were challenged to align the Pisa tower to the lesson goals, the pre-service teachers decided not to use it. They chose to use the Cartesian plane instead. This suggested that the selection of representation involves the interaction of students’ prior knowledge (CON – AOC), lesson goals (FOU – AOP), and the concept of gradient (FOU – OSK).

In the first research lesson, the effectiveness of representation was tested. The pre-service teachers’ Foundation Knowledge was observed when they could identify students’ errors or difficulties when working with the Cartesian plane. In the first post-lesson discussion, these difficulties were discussed and resulted in the refinement of the representation through using mirroring of the lines. In other words, the Foundation dimension (FOU – IER) influences the refinement of the Transformation dimension (TRA – COR).
However, Yanti – the second lead teacher did not use the mirroring of the lines to construct students understanding about positive and negative gradients. She used a procedural approach that is telling the students the convention that if the line goes up, then the gradient is positive and vice versa. This indicates that even though the pre-service teachers were equipped
with a well-designed representation, the absence of knowledge about using the representation properly hindered them to use the representation to support students’ understanding. This absence is represented by the dashed arrow and dashed boxes on the diagram. Choosing an appropriate representation for teaching mathematics does not necessarily mean that the pre-service teachers know how to use it to build students’ understanding. Without the knowledge of using the representation appropriately, they would likely go back to their initial teaching beliefs – the traditional procedural teaching.

Even though Yanti used procedural teaching, the students understood it and they could tell the positive and negative gradients by looking at the lines. Because of this, the second post-lesson discussion did not discuss Yanti’s use of the mirroring lines. It focused on Yanti’s use of language to introduce the concept of gradient. She used “landai” (flat) and “curam” (steep) when introducing slope, indicating her Connection (CON) Knowledge. This was a good approach to help students understand the idea of the need of a notion to indicate how steep something is. However, she did not extend the discussion to the concept of gradient. This suggested a lack of the Contingency (CNT) Knowledge when she missed the opportunity to extend everyday language use into the concept of gradient.

Moreover, the data showed the interconnection between the Connection dimension and the Foundation dimension. The findings from C1D7 (Sections 5.1.3.1 and 5.1.4.1) and C2D8 (Section 5.1.1.1) suggested the interconnection between the Connection dimension, especially the Anticipation of Complexity (CON – AOC) and the Foundation dimension, especially the concentration on procedure (FOU – COP). Data from C2D8 showed that when the pre-service teachers were able to predict students’ responses and were equipped with handy prompts, they would respond or help students resolve their difficulties using a more student-centred approach, lowering the reliance of the procedural approach. On the contrary, data from C1D7 showed that when predictions did not come with the prompts, the pre-service teachers would use procedural teaching. Both cases suggest that anticipating of complexity and developing prompts contributed to the way pre-service teachers react in the moment. When they did not prepare the prompts, they would more likely to go back to procedural teaching and vice versa. This shows that lesson study enables the Connection dimension contributes to the Foundation dimension.
5.2.2 Observing Research Lessons

This section focuses on the participants as the observers of the research lessons. It addresses what they observed, how they discussed their observation in the post-lesson discussions, and how it contributed to the pre-service teachers’ PCK development. It presents the findings from analysis of video data, interviews, and observation sheets.

Before the research lessons, there were no pre-research lesson briefings and no protocol for observing research lessons. However, the observers namely the pre-service teachers, the mentor teachers, and the lecturers were given a copy of the lesson plan and an observation sheet. The observation sheets (Appendix 3) were intended to help the observers making notes of the important moments in the lessons. It has two sections, the first section has three columns: time, teacher’s activity, and students’ activity. This section was intended to help the observers record the order of the activities in the lesson and the teacher-students interaction. The second section of the observation sheet was intended to record the students’ mathematical thinking especially while they were working on specific tasks. During the research lesson, the observers stood at different locations in the classroom. Each of them stayed in the same location and observed the same group of students throughout the lesson. They made notes on their observation sheet.

In general, the observers only noticed general aspects of the lessons in the first section of their observation sheets and more detailed students’ work in the second section. For example, Figure 5.20 is Diana’s observation sheet taken from Raya’s research lesson in C1D7. In the first section, she only wrote the time with the corresponding teachers’ and students’ activities. In the teachers’ activity column, she wrote short descriptions of the activities. For example, the circled text in Figure 5.20 is “explaining the correct answer” under the teacher’s activity column without any detail of the problem, the correct answer, and how the teacher explained it. In the students’ activity column, Diana simply wrote “listening to the teachers’ explanation”.

In the second section, the observers provided more detailed information of the students’ work. They made notes about students’ mathematical strategies, difficulties, and mistakes. As shown in Figure 5.20, Diana wrote the students’ strategies step-by-step such as guess and check and it changed after the teacher helped them. Similar findings were also found for most pre-service teachers. There was one exception: Jamal from Group E8 only wrote general comments on students’ work such as “students tried to find the solution of Problem 2 using the...
mathematical model”. He did not record the students’ mathematical strategies in detail. Jamal’s case will be presented in detail later in this section.

The mentor teacher of Group D8 – Hani, E8 – Nani, and E7 – Irwan only wrote general comments about the teacher’s and students’ activities on both pages of the observation sheets. Rusdi, the mentor teacher of Group E7 wrote about students’ difficulties but his comments also lacked of depth and detail. For example, he wrote “almost all students had difficulty on understanding the word problem”. He did not identify the part of the word problem that was difficult for the students or the kind of struggles they had when working on the problem. Nur from Group D7 noted the students’ mathematical strategies in a more detailed manner.

Data of the lecturers’ observation sheets were limited. Siti, the lecturer of Group D7 and D8, only attended one research lesson – Pipit’s with D8. Her observation sheet showed that she only noticed general things from the teachers’ and students’ activities. Farida, the lecturer of Group E7 and E8 only attended Vina’s and Yanti’s research lessons. Her observation sheet showed her noticing only on the general teachers’ and students’ activities and some mistakes found in students’ work.

The observers brought their observation sheet to the post-lesson discussions. What the pre-service teachers talked about in the post-lesson discussions and how it linked to the observation sheet is presented below. Figures 5.21 and 5.21 show the pre-service teachers’
observations in the research lessons taken from their observation sheets, and what they talked about in the post-lesson discussions as observed in the video data for Schools D and E, respectively. The columns are arranged to present the research lessons and post-lesson discussions chronologically. For example, in Figure 5.21, the first column – C1D8 RL1 PD8P indicates the first research lesson in the first cycle of group D8, which was taught by Pipit (PD8P). The next column indicates the post-lesson discussion following the research lesson. This chronological order allows the researcher to see the change of the pre-service teachers’ observation over time.

The pre-service teachers’ observations are grouped into three categories: classroom management, pedagogical issues, and students’ mathematical thinking. Classroom management concerns issues such as time, students’ behaviour, students’ grouping, and teachers’ voice. Pedagogical issues include for example, the language or terminology that the lead teacher used in the lesson, the sequence of activities, and the mathematical representations. And lastly, students’ mathematical thinking was the focus when the pre-service teachers provided information about students’ mathematical strategies, difficulties or mistakes.

The data in Figure 5.21 and 5.22 show that almost all pre-service teachers were concerned about classroom management throughout the research lessons and post-lesson discussions. In addition to that, a few of them, PD7I (Ida), PD7R (Raya) and PE8V (Vina), noticed the students’ mathematical thinking from the very beginning of the lesson study. Other pre-service teachers such as Ida, PD8P (Pipit) and PE7U (Umar) shifted their attention from only being concerned about classroom management to including students’ mathematical thinking after a few sessions of research lessons and post-lesson discussions.

At the beginning of the lesson study, the pre-service teachers were unaware of the focus of the observation during the research lessons. Out of the four groups, D8 was the first group that conducted the lesson study. Data from Group D8 showed that the observers mostly focused on the classroom management and the dynamic of the students’ group work rather than students’ mathematical thinking. The following excerpt shows the observers’ comments in the post-lesson discussion of Pipit’s research lesson.
Figure 5.21 School D’s pre-service teachers’ observation

Key

D. Diana (PD7D)  I. Ida (PD7I)  R. Raya (PD7R)  P. Pipit (PD8P)  G. Gina (PD8G)

Shaded column: The lead teacher in the research lesson

SMT: Students’ mathematical thinking  
PI: Pedagogical issues  
CM: Classroom Management

•: This aspect was observed by the pre-service teacher

×: The pre-service teacher did not attend the research lesson and the post-lesson discussion

*: The observation sheet was not available
<table>
<thead>
<tr>
<th>Aspect observed</th>
<th>C2E8 RL1 PE8V</th>
<th>C2E8 PLD1 PE8V</th>
<th>C2E7 RL1 PE7D</th>
<th>C2E7 PLD2 PE7D</th>
<th>C2E7 RL2 PE7J</th>
<th>C2E7 PLD3 PE7J</th>
<th>C2E7 RL3 PE7U</th>
<th>C2E7 PLD3 PE7U</th>
<th>C2E8 RL1 PE8Y</th>
<th>C2E8 PLD1 PE8Y</th>
<th>C2E8 RL2 PE8Y</th>
<th>C2E8 PLD2 PE8Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>•</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>•</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>•</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Aspect observed</th>
<th>C2E7 RL1 PE7U</th>
<th>C2E7 RL2 PE7J</th>
<th>C2E7 PLD2 PE7D</th>
<th>C2E7 RL3 PE7D</th>
<th>C2E7 PLD1 PE7J</th>
<th>C2E7 PLD3 PE7J</th>
<th>C2E8 RL1 PE8V</th>
<th>C2E8 PLD1 PE8Y</th>
<th>C2E8 RL2 PE8Y</th>
<th>C2E8 PLD2 PE8Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>•</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>•</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>•</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Key**

- **D**: Deni (PE7D)  
- **J**: Jamal (PE7J)  
- **U**: Umar (PE7U)  
- **V**: Vina (PE8V)  
- **Y**: Yanti (PE8Y)

**Shaded column**: The lead teacher in the research lesson.

<table>
<thead>
<tr>
<th>SMT</th>
<th>Students’ mathematical thinking</th>
<th>PI</th>
<th>Pedagogical issues</th>
<th>CM</th>
<th>Classroom Management</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>•</strong></td>
<td>This aspect was observed by the pre-service teacher</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>×</strong></td>
<td>The pre-service teacher did not attend the research lesson and the post-lesson discussion</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>∗</strong></td>
<td>The observation sheet was not available</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.22 School E’s pre-service teachers’ observation
Gina: In general, Pipit’s classroom management is good. But Pipit needs to move around the classroom more when explaining something. Students at the back did not pay attention, because Gunawan was there. He talked to others while Pipit is explaining at the front. When one student drew the graph, actually it’s correct, but the line is not straight, there was no response from other students. I think it needed to be corrected, the line should be a straight line.

Raya: The students at the back, only some of them paid attention, the others were busy doing something else.

Diana: When Pipit helped a group, she only maintained interaction with the students who asked her the questions. The other students in the groups are kind of neglected. The work in the groups is not equal, not all worked on the problem, and some students in the group did something else then the worksheet.

Ida: In one group, a student did repeated subtraction. Another student in the same group, used multiplication. They ended up with different answers, but they did not communicate their work. There was no discussion. When a student asked questions to the teacher and the teacher helped him, he did not help the other students in the group. Pipit needed to ask the student to explain to his group.

As observers, the pre-service teachers focused on the classroom management such as students did not pay attentions and the dynamic of the groups. Only Gina (PD8G) and Ida mentioned the students’ mathematical work. Gina talked about the graph and Ida talked about the strategies. As the knowledgeable other, I discussed students’ work and encouraged the pre-service teachers to pay attention to students’ work and their mathematical thinking. For example, during the post-lesson discussion of Ida’s first research lesson, I asked Diana to elaborate on the mathematical strategies that the students she observed had used.

Diana: I observed students at the back, Naldo, Abi, etc. One of them did the work, the others looked and gave comment and ideas when needed. One of them wanted to ask Ida but Ida didn’t hear them. When Ida finally heard them, she was aware of the time, and she rushed in finishing the lesson.

Meili: Could you tell more about their mathematical strategies?
Diana: They counted the candies in the picture.
Meili: How did they count the candies?
Diana: Counting one by one.
Meili: What did they write?
Diana: I did not keep note of what he wrote.
Meili: Who’s going to observe the next research lesson? Please pay attention on the mathematical thinking and strategies.

Diana’s attention in the next research lessons shifted to students’ mathematical strategies. She took notes of students’ mathematical strategies. In the first cycle, particularly, during Raya’s post-lesson discussion, Diana reported examples of students’ mistakes as shown in the transcript below.
Diana: Sofia and Aldy, they did not understand, they only counted the candies in the picture and wrote 20 underneath it. Then Sonia wrote the same as Feri: 4 times $2 = 8$.

Aldy wrote $k = 20$.

$k = 20 - 16 = 4$

Sonia worked on the exercise. It should be $3 + 2k = k + 8$. She knew the final answer was 5. She wrote $3 + 2k = 13$. But I do not know how she got 13.

$2k = 13 - 3$

$2k = 10$

$k = 5$

Does shifting their focus to students’ mathematical thinking impact the pre-service teachers’ PCK development? Data from the post-lesson discussions might provide some evidence to answer this question. Ida (PD7I), Diana (PD7D) and Jamal (PE7J) are chosen to illustrate the connection between the pre-service teachers’ observations and their PCK development. These pre-service teachers are chosen because they started and ended the lesson study by noticing different aspects of the lessons. Ida showed attention to students thinking from the beginning and throughout the lesson study. Diana showed a shift in her attention towards students thinking after few lesson study meetings, and Jamal only showed his attention to students’ thinking toward the end of lesson study implementation.

Ida noticed and made notes of students’ various mathematical strategies in her observation sheets, and then talked about it in the post-lesson discussions. Figure 5.23 shows Ida’s observation sheet taken from Gina’s research lesson in C1D8. She wrote down different strategies that the students used to solve the problem such as multiplicative strategy and using an equation. In particular, she wrote that students made mistakes in scaling the coordinates at the $y$-axis. Then in the post-lesson discussion, she commented on students’ difficulties in understanding the word “longkap” (jump). Ida said: “Zarah and Chika were struggling when making the graph. They did not understand the term "longkap" (scale)”. Because the problem used large numbers, up to 200,000, Gina, the lead teacher had been telling the students to use scaling in the Cartesian plane. She encouraged the students to make “jumps” in the axis. Ida highlighted students’ difficulties in understanding the word “jumps” when working with Cartesian plane. Ida could make a link between the students’ mistake and the language use that might have caused it. Identifying errors and the Use of Terminology are under the Foundation dimension, therefore, this suggests that through observing lesson, Ida demonstrated a Foundation Knowledge.
As described above, the knowledgeable other’s suggestion resulted in Diana’s attention shifting towards students’ mathematical thinking. Diana showed this in her attention to students’ strategies and difficulties. Figure 5.20 is Diana’s observation sheet taken from Raya’s research lesson (C1D7). She made notes of students’ strategies such as guess and check. She commented on these strategies in the post-lesson discussion. However, she did not make any links between the students’ thinking and the pedagogical aspects of the lessons. This indicates that even though Diana showed development of the Foundation dimension, in particular the Identifying Errors, there is no evidence that this development impacted on the development of the other KQ components.

Jamal’s observation sheets showed that he only noticed classroom management throughout the lesson study. As the knowledgeable other, I always commented on students’ mathematical thinking but I did not specifically inform Group E7 about focusing on students’ thinking when observing the research lessons. This might have been the reason why Jamal did not pay attention on students’ thinking. It was only during his last post-lesson discussion that he commented on students’ mathematical thinking.

Jamal: It ran as planned. First, it was about whole/partial/unit price. There were no difficulties. When working on the worksheet, some students made mistakes when working with different unit measurements such as dozen, they did not convert it into unit items.
The second worksheet about profit and loss. I felt like I did not explain them enough. I should have explained about the cost price, selling price. When they worked on the second worksheet. The problem was.
The price of … = The price of …
I expected them to write selling price – cost price, but instead, they wrote the amount of money.
In this excerpt, Jamal reflected on his own teaching. He was aware of students’ misunderstanding revealed when working on the second worksheet. He suggested that it was because he did not explain the cost price and selling price well enough. He did not make any associations between the instruction in the worksheet and the students’ unfamiliarity with such problems.

Out of the three pre-service teachers, only Ida could make links between students’ work and the pedagogical aspects of the lesson. The other two pre-service teachers did not demonstrate an understanding of making that links. This suggests that observing lessons with a focus of student thinking does not necessarily stimulate the pre-service teachers to rethink and analyse the pedagogical aspects of the lesson. They need prompts to make sense of the students work in light with the teacher pedagogical decisions.

It is also important to look at the lead teachers’ comments on their post-lesson discussions to understand the impact of observing others’ lessons on their teaching. According to Figure 5.21 and 5.22, all of the pre-service teachers commented on classroom management, with only few of them talking about pedagogical issues and students’ mathematical thinking. This suggests that the pre-service teachers struggled with maintaining the students’ attention and keeping the time. When teaching, classroom management was still their main priority that might have hindered them from focusing on the students’ mathematical thinking.

While the majority of the pre-service teachers only focussed on classroom management, a few of them showed some attention to students’ mathematical thinking. For example, Ida commented about her students’ difficulties during her post-lesson discussion in C1D7.

Ida: I missed the students at Gabriel’s row. I really did not hear them calling me. Some students have not understood the problem, so I decided to discuss them. It took more time than what I expected. Thus I was rushing at the end of the lesson. At the end of the lesson, I forgot to summarise the lesson. I only reminded students to do their assignments and homework. I was confused, there’s a group where a student was able to do the work, but the other student who sat next to him was not. Naldo, from the beginning he knew the answer, but until the end of the lesson, he did not write the equation, he only wrote the calculation.

Meili: Why do you think he did not write the equation?

Ida: Naldo is too lazy to write the process. Evan and Amir, they only focused on the bags, since the bags are the x. They missed that there are some candies there. Aldo, Omar and Safira, I don’t know about them. I asked them since the beginning if they understood the problem, and they said yes. But they didn’t finish the work until the end. I focused on students who almost found the correct answer, and not too much on the students who didn’t know at all. I only explained them the instruction of the problem, I didn’t guide them.
Ida’s focus was on classroom management issues such as time management and students’ attention, completion of the lesson plan, such as her failure to summarise the lesson, and students’ difficulties. Her comments on students’ difficulties revealed that she struggled to understand the students’ mathematical thinking. Ida could not make sense of why Naldo could find the solution to the problem but could not write the equation. She might have referred to the balance problem. When prompted by the knowledgeable other, she said that Naldo did not write the equation simply because he was too lazy. This suggested her unawareness that the problem was too easy for Naldo that he did not need to write his thinking down. Ida’s ability to notice students’ mathematical thinking was only a starting point for her to understand students’ mathematical thinking and improve her teaching.

In general, even though being an observer enabled the pre-service teachers to focus on students’ mathematical thinking, their main attentions were still on classroom management issues when they were teaching. Data from the interviews confirm these findings. The interview asked the pre-service teacher about what they learned from observing research lessons. Ida (PD7I) confessed:

I learned to look closely at students’ thinking. This is very difficult to do when we are the teacher … I became more aware of my students’ thinking. Now I always ask for their scrap papers to see their work and thinking of the problems. Before, I usually gave them the answer directly without looking at their work. (Ida, interview)

Table 5.12 presents the pre-service teachers’ responses to the interview. Six pre-service teachers said that the research lessons had taught them to focus on students’ thinking. For example, Jamal (PE7J) said “I became more aware that the students need guidance when working on the exercise. As the teacher, we can’t just let them work, we need to give them clear instructions and help when needed”.

Seven pre-service teachers said that while observing others, they reflected on their own teaching. For example, Raya (PD7R) said “I observed how they teach, compared it to my own teaching. If they made mistakes, I tried not to do that in my class”. Vina (PE8V) said “from observing Yanti’s lessons, I reflected on my own teaching. I should have been like that, or I should have done that”. These responses indicate that observing research lessons provided opportunity for the pre-service teachers to reflect to their own teaching which, from the pre-service teachers’ perspectives, is learning from others’ mistakes.
Table 5.12 Pre-service teachers’ learning from the research lessons

<table>
<thead>
<tr>
<th>Pre-service teachers’ responses</th>
<th>Pre-service teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students’ responses – students’ mathematical thinking</td>
<td>PD8G, PD7R, PD7I, PE7D, PE7J, PE8Y</td>
</tr>
<tr>
<td>Reflect on their own teaching</td>
<td>PD8P, PD7R, PE7J, PE8V, PD7D*, PE7U*, PE8Y*</td>
</tr>
</tbody>
</table>

* The pre-service teachers reflected on classroom management issues

Three pre-service teachers reflected on the classroom management issues. They said that after observing research lessons they were more aware of classroom management issues. Umar (PE7U) said “I learned that students in different classes are different … I need to learn a lot about to manage the students so that they are not noisy”. Diana (PD7D) said “I observed what the teacher taught and I made notes for myself. What I should or should not do in my class. I learned about students’ characters are different in each class, about classroom management and handling the students”. Yanti (PE8Y) said “I think my problem is in time keeping”. These pre-service teachers were always very conscious about their classroom management skills. Their responses indicate that observing research lessons made them more aware of their weaknesses.

In summary, initially the observers did not use the observation sheets to record students’ mathematical thinking. This was probably because the structure of the observation sheet did not specifically order them to do so. Redesigning the observation sheet with a more operational instructions focusing on students’ thinking may help improve the effectiveness of the observation. Another reason is that the observers did not have a pre-lesson briefing before the research lesson. The discussion of the technical issues and protocol of the observation only took place at the planning of the first cycle of each group. Thus, the observers might not have had a clear understanding of what they should observe.

The video data and the interview confirmed that the pre-service teachers became more focused on students’ thinking while observing research lessons. More specifically the video showed that their attention on students’ thinking shifted from general student activity to the
students’ mathematical strategies and difficulties. This was a positive change in their teaching practicum. However, they could not make sense of why the students made a particular mistake. They did not make any connection between the mistakes and the problem posed, the examples given, or other pedagogical aspects of the lessons. For the pre-service teachers, when they could not understand why students make a particular mistake, they would explain the way to find the correct solution and repeat the same explanation until students could perform the procedure. This explains why they always used a procedural approach when helping students. This suggests that identifying students’ mistakes does not directly contribute to the pre-service teachers’ PCK development. However, this study showed, as discussed in previous sections, the post-lesson discussions provided a rich learning opportunity for the pre-service teachers to discuss students’ mistakes and make links with the problems posed or the examples used.

The video data also showed that the pre-service teachers tended to notice more about classroom management when they were the lead teachers. Moreover, the interviews revealed that the pre-service teachers used observing research lessons as an opportunity for reflecting on their own teaching. However, their reflection seemed to be limited to learning from other’s mistakes and classroom management issues.

5.2.3 Support from the Mentor Teachers and the Knowledgeable Other

To understand the role of the mentor teachers during lesson study, this section addresses findings from the pre-service teachers’ interview and video of the lesson study. The data did not show any substantial contribution by the university lecturer to the pre-service teachers’ PCK development, therefore their contribution will be discussed in the next chapter.

Data from the pre-service teachers’ interviews revealed the pre-service teachers’ opinion of the mentor teachers’ role (Table 5.13). The pre-service teachers were asked for their opinion about the mentor teachers’ role during lesson study. They acknowledged the mentor teachers’ support in giving them advice about the teaching and teaching materials, managing students’ behaviour, and classroom management.

One pre-service teacher noted that the mentor teacher only gave advice when asked. The pre-service teachers needed to be pro-active in initiating a discussion with the mentor teacher. Two pre-service teachers noted the mentor teachers’ concern about the timeline of the lesson and that all lessons should be on schedule so they completed the curriculum on time.
However, the pre-service teachers’ responses were related to the teaching practicum in general, and not to the lesson study specifically.

Table 5.13 The pre-service teachers’ opinion about the mentor teachers’ contribution

<table>
<thead>
<tr>
<th>Pre-service teachers’ responses</th>
<th>Pre-service teachers</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Providing feedback of the teaching materials</td>
<td>PE7U, PE7D, PD7R</td>
<td>3</td>
</tr>
<tr>
<td>Time line</td>
<td>PE7D, PD8P</td>
<td>2</td>
</tr>
<tr>
<td>Advice on handling students’ behaviour</td>
<td>PE7D, PD7D, PD7I,</td>
<td>4</td>
</tr>
<tr>
<td>Advice on classroom management</td>
<td>PD7D, PD7I, PE8Y</td>
<td>3</td>
</tr>
<tr>
<td>Only give support when asked. Mentor teacher did not initiate any discussion</td>
<td>PD8G</td>
<td>1</td>
</tr>
<tr>
<td>General topics</td>
<td>PE7J</td>
<td>1</td>
</tr>
</tbody>
</table>

Providing advice about the teaching materials and classroom management was also found in the video data. The mentor teacher advice about teaching materials included the connection between examples and problem in the exercise. This evidence was found in the planning of C1D7. The mentor teacher – Nur initiated a discussion on the sequence of the activities. The pre-service teachers planned to give students an exercise following the worksheet activity. The exercise would include word problems. Nur suggested that the worksheet should link to the exercise:

If you want to give them this type of problems, you must give similar problems in the worksheet. One or two problems using illustration (balance model), and one or two problems are similar to this (word problems). You have to align the worksheet with the exercise. (Nur, Video of Planning C1D7)

Not only did Nur initiate the discussion about the sequence of the activity, she also highlighted the importance of having clear links between the activities. Nur’s suggestion gave insights into the pre-service teachers’ Connection Knowledge.

