Design Summary of State-Observers for Linear Positive Time-Delayed Systems with Disturbance

by

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I am the author of the thesis entitled Design Summary of State-Observers for Linear Positive Time-Delayed Systems with Disturbance

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Abstract
This project gives the design aspects and disturbance estimation of a positive linear time delayed state observers based on Linear Programming LP. The positivity of a system is throughout maintained which gives all the necessary and sufficient conditions while designing positive observers. The schematics of the proposed architecture is given to start up with the design procedures. The error dynamics derived using the mathematical equations are subjected to an asymptotically stable constrains and then the existence of observers of this kind is confirmed using Linear Programming LP method. Also, the deep explanation of how to derive the conditions correctly give the viewer a correct perception of how this is getting done as a designer.
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1. Introduction

Control engineering is one of the most advanced and progressing, innovative field of study in different branches of engineering. For instance, in areas such as economics, sociology, communications and biology the importance of control engineering and its application is more worth as physical state of the system can be determined from controller design [2]. In the year 1960 introduction of Kalman filter was the major invention which later contributed the pulse to the field of study in control systems. The design of a new process and control of the same all are done based on the branch of study control engineering. If something wrong happens to the process, a parameter uncertainty or a different input change; all are needing to be compensated without affecting the process output product. This is where the control engineering surprises. The more complex is the process more complex will be controlling. To design something to reduce the complexity, irrespective of how complex the system is; always a challenge in control engineering. This is where new ideas and inventions emerged and executed. Depends on the nature of the process control engineering is divided into different sub branches. For instance, positive systems/ non-negative systems or negative systems, linear or non-linear systems, static and dynamic systems, delayed or systems with no time delay and homogenous and non-homogenous systems etc.

1.1 Positive Systems

Positive systems are one of the broad fields of study in which some examples can be quoted straight away from our surroundings for example money, blood sugar, queues, plant populations, the concentration of substances, goods quantity etc. The examples in which the state variables are always positive irrespective of the initial conditions. There is such a relevance in why considering the positive system as a model only because of its important applications in the real world.

Positive systems are the systems whose outputs and states are always non-negative when the initial conditions are non-negative [1].

1.2 Observer Layout and Relevance

The observer is defined that the auxiliary system which is like the real one in its behaviours and characteristics. The auxiliary system is excited with the same input and output, but it can give information about the internal states of the system. There are two classifications of observers, state observers and functional observers. State observer estimates the states of the system whereas functional observer does observe the function of states. But at the conclusion, the functional observers can be reduced as state observer if the functional matrix is given by the value of the identity matrix.

Consider the state space representation of the linear system

\[ \dot{x}(t) = Ax(t) + Bu(t) \]  

(1.1)
\[ y(t) = C x(t) \quad (1.2) \]

Where \( A \in \mathbb{R}^{n \times n} \), \( B \in \mathbb{R}^{n \times m} \), and \( C \in \mathbb{R}^{p \times n} \) are constant matrices. \( x(t) \in \mathbb{R}^{n} \), \( u(t) \in \mathbb{R}^{m} \), and \( y(t) \in \mathbb{R}^{p} \) are state vector, input vector, and output vector. The state vector \( x(t) \) plays an important role in state space representation as it contains all the basic information about a system. Therefore, when it comes to design of a controller the input \( u(t) \) can be related to the state vector as follows.

\[ u(t) = -K x(t) \quad (1.3) \]

where \( K \) the constant matrix and \( K \in \mathbb{R}^{m \times n} \).

A compelling control framework relies upon criticism from the deliberate factors, which are then compared with the reference (or set) values and the error sustained to the controller to make a remedial move. This may have a few difficulties when managing signals that are distant (or inaccessible). The inside and outside unsettling influences experienced in plant and unknown errors in the yield estimations can appear as unmeasurable inputs. This features the principal need to plan powerful control frameworks to improve the framework's dependability.

Numerous physical frameworks include amounts whose esteem are limited to a locale characterized by last and most extreme limits. This is to guarantee that the framework works in a protected or achievable area. A large portion of these amounts is intrinsically positive. For example, current moving through a circuit, weight or temperature in reactors, the volume of fluid in tanks, the populace of a city et cetera. They give more significant data when displayed by positive esteems. It is unreasonable to discuss the negative estimations of such parameters. The controller intended for such frameworks must consider the energy imperatives to guarantee that the framework works inside the doable or safe area when the controller is actualized in the genuine plant.

The observer design is important when the linear variable estimation is needed for the control, stabilization and monitoring purposes must be done. Even if this project describes mainly the system with disturbance it is also paid some attention to the systems like with delay and without delay. That means a general conclusion comes only after discussing different types of system along with system with disturbance. To reduce the complexity and order the project can be extended to functional observer easily by changing the error dynamics and do the same design again with a functional constant vector \( F \). The only factor that needs to be considered in here is the surety of the positivity. When the design progress consideration of positivity plays a major role. It leads to some necessary and sufficient conditions which is considered during the design and coding.