The mentor teachers’ advice about teaching materials also concerned the level of students’ mathematical ability. Evidence was found in the planning and post-lesson discussions when the mentor teachers suggested the pre-service teachers to modify the problems because they were too difficult for the students. For example, in the planning of C2E8, the pre-service teachers were discussing a problem for applying the Pythagorean Theorem.
Pipit: We have an idea of giving a problem. There are two cars heading to different direction, and then we ask the distance of the two cars. But we are worried it is too difficult. So we picked a problem from the textbook.

A fishing boat sails from a harbor to the sea. It usually travels 12 km to the west then 35 km to the south to catch tuna. This time the boat wants to take a short cut to the tuna. How far does the boat have to travel to catch the tuna?

Hani: This is a word problem, I am afraid it is too difficult for our students, because it includes directions west, south, etc. Use simple questions.

This excerpt shows Hani’s prediction of students’ responses to the problem proposed by Pipit. Hani’s knowledge about the students’ mathematical ability contributed to the anticipation of students’ thinking. In this case, Hani thought the problem was too difficult and suggested the pre-service teachers should use simple problems instead. Unfortunately, Hani did not provide ideas of the kind of problems that would be more appropriate for the students. The pre-service teachers continued the discussion and decided to provide an illustration (Figure 5.1) to help the students understand the problem. This was evidence of Hani’s (mentor teacher) prediction of students’ responses, a component of the Connection dimension. The mentor teacher’s prediction of students’ responses has prompted the pre-service teachers (Pipit and Gina) to use pictorial representation, a component of the Transformation dimension.

Similar evidence was also found in the post-lesson discussion of C1E7. The group taught linear equations. Similar to Group D7, this group also used the balance model as an introduction of the concept, then they used word problems for enhancing students’ skills in writing the equation. In the post-lesson discussion of the second research lesson, the mentor teacher – Rusdi, commented on students’ difficulties in writing the equation.

Rusdi: During the group work, like yesterday, many students could solve the problem, but not write the equation. We then can conclude that the construction of mathematical model need to be discussed more in the introduction.

He noticed that students in this classroom and the previous one had the same problem – they could not write the equation. Therefore, he suggested the pre-service teachers to discuss the process of modelling in the introduction of the lesson. Rusdi used his observation of the students’ difficulties in writing the equation and connected it to the structure of the lesson. This example illustrates his contribution as a mentor teacher to pre-service teachers’ PCK, especially the Connection dimension – the Decisions about Sequencing component.

Another important contribution of the mentor teachers was on topic selection. The evidence for this was found in C1D7. In the planning, the group selected the topic for the
research lesson. Initially, pre-service teachers planned to have the research lesson at the first meeting of linear equation unit, which was about open and closed number sentences. The pre-service teachers presented their ideas of teaching it. As the knowledgeable other, I asked the pre-service teachers about the connection between open and closed number sentences and solving linear equations. Responding to this, the mentor teacher – Nur suggested the pre-service teachers to look back at the curriculum. The pre-service teachers then looked through the syllabus of this unit and they started to doubt whether open and closed sentences was the best topic for the lesson study. Nur suggested that solving linear equation would be better for the lesson study. She argued that the topic provides students with more opportunity to do reasoning. Following the mentor teacher’s suggestion, finally the group decided to reschedule the research lesson to match with linear equations.

The interview also asked the pre-service teachers for their opinion on the role of the knowledgeable other. The pre-service teachers’ interview responses (Table 5.14) only revealed their general opinion of the knowledgeable other’s contribution during the lesson study.

Table 5.14 The pre-service teachers’ opinion about the knowledgeable other’s contribution

<table>
<thead>
<tr>
<th>Pre-service teachers’ responses</th>
<th>Pre-service teachers</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Providing feedback</td>
<td>PE7U, PE7D, PD7D, PD8P, PD7R, PE8Y</td>
<td>6</td>
</tr>
<tr>
<td>Involving in everything</td>
<td>PD8G</td>
<td>1</td>
</tr>
<tr>
<td>Deepening the pre-service teachers’ mathematical knowledge</td>
<td>PD8P</td>
<td>1</td>
</tr>
<tr>
<td>Providing advices about classroom management</td>
<td>PD8P</td>
<td>1</td>
</tr>
<tr>
<td>Initiating the lesson study</td>
<td>PE7J, PD7R</td>
<td>2</td>
</tr>
<tr>
<td>Advice in designing tasks and problem</td>
<td>PE8V</td>
<td>1</td>
</tr>
</tbody>
</table>

Section 5.1 has addressed the pre-service teachers’ PCK development in which some might have included the knowledgeable other’s contribution. It highlighted that the lesson study discussions revolved around students’ thinking and its contribution to the development of the pre-service teachers’ Foundation and Transformation Knowledge. The knowledgeable other had an essential role in maintaining students’ thinking as the focus of the lesson study discussions. In the planning, I encouraged the pre-service teachers to predict students’ learning and anticipate students’ responses. In the research lessons, I asked the observers to pay attention on students’ mathematical thinking. In the post-lesson discussion, I focused on
students’ actual work and linked them to the pre-service teacher’s mathematical knowledge, the problem posed, and the mathematical representation used.

Another contribution of the knowledgeable other is creating contingency and using it as an opportunity to enhance the pre-service teachers’ learning. For example in C2D8, the knowledgeable other’s role was very important. First as the knowledgeable other, I created the contingent moment that is by intentionally challenging the pre-service teachers to use large square numbers in their problem. This contingent moment revealed the pre-service teachers’ lack of mathematical knowledge about finding square roots. Then I discussed this and showed the pre-service teacher another strategy of finding square roots. By discussing the contingent moment in the post-lesson discussion, the pre-service teachers learned about another way of helping the students to find square roots and gained new mathematical knowledge about the process of finding square roots. Gina expressed her Foundation Knowledge development during her interview “You [were] involved in everything. I tried to use your suggestion. For example, finding the square root. I tried that with students and they could follow it”.

5.3 Conclusion

This chapter has presented the findings related to Research Question 1 and 2. Research Question 1 examines evidence of the development of the pre-service teachers’ PCK during the implementation of lesson study. Given that the groups taught different topics in Cycle 1 and Cycle 2, it was difficult to examine the development of one particular KQ component across cycles. The development of the pre-service teachers’ PCK was varied in each group’s lesson study cycles.

The pre-service teachers’ development of the Foundation Knowledge was evident in C1D8 and C2D8. The findings from these cases showed evidence of shifts in the pre-service teachers’ beliefs about teaching in Cycle 1, and that the pre-service teachers improved their subject knowledge in Cycle 2. Data from C1E8 suggests the pre-service teachers’ developed the Transformation Knowledge, especially the Choice of Representation. Moreover, the data showed that choosing an appropriate representation did not necessary mean that the pre-service teachers understood how to use it properly, suggesting that knowledge of using the representation is equally important. Analysis of the data from C1D7 showed that the pre-service teachers anticipated the students’ thinking in the planning. They considered students’ prior learning and levels of mathematical ability when making predictions of students’ responses. Moreover, analysis of the data from C2D8 and C1D7 showed that anticipating
students’ responses alone did not help the pre-service teacher to react in the moment. Without having handy prompts to help the students, the pre-service teachers would be more likely to use a procedural approach.

The second section of this chapter addressed Research Question 2 – the characteristics of lesson study that contribute to the development of the pre-service teachers’ PCK. Data from this study have shown that the discussion during the lesson study meetings enabled the pre-service teachers to enact different components of the KQ. This study examined the different knowledge enacted by the pre-service teachers throughout the lesson study meetings. It showed the interconnection of the KQ components in the planning, research lessons and post-lesson discussions. The interconnection of the KQ components enabled one component to contribute to the development of the others or the absence of one component hindered the development of the others.

Observing lessons is a major part of lesson study. This study revealed what the pre-service teachers observed during the research lessons and how it contributed to their PCK development. Initially the pre-service teachers only noticed the classroom management aspects. Gradually during the lesson study, they also noticed students’ mathematical thinking. They were able to identify students’ mistakes and difficulties. However, their comments in the post-lesson discussions suggested that they could not make links between the students’ difficulties and mistakes and the pedagogical issues in the lessons.

Lastly, this chapter has addressed the support from the mentor teachers and the knowledgeable other during the lesson study. The mentor teachers with their experience and knowledge of the students mainly contributed in helping the pre-service teachers anticipate students’ responses. The knowledgeable other contributed in creating contingency and maintaining students’ thinking as the focus of the discussions and observation during the lesson study.
6 Lesson Study in the Teaching Practicum

This chapter addresses Research Question 3 – What are pre-service teachers’, mentor teachers’, and university lecturers’ views about the incorporation of lesson study into pre-service teachers’ mathematics teaching practicum? And Research Question 4 – What are some of the affordances and constraints relating to the implementation of lesson study in pre-service teachers’ mathematics teaching practicum?

6.1 Participants’ Views of Lesson Study in the Teaching Practicum

Chapter 5 has addressed the pre-service teachers’ Pedagogical Content Knowledge (PCK) development during the lesson study implementation in the teaching practicum. Considering the potential implementation of lesson study in future teaching practicums, it is important to take into account the participants’ view on the implementation of lesson study in the teaching practicum. This section presents the findings from interviews. The participants’ views are narrowed down into what they perceived as the benefits and difficulties of participating in lesson study. More specifically, this section discusses pre-service teachers’ views on anticipating students’ responses and all research participants’ views on participating in lesson study.

6.1.1 The Pre-service Teachers’ Views on Anticipating Students’ Responses

Anticipating students’ responses is one of new practices that pre-service teachers learned from participating in lesson study during their teaching practicum. It involves predicting students’ responses and developing prompts to help students’ learning. Chapter 5 has elaborated on a case, Cycle 1 Group D7 (C1D7) in which the pre-service teachers demonstrated some effort in anticipating students’ responses. It has shown on what and how they focused on while anticipating students’ responses. This section unpacks what all pre-service teachers perceived as the benefits and difficulties of anticipating students’ responses based on the interview data.

The pre-service teachers were asked what benefits they get from anticipating students’ responses. Their responses are summarised in Table 6.1. Five pre-service teachers said they were more prepared in responding to students’ questions during their teaching. For example, Diana said “the prediction makes us more prepared when students ask questions”. Umar said
that the prediction helped him anticipate the students’ questions better. Raya said that predicting students’ responses helped her provide more appropriate support for students.

Table 6.1 Benefits of anticipating students’ responses according to the pre-service teachers

<table>
<thead>
<tr>
<th>Benefits</th>
<th>Pre-service teachers</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>More prepared</td>
<td>PD7D, PD7I, PD8G, PE7J, PE8V</td>
<td>5</td>
</tr>
<tr>
<td>Helps support the students</td>
<td>PD7R</td>
<td>1</td>
</tr>
<tr>
<td>Better anticipation</td>
<td>PE7U, PE7D</td>
<td>2</td>
</tr>
</tbody>
</table>

Even though the pre-service teachers said that they were well prepared to anticipate students’ responses, the video data presented in Chapter 5 showed that the pre-service teachers’ anticipations of students’ responses did not always occur in the lesson, and the anticipations of students’ responses did not always come with appropriate prompts. From the interview data, only one pre-service teacher reflected on this. Gina’s responses suggest that despite her feeling more prepared for the lesson, in the actual lesson, she was still not prepared to respond to follow-up questions raised by students.

Gina: We are more prepared for the lesson. But I was confused when students asked the questions that we predicted, and we answered with our predicted answer. We hoped students would understand but they asked more questions we did not predict.

The interviews also revealed the challenges and difficulties involved in anticipating students’ responses (Table 6.2) such as providing detailed anticipations, especially in cases that involve students’ responses to hands-on activities and additional details in the lesson study.

Table 6.2 Difficulties of anticipating students’ responses according to the pre-service teachers

<table>
<thead>
<tr>
<th>Difficulties</th>
<th>Pre-service teachers</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detailed anticipations</td>
<td>PD7D, PD8P, PE7D, PD7R</td>
<td>4</td>
</tr>
<tr>
<td>Anticipating students’ responses to hands-on activities</td>
<td>PD8G</td>
<td>1</td>
</tr>
<tr>
<td>Additional paper work</td>
<td>PE7U</td>
<td>1</td>
</tr>
</tbody>
</table>

Diana and Ida commented on the difficulties of incorporating detailed predictions. Raya said that it was difficult to put herself into the students’ heads.

Raya: I categorised the students, high achiever, middle and low achievers. I usually predicted the mistakes of the calculation, but could not really predict the misconceptions. It is difficult to put myself in their head. What happened in the actual lessons was often different from what I predicted.
These responses confirm the findings from the video data as elaborated on Chapter 5, that is the pre-service teachers used their knowledge of students’ mathematical achievements – the high, middle and low achievers – to predict their calculation errors. Given that the pre-service teachers had no prior teaching experiences before the teaching practicum, these difficulties were understandable. Moreover, there was no subject/unit in in the teacher education curriculum that focused on the development of this skill.

Another difficulty in anticipating students’ responses occurred when dealing with hands-on activities. Gina commented on different levels of difficulties in anticipating students’ responses based on different types of activities, that is, hands-on activity versus a worksheet. She noticed the students were more challenged when proving the Pythagorean Theorem using the puzzle-like activity in the second cycle. Hands-on activities in mathematics lessons are not common practice in Indonesian schools, so teaching mathematics using hands-on activities was probably new for Gina. Hence, anticipating students’ responses to hands-on activity is more challenging for Gina due to the novel nature of this activity for pre-service teachers. Moreover, students’ thinking while working on a hands-on activity might not be clearly presented in their written work.

The additional paperwork required while anticipating students’ responses was identified in the data. For Umar (PE7U), creating worksheets and a more detailed lesson plan in both cycles was an extra work when compared to his regular teaching. The lecturer, Farida (LEF), confirmed this commenting on the lack of attention to planning in Umar’s group (E7). She reported that all pre-service teachers in Group E7 did not prepare lesson plans, and based their teaching mainly on the textbook. This suggests that Umar did not value the planning as an important element in his teaching. Unfortunately, this study did not collect any data from the pre-service regular teaching. Therefore it cannot show how a lack of detailed in the planning impacts the pre-service regular teaching.

The pre-service teachers’ interview responses revealed that they only see anticipating students’ responses merely as a part of the planning phase and is not connected to other phases of lesson study. As elaborated on in Chapter 5, anticipating students’ responses is an ongoing process that evolves during the planning, research lessons, and post-lesson discussions. Even though some of the pre-service teachers’ responses show some connection of their anticipation of students’ responses in the planning to the actual students’ work, they did not indicate that they used the actual students’ responses retrospectively to anticipate in their future lessons.
None of the pre-service teachers saw developing appropriate prompts as a part of anticipating students’ responses, despite the fact that the knowledgeable other encouraged them to plan for appropriate prompts in their planning. Developing appropriate prompts is equally important as predicting students’ responses because the prompts help pre-service teachers to respond and extend students’ thinking. However, this proved to be quite challenging for pre-service teachers as discussed in Chapter 5. The reason for this lack of attention to include appropriate prompts might indicate that the pre-service teachers did not understand the importance of developing prompts as a part of anticipating students’ responses.

Despite the pre-service teachers’ claims that they felt more prepared when having anticipations of students’ responses, the interview revealed that they did not do carry out this practice in their regular teaching. Several reasons for this were identified, including the school’s lesson plan format (which did not include anticipations of students’ responses), the pre-service teacher’s priority over classroom management issues, and time constraints. Deni was reluctant to embed anticipations of students’ responses in the lesson plan because it was not in the school’s lesson plan template. Deni said “I am worried that it would affect the assessment. I would rather follow the regular lesson plan because that is how the school does it”. Since the lesson study was embedded in the teaching practicum, Deni was worried that changing the regular lesson plan might impact negatively on his teaching practicum mark.

While Diana acknowledged the importance of anticipating students’ responses, due to her struggles with classroom management she did not seem to prioritise anticipating students’ responses in her regular teaching. She said “I am more focused on how to teach the lesson and how to be in the same speed with the other two classes”. Three pre-service teachers (Raya, Jamal and Vina) said that even though they did not incorporate anticipation of students’ responses in the lesson plans, they were aware of it. For example, Raya said “I did not do it because it took too much effort. I only thought about it but did not write [it] in the lesson plan”. This suggests that the pre-service teachers have developed an awareness of the need to anticipate students’ responses in their teaching. However, due to time constraints, they did not incorporate it in their lesson plans.

In conclusion, anticipating students’ responses is one of the new practices that the pre-service teachers experienced during lesson study. They identified some benefits of this practice such as feeling better prepared for the lesson and helping them to better support their students. Some difficulties in anticipating students’ responses includes: providing detailed predictions and appropriate prompts especially for hands-on activities, and additional paper work. The pre-
service teachers perceive anticipating students’ responses only as predicting students’ solutions. They did not perceive developing prompts as a part of anticipating students’ responses. This implies that for future lesson study implementation, it is important to make it clear that anticipating students’ responses includes developing appropriate prompts. Moreover, the pre-service teachers seemed to only think of anticipation students’ responses as an element of planning but not an on-going process throughout the lesson study. While the pre-service teachers have demonstrated a developing awareness of anticipating students’ responses, they seemed reluctant to incorporate these in their regular teaching due to their concern of aligning with the school’s practice, and their struggles with classroom management issues.

6.1.2 The Participants’ Views about Lesson Study in the Teaching Practicum

This section discusses participants’ views of the implementation of lesson study in the teaching practicum particularly what they perceived as the benefits and challenges they encountered during lesson study in their teaching practicum. Data from interviews with the participants were used in the analysis. Table 6.3 presents what the participants see as the benefits of participating in the lesson study. Each of these aspects will be discussed.

Table 6.3 Benefits from participating in lesson study according to the participants

<table>
<thead>
<tr>
<th>Benefits of lesson study</th>
<th>Pre-service teachers</th>
<th>Mentor teachers</th>
<th>Lecturers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Getting feedback</td>
<td>PD7D, PD7R, PE7J</td>
<td>MD7N, MD8H, ME7R</td>
<td></td>
</tr>
<tr>
<td>Collaboration</td>
<td>PD7I, PD8P, PE7U, PE7D, PE8V</td>
<td>MD7N</td>
<td>LEF</td>
</tr>
<tr>
<td>Better lesson planning</td>
<td>PD7I, PD7R, PD8G, PD8P</td>
<td>MD7I, MD8H</td>
<td>LEF</td>
</tr>
<tr>
<td>Learning new knowledge</td>
<td>PD7D, PD8P, PE8Y</td>
<td>ME7I, MD7N, MD8H</td>
<td></td>
</tr>
<tr>
<td>Reflection of own-teaching</td>
<td>PE7D</td>
<td>MD7N</td>
<td>LDS</td>
</tr>
<tr>
<td>Getting help from the knowledgeable other</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Awareness of students’ learning</td>
<td></td>
<td>MD8H</td>
<td></td>
</tr>
</tbody>
</table>

Receiving feedback from other members of the group was seen a benefit for most of the participants. Four of the pre-service teachers appreciated the feedback on the strengths and weaknesses of their teaching. For example, Raya (PD7R) stated “The post-lesson discussions informed us about our strengths and weaknesses [in our teaching]”. The pre-service teachers valued the post-lesson discussion as an opportunity to learn from others as well as reflect on their own lessons. They seemed to focus mainly on others’ and their own mistakes during their teaching. It is understandable that they strived to teach to their best ability and avoid making mistakes when they were observed. Even though the aim of lesson study is not to point out pre-
service teachers’ mistakes, they had pre-conceived ideas about the evaluation of their teaching attached to lesson study.

Three mentor teachers also valued the feedback from the post-lesson discussion as an insightful process. They found that getting comments and feedback were very useful for pre-service teachers. For example, Rudy (ME7R) said “The post-lesson discussions provided good feedback for the pre-service teachers. They get to know their mistakes and how to improve them”. Hani (MD8H) found that getting feedback was also beneficial: “sharing the comments and feedback is a positive things. We got constructive feedback, [it] made us aware of our weakness and how to improve it. Each observer might tell different aspects, thus we get so much valuable feedback”. Even though Hani was not the lead teacher, she appreciated the post-lesson discussions and the feedback she received. This suggests that Hani might have used the feedback to reflect on her contribution to the group during lesson study. However, the mentor teachers did not indicate that their feedback to the pre-service teachers is a part of the responsibilities of mentor teachers.

The participants valued the collaboration and group work during the lesson study. For example, Pipit (PD8P) said that the collaboration in the group gave her more ideas when making the lesson plans, it was “very helpful, especially in making the lesson plans, by doing it in group we get more ideas and get advice from the mentor teacher and the lecturer”. The mentor teachers and the lecturers also acknowledged the collaboration in the group as one of the benefits of lesson study.

Better lesson planning was seen as one of the benefits of lesson study for the pre-service teachers. Better lesson planning refers to in-depth and detailed planning. In contrast to their regular planning, the pre-service teachers did not plan in a detailed way (e.g. there was no anticipation of students’ responses). As a result of in-depth planning, the pre-service teachers felt more confident and well-prepared when teaching with the well-designed lesson plan. For example Ida (PD7I) said “we learn to make a good planning on the maths topic, teaching techniques and predictions of students’ answers”. The lecturer, Farida (LEF), confirmed this: “pre-service teachers were more well-prepared. The lesson plans and teaching materials were collaboratively designed. When they teach, they are more confident”. The data suggested that the pre-service teachers, mentor teachers, and lecturers agreed that detailed planning is beneficial for the pre-service teachers.
The participants gained new knowledge from participating in lesson study such as knowledge about content for pre-service teachers, and knowledge about using technologies in teaching for the mentor teachers. Three pre-service teachers learned new knowledge from the lesson study. For example, Diana and Pipit specifically said that they learned a new strategy of finding square roots. Diana’s responses was “I learned about finding the square roots and I taught my students that”. This suggests that lesson study contributes to the pre-service teachers’ content knowledge and this advantage is passed onto their students.

The mentor teachers also learned new knowledge by participating in lesson study. Nur (MD7N) said that she learned about making lessons more interesting for the students, “I learned that we can make the lessons interesting and innovative … the students learned better when learning with different approaches”. Irwan (ME7I) said that he learned about new technologies and different strategies to solve problems, “When discussing with the pre-service teachers, I learned new knowledge, for example using technology and the pre-service teachers have different strategies to solve problems”. Note that School E where Irwan teaches has limited technological support. However, in the lesson study, the pre-service teachers used technologies such as PowerPoint Presentations and various hands-on activities. Irwan observed and learned from this practice. This suggests that even though the lesson study was not designed for the mentor teachers’ learning, it also provided learning opportunities for them.

Even though in the post-lesson discussions the lead teachers were given the opportunity to reflect on their own teaching, self-reflection was not explicitly encouraged for the pre-service teachers. This is probably why, in the interview, only one pre-service teacher highlighted self-reflection as the benefit of lesson study. Deni (PE7D) said, “I get to know about myself, how I teach, and how I should teach better in the future”. The fact that only one pre-service teacher talked about self-reflection suggests that the other pre-service teachers probably did not focus on self-reflection.

Only one participant (i.e. Hani (MD8H), the mentor teacher of Group D8) found that awareness of students’ learning was an insightful experience. She highlighted her awareness of the students’ learning, “by [being] involved in the lesson study, I became more aware about students’ learning”. This might have been the result of observing the research lessons and discussing evidence of students’ learning in post-lesson discussion. Given that focusing on students’ thinking is not common in Indonesia, this indicates that Hani might not have been aware of the students’ learning before participating in lesson study.
Siti (LDS), the lecturer in School D, acknowledged the important role of the knowledgeable other. She said that the knowledgeable other helped her in supporting the pre-service teachers. With the full involvement of the knowledgeable other, Siti felt that some of her responsibilities were taken care of by the knowledgeable other. This could have been the reason for her minimal involvement in the lesson study. The lecturers’ minimal support is considered as one constraint of the lesson study, therefore it will be discussed further in 6.2.2.

The interview also revealed what the participants perceived as the difficulties of participating in lesson study. Table 6.4 presents what the pre-service teachers, mentor teachers and lecturers perceived as the difficulties of participating in lesson study. These difficulties include time, detailed preparation, school factors and classroom management, and lastly lesson study organisation.

Table 6.4 Difficulties of participating in lesson study according to the participants

<table>
<thead>
<tr>
<th>Difficulties of lesson study</th>
<th>Pre-service teachers</th>
<th>Mentor teachers</th>
<th>Lecturers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>PD7I, PD8G</td>
<td>ME7I, ME7R, ME8N, MD7N, MD8H</td>
<td>LEF</td>
</tr>
<tr>
<td>Detailed lesson preparation</td>
<td>PD7D, PD7I, PD7R, PE7D, PE7U, PE8V</td>
<td>ME7R</td>
<td>LDS</td>
</tr>
<tr>
<td>School factors and classroom management</td>
<td>PE7D, PE8Y</td>
<td>ME8N</td>
<td></td>
</tr>
</tbody>
</table>

All mentor teachers agreed that time is a major constraint in implementing lesson study during the teaching practicum. They raised concerns about the time needed to plan the lessons, and the lesson study meetings that clashed with their teaching schedules. For example, Rusdi (ME7R) commented, “the planning took so much time. We must prepare the teaching materials before the lesson, otherwise it would take time in the lesson”. Similarly, the pre-service teachers were also concerned about the time needed for preparing the lesson. Gina (PD8G) said, “since we wanted to teach a good lesson, we invested more time in designing the lesson plan”. The mentor teachers and the pre-service teachers were aware of the amount of time needed to develop a detailed lesson plan. Farida (LEF) also commented on the challenges that the pre-service teachers faced in planning together. For example, each pre-service teacher might have different schedules and they need to juggle the teaching requirements during their practicum and their university classes. She stated, “the time needed to make the lesson plans
and the teaching materials, moreover with the new teaching practicum, each pre-service teachers might have different schedule”.

Another time constraint found in School E, was regarding the mentor teachers’ schedule. School E has eight parallel classes for each year level, therefore providing time for post-lesson discussion is difficult. Irwan (ME7I) said, “the research lessons of [the] Year 7 group had their allocated times which fitted my schedule but the post-lesson discussion did not [fit my schedule]”. In some post-lesson discussion meetings, Irwan and Rusdi had to leave the discussion earlier because they had classes to teach.

Designing a detailed lesson plan and teaching materials that support students’ reasoning were challenging for most of the pre-service teachers. For example, Deni (PE7D) commented, “when making worksheets and problems, it is difficult to predict students’ responses. We need to think of what kind of problems that interest them”. Vina (PE8V) also commented similarly, “making the worksheet is difficult, because we usually used problems from [the] text book. In the worksheet we used different problems, higher-level problems. It makes me afraid whether the students understand it”. The lesson study encouraged the pre-service teachers to teach in a student-centred environment and make students’ thinking visible. Clearly this was a new practice for the pre-service teacher. The reason for this is probably because the pedagogical units they took before the teaching practicum focus more on the teaching and do not have sufficient planning components. The researcher’s observation showed that the pre-service teachers did not have access to a wide variety of quality teaching resources. This might have also caused the difficulties for the pre-service teachers to produce detailed lesson plans.

School factors and classroom management issues were identified as problems by two pre-service teachers and the mentor teacher in School E. Deni from Group E7 taught in the elementary school building. He said, “In my classroom, the lightning is not good. It did not support the use of the projector. I wanted to use video or interactive software in the lesson, but I could not because of that”. Deni’s classroom was exposed to sunlight, with no curtains to adjust the light. Therefore when he used a projector, the screen was not clear enough. This might pose a problem not only for lesson study but also in his regular teaching. However, since lesson study encouraged the pre-service teachers to teach more student-centred approach which sometimes required technological tools, this problem arose. Moreover, the mentor teacher of Group E8, Nani (ME8N), argued that the large classroom was difficult for the pre-service teachers to handle: “Too many students in one class: 36 students”.
In summary, the data showed what the participants perceived as the benefits and challenges to the implementation of lesson study in the teaching practicum. The benefits identified in the data are: receiving feedback about teaching, collaboration, better lesson planning, learning new knowledge, reflection of own-teaching, support from the knowledgeable other, and awareness of students’ learning. The challenges include: time, detailed preparation, and the school factor and classroom management.

6.2 Affordances and Constraints Relating to the Implementation of Lesson Study

This section addresses some issues found during the fieldwork that contributed and hindered the implementation of lesson study in the teaching practicum. Data are taken from videos, interview, and field notes.

6.2.1 Dual Role of the Researcher

As mentioned in Chapter 3, the researcher acted as the knowledgeable other in the lesson study. This section discusses some affordances and constraints of the researcher’s dual role for the lesson study underpinned by the findings from the interview and field notes.

The first affordance from the dual role is having a ‘full access’ throughout the lesson study. As the researcher, I had to attend and observe all lesson study meetings at the two schools. This was an advantage for the lesson study because it allowed myself as the knowledgeable other, to participate in the planning, research lessons, and post-lesson discussions of all groups in both schools. By participating throughout the whole process, I was able to follow and support the pre-service teachers’ learning, and intervened when needed. This full involvement of the knowledgeable other is not common in lesson study. It has enabled me as the knowledgeable other to maintain the focus of lesson study throughout each phase.

Moreover, as the knowledgeable other at the two schools, I benefit from sharing the learning of one group to another group across the schools. Cycle 1 of Group D7 (C1D7) taught about linear equation using the balance model to introduce the concept of equivalence to the students. One week later, Group E7 had their first cycle (C1E7). The goal of the lesson was to transform problems into mathematical equations and solve the equations. In the planning, Group E7 discussed the development of problems to help students achieve these goals. Because the lesson was similar to D7’s, I shared what happened in Group D7.

Meili: I would like to tell you what happened last week at the other school. They did a similar topic. Students were able to find the answer of such questions, but they were unable to
write the mathematical equation. They solved the problem using simple arithmetic calculation. My point is, it is not easy to bring students to the understanding of using algebraic notation in solving problem. The pre-service teachers there used balance problems, students were able to find the answer but not to write the equation.

Deni : We did that before.
Meili : How was it?
Deni : [Referring to the problem in the text book]. This lesson was about solving x, not writing the equation. But they understood that both sides are equal.
Meili : If we start from there, students already understood that both sides have to be equal, and then they need to symbolise the unknown with a variable. Have they learned this before?
Deni : They directly used the variable.
Meili : I think you can use this problem again. You repeat what they have learned and at the same time build a new understanding about [the] variable and after that moving forward using more other problems.

The group followed my advice and used the balance model. Even though they did not participate in Group D7’s lesson study, they learned from what I told them happened in D7. This resulted in some changes of the use of the balance model. Group E7 used several balance problems, each with a different level of difficulty.

While the dual role have brought some affordances in the lesson study, it also has brought constraints. The first constraint was related to the participants’ misunderstanding and unclear expectation of my role. Siti (LDS), the lecturer of School D, misunderstood my role as the knowledgeable other. She thought my position as the knowledgeable other/researcher in the lesson study could replace her. As addressed in 6.1.2, she said that my involvement had helped her as the lecturer. This might have justified her minimal support during the lesson study. Detailed discussion of the lecturers’ role will be presented in a later section (6.2.2).

The second constraint was that my role as the researcher overshadowed my role as the knowledgeable other. I often focused too much on the organisation and logistics of the lesson study. It took much effort and energy to make sure the lesson study ran well. This organisational work included setting up the schedules of the lesson study meetings to fit the pre-service teachers’, mentor teachers’, and lecturers’ schedules. I also focused on preparations for the lesson study meetings, such as making sure the video and audio recorders were ready, and reminding the mentor teachers and lecturers to come to the planning. Consequently, I did not prepare myself with mathematical and pedagogical material to support the pre-service teachers.

As a summary, taking up the dual role as the researcher and the knowledgeable other has created some benefits for the lesson study. As the knowledgeable other, I participated throughout the whole process of the lesson study, which allowed me to fully support the pre-service teachers. Furthermore, this allowed me to share the learning from one school to another school. As discussed, the dual role also created an extra challenge for me to maintain my focus
on the lesson study due to competing demands to manage the logistics of the research. However, one lecturer misinterpreted the researcher’s role as the knowledgeable other which resulted in her minimal involvement in supporting pre-service teachers during their practicum. This will be discussed in more detail in the following section.

6.2.2 The Role of the University Lecturers

The lecturers are expected to be the experts and thus able to help the pre-service teachers’ learning during the teaching practicum. This section addresses the role of the lecturers in the lesson study aligned with the affordances and constraints. The findings are taken from field notes, interviews, and video data.

The data from this study shows that the involvement of the lecturers at both schools were very limited, therefore in general it is a constraint for the lesson study. Table 4.5, 4.6, 4.7 and 4.8 show the lecturers’ attendance during the lesson study meetings. In School D, the lecturer, Siti, attended only the planning in Cycle 1, Group D7 (C1D7), the planning in both cycles of Group D8 (C1D8 and C2D8), and the first research lesson in C2D8. In School E, the lecturer, Farida, did not attend any of lesson study meetings in Group E7. For Group E8, she attended Vina’s research lesson and post-lesson discussion in C1E8 and Yanti’s in C2E8.

The video provided data of the lecturers’ participation in the lesson study meetings they attended. Data from C1D7 shows that even though Siti attended the whole planning session, she did not contribute actively in the discussion. She asked about the curriculum at the beginning of the meeting, confirming if the school uses the most recent curriculum, then she stayed quiet throughout the meeting, and reminded the pre-service teachers about the structure of the lesson at the end of the meeting: “do not forget to review [the] previous lesson in the opening and draw conclusion[s] in the closing of the lesson”. This was not a substantial contribution to the pre-service teachers’ PCK development. Moreover, due to her other commitments, Siti could not attend any of the research lessons in Group D7.

In Group D8, Siti attended the planning and both research lessons in C2D8. In this planning she did not contribute actively into the discussion either. She had to leave midway through the planning. Before leaving, she made sure of the research lesson schedule. In the first research lesson, with Pipit as the lead teacher, Siti observed only half of the lesson. Again she had to leave midway through the lesson because of her other commitments. However, she made some notes on her observation sheet. Her observation sheet showed that she only noticed the
student activities such as: “Students were active and enthusiastic. Only Dodi is very quiet”. In the second research lesson, Siti also attended it halfway. However, her observation sheet was unavailable for this research lesson. Before she left, she asked me to remind the pre-service teachers about the position of the triangles. “In a right angled triangle, please be careful when using the words ‘base’, ‘height’ and ‘hypotenuse’. When the triangle is rotated, the hypotenuse could be seen as the base”. This is important feedback concerning students’ common misunderstanding of determining the hypotenuse of a right angle triangle. I discussed this with the pre-service teachers in the post-lesson discussion.

In School E, due to her busy schedule, the lecturer Farida only attended the research lessons in Group E8. In C1E8, she attended Vira’s research lesson and post-lesson discussion. She commented about students’ mistakes, connected to the notion of gradient.

Farida: The lesson started at 12.30 pm. First Vina gave examples of ‘slope’ using many pictures. I think this is very good. At 12.35 pm, Vina distributed the first worksheet. Vina defined the slope as the difference of the height over the difference of the horizontal. There were no problem[s] in the first worksheet. But my concern is, the slope is not always an integer. Since it is a division, it might resulted in [a] fraction or decimal numbers. It will be better if you have a convention about what to use either fraction or decimal. If you are going to use the decimal, if you want to round it, please follow the correct procedure of rounding. For example: 8/11, you wrote 0.72 when in fact it is 0.727, so if you’re going to use two decimal numbers, it’s supposed to be 0.73.

In the second worksheet, I found some students did not know where (0,0) is. Many of them did not know how to plot the coordinate, they did not understand the positive and negative part of the Cartesian plane. Actually, from the graph only, students should have been able to tell whether the gradient is positive or negative, no need to use the formula. But most students thought the gradient is always positive.

Timing is good, lesson structure is completed.

Farida’s comments suggested that she observed students’ work during the research lesson. She commented on students’ use of decimal of fraction numbers in expressing the gradient. She reminded Vina to pay attention to the rounding. In the second research lesson, Yanti incorporated this feedback into her lesson. When students struggled to convert a fraction into a decimal, Yanti told them to use a fraction. Farida also noticed students’ difficulties when plotting points in the Cartesian plane and their misunderstanding that the gradient is always positive.

Farida also observed Yanti’s research lesson in the second cycle of the lesson study (C2E8). The topic of the lesson was about the gradient of parallel and perpendicular lines. In the post-lesson discussion, Farida commented on time keeping issue, students’ difficulties working with a Cartesian plane, and she corrected the notation of coordinates.
Farida: The opening was good, [it was] on time and fitted the lesson plan. But you took too much time on discussing the first worksheet. More time than the allocation in the lesson plan, thus you were rushing when doing the second worksheet. You should keep the time according to the lesson plan. Most students did not know how to pick any two points on the line, and were confused about writing the coordinate.

Yanti: The graph is also very clear with the axis, I do not know why they did not get it. Farida: They misplace the \( x \) and \( y \). It is very difficult for them to understand. They did not understand [that] the \( x \)-axis lies horizontally, nor [that] the left side is negative and the right side is positive. They did not understand [the] \( y \)-axis either. Also, when they knew the coordinate, they mistaken the substitution in the formula. Which one is the \( x_1, x_2, y_1, \) and \( y_2 \). This needs following up. You need to keep helping them on this. Other than that, the notation of coordinates. It should be \( A(x, y) \). They wrote \( A = (x, y) \). The correct one is without a = sign. When doing the second worksheet, they had difficulties working with negative numbers. For example, \(-3 - 6 = 3\).

The interview asked the pre-service teachers about the role of the lecturers in the lesson study. The pre-service teachers were very careful in avoiding negative responses for this question because the lecturers are respected figures for them. Moreover, the lecturers would give the pre-service teachers marks for the teaching practicum. The pre-service teachers’ responses are more general to the lecturers’ role in the teaching practicum, not specifically in the lesson study.

Table 6.5 summarises the pre-service teachers’ responses to the interview question about the lecturer’s role in the teaching practicum. Four pre-service teachers from School E and only one from School D said that the lecturer provided good feedback for them.

Table 6.5 The pre-service teachers’ opinion about the lecturer’s role in the teaching practicum

<table>
<thead>
<tr>
<th>The pre-service teachers’ responses</th>
<th>School D</th>
<th>School E</th>
</tr>
</thead>
<tbody>
<tr>
<td>The lecturer was busy</td>
<td>PD8G n = 3</td>
<td>PE7U n = 3</td>
</tr>
<tr>
<td></td>
<td>PD8P</td>
<td>PE7D</td>
</tr>
<tr>
<td></td>
<td>PD7D</td>
<td>PE8Y</td>
</tr>
<tr>
<td>The lecturer provided positive feedback</td>
<td>PD8P n = 1</td>
<td>PE8Y</td>
</tr>
<tr>
<td></td>
<td>PE8V</td>
<td>PE7D</td>
</tr>
<tr>
<td></td>
<td>PE7J</td>
<td></td>
</tr>
<tr>
<td>The lecturer did not give enough support</td>
<td>PD7R n = 2</td>
<td>PE7U n = 1</td>
</tr>
<tr>
<td></td>
<td>PD7I</td>
<td></td>
</tr>
</tbody>
</table>

Umar (PE7U), Deni (PE7D), Yanti (PE8Y), Diana (PD7D), Pipit (PD8P) and Gina (PD8G) (\( n = 6 \)) said that their lecturer was busy. Moreover, three (Umar, Diana, and Gina) of them said that because their lecturer was busy, they did not get enough support during the teaching practicum. On the contrary, the other three of them (Deni, Yanti, and Pipit) confessed that even though the lecturer was busy, she still had time and provided good feedback for the
pre-service teachers. Pipit commented, “She did not come often. When she came, she commented on the right-angled triangle, and it was a good point. I explained it to students in the next lesson”. Pipit valued the feedback from the lecturer and incorporated it in her teaching.

Vina (PE8V) and Jamal (PE7J) (n = 2) said that their lecturer helped them improve their teaching. Vina mentioned that she became aware of how to improve her lesson delivery, while Jamal mentioned that the lecturer provided feedback for his improvement. On the other hand, two pre-service teachers from School D, namely Ida (PD7I) and Raya (PD7R) wished the lecturer would have come more often and given them more feedback.

The lecturers were also interviewed and asked what they thought about their contribution to the lesson study. Siti appraised the effort from the pre-service teachers and the mentor teacher when planning the lessons and research lesson:

Siti: I commented on the lesson plans, I see that the mentor teacher and the pre-service teachers are very creative and serious in planning the lesson. I did not come to the research lessons and post-lesson discussions often. But when I came to the research lessons, I think it is good. The only problem is the students, but students these days are like that.

As presented earlier in this section, Siti attended few planning meetings. When Siti said that she commented on the lesson plans in the planning, it was probably when she reminded the pre-service teachers to start the lesson with a review of the previous lesson, and end the lesson with a conclusion. Siti admitted that she did not come to the research lessons and post-lesson discussions, but when she came to the research lesson, she thought they were good. Her only concern was about the students, most probably in relation to the students’ behaviour and attention during the lesson.

Farida, the lecturer of School E, was more concerned with the students’ learning. She focused more on the students and pointed out that even though the pre-service teachers might have followed the lesson plan well, they still need to ask whether the students understand the concepts taught.

Farida: I focused on the students and the teacher, how they learned, did they understand the lesson? I think Yanti and Vina taught well, in terms of following the lesson plans. However, the students did not follow the lesson well, they did not understand it.

From these responses, even though Siti and Farida have had some lesson study experiences, they have different understandings of lesson study. For Siti, lesson study might seem an artificial program that she followed simply as a part of her responsibilities as lecturer. She did not use the opportunity to enhance the pre-service teachers’ learning nor her own
learning. Farida seems to have a better understanding of lesson study. She understood the focus on students’ learning in lesson study. She used evidence of students’ learning in the post-lesson discussion to raise some teaching issues.

When asked about the difficulties in implementing lesson study in the teaching practicum, Siti mentioned the different lesson plan templates used by the pre-service teachers. “There are differences in the lesson plan template from us and from the mentor teachers. It is not a big problem, because it is only for administrative purposes”. Farida highlighted the time consumed by lesson study, especially for the pre-service teachers. “The time needed to make the lesson plans and the teaching materials is enormous. Moreover, with the new system, pre-service teachers might have different schedules”. Farida was concerned about the pre-service teachers’ workload with lesson study and the new teaching practicum setting. She might see this as a potential problem because lesson study requires more time and the teaching practicum only allows the pre-service teachers to spend a maximum of three days at a school. Siti and Farida did not comment on any problems associated with themselves and did not seem aware that they were expected to contribute more in the lesson study.

To sum up, the data showed the constraint from the limited contribution of the lecturers. The lecturers’ busy schedules and other work commitments might have caused their lack of visits during the teaching practicum. The interview with the pre-service teachers revealed that even though some of them appreciated the support given by the lecturers, they expected more. The interview with the lecturers indicates that they were not aware of this expectation. It implies that for future implementation of lesson study, it is necessary to build the lecturer’s understanding about their expected contribution and negotiate the organisation so that they can fulfil these expectations.

6.2.3 Dynamics of the Groups

Chapter 5 has shown some lesson study groups in which the pre-service teachers had opportunities to develop their PCK. All groups except Group E7, showed PCK development during the lesson study. This section discusses the dynamics of the groups that impacted on the development of the pre-service teachers’ PCK. Data are taken from interviews, field notes, and videos.

The pre-service teachers’ participation in the lesson study impacted on the success of the groups. The groups with active participation showed development of PCK while the group
with lower participation did not showed development of PCK. The pre-service teachers’ attendance in the lesson study meeting can provide insight into their participation. Chapter 4 has presented the participants’ attendance in the lesson study meeting. Members of Group D7, D8 and E8 attended and participated actively in all the planning (see Table 4.5, 4.6, 4.7 and 4.8). Absence was only found in C2D7, Ida (PD7I) was absent at the first research lesson and post-lesson discussion, and Raya (PD7R) was absent at the third research lesson and post-lesson discussion. In contrast, the pre-service teachers of Group E8 had the most absence. In the first cycle (C1E7), Umar (PE7U) was late to the planning, and absent from the first research lesson and post-lesson discussion. Deni (PE7D) did not attend the second and third research lesson and post-lesson discussion. In the second cycle (C2E7), Umar did not attend the planning, the second and third research lesson, and the second post-lesson discussion. Deni and Jamal (PE7J) were absent in the first research lesson.

Health issues and other commitments were identified as the cause of the pre-service teachers’ absence. The reason for Ida’s, Raya’s and Deni’s absence was their health condition. During the semester of the teaching practicum, the pre-service teachers took other units at the university. The interview revealed that they could manage the time well for classes and the teaching practicum. However, Umar (PE7U) had other priorities besides his teaching practicum and classes. He was the president of the student union. He admitted that his commitment in the student organisation made it difficult for him to spare time for the lesson study group: “yes, because my responsibility at the student organisation, my schedule changes a lot”. This explained his absence during some lesson study meetings.

The pre-service teachers’ participation is not only shown by their attendance in the lesson study meetings but also by their contribution to the meetings. All groups distributed the work to different members of the group. However, Group E7 did not perform as good as the other three groups. This was especially evident in the quality of the teaching materials they produced. The difference in quality seems to be connected to the pre-service teachers’ participation in the planning. The pre-service teachers in Group D7, D8 and E8 attended the planning sessions and shared an understanding of the lessons, therefore even though they distributed the work, they produced high quality teaching materials. On the contrary, in Group E7, because Umar missed the planning session, the group did not have the same shared understanding of the lesson plans and teaching materials.
Video data of Umar’s research lesson showed that missing the planning impacted the teaching. Umar did not attend the planning of the second cycle therefore, he missed the information about the lesson structure. This impacted the teaching and the students’ learning. In the planning, Jamal and Deni developed two worksheets, each should be given separately so that students would discuss it in pairs. Umar did not know about this arrangement. He gave the students the two worksheets all together. Consequently, instead of discussing the worksheet in pairs, the students worked on each worksheet individually. I raised this issue in the post-lesson discussion.

Meili : The Worksheet 1 and Worksheet 2 are done at the same time, why?
Umar : I usually do that, first explaining the topics, examples, and then student work. But since I did not make the worksheets, so I gave them all together. If I had made the worksheet, I would put all of it in one worksheet.
Meili : I see some ‘miscommunication’ in this group. Umar did not attend the planning, you missed out the information and the agreement. Who made the lesson plan?
Umar : I did.
Meili : Who made the worksheets?
Jamal : I did, but I showed it to Umar.
Meili : Umar, did you see the worksheets before?
Umar : Yes I did.
Meili : Since the worksheets were given at the same time, like Rusdi said, there was no discussion.

This excerpt suggests that there was a lack of collaboration in Group E7. The pre-service teachers did not share sufficient information among themselves. Umar did not seem to know that the worksheets were designed to be completed separately. Moreover, Umar’s misunderstanding impacted the structure of the lesson and the students’ learning. Because he was the first lead teacher, it suggests that he taught the lesson only based on his interpretation of the shared information from other group members.

During the interview, Umar and Jamal explained their group work. Umar admitted that the group did not write the lesson plan together, they distributed the work instead. Moreover, because he did not attend the planning, he did not get the idea of the lesson.

Umar : I got misunderstood. Especially, in the planning. Our group did not write the lesson plan together. We divided the work-load. One did the lesson plan, one did the worksheet, and the other one did the PowerPoint presentation.
Meili : What was your contribution in the planning?
Umar : I did not come the planning, so I did not contribute. But I did my part, when asked to make the worksheet, I did it. I asked the other what we are going to do then I wrote the worksheet or lesson plan.
On the last cycle, Jamal and Deni agreed to make two worksheets, but I did not know that. I wrote the lesson plan and it did not separate the worksheets. I missed that one.
Umar’s interview suggests that Group E7 did not establish good communication among the pre-service teachers. Umar did not show any effort to catch-up on what he missed in the planning. Jamal and Deni did not share enough information about the lesson for Umar to catch-up on what he missed in the planning and for him to be able to produce suitable teaching material. Jamal confirmed this in the interview. Moreover, he explained that time constraints seemed to hinder them from working together.

Meili : How was the work distribution in the group?
Jamal : In the first planning, I prepared the lesson plan and the problems for the worksheet before the planning. So that the discussion in the planning is for finalising the worksheet and lesson plan.

We only agreed the topic for Cycle 2 but we did not prepare anything. Deni did the PowerPoint presentation, I made the worksheet, and Umar made the lesson plan. We did not work together, thus the lesson plan and the lesson were not aligned.

Meili : Why did you not discuss it together?
Jamal : We could not find the right time to meet, all of us only come to school together on Monday. We did not have much time from the planning to the research lesson. We were having the exam week during the second cycle, so that we did not work together on the lesson.

To sum up, the pre-service teachers’ participation in the planning and collaboration are important for the lesson study’s success. In cases where all the pre-service teachers attended and actively participated in the planning, they shared the same understanding of the lesson, therefore the lesson plan and learning materials they produced are relatively aligned. In contrast, in E7, Umar who missed the planning, did not get the overarching idea behind the lessons. Consequently, the lesson plan and the teaching materials were not aligned. More importantly, it impacted Umar’s lesson and the students’ learning. In short, the dynamic of the groups can be seen as an affordance when the pre-service teachers are actively contributing in the lesson study, and as a constraint when the pre-service teachers did not establish good collaboration amongst themselves.

6.2.4 The Organisation of Lesson Study

This section elaborates on the aspects of the organisation of the lesson study based on the researcher’s reflections. The findings are derived from field notes and videos. Some aspects of the organisation were identified as constraints for the implementation of lesson study. Identifying these aspects provides insights that are valuable for the design and future implementations of lesson study in the teaching practicum.
The first aspect identified as a constraint for the lesson study implementation during the teaching practicum was the coordination between the mentor teachers, and the pre-service teachers when deciding on the topic taught during their research lessons. In this study, the lesson study groups decided on the topic taught during the planning. The pre-service teachers and the mentor teachers may have had a conversation about the selection of the topic before the planning but this information was not passed on to the knowledgeable other and the university lecturers. In many cases, the pre-service teachers came to the planning with ideas of the lessons and the tasks for students but the university lecturers and the knowledgeable other did not know in advance of the selected topic. Because of that, the lecturers and the knowledgeable other were not prepared with appropriate resources to support pre-service teachers’ planning on the selected topics.

In the planning, the pre-service teachers presented their teaching ideas, but most of the times their ideas were too broad. Consequently, the planning sessions were spent on re-shaping or developing new ideas. Without enough time to prepare, the lecturers and the knowledgeable other could only provide limited support. Moreover, as the knowledgeable other, I often found contingent moments during the planning sessions where I had to act in the moment. For example, in the planning of Cycle 2 Group E8 (C2E8), the pre-service teachers struggled to find ways to teach gradients of perpendicular lines. The overarching question was ‘Why $m_1 \times m_2 = -1$ ?’. The group discussed how to find a proper way to prove this formula. At the time I had a few ideas, such as using rotation of the line, but that was not an easy explanation for the students. This teaching approach was considered too challenging for students and hence the group decided not to use this approach. As the knowledgeable other, if I were to have had more time and information about the topic, I would have done more research prior to the planning sessions to offer the pre-service teachers with appropriate ways of proving the formula.

For future implementation of lesson study in the teaching practicum, it is important for all participants to be informed of the topic prior to the planning. The goal of the lesson and appropriate teaching resources could then be discussed in-depth during the planning. This would make the planning more productive and time-efficient because the pre-service teachers could bring their ideas of the lesson and the tasks to the planning. This would also help the mentor teachers, lecturers and the knowledgeable other with relevant resources that support the learning of the chosen topic.
The second constraint is the lack of protocol to maintain the focus of discussions in the planning sessions and post-lesson discussions. For example, Cycle 1 of Group E8 (C1E8) is chosen to illustrate this problem. In this cycle, the group taught lessons on the Gradient. They followed the textbook and defined the gradient as the vertical differences over the horizontal differences or \[ m = \frac{y_2 - y_1}{x_2 - x_1}. \] As discussed in Chapter 5, the students counted the grids to find the difference, consequently, they did not get the idea of positive and negative gradients. In the planning, as the knowledgeable other, I mentioned the possibility of students counting the grids. Unfortunately, the discussion did not continue with finding ways to help students.

Meili : What if the students understand it as the distance of the \( x \) and the \( y \). They only count the grids without knowing the positive or negative value?
Yanti : We will tell them.
Meili : What are your learning goals?
Pre-service teachers looked at the syllabus.

The excerpt above shows that the focus of discussion in the planning jumped from possible students’ mistakes to the lesson goals. Because there was no protocol for the planning, it was difficult to maintain focus and structure in the planning sessions. Even though discussing possible students’ mistakes and the lesson goals were both important, having a structure in the planning would help to maintain the focus of the discussion.

The data showed that the pre-service teachers, the mentor teachers and the lecturers often talked about classroom management issues especially during the first cycle of lesson study. For example, in the post-lesson discussion following Raya’s lesson, of the first lesson study cycle of Group D7 (C1D7), Raya as the lead teacher commented on her classroom management issues such as students’ attention and timing.

Raya : During the presentation, it was difficult to ask them to write their answers on the board. They were shy and unconfident. One student who came and wrote his answer on the board, could not explain how he got his answer, this is because he copied the answer from his friend. Students who understood the concept, only understood it for themselves, they cannot explain their understanding to others. I gave them 15 - 20 minutes, they should have been able to finish it in time, but since they were reluctant, they did not want to do it in class and wanted it to be a homework instead. They will come up with many excuses not to do the homework.

Other pre-service teachers were also found prioritising classroom management in their discussion at the planning and post-lesson discussions. It was not a surprise that classroom management was the main concern for the pre-service teachers. However, classroom management was not the focus of the lesson study. The goal of the lesson study was on developing the pre-service teachers’ PCK, with a focus on the students’ mathematical thinking.
The findings indicated that without the focus of discussion, the participants’ attention easily drifted away from the focus of the lesson study. Hence, it is important for all participants of lesson study to build the same understanding of the focus of lesson study. Moreover, to maintain the focus of the discussion, it is necessary to establish a discussion protocol so that all participants can contribute more specifically towards the focus of the lesson study.

### 6.3 Conclusion

This chapter has shown that from anticipating students’ responses, the pre-service teachers felt more well prepared in the lesson. When they have anticipated students’ difficulties, they could prepare ways to help students overcome these difficulties. The data also showed what the pre-service teachers think as difficulties in anticipating students’ solutions: providing detailed anticipation and the additional paper work related to it. Especially anticipating students’ responses to hands-on activities was found to be difficult for the pre-service teachers. Moreover, the data indicated that the pre-service teachers were not aware of the retrospective anticipation of students’ responses that it evolves throughout the planning, research lessons, and post-lesson discussions. The pre-service teachers did not always prepare appropriate prompts. Lastly, the pre-service teachers did not incorporate anticipating students’ responses into their regular teaching. The pre-service teachers, mentor teachers, and university lecturers highly valued the collaboration in lesson study. More specifically, the pre-service teachers highly appraised the feedback they received during post-lesson discussions. However, they also reported some difficulties in participating in lesson study such as the time commitment and detailed lesson preparation required.

This chapter has also addressed the affordances and constraints in the implementation of lesson study in the teaching practicum. While the university lecturers were expected to provide active supervision in the teaching practicum, their involvement was constrained by their lack of time availability. The active involvement of the knowledgeable other served as an affordance because the knowledgeable other maintained the focus of lesson study on student learning, shared the lesson study experience from one school to the other. It also served as a constraint, causing a misunderstanding from the lecturer which resulted in her lack of involvement. The data showed that the dynamic of the group influenced the success of the lesson study group. The group whose members contributed actively in their lesson study group meetings demonstrated development in their PCK. On the contrary, the group whose members did not participate actively in their lesson study group meetings did not demonstrate apparent
development of their PCK. Lastly, the structure and organisation of the lesson study was seen as a constraint for several reasons. Firstly because lesson study groups decided on the topic taught for the lesson study in the planning, the university lecturers and knowledgeable other did not have an opportunity to prepare for that specific topic, consequently they could only provide limited support. The other reason is that because there was no protocol in the planning, it was difficult to maintain the focus of discussion.
7 Discussion and Conclusion

This study investigated the development of pre-service teachers’ PCK during the implementation of lesson study in the teaching practicum. The following research questions guided the investigation:

RQ 1 What changes are evident in pre-service teachers’ PCK during lesson study in their teaching practicum?

RQ 2 What characteristics of lesson study contribute to the development of pre-service teachers’ PCK?

RQ 3 What are pre-service teachers’, mentor teachers’, and university lecturers’ views about the incorporation of lesson study into pre-service teachers’ mathematics teaching practicum?

RQ 4 What are some of the affordances and constraints relating to the implementation of lesson study in pre-service teachers’ mathematics teaching practicum?

The Knowledge Quartet (KQ) framework (Rowland et al., 2009) was used in the analysis to identify the pre-service teachers’ PCK. Moreover, this study has developed a KQ Rubric to mark the changes in the pre-service teachers PCK during the planning, research lessons, and post-lesson discussions of lesson study. The last two chapters have presented the findings aligned with the research questions. This chapter presents the answers to the research questions and discusses the position of this study in the wider literature. The summary of the findings, significance, implications, and limitations of the study will be presented at the end of the chapter.

7.1 Examining Changes in Pre-service Teachers’ PCK using a KQ Rubric

Chapter 5 has presented the evidence of the development of the pre-service teachers’ PCK during lesson study. The development of the pre-service teachers’ PCK was indicated by the changes in their PCK. Change in this study was defined as a shift in the quality of KQ components demonstrated by the pre-service teacher. A KQ Rubric was developed as a methodological tool to identify changes in the pre-service teachers’ PCK over different phases of lesson study.
Moreover, the KQ Rubric is used as a tool to indicate not just the presence but also the quality of pre-service teachers’ PCK. Earlier studies (e.g., Rowland et al., 2005; McAuliffe & Lubben, 2013) only focused on the presence of the KQ components demonstrated by pre-service teachers in their teaching. This approach is not sufficient to capture the development of pre-service teachers’ PCK over an extended period of time. Weston (2013) argues that identifying the presence of any KQ components does not necessarily give insight into the relative quality of the KQ components. For example, showing the presence or absence of the Choice of Representation – one component under the Transformation dimension in a lesson does not indicate the pre-service teachers’ knowledge of that component. It is more important to understand the reasons that underpin the pre-service teachers’ choice of the representation and how they use the representation in the lesson.

Some scholars used rubrics to measure the quality of an individual’s ability (see for example Tuñón & Brydges, 2006). However, there are different views in the literature regarding assessing the quantity of the individual’s ability. Weston’s (2013) coding protocol includes both quantity and quality of the KQ components. On the contrary, Leavy (2015) did not recommend using a rubric because of the misleading use of applying the quantity of the knowledge observed. Furthermore, Leavy (2015) argues that a low frequency of the knowledge demonstrated by the pre-service teachers is no less important than the higher frequency. This applies especially for some of the KQ components with a negative connotation, for example, concentration on procedures under the Foundation dimension. In this study, when the pre-service teachers used procedural teaching more frequently, they demonstrated a lack of the Foundation Knowledge. Therefore, to avoid confusion, the KQ Rubric designed for this study disregarded the quantity of the presence of the KQ components. Instead, it focused on the quality of the KQ components.

In the development of the KQ Rubric, it was challenging to clarify the boundary of each of the KQ components. For example, in order to understand the pre-service teachers’ Choice of Representation, it is important to find out the reason behind their choice of the representation, and how they used the representation in the lesson. The pre-service teachers’ reasons may be found in different KQ components such as their Overt Subject Knowledge and consideration of students’ prior knowledge. This raised a challenge in developing a clear descriptor of each level of the KQ components. The process of developing the descriptors is illustrated below.
First, the researcher interpreted Rowland’s et al. (2009) definition of the KQ components. The next step was to specify the descriptors of each level of the KQ components by aligning the KQ definitions and the data from this study. There were times when the descriptor of one component overlapped with the other. For example, initially, the researchers’ interpretation of Responding to Students’ Ideas component was how the pre-service teachers responded to unanticipated students’ ideas. Informed by the data, the pre-service teachers’ responses could be categorised into three groups: they did not respond, they responded with procedural teaching, and they responded with a more student-centred teaching. These descriptors were overlapped with the concentration on procedure component. Thus, the descriptors of Responding to Students’ Ideas were changed into: Level 1 – pre-service teachers did not respond, Level 2 – the pre-service teachers responded inappropriately, and Level 3 – the pre-service teachers responded appropriately. Even though the overlapping descriptors revealed the challenge of disentangling the KQ components, it suggested that a particular situation in the teaching might reveal several KQ components.

Having a dynamic view of PCK, this study used the lesson observation as the main data for investigating pre-service teachers’ PCK, in which the pre-service teachers were observed while enacting their knowledge in the teaching (Alonzo & Kim, 2016; Petrou & Goulding, 2011). Moreover, employing lesson study, this study argues that teaching cannot be separated from the planning and the post-lesson discussion. Therefore, the KQ Rubric was used to identify the development of the pre-service teachers’ PCK throughout the planning, research lessons, and post-lesson discussions. However, the KQ Rubric was not used to analyse the written test because the mathematical topics in the written test were different from the mathematical topics taught during lesson study. On the other hand, some of the interview questions were intended to gain information about the pre-service teachers’ learning during lesson study. Some of the pre-service teachers’ responses referred to a specific moment in the lesson study cycles. The KQ Rubric was used in some cases where the pre-service teachers’ interview responses indicated their PCK development. Because a lesson is multifaceted, Henze and Van Driel (2015) point out that PCK develops over time. In line with Henze and Van Driel (2015), this study has used the KQ Rubric to identify the pre-service teachers’ PCK development over different phases of lesson study.

The KQ Rubric offers an addition to the wide range of methodological tools used to capture teachers’ or pre-service teachers’ PCK development and its complexity. Studies have
shown different research instruments for identifying teachers’ or pre-service teachers’ PCK development, such as coding protocol (Weston, 2013), teachers’ self-report and video stimulated interviews (Alonzo & Kim, 2016; Henze & Van Driel, 2015). Nevertheless, the KQ Rubric has some limitations. It was not created as an assessment tool but a methodological tool to analyse data. Therefore, the KQ Rubric is specific to the data of this present study. Even though the descriptors in the rubrics were designed to be general enough for future use, it is subject to the researcher’s interpretation of the KQ components. Thus, future users may want to consider the reliability and validity of the KQ Rubric as an assessment tool or a methodological tool.

7.2 The Development of Pre-service Teachers PCK

This study aimed to investigate the development of pre-service teachers’ PCK during lesson study. Research Question 1 particularly asked “What changes are evident in pre-service teachers’ PCK during lesson study in their teaching practicum?” This study found some changes of the pre-service teachers’ PCK that indicate their PCK development during the lesson study. The pre-service teachers demonstrated some development in some components of the Foundation, Transformation, Connection, and Contingency dimension.

This study found evidence of the development of the pre-service teachers’ PCK in different data. The written test responses provided some insights about the pre-service teachers’ PCK before the implementation of lesson study. The video of the lesson study meetings and lesson study artefacts brought about the findings on the development of the pre-service teachers’ PCK during lesson study. The interviews provided additional information about the pre-service teachers’ PCK development.

In line with earlier research on PCK (Hill, Ball, et al., 2008; Lee, 2010; Van Driel & Berry, 2012; Van Driel, Verloop, & de Vos, 1998), this study followed the pre-service teachers’ PCK development over one specific mathematical topic. Even though each group taught different topics over two lesson study cycles, it was difficult to make sense the development of the pre-service teachers’ PCK over different mathematical topics. Therefore, the pre-service teachers’ PCK development was observed over a particular mathematical topic in a particular lesson study cycle.
7.2.1 Foundation

This study found that the pre-service teachers demonstrated some development of their Foundation Knowledge, especially in two components – Overt Subject Knowledge and the Theoretical Underpinning of Pedagogy. The first component, Overt Subject Knowledge, is concerned with the pre-service teachers’ mathematical knowledge. This study found that because mathematics consists of different topics that are connected, sometimes the focus in the research lesson may deviate from the planning. The mathematical topics focused on by the lesson study groups were often driven by students’ learning. The findings of this study indicated that the students’ learning in the research lesson directed the focus of the following post-lesson discussion. Hence, this focus influenced the development of the pre-service teachers’ Overt Subject Knowledge. This study found that even in one lesson, the focus of the lesson study may vary. In Cycle 2 of Group D8, the pre-service teachers discussed verifying Pythagorean Theorem in the planning, but the research lesson and the post-lesson discussion revealed the pre-service teachers’ lack of knowledge about square roots. While the mathematical focus in one cycle may vary, the development of the pre-service teachers’ Overt Subject Knowledge in this study was observed for one particular topic – strategies to find square roots.

Overall, this study found that the development of the pre-service teachers’ mathematical knowledge was driven by observing students’ learning in the research lessons, and discussing students’ difficulties in the post-lesson discussions. Through these activities, the pre-service teachers were encouraged to revisit and rethink the mathematics involved and their own mathematical knowledge, which enabled them to learn new knowledge or deepen their existing mathematical knowledge (Leavy, 2015). This study concurs with previous research that claimed lesson study contributes to teachers’ and pre-service teachers’ learning of mathematical knowledge (Leavy, 2015; Lewis et al., 2012).

The second component was the Theoretical Underpinning of Pedagogy. It is concerned with pre-service teachers’ beliefs about teaching mathematics. Some studies showed that teachers’ beliefs are associated with their practice. For example, Stipek et al. (2001, p. 223) found that “teachers who held the more traditional beliefs also gave students relatively less autonomy and maintain a social context in which mistakes were something to be avoided”. Moreover, Blömeke, Buchholtz, Suhl, and Kaiser (2014) investigated the relationship between teachers’ knowledge and their beliefs, and found that teachers’ knowledge informs the
teachers’ beliefs about mathematics teaching and learning. To identify the shift of pre-service teachers’ beliefs, this study assumed that the pre-service teachers’ beliefs were reflected in and consistent with their practice.

Rowland et al. (2009) argue that pre-service teachers’ beliefs about mathematics, why and how mathematics is learned affects their classroom practice. This study interpreted Rowland’s (2009) conception of beliefs as to how pre-service teachers think about teaching mathematics. This study examined the pre-service teachers’ responses to the written test, activities during lesson study phases and interviews to seek evidence of some changes in their beliefs. Since the pre-service teachers’ beliefs were captured in their practice, many of the evidence was also coded under other KQ components.

The findings from the written test showed that the pre-service teachers had beliefs about traditional teaching prior to participating in the lesson study. Nevertheless, the written test did not explicitly ask about the pre-service teachers’ beliefs, instead, it asked for the pre-service teachers’ responses to students’ common mistakes in algebra. Their responses revealed their beliefs about teaching mathematics. Thus, it was coded as the Theoretical Underpinning of Pedagogy, more specifically on pre-service teachers’ beliefs.

During the lesson study, the pre-service teachers demonstrated a shift in their beliefs towards a more student-centred teaching. In the planning session, the pre-service teachers incorporated anticipation of students’ responses in the lesson plans. In the research lessons, the pre-service teachers used better-designed teaching materials and a more student-centred teaching approach. This was particularly evident when the pre-service teachers used problems to stimulate students’ reasoning. In the post-lesson discussions, the pre-service teachers discussed evidence of students’ learning and used that to improve the next lesson. Consistent with earlier research (Inprasitha & Changsri, 2014; Yakar & Turgut, 2017), these findings suggested that lesson study allowed teachers or pre-service teachers to shift their beliefs toward a more student-centred teaching and learning.

However, this study found that these changes were not consistent. Especially when the pre-service teachers were challenged by contingent moments, they were more likely to teach traditionally by using a procedural approach. This suggests that the pre-service teachers would more likely back to their traditional beliefs when they encountered unanticipated situation in the lessons. Given that Indonesian teachers typically employ a teacher-centred approach with heavy reliance on textbook (Kusanagi, 2014) and use directive teaching by telling the students.
what the problem is and immediately correcting students’ mistakes (Wijaya, van den Heuvel-Panhuizen, & Doorman, 2015), this finding suggested that the pre-service teachers’ beliefs were influenced by their experience as students. This study confirms earlier studies which showed that pre-service teachers’ beliefs are influenced by their experience as students (Grootenboer, 2008; Raymond, 1997). In line with Grootenboer (2008), this finding suggested that change in pre-service teachers’ beliefs is a complex process. Even though this change was not prominent over the two cycles, lesson study has enabled the pre-service teachers to change their practice. These findings resonate with Benbow (1995) who found that pre-service teachers did not change their mathematical beliefs after an intervention program, however, their decision making of the instructional program was changed.

7.2.2 Transformation

The pre-service teachers demonstrated some improvement in the Transformation dimension especially in the Choice of Representation component. Findings from the written test suggested that prior to participating in lesson study, the pre-service teachers’ Choice of Representation was underpinned by their existing knowledge of types of mathematical representation and their connection to the mathematical topics. This study is in line with Ball (1990) who found that the pre-service teachers’ prior knowledge informed their decision making for selecting mathematical representation for fractions.

Furthermore, data from the lesson study implementation showed that the pre-service teachers considered students’ prior knowledge and the practical aspects of the representation when selecting or designing mathematical representation. They tended to avoid selecting representation that requires more work. Their Transformation Knowledge improvements were found in the planning and post-lesson discussions; in the planning where they rethink the mathematics and lesson goals, and in the post-lesson discussions where they discussed students’ responses to their Choice of Representation and whether the representation helped the students to achieve the lesson goals. These findings confirm earlier studies on lesson study to support teachers’ understanding and use of mathematical representation (Murata, Bofferding, Pothen, Taylor, & Wischnia, 2012).

7.2.3 Connection

This study showed that lesson study supports the development of pre-service teachers’ Connection Knowledge especially in the Anticipation of Complexity components. Anticipation
of Complexity in this study refers to the pre-service teachers’ anticipation of students’ responses, which includes prediction of students’ responses and developing prompts to support students. This finding is similar to Shuilleabhain (2016, p. 222) who showed that by “participating in lesson study, teachers develop their ability to anticipate and interpret mathematical content through the eyes of the student”.

In this study, anticipating students’ responses was most prominent in the planning. The findings showed that initially the pre-service teachers only considered students’ prior mathematical learning and behaviour for their prediction of students’ responses. Their predictions of students’ responses were vague and did not include students’ reasoning or misconceptions. The pre-service teachers’ anticipation of students’ responses in this study lacked depth and detail. Meyer and Wilkerson (2011) found that teachers needed to spend a significant amount of time to produce detailed anticipation of students’ responses which includes discussing possible areas which are difficult for students and predicting the students’ questions and responses that may arise from the task. In this study, the lack of depth and detail in the pre-service teachers’ anticipations of students’ responses might have been attributed to the limited time spent on discussing such issues in the planning.

It should be noted that anticipating students’ responses was a new practice for the pre-service teachers. The novelty of anticipating students’ responses created an extra challenge for the pre-service teachers to embrace this fully in their first-hand experience of teaching mathematics in secondary schools. Given that the pre-service teachers had little prior knowledge and experience in anticipating students’ responses, it was difficult for pre-service teachers to develop this new skill without support. Earlier studies suggested that by involving the teachers in solving the mathematics problems prompted them to think about how their students would respond to similar problems (Moss, Hawes, Naqvi, & Caswell, 2015; Widjaja et al., 2017). Having the pre-service teachers solve the mathematical problems themselves in the planning phase would help them develop their readiness to anticipate students’ responses. The knowledgeable other and the mentor teachers in this study provided the support needed for the pre-service teachers. This study found that the mentor teachers’ comments triggered the pre-service teachers to incorporate prompts with the predictions. The prompts from the knowledgeable other stimulated the pre-service teachers to provide more detailed anticipations. This suggests that lesson study, especially support from the mentor teachers and the knowledgeable other, helped the pre-service teachers develop the Connection Knowledge on anticipating students’ solutions.
7.2.4 Contingency

Rowland et al. (2015) claimed that contingency is triggered by unanticipated students’ responses, the teachers themselves and availability or unavailability of tools or resources. Consistent with Rowland et al. (2015), the contingent moments in this study were typically triggered by unanticipated students’ responses. Moreover, the findings showed that when the pre-service teachers were faced with unanticipated students’ difficulties, they would be more likely to use a procedural approach or ignore it instead of exploring the students’ thinking.

Hallman-Thrasher (2017) investigated pre-service teachers’ responses to unanticipated students’ incorrect solutions to mathematics problem-solving tasks. The participants of her study were pre-service teachers with strong mathematical and communication skills. She found that when pre-service teachers had unanticipated students’ incorrect solutions, they elicited students’ strategies by using questioning. On the contrary, this study found that instead of asking questions to elicit students’ strategies, the pre-service teachers directly guided the students to the correct solutions by using a procedural approach. The reason for this is probably because the pre-service teachers in this study were not trained to elicit students’ thinking. They tended to ignore students’ misconception and went directly to show them the correct procedure. The pre-service teachers in this study guided their students to use a procedural approach. The reason for this is probably because they referred to their own learning experience – how they were taught (Ball, 1990; Grootenboer, 2008; Raymond, 1997).

There are some possibilities as to why teachers decided not to respond to students’ difficulties or strategies. The first possibility is that teachers do not notice students’ difficulties and strategies, and hence do not attend to these. The second possibility is even though teachers attend to students’ difficulties, they may decide to respond or not (Jacobs, Lamb, Philipp, & Schappelle, 2011). When investigating teachers’ decision making during their teaching, it is difficult to capture the reason behind the teachers’ decisions. This study showed that lesson study offers ways to understand teachers’ decisions in contingent moments. The findings showed that in the post-lesson discussions, the pre-service teachers were encouraged to reflect and clarify their decisions. Asking the pre-service teachers to clarify their teaching provides opportunities for the researcher to unpack their PCK. This study found that the pre-service teachers’ inability to respond appropriately in contingent moments was typically caused by their lack of the Foundation Knowledge.
Earlier studies found that pre-service teachers learn best about contingency during their teaching practicum (Smith & Lev-Ari, 2005). However, dealing with contingency is difficult, as this study showed that the pre-service teachers tended to rely on procedural teaching when they dealt with contingent moments. Moreover, this study also showed that lesson study provides structure for pre-service teachers, mentor teachers, university lecturers and the knowledgeable other to discuss these contingent moments. This discussion enabled the pre-service teachers to reflect on and rethink these moments to find ways to react better to such moments in future lessons.

7.3 The Characteristics of Lesson Study That Contribute to the Development of the Pre-service Teachers’ PCK

This section answers Research Question 2 – What characteristics of lesson study contribute to the development of pre-service teachers’ PCK?

7.3.1 Lesson Study Phases

The goal of lesson study is to improve instruction (Fernandez & Yoshida, 2004; Lewis et al., 2009; Watanabe et al., 2008). Some researchers argue that this was made possible through teachers’ development of knowledge, interpersonal relations, and personal disposition (Lewis et al., 2009; Stigler & Hiebert, 2016). This study focused on developing teachers’ knowledge using lesson study.

This study modified Lewis’s et al. (2009) lesson study phases. With only cycles of planning, research lessons, and post-lesson discussions, this lesson study model is similar to what is widely understood in Indonesia (Kusanagi, 2014; Suratno & Cock, 2009). This study did not start with a question that transforms into the theme of lesson study as commonly practiced in Japanese Lesson Study (Fujii, 2016). Furthermore, Lewis (2002) associated this theme with the long-term goal of Japanese Lesson Study. However, the focus on a long-term goal is not a common practice in other countries, for example in the USA (Lewis, 2002), and in Indonesia (Kusanagi, 2014).

The planning in this study is different from the traditional Japanese Lesson Study which can take months and results in a detailed lesson proposal (Fujii, 2016; Fernadez & Yoshida, 2004). In this study, the planning took about 60 minutes and was only carried out once every cycle. The structure of the discussion in the planning is different from Japanese Lesson Study
where the teachers are engaged in a detailed discussion of the tasks such as determining the numbers (Fujii, 2016). In this study, the focus of the planning was to design the lesson plan. However, because of the time constraints, the lesson study groups were unable to produce a detailed lesson plan as in Japanese Lesson Study.

Even though the planning was short, the pre-service teachers were able to demonstrate some elements of kyozaikenkyu such as anticipating students’ responses. This was proven as a rich learning opportunity for the pre-service teachers. It required the pre-service teachers to justify their pedagogical decision (Leavy & Hourigan, 2016). Anticipating students’ responses stimulates the pre-service teachers’ enactment of different KQ components. For example, when discussing how students would respond to a task, the pre-service teachers unpacked the mathematical concepts and the students’ previous learning.

This study showed that the pre-service teachers enacted some KQ components during the research lessons. As discussed in the previous section, the research lessons revealed the pre-service teachers’ enactment of the Foundation, Transformation, Connection, and Contingency dimensions. This study also found that observing the research lessons offered learning opportunities for the observers as they developed awareness of students’ thinking (Murata et al., 2012). Detailed discussion on the observation will be presented in Section 7.3.2.

In line with Murata and Pothen (2011), this study found that the post-lesson discussions encouraged the pre-service teachers to critically discuss students’ learning. Moreover, in this study, the post-lesson discussions served as a vehicle to discuss the lead pre-service teachers’ reasons and justifications of the decisions they made in the teaching as well as how these decisions affected the students’ learning. Open and constructive discussion about the pre-service teachers’ teaching provides a supportive learning environment for the pre-service teachers (Murata et al., 2012; Warwick et al., 2016). In this study, the pre-service teachers highly appreciated the comments and feedback they received during the post-lesson discussion. This positive and open attitude allowed them to learn and improve their teaching.

Overall, the lesson study planning, research lessons, and post-lesson discussion facilitated the pre-service teachers engagement in a rich discussion focusing on students’ thinking (Fernandez, 2002; Lewis, Perry, Hurd, et al., 2006; Marble, 2006). In this study the pre-service teachers were engaged in focusing students’ thinking through anticipating students’ responses in the planning, collecting data about students’ work in the research lessons, and discussing evidence of students’ learning in the post-lesson discussions. Within this context,
the pre-service teachers enacted their PCK. This study has captured the development of their PCK throughout planning, research lessons, and post-lesson discussion. It was found that the interaction of the KQ components during lesson study phases supported the development of pre-service teachers’ PCK. More importantly, the development of the pre-service teachers’ PCK was embedded in the context of teaching so that the pre-service teachers could immediately apply their new knowledge in their teaching (Chokshi & Fernandez, 2004).

7.3.2 Observing Lessons

Observing lessons is one of the unique characteristics of lesson study. Studies recommended the need to observe students’ learning carefully (Lewis, 2002; Murata et al., 2012). This includes collecting data about students’ written work or even sometimes videotaped evidence of students’ engagement and the quality of students’ group discussions (Lewis, 2002). This study encouraged the pre-service teachers, especially the observers to focus on students’ learning during research lessons. An observation sheet was developed to help the observers record the students’ learning. Moreover, the observation sheet provides evidence of the observers focused on during the research lessons.

The findings showed that the pre-service teachers’ observation could be categorised into three groups – classroom management, pedagogical issues, and students’ mathematical thinking. Analysis of the pre-service teachers’ observation over time suggested that at the beginning of the lesson study implementation, the pre-service teachers’ observation was mainly about classroom management. This is not surprising as earlier studies on lesson study for pre-service teachers reported the same findings (Amador & Weiland, 2015; Meiliasari, 2013; Post & Varoz, 2008). This lack of attention on students’ thinking at the initial stage of lesson study was also found in a study of in-service teachers (Tepylo & Moss, 2011).

The reason the pre-service teachers in this study did not focus on students’ learning in their observation especially at the beginning stage of lesson study is probably that the pre-service teachers have little or no teaching experience (Burroughs & Luebeck, 2010; Murata & Pothen, 2011). With little or no teaching experience, it was reasonable that the pre-service teachers struggled with classroom management issues, hence it attracted their attention more than the students’ learning. Another reason could be due to the fact that the pre-service teachers did not understand the aim of observing lessons in lesson study. The participants’ lack of understanding of the lesson study process might risk the outcomes of lesson study (Fernandez,
Cannon, & Chokshi, 2003; Meyer & Wilkerson, 2011; Murata, 2011). The pre-service teachers in this study did not fully understand the focus of the observation in the research lessons. The structure of the observation sheet did not help the pre-service teachers to understand that they were expected to record students’ learning. Therefore, this implies a need of having a more structured observation tool to direct the observers’ attention more explicitly to focus on students’ learning (Sims & Walsh, 2009). ‘Professional noticing’ may be a promising means for a framework of observation in lesson study as recommended by some studies (Anthony, Hunter, & Hunter, 2015; Amador & Carter, 2018; Larssen et al., 2018).

As the lesson study continued, the majority of the pre-service teachers shifted their focus to students’ learning. This shift was driven by the knowledgeable other who continued emphasising the need to provide evidence of students’ learning in the post-lesson discussion (Amador & Carter, 2018). These findings align with previous studies that found that lesson study supports teachers’ focus on students’ learning (Alston, Pedrick, Morris, & Basu, 2011; Fernandez, 2005; Shuilleabhain, 2016). However, the findings also showed that when the pre-service teachers acted as the lead teacher, their attention was mostly on classroom management issues such as completing all activities in the lesson plan, juggling time, and orchestrating the lesson.

As discussed in Chapter 5, the data from this study showed that even though the pre-service teachers focused on students’ mathematical thinking, they could not connect the students’ thinking to the pedagogical aspects of the lesson. Amador and Carter (2018) investigated pre-service teachers’ noticing during lesson study and found that pre-service teachers’ noticing was driven by prompts from the facilitator. They argue that attending to students’ thinking is a component of teacher noticing alongside interpreting students’ thinking and responding to students’ thinking. Similarly, Anthony et al. (2015) employed teaching rehearsal activities to support pre-service teachers’ noticing of students’ mathematical thinking and found that the teacher educators played a vital role in encouraging the pre-service teachers to link students’ mathematical thinking with the lesson goals. Even though this study did not focus on the pre-service teachers’ professional noticing, the findings were consistent with Amador and Carter (2018) and Anthony et al. (2015). In this case, the knowledgeable other, as the facilitator of lesson study directed the post-lesson discussions to focus on students’ mathematical thinking and prompted the pre-service teachers to reflect on their choice of pedagogical strategies that correspond to students’ mathematical thinking.
Earlier studies found that focusing on students’ thinking through professional noticing supports in-service and pre-service teachers’ PCK (Jacobs, Lamb, & Philipp, 2010; Philipp et al., 2007). This study did not use professional noticing for the pre-service teachers’ observation. Nevertheless, it was found that through discussing their observation of students’ thinking in the post-lesson discussions, the pre-service teachers were able to improve their PCK. These findings were consistent with earlier studies about the role of observers in lesson study. For example, Lewis (2009) found that observers develop knowledge about how to have a productive and grounded discussion about teaching. Alston et al. (2011) found that in post-lesson discussions, teachers increased their awareness of students’ mathematical thinking and how the teachers’ actions in the teaching affect the students’ mathematical thinking.

Larssen et al. (2018) pointed out a gap in the literature, especially on the lack of details in the literature about how the observation is conducted in lesson study. This study has lessened that gap by providing detailed descriptions of the observation process, evidence of the pre-service teachers’ observations, and most importantly, the findings on what the pre-service teachers’ observed and how their observations changed during lessons study.

7.3.3 The Support from the Knowledgeable Other, Mentor Teachers and University Lecturers

In lesson study for pre-service teachers, the mentor teacher and the university lecturers play an important role (Chichibu, 2016). The mentor teacher or the university lecturer could act as the knowledgeable other in the lesson study (Amador & Weiland, 2015; Bjuland & Helgevold, 2018). In this study, the researcher acted as the knowledgeable other, while the mentor teachers and university lecturers participated as members of the lesson study groups. This section discusses the contribution of the mentor teachers, university lecturers, and the knowledgeable other in supporting the development of the pre-service teachers’ PCK.

Aydin et al. (2015) found that mentoring played a vital role in pre-service teachers’ PCK development. This study found that the mentor teachers contributed to the development of the pre-service teachers’ PCK by providing advice during lesson study meetings. Their advice included the selection of the topic for lesson study. In this study, the pre-service teachers followed the school curriculum closely. Most of the time the mentor teachers acted as the authorised persons who determine the order of topics in each semester. Therefore, the mentor teachers played an important role in deciding the topic for lesson study, this reflected their expertise in the school curriculum.
The mentor teachers also provided advice on students’ prior knowledge and possible difficulties for the consideration when selecting mathematical representations and teaching materials. This study found that the pre-service teachers’ selection of teaching material did not incorporate possible students’ learning. The mentor teachers provided help in this area. With their teaching experience and knowledge of students’ characteristics, the mentor teachers helped selecting appropriate teaching materials by providing information of students’ prior knowledge and possible students’ difficulties (Chichibu, 2016).

While the mentor teachers provided substantial support in the lesson study, this study found the university lecturers’ support was very limited. The university lecturers only attended a few lesson study meetings because it was difficult to find time in their busy schedules. However, the data also showed that when they attended the lesson study meetings, they did not always contribute actively. One of the university lecturers in this study seemed to misunderstand the role of the knowledgeable other, thus she only participated passively in the lesson study.

Researchers have pointed out the important role of the knowledgeable other in maintaining the effectiveness of lesson study (Corcoran, 2011; Lewis, Perry, Hurd, et al., 2006; Takahashi, 2014). Takahashi (2014) investigated the knowledgeable other’s final comments in Japanese Lesson Study and found that the knowledgeable other helps teachers in making connections between theory and practice, developing teacher knowledge about teaching and encouraging reflection on their teaching. This study was situated in the context of lesson study for pre-service teachers during the teaching practicum. Hence, the knowledgeable other’s contributions were different from the school-based lesson study reported by Takahashi (2014). In this study, not only did the knowledgeable other provide final comments but also contributed throughout the planning, research lessons and post-lesson discussions. However, the knowledgeable other in this study did not focus on encouraging the pre-service teachers to connect theory and practice. Even though this study encouraged the pre-service teachers to reflect on their teaching, it was not the main focus of the knowledgeable other.

This study found that the knowledgeable other supported the development of the pre-service teachers’ PCK by provoking contingency and maintaining students’ thinking as the focus of discussions. Hurst (2017) suggested the need to expand contingency from only as acting in the moment to also include provoking contingency. He argues provoking contingency involves designing challenging tasks that invoke students’ reasoning and reveal their
misconceptions. He argues that by provoking contingency, teachers will be able to better plan their lesson. Similar to Hurst (2017), the knowledgeable other provoked contingency by prompting the pre-service teachers to give students a challenging task. However, in this study, the provocation by the knowledgeable other revealed not only students’ difficulties but also the pre-service teachers’ lack of the Foundation Knowledge.

This study also found that another critical role of the knowledgeable other was to maintain the focus of the lesson study discussions on students’ thinking. Given that the pre-service teachers in this study had no teaching experience prior to the teaching practicum, it was natural that their attention was drawn more into classroom management. The knowledgeable other guided them to focus more on students’ thinking through questioning, giving examples, and prompts. This study supports earlier studies on the role of the knowledgeable other in encouraging conversation and observation on students’ thinking in lesson study (Amador & Carter, 2018; Sims & Walsh, 2009).

7.4 Interconnections of the Knowledge Quartet Components

The previous two sections have discussed the answers to Research Question 1 and 2. While analysing the data to answer these two research questions, an important finding was the interconnection of the KQ components. This section discusses it in detail.

As discussed earlier, the KQ Rubric captured the quality of the PCK demonstrated by the pre-service teachers. Tracing the pre-service teachers’ development throughout the planning, research lessons, and post-lesson discussions gave insights into understanding how the pre-service teachers’ PCK developed. This study found that one of the driving forces of the pre-service teachers’ PCK development is the interconnection of KQ components. It refers to how the absence or presence of one KQ component may influence the development of other components.

In general, this study confirms Rowland et al. (2009) in that the Foundation dimension is manifested in the other three dimensions. The findings showed that the Foundation dimension, more specifically the Overt Subject Knowledge component, underpinned the pre-service teachers’ decision making in the planning. Their decision making related to the selection of mathematical representations used in the lessons, developing tasks and anticipating students’ responses which are under the other three dimensions. Moreover, in the research lessons, the pre-service teachers’ Foundation Knowledge also underpinned their decision
making in the moment of teaching. Given that the content knowledge underpins the planning and the teaching, this study agrees that to some extent content knowledge influences the teachers’ PCK (Hill, Ball, et al., 2008; Hill et al., 2005; Krauss et al., 2008).

Moreover, Rowland’s et al. (2005, 2009) and Weston’s (2013) showed that the Contingency dimension is informed by the Transformation and Connection dimensions (see Figure 2.2). This study extended the interconnections by adding three more connections: (1) the Connection Knowledge contributes to the Transformation Knowledge; (2) the Connection Knowledge contributes to the Foundation Knowledge; and (3) the Contingency Knowledge contributes to the Foundation Knowledge. Figure 7.1 is underpinned by Rowland et al. (2005, 2009) and Weston’s (2013) model, shown by the bold arrows indicating the connections of the KQ dimensions. The new connections found in this study are shown using dashed arrows.

The first interconnection found in this study is the contribution of the Connection Knowledge to the Transformation Knowledge. This took place in the planning, particularly when the pre-service teachers selected the mathematical representation under the Transformation dimension. When the pre-service teachers selected the mathematical representation, they considered some aspects under the Connection dimension such as possible students’ responses, the sequence of the activities, and the connections between the mathematical concepts.

Leavy (2015) showed, in lesson study context, that the development of one PCK subdomain can contribute to learning in other subdomains. Using the MKT framework, Leavy (2015) showed the connection between knowledge of content and students (KCS), and
knowledge of content and teaching (KCT). Similarly, this study found that the pre-service teachers’ knowledge of students’ prior knowledge informs their Choice of Representation. This connection was observed when the pre-service teachers’ Choice of Representations for the lesson was greatly influenced by the students’ mathematical ability. This study found that besides students’ mathematical ability, the pre-service teachers’ Choice of Representation was also influenced by several other factors. At first, they only considered the realistic context of the representation. This is suggestive of the pre-service teachers’ pre-knowledge of using mathematical representation and models. However, the prompts from the knowledgeable others encouraged the pre-service teachers to look at the lesson goals, the possible students’ responses, and the students’ prior knowledge. Thus, this study argues that, while the decision of the mathematical representation was initially underpinned by the pre-service teachers’ subject knowledge, lesson study enables the discussion for the pre-service teachers to extend their consideration by including lesson goals, students’ prior knowledge and anticipation of students’ responses.

Next, this study found that the research lessons revealed the second interconnection – between the Connection dimension and the Foundation dimension. It highlights the contribution of the Connection dimension especially the Anticipation of Complexity component to the Foundation dimension, more specifically the concentration on procedure component. This study interpreted Anticipation of Complexity as anticipation of students’ responses. Furthermore, this study found that when the pre-service teachers have anticipated students’ responses and are equipped with prompts, they could help the students in a more student-centred way. On the contrary, when the pre-service teachers only anticipated students’ difficulties without the prompts, they were more likely to demonstrate traditional teaching. These findings align with previous research that claimed anticipating students’ responses contributes to the development of teachers’ knowledge for teaching mathematics (Meyer & Wilkerson, 2011; Vale, Widjaja, Doig, & Groves, 2018).

Hurst (2017) argues that teachers could benefit from planning and provoking contingent moments rather than only responding to them. He explained that provoking contingency includes, for example, developing challenging tasks that could potentially reveal students’ misconception. He argues that by anticipating this, teachers could perform “powerful teaching” (Hurst, 2017). In this study, the knowledgeable other provoked contingent moments by asking the pre-service teachers to give students a challenging task. In line with Hurst (2017), the
challenging tasks revealed the students’ misconceptions, and more importantly, it revealed the pre-service teachers’ lack of the Foundation Knowledge, creating the third interconnection.

The post-lesson discussions revealed the third interconnection – between the Contingency dimension and the Foundation dimension. In this study, the post-lesson discussions have enabled the pre-service teachers to discuss their contingent moments, especially on Responding to Students’ Ideas. This discussion unpacked their reasoning behind their responses to students’ ideas. This study found that the inability of the pre-service teachers to respond properly in the contingent moments was caused by their lack of the Foundation Knowledge, and more specifically the Overt Subject Knowledge component. This finding suggests that discussing contingency, especially Responding to Students’ Ideas can support pre-service teachers’ Foundation Knowledge particularly the Overt Subject Knowledge component.

Clarke and Hollingsworth (2002) argue that the development of teacher knowledge is underpinned by teachers’ prior knowledge from different domains, and built upon experiences, and interaction with students and colleagues. In line with Clarke and Hollingsworth (2002), this study has shown that the pre-service teachers enacted different KQ components while planning, teaching, and discussing lessons. Moreover, it showed that the KQ components interact in a complex way. It is important to understand the interaction of PCK components and how the interaction influences teaching (Nilsson, 2008). This study has shown the interconnection of the KQ components, that one KQ component may contribute to the development of other KQ components. Nevertheless, it is important to note that the interconnection of the KQ components found in this study are specific to one particular component under a KQ dimension to another particular component under another KQ dimension.

7.5 The Participants’ Views about Lesson Study in the Teaching Practicum

This section answers Research Question 3 – What are pre-service teachers’, mentor teachers’, and university lecturers’ views about the incorporation of lesson study into pre-service teachers’ mathematics teaching practicum? This study narrows down the participants’ views into two themes. The first theme is about what the pre-service teachers think are the benefits and challenges of anticipating students’ responses. The second theme is what the participants, (that is the pre-service teachers, mentor teachers and university lecturers) think are the benefits and difficulties from participating in lesson study.
The findings showed that the pre-service teachers became more prepared and more confident when they have anticipated students’ responses. These findings confirm earlier studies that found that the pre-service teachers grew self-confidence from participating in lesson study (Cajkler & Wood, 2016a; Corcoran, 2011; Lamb, 2015). However, the pre-service teachers in this study did not relate the process of anticipating students’ responses with their learning of the content knowledge.

The pre-service teachers in this study found that anticipating students’ responses required detailed predictions of students’ solutions. Since anticipating students’ responses was a new practice that the pre-service teachers’ experienced during lesson study, for most of the pre-service teachers, this was difficult. Especially when it involved anticipating students’ responses to hands-on activities. These findings confirm an earlier study about the pre-service teachers’ lack of teaching experience which hindered them in addressing students’ common misconceptions and in predicting students’ responses (Burroughs & Luebeck, 2010).

Even though the pre-service teachers admitted that they gained some benefits from anticipating students’ responses, they did not carry this out in their regular teaching. This study found some reasons for this. First, the pre-service teacher reported that anticipating students’ responses requires detailed work and substantial extra time. This finding aligns with Meyer and Wilkerson (2011) who reported that their participating teachers spent a significant amount of time to anticipate students’ responses. Second, anticipating students’ responses is not the schools’ regular practice. When the new practice is not in the school system, an individual-driven innovation is very unlikely to occur especially when power interaction and cultural tensions are involved (Corcoran, 2011). In this study, the pre-service teachers did not have any power or authority to initiate a change. They were more likely to follow the existing practice and get a good result rather than changing the practice and risk their result. The last reason was that the pre-service teachers prioritised classroom management issues as they were still struggling with these.

The second theme is the participants’ opinions of the benefits and challenges of participating in lesson study. The participants in this study acquired new knowledge about the mathematics, teaching mathematics, and students’ learning. These findings support previous studies that found that lesson study supports pre-service teachers’ improving their knowledge about mathematics and teaching mathematics (Leavy, 2015; Murata & Pothen, 2011).
This study shows that even though the lesson study was not specifically designed for the mentor teachers, they still learned about teaching mathematics and students’ learning by participating in lesson study. This finding confirms earlier research by Chassels and Melville (2009) that found that both pre-service teachers and mentor teachers gained some insights into students’ learnings and curriculum. Another finding of this study is that the benefits of lesson study from the participants’ point of views are better lesson planning and reflection of their own teaching. These findings support previous studies that found that the pre-service teachers gained benefits from planning and observing lessons that stimulate their reflection (Chassels & Melville, 2009; Lamb, 2015).

Most participants in this study claimed that time was a major challenge in lesson study. Time relating to lesson study organisation has been reported in earlier studies (Alston et al., 2011; Chassels & Melville, 2009; Post & Varoz, 2008; Susanta, 2012). In this study, the schedule of the post-lesson discussion created a challenge for the mentor teachers to keep up with their own teaching responsibility. Because the post-lesson discussion was held as soon as possible after the research lesson, sometimes the schedule clashed with the mentor teachers’ teaching schedule. In such cases, the mentor teachers only attended half of the post-lesson discussion.

### 7.6 Affordances and Constraints of the Implementation of Lesson Study in the Teaching Practicum

Research Question 4 asked – What are some of the affordances and constraints relating to the implementation of lesson study in pre-service teachers’ mathematics teaching practicum? This section discusses the answers to this question as derived from the themes that emerged in the findings. This study found three factors that could be affordances or constraints: (1) the dual role of the knowledgeable other, (2) the dynamics of the groups, and (3) the lesson study organisation.

The first factor is the dual role of the knowledgeable other. In this study, it served as an affordance and a constraint. It was an affordance because it allowed the knowledgeable other to participate fully in all lesson study meetings of all lesson study groups. The knowledgeable other’s role is typically observing the research lessons and providing comments and feedback in the post-lesson discussion (Fujii, 2016; Takahashi, 2014). Sometimes the knowledgeable other attended the planning, however, this is not a common practice (Fujii, 2016). In this study,
being the researcher allowed the knowledgeable other to attend all the planning, research lessons, and post-lesson discussions of all lesson study groups. Moreover, the knowledgeable other actively participated in the lesson study meetings by facilitating the lesson study meetings and providing prompts and advice when needed. The knowledgeable other also shared the lesson study experience of one school to the other. This has contributed to the outcomes of the lesson study (Amador & Carter, 2018), in this case, the development of the pre-service teachers’ PCK.

However, the dual role of the knowledgeable other also served as a constraint. Maintaining the balance of actively supporting the pre-service teachers and allowing space for the participants to participate actively was challenging. The knowledgeable other in this study seemed to dominate the discussions in the lesson study meetings, which may weaken the autonomy of the pre-service teachers (Amador & Carter, 2018). This might have created tensions of power in the relationships and interaction between the knowledgeable other and the participants (Corcoran, 2011).

One possible cause of the knowledgeable other’s dilemma in this study may lie in the Indonesian hierarchical society. Ebaeguin and Stephens (2014) argue that the success of lesson study adaptation may be challenged by a highly hierarchical society where subordinates are reluctant to initiate actions. As shown by Kusanagi (2014), Indonesian teachers’ teamwork in lesson study was promoted by a system of hierarchy and seniority. The other constraint from the dual role of the knowledgeable other is that the researcher’s role most of the time overshadowed the knowledgeable other. Maintaining focus on the lesson study was difficult when the research demands were more urgent, such as making sure the invitation of the lesson study meetings were sent, and video and audio recorders were ready.

The next factor is the dynamics of the groups. Earlier studies found that collaborative practice is highly appreciated by the pre-service teachers participating in lesson study (Murata & Pothen, 2011). Moreover, Murata and Pothen (2011) argue that lesson study engages the teachers to collaborate meaningfully involving all members of the group in active participation. This study found that when the pre-service teachers were involved in such meaningful collaboration in their group, they showed some development in their PCK. Moreover, this study also showed a case when meaningful group collaboration did not occur. The pre-service teachers who did not collaborate well in their group showed less PCK development. This suggests that the collaboration in the lesson study groups contributes to the pre-service
teachers’ PCK development. Earlier studies argue that lesson study offers teachers opportunities to engage in collaboration in a professional community (McMahon & Hines, 2008; Warwick et al., 2016). This study found that establishing good team collaboration could be challenging, especially when each individual in the group has their own personal priorities.

The last factor is the organisation of lesson study. This study found that the organisation of the lesson study affected the process and outcomes of the lesson study. Rather than focusing on designing lessons that supported students’ learning (Fujii, 2016), the planning meetings were sometimes spent on discussing the logistics of the research lessons and post-lesson discussion, such as the schedule of the research lessons. Earlier studies reported similar findings that when the content of discussion shifted, the focus of the discussion moved away to different topics, sometimes irrelevant topics (Amador & Carter, 2018; Shuilleabhain, 2016). This study is aware of the lack of structure and focus on the planning, which resulted in difficulties in maintaining the focus of the discussion, especially in the planning. This study did not have a planning protocol and the participants did not have a solid understanding of the planning. This differs from Japanese teachers’ planning, where the teachers discuss in detail the selection of the task including the mathematical aspect and the appropriateness to the lesson goals and students’ prior knowledge (Fujii, 2016).

7.7 Conclusion

The previous sections have discussed the findings of this study compared with previous studies. This section provides the summary of the findings, the significance and limitations of the study, and the implications for teacher education and future research.

7.7.1 Summary of the Findings

The findings showed that lesson study supports the development of the pre-service teachers’ PCK. The pre-service teachers demonstrated their PCK development in four KQ dimensions. There are two components under the Foundation dimension that were improved during the implementation of lesson study. The first component was Overt Subject Knowledge, which refers to the pre-service teachers’ mathematical knowledge. The findings showed that the pre-service teachers’ Overt Subject Knowledge development was topic-specific and driven by discussing contingent moments about responding to students’ difficulties in the post-lesson discussion.
The second component was the Theoretical Underpinning of Pedagogy, which refers to the pre-service teachers’ beliefs about teaching mathematics. This study assumed that the pre-service teachers’ beliefs were reflected in their practice. The findings suggested that prior to the teaching practicum, the pre-service teachers held traditional beliefs about teaching. During lesson study, the pre-service teachers showed some shifts in their beliefs towards more student-centred teaching. However, the pre-service teachers would go back to traditional teaching when they encountered contingent moments in the teaching.

In the Transformation dimension, the pre-service teachers demonstrated some improvement in their Choice of Representation component. This study found that lesson study supports pre-service teachers’ decision making in the Choice of Representation. In the planning, pre-service teachers’ Choice of Representation were enacted when they considered the mathematical concepts, lesson goals, and students’ prior knowledge in their selection of the mathematical representations. The pre-service teachers used students’ actual work on the representation and discussed whether the representation helped the students learning. This resulted in the refinement of the mathematical representation.

The development of the pre-service teachers’ Connection Knowledge was evident in the Anticipation of Complexity component. It is about anticipating students’ responses, which includes making predictions of students’ responses and developing prompts to support student learning. For the pre-service teachers in this study, anticipating students’ responses was a new practice. Therefore the findings showed that the pre-service teachers’ anticipations of students’ responses were vague and lacked detail. Limited time in the planning might also have caused this. The prompts from the mentor teachers and the knowledgeable other have helped the pre-service teachers make more detailed anticipations of students’ responses.

The pre-service teachers demonstrated a development in the Responding to Students’ Ideas component under the Contingency dimension. The contingent moments in this study were typically caused by unanticipated students’ responses. The findings showed that in responding to unanticipated students’ difficulties, the pre-service teachers tended to ignore them or use procedural teaching. The post-lesson discussions encouraged the pre-service teachers to reflect on their teaching especially in their contingent moments. Discussing contingent moments revealed the pre-service teachers’ PCK and helped them prepare more appropriate ways to respond to the students’ difficulties in future lessons.
This study found that the development of the pre-service teachers’ PCK was driven by some contributing factors of lesson study. The first contributing factor was cycles of planning, research lessons, and post-lesson discussions that focused on students’ thinking. The second was observing research lessons with a focus on students’ thinking. By focusing on students’ thinking while observing the research lesson, the pre-service teachers collected evidence of students’ learning. Discussing this evidence in the post-lesson discussions enabled them to rethink and reflect on their teaching. Lastly, support from the mentor teachers and the knowledgeable other was essential in the pre-service teachers’ PCK development. In this study, the mentor teachers contributed by providing advice about the selection of the topic taught in the lesson study and information about students’ characteristics that were useful for anticipating students’ responses.

The knowledgeable other prompted the pre-service teachers to give their students challenging tasks which created contingent moments. The challenging tasks revealed unanticipated students’ difficulties, provoking the pre-service teachers to respond in the moment. Moreover, this study found that the pre-service teachers’ responses to contingent moments revealed their PCK, particularly their Foundation Knowledge. The knowledgeable other in this study also contributed in maintaining students’ thinking as the focus of the lesson study discussions.

One important finding of this study is that lesson study enabled the interconnection of the KQ components as one of the driving forces of the development of the pre-service teachers’ PCK. This study found that the planning allowed the pre-service teachers to enact their Connection Knowledge, such as anticipating students’ responses and deciding on the sequence of the activities, while making decisions on the selection of the mathematical representation. Therefore, in lesson study context, the Connection Knowledge contributes to the Transformation Knowledge. Moreover, this study found that the pre-service teachers’ anticipation of students’ responses informed their teaching. When the pre-service teachers’ included prompts in their anticipation of students’ responses, they would support the students with more student-centred teaching. On the contrary, when the pre-service teachers did not have the prompts, they would use a procedural approach to help the students. Hence, this study suggests that the Connection Knowledge informs the Foundation Knowledge. Lastly, this study found that by discussing contingent moments in the post-lesson discussions, the pre-service teachers were encouraged to rethink of the mathematics, revealing the pre-service teachers’ Overt Subject Knowledge. Moreover, the post-lesson discussions provided support for the pre-
service teachers to deepen their subject knowledge. This study suggested that in lesson study context, the Contingency Knowledge contributes to the development of the Foundation Knowledge.

This study found that the participants gained some benefits from taking part in lesson study such as: the way in which the pre-service teachers and mentor teachers improved their mathematical knowledge; the way in which the pre-service teachers became more confident while teaching; and lastly, the constructive feedback and reflection opportunities the pre-service teachers received. Some of the challenges of participating in lesson study, according to the participants, were the extra work related to producing detailed lesson plans and teaching materials, and the extra time needed for lesson study meetings.

Lastly, the findings showed that the dual role of the knowledgeable other, the dynamics of the lesson study groups, and the organisation of lesson study brought some affordance and constraints. The dual role of the knowledgeable other enabled the knowledgeable other to provide support throughout the whole lesson study process, and to share the lesson study experience from one school to another. However, the dual role of the knowledgeable other also created some constraints to the implementation of lesson study. The dominant role of the knowledgeable other weakened the autonomy of the lesson study participants. The other constraint was that the knowledgeable other’s role was often under-prioritised due to the urgency of preparing the logistics of the research. This study found that the collaboration in the lesson study group contributed to the development of the pre-service teachers’ PCK, however establishing good collaboration in a lesson study group was not easy. Lastly, the findings showed that the organisation of the lesson study influenced the process and outcomes of the lesson study.

7.7.2 Significance of the Study

This study contributes to the understanding of pre-service teachers’ PCK development in a lesson study context during the teaching practicum. This study has identified the pre-service teachers’ PCK development in each phase of lesson study and the characteristics of lesson study that contribute to the pre-service teachers’ PCK development. Moreover, this study also contributes to the implementation of lesson study in teacher education outside Japan, especially in Indonesia.
The KQ Rubric contributes to the literature by offering a methodological tool to identify and analyse the quality of the teachers’ or pre-service teachers’ PCK over time. The KQ Rubric has enabled the researcher to identify the development of the pre-service teachers’ PCK by analysing the change in the quality of the PCK demonstrated by the pre-service teachers over time and over different sets of data. The KQ Rubric was used to analyse the pre-service teachers’ PCK over different phases of lesson study and interview after the lesson study. This suggests the possibility of using the KQ Rubric for further research on PCK especially with the KQ framework, beyond lesson study context.

Moreover, this study has provided empirical evidence of the complexity of the development of pre-service teachers’ PCK during lesson study in the teaching practicum. Earlier studies have shown that the Foundation Knowledge informs teachers’ Transformation, Connection and Contingency Knowledge, furthermore, the Transformation and Connection Knowledge underpin teachers’ Contingency Knowledge (see Figure 7.1) (Rowland et al., 2005; Rowland et al., 2009; Weston, 2013). Using the KQ framework, this study has shown the interconnections of KQ components in planning, teachings, and post-lesson discussions. It has shown that discussing contingency in the post-lesson discussion resulted in the development of the pre-service teachers’ Foundation Knowledge; and that anticipation of students’ responses under the Connection Knowledge informs the pre-service teachers’ Transformation Knowledge, especially Choice of Representation. Moreover, anticipation of students’ responses also underpins the pre-service teachers teaching approach – when they were ready with prompts, they were able to support students’ in a more constructive way, otherwise when they did not have the prompts, and they were more likely to use a procedural approach. In other words, the anticipation of students’ responses underpins the pre-service teachers’ Foundation Knowledge, in this case concentration on procedures. This finding contributes to a better understanding of how KQ components are interconnected and can be developed. This understanding may provide insights for teacher educators and curriculum developers of teacher education to design better ways of supporting pre-service teachers to develop their PCK.

This study showed the important role of the knowledgeable other in lesson study for pre-service teachers especially in a teaching practicum context. It contributes to the literature about the role of the knowledgeable other in several ways. First, it extends the understanding of the contributions and challenges faced by the knowledgeable other in the planning, research lessons and post-lesson discussions. Second, it highlights a new role of the knowledgeable other in lesson study, that is provoking contingency, in addition to the three other roles reported
by Takahashi (2014). Lastly it describes the affordances and constraints of the dual role of the knowledgeable other in the implementation of lesson study during the teaching practicum.

Given the growing implementation of lesson study worldwide, this study contributes to the literature about implementing lesson study in Indonesia more specifically in a teacher education context. This study employed lesson study in the teaching practicum for pre-service teachers. It has provided detailed descriptions of the lesson study design, showed the benefits and challenges for the participants, and discussed the affordances and constraints of the lesson study implementation in the teaching practicum. Moreover, this study offered an insight into improving the quality of pre-service teachers’ learning in their teaching practicum through lesson study.

7.7.3 Limitations of the Study

While this study has contributed to the literature on pre-service teachers’ PCK and lesson study, it has some limitations. The first limitation is due to the small number of participants. This is common for case study research as it aims to understand a phenomenon within its context (Yin, 2014). This study aimed to understand the development of pre-service teachers’ PCK during the implementation of lesson study in their teaching practicum. To achieve this goal, this study ensured the good quality of the lesson study as well as the data collection. The desired level of quality could only be achieved with a small number of participants. Having two participating schools with ten pre-service teachers, five mentor teachers, and two university lecturers, enabled feasible lesson study organisation and data collection. Having larger number of schools would have not been feasible due to the dual role of the researcher and the limited time available for this study.

The second limitation resulted from the dual role of the researcher/knowledgeable other. This study has provided a detailed description of the dual role of the researcher/knowledgeable other in the lesson study (see sections 3.5, 6.2, and 7.6). While this has benefitted the lesson study, there still remains a lack of objectivity with this approach (Iacono, Brown, & Holtham, 2009).

One of the findings of this study was that the pre-service teachers demonstrated some shift in their beliefs – from beliefs about traditional teaching to beliefs towards a more student centred teaching. It is important to note that this study did not intentionally focus on pre-service teachers’ beliefs. Therefore, the instruments used and the data analysis were not specifically
designed for the pre-service teachers’ beliefs. This might have weakened the study’s interpretation on the pre-service teachers’ beliefs.

By the time this study was conducted, the university implemented a new curriculum in which the teaching practicum was given less credit. In this curriculum, the teaching practicum was allocated two credits less than in the previous curriculum, which was four credits. This impacted on the time spent by the pre-service teachers for their teaching practicum at the school. They spent a maximum of three days per week at the school. When they were not at the school, they undertook coursework at the university campus. With lesser time at the school, there was less flexibility in the time organisation of the lesson study. Within the available time, this study was only able to organise two lesson study cycles for each lesson study group. Moreover, the pre-service teachers’ coursework might have affected their level of engagement in the lesson study.

7.7.4 Implications for Teacher Education Program

While this study found that lesson study supports the pre-service teachers’ PCK development, it is also important to align the teaching practicum and the courses in the teacher education curriculum. This study provides insights about the quality of the pre-service teachers PCK prior to the teaching practicum. Therefore, it suggests what they have learned in the courses. This study found that prior to the teaching practicum, the preservice teachers were typically lacking knowledge of mathematics teaching, such as critically selecting appropriate mathematical representations and anticipating students’ responses. Some scholars highlighted the gap between coursework and the teaching practicum – because subject matter knowledge and pedagogical knowledge are taught separately, the pre-service teachers struggle in finding ways of enacting various knowledge in the context of teaching (Nilsson, 2008; Sims & Walsh, 2009). This implies that the teacher education needs to pay more attention to their curriculum to offer more balanced programs that incorporate PCK before the teaching practicum.

Indonesian teacher education curriculum allocates very little time to the teaching practicum. More particularly in this study, time was a constraint in the implementation of lesson study during the teaching practicum. One of the time limitations is the duration of the planning. Unlike Japanese Lesson Study, the planning sessions in this study only took about 60 minutes. Therefore, it is very difficult to do kyozaikenkyu – study of the curriculum and teaching materials. Livy (2014) found that Australian pre-service teachers get opportunities to
develop their content knowledge from coursework and the teaching practicum. This study suggests that incorporating some of the kyozaikenku in the content of pedagogical units prior to the teaching practicum may help the pre-service teachers develop skills in selecting learning resources critically, anticipating students’ solutions. Some studies employed lesson study in on-campus units during teacher education programs (Fernández, 2010; Mostofo & Zambo, 2015). This can be a promising means to incorporate kyozaikenkyu in teacher education curriculum.

*Who is going to be the knowledgeable other?*

This study has shown the central role of the knowledgeable other such as maintaining the focus of the lesson study on students’ thinking. This includes prompting the pre-service teachers to anticipate students’ responses, pay attention to students’ work and critically discuss students’ thinking. To perform this role, the knowledgeable other is expected to be able to attend the lesson study meetings. Consequently, there are at least two major criteria for the knowledgeable other. The first criterion is that the knowledgeable other must have adequate knowledge about lesson study, mathematics, and mathematics teaching. The knowledgeable other is responsible for connecting theory to practice, helping teachers gain new knowledge from research and curriculum, and lastly helping teachers to reflect on their own teaching and learning (Takahashi, 2014). Moreover, Warwick et al. (2016) argue that dialogue moves (e.g. questioning, negotiating meaning, building on each other’s ideas, and establishing agreements) in lesson study meetings are important to warrant teachers’ learning. This implies that the knowledgeable other must have the skills needed to orchestrate those dialogue moves. Furthermore, this includes providing guidance and at the same time giving opportunities for the pre-service teachers and mentor teachers to become assertive in the lesson study meeting. Such balance requires a noticing skill of the knowledgeable other (Amador & Carter, 2018). The second criterion is the time availability of the knowledgeable other. The knowledgeable other is expected to attend the research lessons and post-lesson discussions.

For future implementations of lesson study in the teaching practicum, who would be the knowledgeable other remains a question. Earlier studies have reported the role of the mentor teachers as the knowledgeable other (Amador & Weiland, 2015) while others reported the involvement of university lecturers as the knowledgeable other in lesson study for pre-service teachers (Corcoran, 2011). Moreover, Corcoran (2011) found that there was tension in the
power distribution of the dual role of lecturer and knowledgeable other, and also the power interaction between the lecturer as the knowledgeable other and the pre-service teachers.

Considering the knowledge criteria, with their teaching experience, the mentor teachers possess knowledge related to school curriculum and students’ characteristics and learning. However, typical Indonesian teachers are not updated with current research in education. Therefore, they may not be able to bring new knowledge from research into their teaching. On the other hand, while the university may not have profound knowledge about students’ characteristics, they are more updated with current research than the mentor teachers. Therefore, the university lecturers may be better knowledgeable others than mentor teachers. However, this study found that the university lecturers’ participation was hindered by their time availability. This gives insight into the limited time availability from the university lecturers for future implementation of lesson study in the teaching practicum. Some studies reported that the knowledgeable other was performed by an external expert (Fujii, 2016). In Japanese Lesson Study, these experts are typically experienced teachers or principals, university professors with school teaching backgrounds, and school superintendents (Takahashi, 2014). Involving external experts in Indonesian universities for lesson study in the teaching practicum seems difficult because the university must spend more from a limited budget.

Sustainability

The teaching practicum is a mandatory unit in the curriculum of Indonesian teacher education. That means the university must establish a sustained partnership with schools. While the schools’ availability for teaching practicum may not be a problem, involving them in lesson study during the teaching practicum will not be easy. Involving more partner schools to conduct lesson study in the teaching practicum is difficult for several reasons. The first reason is the schools’ or teachers’ unfamiliarity with lesson study. In Indonesia, lesson study was established through university-school partnerships (Saito et al., 2007; Suratno & Cock, 2009). Therefore, the number of schools involved were limited and only located in those areas where universities are involved as a driving force of lesson study. Given that the mentor teachers are expected to play an active role in lesson study in the teaching practicum, they need to have a good understanding of lesson study. A significant amount of time and teacher professional development on lesson study will be needed prior to the implementation of lesson study in the teaching practicum.
The second reason is the difficulties of getting voluntarily involvement from schools and mentor teachers especially when there is no incentive for them. As this study found, participating in lesson study during the teaching practicum required more time and extra work from the mentor teachers. Therefore, to motivate the teachers in participate in lesson study in the teaching practicum requires resources and funding (Susanta, 2012). Teacher incentives can be incorporated into professional development programmes. Universities, schools, and policymakers need to establish an understanding of how this extra workload can benefit the mentor teachers.

7.7.5 Implications for Future Research

This study has shown the development of the pre-service teachers’ PCK during the teaching practicum and revealed the interconnection of the KQ components. However, the development of the pre-service teachers’ PCK was limited to the specific mathematical topics. Thus, the interconnections of the KQ components found in this study are topic specific. More studies need to be done to find out whether similar interconnections of KQ components apply to other mathematical topics, and whether there are more interconnections of KQ components.

Furthermore, because of the organisation of the lesson study, the development was defined within a particular lesson study cycle of a lesson study group. The organisation of the lesson study, one pre-service teacher taught one lesson in each lesson study cycle, it did not allow this study to follow the development of each individual pre-service teacher. Future research can be designed to specifically follow individual pre-service teacher’s PCK development.

While the pre-service teachers showed improvements in their PCK in lesson study, some questions remain – were those improvements prominent in their regular lessons during the teaching practicum? What impact may lesson study have on the pre-service teachers’ future teaching beyond the teaching practicum? This study did not examine whether lesson study in the teaching practicum affects the pre-service teachers’ regular teaching and beyond their teaching practicum. Future studies focusing on the impact of lesson study in the teaching practicum on the pre-service teachers’ short-term and long-term teaching are needed.

The aim of lesson study is to improve instruction (Lewis et al., 2009), which ultimately improve students’ learning. While this study showed that lesson study supports the development of pre-service teachers’ PCK which was evident through their improved
instructions, this study did not focus on the students’ learning. This calls for future studies on lesson study in the teaching practicum focusing on its influence on students’ learning.

One of the findings of this study is that, by participating in lesson study for pre-service teachers, the mentor teachers developed their mathematical knowledge and learned about integrating technologies into teaching mathematics. This is an important finding because even though the study was not designed for the mentor teachers’ learning, they also benefitted from their participation. However, the study did not explore in depth of the mentor teachers and the university lecturers’ learning in lesson study for pre-service teachers. This calls for future research focusing on the mentor teachers’ learning during lesson study in the teaching practicum.

Consistent with earlier studies (Takahashi, 2014), this study has shown the central role of the knowledgeable other in lesson study for pre-service teachers. It is imperative that the knowledgeable other possesses good noticing skills in order to support pre-service teachers and teachers’ learning during lesson study. Noticing skills are critical, not only during the research lesson to facilitate productive post-lesson discussions, but also during the planning as in this study the knowledgeable other was involved in all phases of lesson study. This is an important aspect that further study may investigate to find ways to support the knowledgeable other’s noticing. This study highlights the dilemma of the knowledgeable holding the dominant role, as it might have weakened the participants’ sense of agency of the lessons. What can the knowledgeable other do to encourage the participants to be more active in lesson study? Further studies are needed to unpack the challenges that the knowledgeable other faces, and find ways to support the knowledgeable other in facilitating productive lesson study.
References


National Health and Medical Research Council, Australian Research Council, & Australian 
Vice-Chancellors’ Committee. (2007). National statement on ethical conduct in 
human research Retrieved from 
https://www.nhmrc.gov.au/_files_nhmrc/publications/attachments/e72_national_state 
ment_march_2014_140327_1.pdf.

knowledge in pre-service education. International Journal of Science Education, 
30(10), 1281-1299. doi:10.1080/09500690802186993

elementary teachers’ pedagogical content knowledge. Journal of Science Teacher 

OECD. (2014). PISA 2012 result in focus what 15-year-olds know and what they can do with 
what they know. Retrieved from https://www.oecd.org/pisa/keyfindings/pisa-2012-
results-overview.pdf

SAGE.

Park, S., & Oliver, J. S. (2008). Revisiting the conceptualisation of pedagogical content 
knowledge (PCK): PCK as a conceptual tool to understand teachers as professionals. 

doi:http://dx.doi.org/10.1016/j.tate.2007.04.003

Petrou, M., & Goulding, M. (2011). Conceptualising teachers’ mathematical knowledge in 
teaching In T. Rowland & K. Ruthven (Eds.), Mathematical knowledge in teaching 
(pp. 9-26). Dordrecht: Springer Science+Business Media B.V.

Chauvot, J. (2007). Effects of early field experiences on the mathematical content 
knowledge and beliefs of prospective elementary school teachers: An experimental 

International (UK) Ltd.

Porter-Magee, K. (2004). Teacher quality, controversy, and NCLB. The Clearing House, 
78(1), 26-29. doi:10.2307/30181989

Teaching Children Mathematics, 472 - 478.

pre-service teachers’ pedagogical content knowledge. Teaching and Teacher 

Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher’s 
mathematics beliefs and teaching practice. Journal for Research in Mathematics 


251-267. doi:10.1007/s10857-010-9155-7

Research in Education 13(1), 31-38.

Rosnick, P. (1981). Some misconceptions concerning the concept of variable. The 
Mathematics Teacher, 74(6), 418-450.


Scotland, J. (2012). Exploring the philosophical underpinnings of research: Relating ontology and epistemology to the methodology and methods of the scientific, interpretive, and critical research paradigms. *English Language Teaching, 5*(9), 9-16.


Appendices

Appendix 1: Ethics Approval

Memorandum

To: Dr Wanti Widjaja
   School of Education

B

cc: Ms Meliasari

From: Deakin University Human Research Ethics Committee (DUHREC)

Date: 21 July, 2015

Subject: Developing Pre-Service Teachers’ Pedagogical Content Knowledge in Algebraic Reasoning through lesson study

Please quote this project number in all future communications

The application for this project was considered at the DU-HREC meeting held on 20/7/2015.

Approval has been given for Ms Meliasari, under the supervision of Dr Wanti Widjaja, School of Education, to undertake this project from 21/07/2015 to 21/07/2019.

The approval given by the Deakin University Human Research Ethics Committee is given only for the project and for the period as stated in the approval. It is your responsibility to contact the Human Research Ethics Unit immediately should any of the following occur:

* Serious or unexpected adverse effects on the participants
* Any proposed changes in the protocol, including extensions of time.
* Any events which might affect the continuing ethical acceptability of the project.
* The project is discontinued before the expected date of completion.
* Modifications are requested by other HRECs.

In addition you will be required to report on the progress of your project at least once every year and at the conclusion of the project. Failure to report as required will result in suspension of your approval to proceed with the project.

DUHREC may need to audit this project as part of the requirements for monitoring set out in the National Statement on Ethical Conduct in Human Research (2007).

Human Research Ethics Unit
research-ethics@deakin.edu.au
Telephone: 03 9251 7123
Appendix 2: Sample of Plain Language Statements and Consent Forms

PLAIN LANGUAGE STATEMENT AND CONSENT FORM

TO: Pre-Service Teacher

Plain Language Statement

Date: 10th June, 2015
Full Project Title: Developing Pre-Service Teachers’ Pedagogical Content Knowledge in Algebraic Reasoning through Lesson Study
Principal Researcher: Dr. Wanty Widjaja
Associate Researcher(s): Assoc. Prof. Susie Groves
Assoc. Prof. Colleen Vale
Meiliasari

We invite you to participate in the study Developing Pre-Service Teachers’ Pedagogical Content Knowledge in Algebraic Reasoning through Lesson Study. Lesson study is a model of teacher professional development through cycles of collaborative planning, teaching and reflection of lesson. A group of teachers set a learning goal and plan their lessons together. It is followed by a research lesson where one teacher will teach the lesson, and the other member of the group observe the lesson. The research lesson may also involve additional observers such as other teachers who are not in the planning group and an external expert. A post-lesson discussion is conducted after the research lesson. In this discussion, the teachers reflect on their teaching and planning in the light of the learning goals. Observers provide feedback based on the evidence of students’ learning they noticed during the research lesson. This cycle is repeated for a different lesson and a different teacher teaches the lesson.

This study will employ lesson study in a teaching practicum to develop pre-service teachers’ Pedagogical Content Knowledge (PCK) in algebraic reasoning. The study will take place in a teaching practicum unit in one university in Jakarta. It will be conducted in two lower secondary schools in Jakarta. In each school, it will involve four to six pre-service teachers, four to six mentor teachers, and one university lecturer.
Pre-service teachers who agree to participate in the study will:

- Attend a one-day workshop of lesson study at the university. This workshop will be videotaped.
- Form a lesson study group based on their school and year level taught. Each lesson study group will consist of two pre-service teachers, two mentor teachers, and one university lecturer. In each group, the participants will collaboratively plan the lessons, conduct research lessons and post-lesson discussion. During the lesson study implementation, you will be involved in four cycles of the planning sessions, research lessons, and post-lesson discussions. You will be expected to teach two of the research lessons. The planning sessions will be audiotaped, and the research lessons and post-lesson discussions will be videotaped.
- Take a written test at the beginning of the project. This test will not be used for any marking of the teaching practicum unit.
- Submit all copies of your lesson plans and teaching materials used in the research lessons.
- Complete a reflection note at the end of each lesson study cycle. This reflection note will not be used for any marking of the teaching practicum unit.
- Undertake an individual interview for about 45 minutes. The interview will be audio-taped.

As a participant in this project you are expected to provide your time for the lesson study meetings and to actively contribute in the group discussions. The personal risks associated with your involvement in this project are no greater than the risks associated with the usual program of teaching experience. Indeed we think that the collaborative nature of this project will benefit you personally, academically and professionally.

All the information gathered from your written test and individual interview, will be confidential and will not be disclosed to the mentor teachers, university lecturers or school principal. The audio-taped planning meeting and the video-taped lessons and post-lesson discussions will not be heard or viewed by anyone other than the researchers.

The results of the study will be reported in a variety of ways for the mathematics education research communities, teacher education community, and the school and teaching community. This will be done, for example, through journal articles, presentations at conferences and in professional development sessions for teachers.

Confidentiality of the university, lecturers, pre-service teachers, school, mentor teachers and students will be maintained throughout the project and pseudonyms will be used during the process of analysing the data and in publications.

We hope that you will consent to your participation in this project and agree to work with us. Please complete the consent form and return to:

Meiliasari  
School of Education  
Deakin University  
221 Burwood Highway  
Burwood VIC 3125  
Mobile: +628129690575  
Email: meili@deakin.edu.au  

If you have any questions please do not hesitate to make contact.

Please also be aware that the school, you and students are able to withdraw from the study at any time and if this occurs the data collected from this participant will be deleted from transcripts or
destroyed (in the case of interviews or work samples) and will not be used in the study or any forthcoming publications.

**Complaints**

If you have any complaints about any aspect of the project, the way it is being conducted or any questions about your rights as a research participant, you may contact:

Mathematics Department,
Faculty of Mathematics and Natural Sciences,
State University of Jakarta
Building Dewi Sartika, 7th Floor
Kampus A UNJ
Jl. Pemuda No. 10 Rawamangun Jakarta Timur
Telephone: +622129266281

The Manager, Research Integrity
Deakin University
221 Burwood Highway
Burwood Victoria 3125
Telephone: +61392517129
Facsimile: +61392446581
Email: research-ethics@deakin.edu.au
PLAIN LANGUAGE STATEMENT AND CONSENT FORM

TO: Pre-Service Teachers

Consent Form

Date:

Full Project Title: Developing Pre-Service Teachers’ Pedagogical Content Knowledge in Algebraic Reasoning through Lesson Study

Reference Number:

I have read and I understand the attached Plain Language Statement.

I freely agree to participate in this project according to the conditions in the Plain Language Statement.

I have been given a copy of the Plain Language Statement and Consent Form to keep.

The researcher has agreed not to reveal my identity and personal details, including where information about this project is published, or presented in any public form. However I understand that there is a possibility of recognition in video excerpts that may be shown in conference presentations or in professional learning or teacher education programs.

Participant’s Name (printed) ………………………………………………………………………

Signature ………………………………………………………  Date ……………………………

Meiliasari
School of Education
Deakin University
221 Burwood Highway
Burwood VIC 3125
Mobile: +628129690575
Email: meili@deakin.edu.au
PLAIN LANGUAGE STATEMENT AND CONSENT FORM

TO: Pre-Service Teachers

Withdrawal of Consent Form

Date:

Full Project Title: Developing Pre-Service Teachers’ Pedagogical Content Knowledge in Algebraic Reasoning through Lesson Study

Reference Number:

I hereby wish to WITHDRAW my consent to participate in the above research project and understand that such withdrawal WILL NOT jeopardise my relationship with Deakin University.

Participant’s Name (printed) ……………………………………………………………………………………

Signature ……………………………………………………… Date ……………………………

Please mail this form to:

Meiliasari
School of Education
Deakin University
221 Burwood Highway
Burwood VIC 3125
Mobile: +628129690575
Email: meili@deakin.edu.au
# Appendix 3: Observation Sheet

**Observation Sheet**

<table>
<thead>
<tr>
<th>Observer’s name:</th>
<th>Teachers’ name:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>School</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Classroom:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date:</th>
<th>Lesson start:</th>
<th>Lesson end:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This observation form is intended to help you record the lesson. Please write your observation in the column below.

Please fill in the table for classroom observation.

<table>
<thead>
<tr>
<th>Time</th>
<th>What the teacher does</th>
<th>What the students do</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Please fill in the table for group observation

<table>
<thead>
<tr>
<th>Group description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Please describe clearly the group you observe.</td>
</tr>
<tr>
<td>Group’s members:</td>
</tr>
<tr>
<td>Group’s seating position:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
<th>Group activity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix 4: Post-Lesson Discussion Protocol

1. The post-lesson discussion is conducted immediately after the research lesson. Exception will be made when it is not possible to conduct immediate post-lesson discussion.

2. The pre-service teachers in the lesson study group, mentor teachers, university lecturer, other pre-service teachers are invited to join the post-lesson discussion.

3. The facilitator/knowledgeable other chairs the post-lesson discussion.

4. All participants must show respect to one another.

5. All participants must provide constructive feedback for the lead teacher and the lesson study group.

6. The facilitator/knowledgeable other may ask some questions, give directions to the participants in order to maintain the focus of the post-lesson discussion.

7. The lead teacher is given the first chance to express their reflection of the research lesson. Then it is followed by the university lecturer, mentor teachers, pre-service teachers in the lesson study group, and pre-service teachers outside the lesson study group who observed the research lesson.

8. The facilitator/knowledgeable other gives the final comments and remarks of the post-lesson discussion.
Appendix 5: Written Test

PART 1: Mathematical Knowledge on Algebra

1. If the graph of linear functions f(x) and g(x) intersect at a point P on the x-axis, prove that the graph of their sum function (f + g)(x) must also go through P.

Modified from TEDS-M secondary 2009

2. A function is defined by the following rule for all real numbers x:

\[ f(x + 1) = f(x) + 5 \text{ and } f(0) = 3 \]

a. Calculate \( f(4) \)
b. What rule could be used to calculate \( f(x) \) for any given \( x \)?

Modified from:

PART 2: Pedagogical Content Knowledge on Algebra

3. A. List three key algebraic concepts in year 7 and 8 mathematics curriculum.
   B. Choose one concept from those three concepts, and explain how it relates to other concepts in the mathematics curriculum.

4. The picture below shows a student’s answer.

![image](http://mathmistakes.org/wp-content/uploads/2013/05/Dist-2.png)
a. What is the students’ mistake and what kind of misconception does it indicate?
b. Why do you think he made such mistake?
c. How would you help him correct his mistake and understand the concept?

5. Look at the problem below.
   A. Write your solution and show your work of the problem
   B. Write some possible students’ answers.
   C. Write some possible students’ mistakes.

A grocery store wants to stack cans in a triangular display just like the one in the picture. How many cans are needed if they want to make a 10 row-high triangular display?

Modified from:
6. The pictures below are excerpts from two year 8 textbooks.

**BOOK 1**

**Table 2.2**

<table>
<thead>
<tr>
<th>Students' Name</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anik</td>
<td>35</td>
</tr>
<tr>
<td>Andre</td>
<td>34</td>
</tr>
<tr>
<td>Gita</td>
<td>30</td>
</tr>
<tr>
<td>Bayu</td>
<td>35</td>
</tr>
<tr>
<td>Asep</td>
<td>33</td>
</tr>
<tr>
<td>Dewi</td>
<td>32</td>
</tr>
</tbody>
</table>

**Figure 2.6** is an arrow diagram that shows the relation (i.e. the weight) of the data in Table 2.2.

From the diagram, we know the following:

a. Each student has a weight. This means that each member of set A has a partner in set B.
b. Each student has exactly one weight. This means that each member of set A has exactly one partner in set B.

Thus, we can conclude that the relation from set A to set B is a special relation that matches each member of set A to exactly one member of set B. This type of relation is called function. Hence, a function of set A to set B is a special relation that matches each member of A to exactly one member of B.

A relation is called a function if it fulfills the following condition:

a. Each member of A has a partner in B.
b. Each member of A is matched to exactly one member of B.

Menahan Ciri-ciri Fungsi

Fungsi merupakan salah satu konsep penting dalam matematika. Dengan mengenal fungsi akan membantu memahami sifat-sifat matematika, khususnya dalam memecahkan soal matematika yang rumit.

Pemenanglah, mari kita pelajari ciri-ciri dari suatu fungsi.

Pola dalam urutan yang akan diikuti adalah:

Amata 1:
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Amata 2:
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Amata 3:
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Amata 4:
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Pola dalam urutan yang akan diikuti adalah:
1. Sebelum
2. Selangka
3. Selambat
4. Si Udin

Cara memahami pola tersebut (balih di ketahui keja terpintas) dengan menurunkan urut yang mengikuti terdahuluk.

<table>
<thead>
<tr>
<th>Kata Asli</th>
<th>Peka Sebagai Kata yang Muncul dalam Buku Menggunakan Amata 1</th>
<th>Amata 2</th>
<th>Amata 3</th>
<th>Amata 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sebelum</td>
<td>A B C D E F G H I J K L M N O P Q R S T U V W X Y Z</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Selangka</td>
<td>A B C D E F G H I J K L M N O P Q R S T U V W X Y Z</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Selambat</td>
<td>A B C D E F G H I J K L M N O P Q R S T U V W X Y Z</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Si Udin</td>
<td>A B C D E F G H I J K L M N O P Q R S T U V W X Y Z</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Pencapaian dengan siswa yang menang setiap kata besar untung. Maknanya: "Apa salah atau kata disandali hanya dengan satu "tanda".

Activity 3.1
Understanding the characteristic of a function

Function is an important concept in mathematics. Understanding function and functional relationship between mathematical elements helps us to understand and solve problems. Thus, understanding function is highly important in mathematics. First, let's learn about the characteristic of a function.

Observe the rule of the following ciphers:

Rule 1:
- A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
- A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
- A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
- A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Rule 2:
- A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
- A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
- A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
- A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Rule 3:
- A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
- A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
- A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
- A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Rule 4:
- A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
- A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
- A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
- A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Look at these words:
1. Sebelum
2. Selangka
3. Selambat
4. Si Udin

Fill out this table (you may work in a different sheet) with the cipher test

<table>
<thead>
<tr>
<th>Plain Text</th>
<th>Cipher Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sebelum</td>
<td>Rule 1</td>
</tr>
<tr>
<td>Selangka</td>
<td>Rule 2</td>
</tr>
<tr>
<td>Selambat</td>
<td>Rule 3</td>
</tr>
<tr>
<td>Si Udin</td>
<td>Rule 4</td>
</tr>
</tbody>
</table>

Look closely at each of the plain text and its cipher text. Is each of the plain text translated to a single cipher text?

a. Write the strengths and weaknesses of the approach used in each book!

b. Which book is best used for teaching the concept of function? Why?
7. You are going to teach about distributive property in algebraic operation

\[ a(b + c) = ab + ac \]

You found two different approaches in helping students understand this property.

1st approach: Using geometric representation with area of a rectangle

\[
\begin{array}{c|c}
\text{Area of the rectangle} & \text{area of blue} + \text{area of orange} \\
\hline
a \quad ab & ac \\
\hline
b & c
\end{array}
\]

2nd approach: Using real word context

How much would you pay if you buy two coffees and two muffins?

This problem can be solved in two ways

1. Price of two coffees : \( 2 \times 3 = 6 \)
   Price of two muffins: \( 2 \times 4 = 8 \)
   Total price : \( 6 + 8 = 14 \)

2. Price of one coffee and once muffin : \( 3 + 4 = 7 \)
   Since you buy two coffees and two muffins, then the total price is \( 2 \times 7 = 14 \)

From solution 1 and 2 we can see that \( 2 \times (3 + 4) = (2 \times 3) + (2 \times 4) \)

A. Write the strength and weakness of each approach?
B. Which approach do you think is the best way to help students understand distributive property? Why?
Appendix 6 : Sample of Student Worksheet

LEMBAR AKTIVITAS SISWA
Penyelesaian Persamaan Linear Satu Variabel

Nama :
Kelas :

1. Perhatikan gambar di bawah ini!

Berapa jumlah permen yang terdapat dalam satu kantong permen? 

20 - 4 = 16 Jika 16 saja 1 kantong berarti 1 kantong berisi 8 buah permen

2 kantong = 2 x 8 = 16 permen

2. Suatu bilangan dikalikan 2 kemudian ditambah satu hasilnya adalah 13, tentukan bilangan tersebut:

n x 2 + 1 = 13

6 x 2 + 1 = 13
Appendix 7 : Sample of Pre-Service Teacher’s PowerPoint Presentation
Appendix 8: Interview Guidelines

1. Interview for pre-service teachers

COURSEWORK
1. How many credits do you take this semester?
2. How do you manage your time between teaching practicum and coursework?

TEACHING PRACTICUM PROGRAM
1. Please explain the work or activities you did in the teaching practicum.
2. What did you learn from the teaching practicum?
3. Please specify which work/activities supported your learning.
4. Please specify which work/activities did not support your learning.

LESSON STUDY
1. Please describe the work/activities you did in the lesson study.
   - What did you do for planning?
   - What did you do in the research lesson?
   - What did you do in the post-lesson discussion?
2. What did you learn from the planning? Research lesson? Post-lesson discussion?
3. What do think about lesson study in the teaching practicum program?
4. What benefits did you have from participating in lesson study?
5. What difficulties did you encounter when participating in lesson study?
6. Could you please describe your mentor teacher’s involvement in the lesson study program?
7. What help did you expect from your mentor teacher and university lecturer in the teaching practicum?
8. What kind of support did your mentor teacher provide? Please explain (in the planning? Research lesson? Post-lesson discussion?)

PCK ON ALGEBRA
1. Let’s take a look at your written test answers.
   Do you want to change or add clarification or explanation to the responses you gave before the teaching practicum experience? If yes, what would you like to change or clarify? Why?
II. Interview for mentor teachers

TEACHING PRACTICUM PROGRAM
1. Please describe how you usually mentor pre-service teachers

LESSON STUDY
2. Were you able to attend all planning meetings, research lessons and post-lesson discussion? If not, what prevented you from going?
3. Please describe what kind of activities you participated in during the lesson study
   - What did you contribute to the team’s planning of the research lessons?
   - What did you do during the research lessons?
   - How did you contribute in the post-lesson discussion?
4. Please explain how this practice differs from your role and activities as a mentor teacher during traditional teaching practicum
5. What were the benefits for you from participating in lesson study?
6. What difficulties or challenges did you encounter while participating in lesson study?
7. What differences were evident between pre-service teachers’ teaching approaches in the lesson study and regular teaching practicum?
8. Do you have any other comments regarding the implementation of lesson study?
III. Interview for university lecturers

TEACHING PRACTICUM PROGRAM
1. Please describe what kind of activities you usually take part in when supervising the pre-service teachers

LESSON STUDY
1. Were you able to attend all planning meetings, research lessons and post-lesson discussion? If not, what prevented you from going?
2. Please describe what kind of activities you participated in during the lesson study
   - What did you contribute to the team’s planning of the research lessons?
   - What did you do during the research lessons?
   - How did you contribute in the post-lesson discussion?
3. Please explain how this practice differs from your role and activities as a teaching practicum supervisor for pre-service teacher during traditional teaching practicum
4. What were the benefits for you from participating in lesson study?
5. What difficulties or challenges did you encounter when participating in lesson study?
6. What differences are evident between pre-service teachers’ performance in the lesson study and regular teaching practicum?
7. Do you have any other comments regarding the implementation of lesson study?
Appendix 9: Lesson Plan of Group D8 in Lesson study Cycle 1 (C1D8)

Rencana Pelaksanaan Pembelajaran

Nama Sekolah : SMPD Jakarta
Mata Pelajaran : Matematika
Kelas / Semester : VIII / ganjil
Materi Pokok : Persamaan Garis Lurus
Alokasi Waktu : 2 Pertemuan (3 x 40 menit)

A. KOMPETENSI INTI

3. Memahami dan menerapkan pengetahuan (faktual, konseptual, dan prosedural) berdasarkan rasa ingin tahu nya tentang ilmu pengetahuan, teknologi, seni, budaya terkait fenomena dan kejadian tampak mata.

B. KOMPETENSI DASAR

3.4 Menentukan persamaan garis lurus dan grafiknya.

C. INDIKATOR PENCAPAIAN KOMPETENSI

1. Mampu menyelesaikan permasalahan sehari-hari yang berkaitan dengan persamaan garis lurus.

D. TUJUAN PEMBELAJARAN

Melalui pengamatan, tanya jawab, dan diskusi kelompok, siswa dapat:
1. Menyelesaikan permasalahan sehari-hari yang berkaitan dengan persamaan garis lurus.

E. MATERI PEMBELAJARAN

Aplikasi persamaan garis lurus dalam kehidupan sehari-hari.
F. PENDEKATAN/ MODEL/ METODE PEMBELAJARAN
Pendekatan : Scientific Approach
Model : Problem Based Learning
Metode : Pengamatan, diskusi kelompok.

G. ALAT DAN MEDIA PEMBELAJARAN
Laptop, LCD, power point, dan LAS (Lembar Aktifitas Siswa).

H. SUMBER PEMBELAJARAN
Buku paket Matematika SMP kelas 8, Yudhistira, Kurikulum 2013.

I. KEGIATAN PEMBELAJARAN

<table>
<thead>
<tr>
<th>Kegiatan</th>
<th>Deskripsi Kegiatan</th>
<th>Waktu</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pendahuluan</td>
<td>1. Guru mengajak peserta didik untuk berdoa.</td>
<td>15 menit</td>
</tr>
<tr>
<td></td>
<td>2. Guru menanyakan kabar dan mengecek kehadiran peserta didik</td>
<td></td>
</tr>
<tr>
<td>Kegiatan Inti</td>
<td><strong>Mengamati</strong> 1. Siswa mencermati masalah sehari-hari yang berkaitan dengan persamaan garis lurus.</td>
<td>95 menit</td>
</tr>
</tbody>
</table>

229
230

Menanya
2. Siswa menanya tentang permasalahan sehari-hari yang berhubungan dengan persamaan garis lurus.

Menalar

Mencoba
4. Siswa menggali informasi tentang penerapan persamaan garis lurus dalam masalah sehari-hari.
5. Siswa menggali informasi untuk memecahkan masalah yang berkaitan dengan persamaan garis lurus.

**Mengomunikasikan**

6. Siswa menyajikan secara tertulis hasil pembelajaran mengenai permasalahan yang berkaitan dengan persamaan garis lurus.

<table>
<thead>
<tr>
<th>Penutup</th>
<th>1. Siswa dan guru membuat kesimpulan mengenai materi yang dipelajari.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2. Guru memberikan pekerjaan rumah kepada siswa berkaitan dengan aplikasi persamaan garis lurus.</td>
</tr>
<tr>
<td></td>
<td>3. Guru menginformasikan materi yang akan dipelajari selanjutnya.</td>
</tr>
</tbody>
</table>

10 menit

**J. PENILAIAN**

1. Prosedur Penilaian

<table>
<thead>
<tr>
<th>No</th>
<th>Aspek yang dinilai</th>
<th>Teknik penilaian</th>
<th>Waktu penilaian</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Pengetahuan</td>
<td>Pengamatan dan hasil diskusi</td>
<td>Setelah diskusi kelompok dan melalui <em>Post-test</em>.</td>
</tr>
<tr>
<td></td>
<td>a. Menyelesaikan permasalahan sehari-hari yang berkaitan dengan persamaan garis lurus.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b. Merepresentasikan pernyelaian masalah sehari-hari ke dalam bentuk grafik persamaan garis lurus.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Instrumen Penilaian
Harits pergi ke sebuah mall menggunakan sepeda motor. Tarif parkir di mall tersebut adalah Rp2000,00 pada satu jam pertama, dan untuk penambahan tiap jam berikutnya sebesar Rp1.000,00.

a. Gambarlah grafik yang menggambarkan kenaikan biaya parkir.
b. Tentukan bentuk persamaan garis dari permasalahan tersebut.
c. Jika Harits berada di mall tersebut selama 6 jam, berapakah biaya parkir yang harus dibayar oleh harits?
d. Jika biaya parkir yang dibayar Harits Rp12.000,00, berapa lama Harits berada di mall tersebut?

3. Prediksi Jawaban

**Prediksi 1**

a. Tidak dijawab.
b. Tidak dijawab.
c. Biaya parkir = 2000 + 1000 + 1000 + 1000 + 1000 + 1000 = 7000

d. 12.000 = 2000 + 1000 + 1000 + 1000 + 1000 + 1000 + 1000 + 1000 + 1000 + 1000 + 1000 + 1000

*iama parkir = 11 jam.*
**Prediksi 2**

a. Tidak dijawab  
b. Tidak dijawab  
c. *Biaya parkir* = 2000 + (1000 \cdot 5) = 2000 + 5000 = 7000  
d. 12.000 = 2000 + (1000 \cdot 10)  
   *Lama parkir* = 11 jam.

**Prediksi 3**

a. Menggambar Grafik

<table>
<thead>
<tr>
<th>x (rupiah)</th>
<th>2000</th>
<th>3000</th>
<th>4000</th>
<th>5000</th>
<th>6000</th>
<th>7000</th>
<th>8000</th>
</tr>
</thead>
<tbody>
<tr>
<td>y (jam)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

b. Persamaan garis

Dititili dua titik (2000,1) dan (3000,2). Persamaannya adalah sebagai berikut.

\[
\begin{align*}
    y - y_1 &= \frac{x - x_1}{x_2 - x_1} \\
    y_2 - y_1 &= x - 2000 \\
    1 &= x - 2000 \\
    1 &= \frac{1}{1000} \\
    1000(y - 1) &= x - 2000 \\
    1000y - 1000 &= x - 2000 \\
    1000y &= x - 2000 + 1000 \\
    y &= \frac{x}{1000} - 1
\end{align*}
\]

Persamaan garis yang menggambarkan keadaan tersebut adalah \( y = \frac{x}{1000} - 1 \).
c. Jika Harits berada di mall selama 6 jam \((y = 6)\)

\[
y = \frac{x}{1000} - 1
\]

\[
6 = \frac{x}{1000} - 1
\]

\[
6000 = x - 1000
\]

\[
x = 7000
\]

Jika Harits berada di mall selama 6 jam, maka biaya parkir yang harus dibayarkan adalah Rp7000,00.

d. Jika biaya parkir yang dibayarkan Harits Rp12.000,00 \((x = 12000)\)

\[
y = \frac{x}{1000} - 1
\]

\[
y = \frac{12000}{1000} - 1
\]

\[
y = 12 - 1 = 11
\]

Jika biaya parkir yang dibayar Harits Rp12.000,00, maka Harits berada di mall selama 11 jam.

**Prediksi 4**

a. Menggambar grafik

<table>
<thead>
<tr>
<th>x (jam)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>y (rupiah)</td>
<td>2000</td>
<td>3000</td>
<td>4000</td>
<td>5000</td>
<td>6000</td>
<td>7000</td>
<td>8000</td>
</tr>
</tbody>
</table>

b. Persamaan garis

Diambil dua titik \((1,2000)\) dan \((2,3000)\). Persamaannya adalah sebagai berikut.
\[ y - y_1 = \frac{x - x_1}{x_2 - x_1} \\
\frac{y_2 - y_1}{x_2 - x_1} = \frac{2000}{x - 1} \\
\frac{3000 - 2000}{x - 1} = \frac{2 - 1}{x - 1} \\
1000 \\
y - 2000 = 1000(x - 1) \\
y - 2000 = 1000x - 1000 \\
y = 1000x - 1000 + 2000 \\
y = 1000x + 1000 \\

Persamaan garis yang menggambarkan keadaan tersebut adalah \( y = 1000x + 1000 \).

c. Jika Harits berada di mall selama 6 jam (\( x = 6 \))
\[ y = 1000x + 1000 \]
\[ y = 1000 \cdot 6 + 1000 \]
\[ y = 7000 \]
Jika Harits berada di mall selama 6 jam, maka biaya parkir yang harus dibayarkan adalah Rp7000,00.

d. Jika biaya parkir yang dibayarkan Harist Rp12.000,00 (\( y = 12000 \))
\[ y = 1000x + 1000 \]
\[ 12000 = 1000x + 1000 \]
\[ 12000 - 1000 = 1000x \]
\[ 1000x = 11000 \]
\[ x = 11 \]
Jika biaya parkir yang dibayar Harits Rp12.000,00, maka Harits berada di mall selama 11 jam.
K. KERANGKA PENILAIAN

1. Aspek Pengetahuan

<table>
<thead>
<tr>
<th>Aspek Penilaian</th>
<th>Rubrik Penilaian</th>
<th>Skor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harits pergi ke sebuah mall menggunakan sepeda motor. Tarif parkir di mall tersebut adalah Rp2000,00 pada satu jam pertama, dan untuk penambahan tiap jam berikutnya sebesar Rp1.000,00.</td>
<td>a. Menggambar grafik</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>b. Persamaan garis</td>
<td>15</td>
</tr>
</tbody>
</table>
| | \[
| \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \\
| \frac{y - 2000}{3000 - 2000} = \frac{x - 1}{2 - 1} \\
| \frac{y - 2000}{1000} = \frac{x - 1}{1} \\
| y - 2000 = 1000(x - 1) \\
| y - 2000 = 1000x - 1000 \\
| y = 1000x - 1000 + 2000 \\
| y = 1000x + 1000 \\
| Persamaan garis yang menggambarkan keadaan tersebut adalah \( y = 1000x + 1000 \) | 5 |
| | c. Jika Harits berada di mall tersebut selama 6 jam, berapakah biaya parkir yang harus dibayar oleh Harits? | 5 |
| | d. Jika biaya parkir yang dibayar Harits | |
Rp12.000,00, berapa lama Harits berada di mall tersebut?

c. Jika Harits berada di mall selama 6 jam 
\( (x = 6) \)
\[ y = 1000x + 1000 \]
\[ y = 1000 \cdot 6 + 1000 \]
\[ y = 7000 \]
Jika Harits berada di mall selama 6 jam, maka biaya parkir yang harus dibayarkan adalah Rp7000,00.

d. Jika biaya parkir yang dibayarkan Harits Rp12.000,00 \((y = 12000)\)
\[ y = 1000x + 1000 \]
\[ 12000 = 1000x + 1000 \]
\[ 12000 - 1000 = 1000x \]
\[ 1000x = 11000 \]
\[ x = 11 \]
Jika biaya parkir yang dibayar Harits Rp12.000,00, maka Harits berada di mall selama 11 jam.

<table>
<thead>
<tr>
<th>Skor maksimal: 100</th>
</tr>
</thead>
</table>
L. MEDIA PEMBELAJARAN

1. Power Point Text, sebagai berikut:

Tujuan Pembelajaran

- Menyelesaikan permasalahan sehari-hari yang berkaitan dengan persamaan garis lurus.
- Merepresentasikan pernyelasan masalah sehari-hari ke dalam bentuk grafik persamaan garis lurus.

Problem 1

Kalau tanda telah mempelajari persamaan garis lurus. Tapi agaklah kalau tahu apa kegunaannya dalam kehidupan? Cobalah jelaskan, kamu diberikan uang oleh jodohmu sebesar Rp200.000,00 untuk satu bulan. Jika kamu menghabiskannya Rp10.000,00 setiap hari, berapa uang uangmu setelah 1 hari? Kegumaman uangmu setelah 4 hari dan 10 hari?

Isilah pada tabel berikut ini:

<table>
<thead>
<tr>
<th></th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sisa uang (Rp)</td>
<td>a)</td>
<td>b)</td>
<td>c)</td>
<td>d)</td>
<td>e)</td>
<td>f)</td>
</tr>
</tbody>
</table>

a) b) c) d) e) f)
• Tabel di atas membantu untuk membuat grafik, buatlah grafiknya dalam kertas berpetak!

• Tuliskan persamaan dari garis pada grafik yang telah dibuat: 

• Jika kamu diminta untuk menentukan berapa sisal uangmu di hari ke-24, bagaimana cara kamu menentukannya?

---

**Answer A**

a. i) Rp 200.000,00 - Rp 6.000,00(2 hari) = Rp 188.000,00
   ii) Rp 200.000,00 - Rp 6.000,00(5 hari) = Rp 178.000,00
   iii) Rp 200.000,00 - Rp 6.000,00(10 hari) = Rp 148.000,00

---

**Answer B**

<table>
<thead>
<tr>
<th>Hari (x)</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sisa Uang (y)</td>
<td>180.000</td>
<td>170.000</td>
<td>160.000</td>
<td>150.000</td>
<td>120.000</td>
<td>90.000</td>
</tr>
</tbody>
</table>

---

**Answer C**

a. Ambil dua batik, seperti (2.180.000) dan (4.210.000), sebagan pada dibuat suatu persamaan

\[
\begin{align*}
170.000 & - 180.000 = 4 - 2 \\
2y & = -10.000 \\
y & = -5.000
\end{align*}
\]

\[
\begin{align*}
2y & = -5.000 = -120.000 + 2y \\
y & = -5.000 + 2r \\
y & = -5.000 + 10.000
\end{align*}
\]

---

**Answer D**

---
LEMBAR AKTIVITAS SISWA

APLIKASI PERSAMAAN GARIS LURUS

Nama :

Kelas :

Diskusikan masalah berikut dengan teman sekelompokmu!

Kalian tentu telah mempelajari persamaan garis lurus. Tapi apakah kalian tahu apa kegunaannya dalam kehidupan?. Coba bayangkan, kamu diberikan uang oleh ayahmu sebesar Rp 200.000,00 untuk satu bulan. Jika kamu menghabiskannya Rp 6.000,00 setiap harinya, berapa sisa uangmu setelah 2 hari? Bagaimana sisanya setelah 4 hari atau 10 hari?

Isilah pada tabel bantuan ini.

<table>
<thead>
<tr>
<th>Hari</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sisa uang (Rp)</td>
<td>a)</td>
<td>b)</td>
<td>c)</td>
<td>d)</td>
<td>e)</td>
<td>f)</td>
</tr>
</tbody>
</table>

Tuliskan caranya!

a) 
b) 
c) 
d) 
e) 
f)

Tabel di atas membantumu untuk membuat grafik, buatlah grafiknya dalam kertas berpetak!

Tuliskan persamaan dari garis pada grafik yang telah dibuat→

Jika kamu diminta untuk menentukan berapa sisa uangmu di hari ke-24, bagaimana cara kamu menentukannya?
Selesaikan permasalahan berikut dengan tepat!

Harits pergi ke sebuah mall menggunakan sepeda motor. Tarif parkir di mall tersebut adalah Rp2000,00 pada satu jam pertama, dan untuk penambahan tiap jam berikutnya sebesar Rp1.000,00.

a. Gambarlah grafik yang menggambarkan kenaikan biaya parkir.
b. Tentukan bentuk persamaan garis dari permasalahan tersebut.
c. Jika Harits berada di mall tersebut selama 6 jam, berapakah biaya parkir yang harus dibayar oleh Harits?
d. Jika biaya parkir yang dibayar Harits Rp12.000,00, berapa lama Harits berada di mall tersebut?
Appendix 10 : Lesson Plan of Group D7 in Lesson Study Cycle 1 (C1D7)

RENCANA PELAKSANAAN PEMBELAJARAN

Mata Pelajaran : Matematika
Kelas/Semester : VII/1
Materi Pokok : Persamaan dan Pertidaksamaan Linear Satu variabel
Alokasi Waktu : 2 x 40 menit

4. KOMPETENSI INTI

1. Menghargai dan menghayati ajaran agama yang dianutnya.
2. Menghargai dan menghayati perilaku jujur, disiplin, tanggung jawab, peduli (toleransi, gotong royong), santun, percaya diri, dalam berinteraksi secara efektif dengan lingkungan sosial dan alam dalam jangkauan pergaulan dan keberadaannya.
3. Memahami pengetahuan (faktual, konseptual, dan prosedural) berdasarkan rasa ingin tahunya tentang ilmu pengetahuan, teknologi, seni, budaya terkait fenomena dan kejadian tampak mata
4. Mencoba, mengolah, dan menyaji dalam ranah konkret (menggunakan, mengurai, merangkai, memodifikasi, dan membuat) dan ranah abstrak (menulis, membaca, menghitung, menggambar, dan mengarang) sesuai dengan yang dipelajari di sekolah dan sumber lain yang sama dalam sudut pandang/teori

5. KOMPETENSI DASAR

1.1 Menghargai dan menghayati ajaran agama yang dianutnya.
3.3 Menentukan nilai variabel dalam persamaan dan pertaksamaan linear satu variabel.
4.2 Membuat dan menyelesaikan model matematika dari masalah nyata yang berkaitan dengan persamaan dan pertidaksamaan linear satu variabel.

6. INDIKATOR PENCAPAIAN KOMPETENSI

1. Mampu menyelesaikan persamaan linear satu variabel melalui cara coba-coba dan kaidah keekuivalenlan (bentuk setara).
2. Mampu menyelesaikan model matematika dari masalah nyata berkaitan dengan persamaan linear satu variabel.
7. **TUJUAN PEMBELAJARAN**

Melalui pengamatan, tanya jawab, dan penugasan individu siswa dapat:

- Menyelesaikan persamaan linear satu variabel melalui cara coba-coba dan kaidah keekuivalenan (bentuk setara).
- Menyelesaikan model matematika dari masalah nyata berkaitan dengan persamaan linear satu variabel.

8. **MATERI PEMBELAJARAN**

Penyelesaian Persamaan Linear Satu Variabel (PLSV) dengan cara coba-coba dan kaidah keekuivalenan (bentuk setara), dan model matematika yang berhubungan dengan PLSV.

9. **PENDEKATAN / MODEL / METODE PEMBELAJARAN**

Pendekatan : *Scientific Approach*

Model : *Problem Based Learning* (Pembelajaran Berbasis masalah)

Metode : Penugasan, diskusi, tanya jawab

10. **ALAT DAN MEDIA PEMBELAJARAN**

Bahan ajar, PPT dan Lembar Kerja Siswa.

11. **SUMBER PEMBELAJARAN**


12. **KEGIATAN PENMBELAJARAN**

<table>
<thead>
<tr>
<th>Kegiatan Pendahuluan</th>
<th>Waktu</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. Guru menyiapkan siswa untuk siap menerima pelajaran.</td>
<td>15 menit</td>
</tr>
<tr>
<td>5. Guru menyampaikan tujuan pembelajaran.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Kegiatan Inti</th>
<th>Pertanyaan Bantuan Guru</th>
<th>Waktu</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aktivitas Pembelajaran</td>
<td>Lihat apa yang diketahui dalam permasalahan?</td>
<td>50 menit</td>
</tr>
<tr>
<td>o Menjelaskan situasi masalah pada Lembar Aktivitas Siswa menggunakan <em>slide presentation</em> (Terlampir).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Penyelesaian Masalah</td>
<td>Pertanyaan dan Saran dari Guru</td>
<td></td>
</tr>
<tr>
<td>----------------------</td>
<td>-------------------------------</td>
<td></td>
</tr>
</tbody>
</table>

**Lembar Aktivitas Siswa Nomor 1**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Siswa tidak mengetahui bagaimana menyelesaikan masalah.</td>
<td>Coba lihat dan baca kembali masalahnya! Apa yang kamu ketahui dari masalah itu? Apa yang harus kamu lakukan?</td>
</tr>
<tr>
<td>Siswa memindahkan 4 permen di ruas kanan, ke ruas kiri timbangan. Namun, posisi timbangan tidak setimbang.</td>
<td>Kenapa kamu memindahkan 4 permen ke ruas kiri timbangan? Jika kamu melakukan itu, apa posisi timbangan tetap setimbang? Bagaimana seharusnya posisi timbangan jika kamu memindahkan 4 permen ke ruas kiri?</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td><img src="image2.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>

Siswa memindahkan 4 permen di ruas kanan, keruas kiri timbangan. Namun, posisi timbangan tidak setimbang.
<table>
<thead>
<tr>
<th>Siswa menyelesaikan masalah ke dalam bentuk kalimat terbuka. Misal: 10 buah permen setimbang dengan 2 kantong permen dan 2 buah permen. 10 buah permen sama dengan dengan 2 kantong permen dan 2 buah permen.</th>
<th>Coba kamu ubah kalimat tersebut ke dalam bentuk persamaan linear satu variabel! Apa yang dimisalkan dengan variabel dalam kalimat tersebut?</th>
</tr>
</thead>
</table>
| Siswa menyelesaikan masalah ke dalam bentuk persamaan linear satu variable. Misal: 

$$20 = 2p + 4; p = 1 \text{ kantong permen}$$

| Coba selesaikan persamaan yang kamu buat! Dapatkan kamu menentukan berapa permen yang terdapat dalam satu kantong? Setelah kamu substitusikan nilai variabel yang kamu dapat, apa persamaan yang kamu buat bernilai benar? |
|---|---|
| Siswa tidak mengetahui permasalahan | Coba lihat dan baca kembali masalahnya! Coba diingat kembali apa yang kamu pelajari tentang PLSV pertemuan sebelumnya! Apa yang kamu ketahui dari situ? Apa yang harus kamu lakukan? |

**Lembar Aktivitas Siswa Nomor 2**

Siswa tidak mengetahui permasalahan
<table>
<thead>
<tr>
<th>Tidak disebutkan dalam masalah, kamu harus mencari tahu bilangan apa itu?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Siswa menyimpulkan bahwa nilai dari variabel adalah bilangan genap.</td>
</tr>
<tr>
<td>Darimana kamu tahu bahwa hasilnya bilangan genap?. Berapakah nilai bilangan genap yang kamu maksud? Coba tuliskan bagaimana caranya kamu mendapatkan hasil bilangan genap!</td>
</tr>
<tr>
<td>Siswa mengubah menjadi persamaan berikut: $b \times (2 + 1) = 13$</td>
</tr>
<tr>
<td>Coba baca kembali masalahnya! Apakah persamaan yang kamu buat sudah benar?</td>
</tr>
<tr>
<td>Siswa mengubah menjadi persamaan berikut: $(b \times 2) + 1 = 13$ atau $2b + 1 = 13$</td>
</tr>
<tr>
<td>Coba selesaikan persamaan yang kamu buat! Setelah kamu subtitusikan nilai variabel yang kamu dapat, apa persamaan yang kamu buat bernilai benar?</td>
</tr>
</tbody>
</table>

Menjelaskan jawaban menggunakan *slide presentation* (Terlampir).

**Kegiatan Penutup**

8. Siswa dan guru membuat kesimpulan mengenai materi yang dipelajari.
10. Guru memberikan pekerjaan rumah kepada siswa mengenai materi yang dipelajari.

| 15 menit |
13. **PENILAIAN**

1. **Prosedur Penilaian**

<table>
<thead>
<tr>
<th>No</th>
<th>Aspek yang Dinilai</th>
<th>Teknik Penilaian</th>
<th>Waktu Penilaian</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Pengetahuan dan keterampilan matematika</td>
<td>Lembar Aktivitas Siswa</td>
<td>Kegiatan inti</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Tugas</td>
<td>Kegiatan inti</td>
</tr>
</tbody>
</table>

2. **Instrumen Penilaian**

**TUGAS**

**PERSAMAAN LINEAR SATU VARIABEL**

9. Perhatikan gambar berikut!

![Gambar keranjang dan bola basket](image1)

Berapa jumlah bola basket dalam satu keranjang?

10. Jumlah dari enam dengan empat kali sebuah bilangan adalah 18. Tentukan nilai bilangan tersebut!

11. \[14b + 6 = 15b - 2\]

12. \[
\frac{2y}{5} - 4 = 0
\]
3. Kunci Jawaban

1. Berapa jumlah bola basket dalam satu keranjang?

   \[2k + 3 = k + 8; k = \textit{satu keranjang bola basket}\]

   \[2k - k = 8 - 3\]

   \[k = 5\]

   Jadi, jumlah bola dalam satu keranjang basket adalah 5 bola.

2. Jumlah dari enam dengan empat kali sebuah bilangan adalah 18. Tentukan nilai bilangan tersebut!

   \[6 + 4x = 18; x = \textit{sebuah bilangan}\]

   \[4x = 18 - 6\]

   \[4x = 12\]

   \[x = \frac{12}{4}\]

   \[x = 3\]

3. \[14b + 6 = 15b - 2\]

   \[14b - 15b = -2 - 6\]

   \[-b = -8\]

   \[-b \times (-1) = 18 \times (-1)\]

   \[b = 18\]

4. \[\frac{2y}{5} - 4 = 0\]

   \[\frac{2y}{5} = 0 + 4\]

   \[5 \times \frac{2y}{5} = 4 \times 5\]

   \[2y = 20\]

   \[y = \frac{20}{2}\]

   \[y = 10\]
14. **KERANGKA PENILAIAN**

1. Aspek Pengetahuan

<table>
<thead>
<tr>
<th>Aspek Penilaian</th>
<th>Rubrik Penilaian</th>
<th>Skor</th>
</tr>
</thead>
</table>
| 2. Soal pada Lembar Aktivitas Siswa. Berapa jumlah bola basket dalam satu keranjang? | \[ \begin{align*} 2k + 3 &= k + 8; \\
    k &= \text{satu keranjang bola basket} \\
    2k - k &= 8 - 3 \\
    k &= 5 \end{align*} \] | 25 |
| | 6 + 4x = 18; \( x = \text{sebuah bilangan} \) | 25 |
| | 4x = 18 - 6 \\
| | 4x = 12 \\
| | \( x = \frac{12}{4} \) \\
| | \( x = 3 \) | |
| 3. Jumlah dari enam dengan empat kali sebuah bilangan adalah 18. Tentukan nilai bilangan tersebut! | \[ \begin{align*} 14b + 6 &= 15b - 2 \\
    14b - 15b &= -2 - 6 \\
    -b &= -8 \\
    -b \times (-1) &= 18 \times (-1) \\
    b &= 18 \end{align*} \] | 25 |
| 4. \( \frac{2y}{5} - 4 = 0 \) | \[ \begin{align*} \frac{2y}{5} &= 0 + 4 \\
    5 \times \frac{2y}{5} &= 4 \times 5 \\
    2y &= 20 \\
    \frac{20}{2} &= y \\
    y &= 10 \end{align*} \] | 25 |

Skor maksimal = 100
15. BAHAN AJAR

Model Matematika yang Berhubungan dengan Persamaan Linear Satu Variabel

Gabungan antara penerapan bagaimana menyusun sebuah persamaan dari informasi yang ada, dan bagaimana cara menyelesaikan persamaan yang dibuat.
Contoh:
9. Kelereng Budi 4 kurangnya dari $\frac{2}{3}$ kelereng Joni. Jika Budi memiliki 16 kelereng, berapa banyak kelereng Joni?
Model matematika dari masalah di atas adalah sebagai berikut:
Misalnya, kelereng Joni = $x$
Maka banyaknya kelereng Budi adalah $\frac{2}{3}x - 4$
Sehingga bentuk persamannya adalah $\frac{2}{3}x - 4 = 16$

\[
\begin{align*}
\frac{2}{3}x - 4 &= 16 \\
\frac{2}{3}x &= 16 + 4 \\
\frac{2}{3}x &= 20 \\
x &= \frac{20 \times 3}{2} \\
x &= 30
\end{align*}
\]
Jadi, Joni memiliki 30 kelereng.

10. Santi akan berumur 4 kali umurnya sekarang setelah 15 tahun. Berapa umur Santi sekarang?
Model matematika dari masalah di atas adalah sebagai berikut:
Misalnya, umur Santi sekarang = $x$ tahun
4 kali umurnya = $4x$
Setelah 15 tahun akan menjadi $(x + 15)$ tahun
Sehingga bentuk persamannya adalah $x + 15 = 4x$

\[
\begin{align*}
15 &= 4x - x \\
15 &= 3x \\
\frac{15}{3} &= x \\
x &= 5
\end{align*}
\]
Jadi, umur Santi sekarang adalah 5 tahun.
LEMBAR AKTIVITAS SISWA
Penyelesaian Persamaan Linear Satu Variabel

Nama :
Kelas :

- Perhatikan gambar di bawah ini!

Berapa jumlah permen yang terdapat dalam satu kantong permen?

- Suatu bilangan dikalikan 2 kemudian ditambah satu hasilnya adalah 13, tentukan bilangan tersebut!
## Appendix 11: List of Codes

<table>
<thead>
<tr>
<th>Codes</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>KQ</td>
<td>Knowledge Quartet</td>
</tr>
<tr>
<td>FOU</td>
<td>Foundation</td>
</tr>
<tr>
<td>AOP</td>
<td>Awareness of purpose</td>
</tr>
<tr>
<td>COP</td>
<td>Concentration on procedures</td>
</tr>
<tr>
<td>IER</td>
<td>Identifying errors</td>
</tr>
<tr>
<td>OSK</td>
<td>Overt Subject Knowledge</td>
</tr>
<tr>
<td>ATB</td>
<td>Adheres to textbook</td>
</tr>
<tr>
<td>TUP</td>
<td>Theoretical Underpinning of Pedagogy</td>
</tr>
<tr>
<td>UOT</td>
<td>Use of Terminology</td>
</tr>
<tr>
<td>TRA</td>
<td>Transformation</td>
</tr>
<tr>
<td>COE</td>
<td>Choice of Examples</td>
</tr>
<tr>
<td>COR</td>
<td>Choice of Representations</td>
</tr>
<tr>
<td>TUD</td>
<td>Teacher use of demonstration</td>
</tr>
<tr>
<td>UTM</td>
<td>Teacher use of teaching materials</td>
</tr>
<tr>
<td>CON</td>
<td>Connection</td>
</tr>
<tr>
<td>AOC</td>
<td>Anticipation of Complexity</td>
</tr>
<tr>
<td>CBC</td>
<td>Connection between concepts</td>
</tr>
<tr>
<td>CBP</td>
<td>Connection between procedures</td>
</tr>
<tr>
<td>DAS</td>
<td>Decision about sequencing</td>
</tr>
<tr>
<td>RCP</td>
<td>Recognition of conceptual appropriateness</td>
</tr>
<tr>
<td>CNT</td>
<td>Contingency</td>
</tr>
<tr>
<td>DFA</td>
<td>Deviation from agenda</td>
</tr>
<tr>
<td>RSI</td>
<td>Responding to Students’ Ideas</td>
</tr>
<tr>
<td>UOA</td>
<td>Use of Opportunities</td>
</tr>
<tr>
<td>1</td>
<td>Level 1</td>
</tr>
<tr>
<td>2</td>
<td>Level 2</td>
</tr>
<tr>
<td>3</td>
<td>Level 3</td>
</tr>
<tr>
<td>MTC</td>
<td>Mentor teachers’ contributions</td>
</tr>
<tr>
<td>Code</td>
<td>Description</td>
</tr>
<tr>
<td>------</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td>ULC</td>
<td>University lecturers’ contributions</td>
</tr>
<tr>
<td>KOC</td>
<td>Knowledgeable other’s contributions</td>
</tr>
<tr>
<td>BNF</td>
<td>Benefits from lesson study</td>
</tr>
<tr>
<td>KLR</td>
<td>Knowledge learned</td>
</tr>
<tr>
<td>SPP</td>
<td>Support from lesson study group</td>
</tr>
<tr>
<td>CFD</td>
<td>Confidence</td>
</tr>
<tr>
<td>DFF</td>
<td>Difficulties</td>
</tr>
<tr>
<td>DTM</td>
<td>Detailed teaching materials</td>
</tr>
<tr>
<td>TMS</td>
<td>Time and schedules</td>
</tr>
</tbody>
</table>