The LP is what is used for the realization of observer. The optimization of observer is performed with the help of linear programming. To know whether the proposed architecture exists or not, the LP plays an important role while subjecting to constraints that are part of the observer design and the positivity constraints are
equally considered while the solution of the linear programme results in a feasible solution where the observer of the suggested form exists.
2. Project Objective and Encouragement

In a real system, the designed observer plays a role which helps to reduce the damage or repair caused by the unexpected variation happening to the system. In a process, there is an output which has been producing for a long period of time. If there is a fault occurs which can badly affect the system and produce millions of losses. If the system can be modelled into state space representation and an observer can estimate all the states of a system, then the change or the variation that would occur in the system can be bypassed through controller without affecting the real system. Which helps to build a consistent process which compensates all the unmeasurable variables through the designed observer-based controller. The objective of this project is to design an observer-based controller that is a compensator, which helps to compensate the unmeasurable inputs by estimates the states of the system with the help of the designed observer. Which in turn develop a robust and dynamic system, also reduces the cost and make it solid.

2.1 Project Outlook

This project is progressing on the design of full order observer for systems with time delay and disturbance. As there are so many other classifications of the system as this project is focusing on one objective other parts are briefly discussed in the literature. The reality of the design is robust as the system architecture which has chosen for this project can be realized through different strategies. And, the future scope of this project can be further extended to reduced order, functional observer or to multiple time delayed system, or even to a discrete system instead of continuous time systems. This is an active research area as it has further extension is possible. The application of the system is considered when it comes to the real world, every system has some sort of disturbance which affects the sensitivity as well as faults, so the design of an observer substitute this problem and make the system insensitive to unknown inputs. All the uncertainties in this type of systems can be solved using this estimation as all the state variables are being made to available for the controller to control the system very effectively. As little attention has been paid to systems having disturbance this work will be an invention to solve all the uncertainties in the system. It is very necessary to cope with systems having time delay as it may lead to instability, also while we consider the positive systems, these types of systems have a lot of application than a negative system. It is easy to concentrate to this type of system because the positivity comes as a constraint while the design progress.

As the existence of this type of observer is found using the mathematical technique called LP, the stability of the observer can be found the solution to the Linear programming problems comes feasible. Instead of doing all the calculations and getting a negative estimation at the end, the problem optimization using LP helps to know if the proposed observer exists or not. If the solution of LP problem comes as infeasible then it’s the responsibility of the designer to change the current
architecture and find new equations to the observer or propose a new architecture. The progression in the literature shows the relevance of this study in control system engineering. The change or the variation that would occur in the system can be bypassed through controller without affecting the real system.
3. Literature Review

3.1 Positive Systems

Positive systems are defined as the exceptionally relevant types of systems in the areas of science and technology. For example, chemical reactors, the concentration of substances in chemicals, storage systems as memories, levels of liquids are very common applications of positive systems (see for example, ([1] - [5])). As a basic definition for a positive system can easily find in different research articles. That is dynamic systems whose state vectors are always non-negative whenever the initial conditions are non-negative, [6 - 7]. Literature about basic control system and positive systems were demanding from the year 1960 itself.

3.2 Basic Control System/ State Variables

Luenberger in 1960s, in his own work explaining the general and basic definitions for the positive systems as

\[ \dot{x}(t) = A x(t) + A_1(t-T) + Bu(t), \quad t \geq 0 \]  

(3.1)

and

\[ y(t) = C x(t), \]  

(3.2)

Where \( x(t) = \) state vector \( u(t) = \) control input vector and \( y(t) = \) output vector. In addition to this, it is mentioned that the above system is positive if and only if A is a Metzler matrix and \( A_1 \) and B are non-negative [20]. For a Metzler matrix M, all off diagonal elements are nonnegative [37]

\[ i.e., m_{ij} \geq 0, \quad i \neq j. \]  

(3.4)

Therefore, positive systems are a class of system which have the property that the state is nonnegative whenever the initial conditions are non-negative [19].

3.3 Time Delayed Positive Systems

The parameter uncertainty and time-delay make the positive system more challenging in control. The change in parameters affect the Properties of the positive systems in such a way that, first it reduces the computation of Eigenvector, secondly it provides a direct correspondence between positive equilibrium point and finally stability and comparative statics which is applicable to stable systems. Every stable system has its own equilibrium point at rest. If a parameter changes the system will move to a new equilibrium point. Comparative statics is the question of how the change in equilibrium points is related to the parameter change that produced it.

3.4 Application of Positive Systems

In the control system, it is very important to deal with systems having time delay since it may lead to instability [6]. The systems having time delay faces the
problems like oscillation, instability and poor characteristics. The stability of the positive real systems, it has been proved that if the system without delay is asymptotically stable then the stability will be independent of constant delays [7]. It is really challenging to do control of a positive system having a time delay. Some valuable studies were done on this type of systems [8]. A dynamic system is known as positive if the trajectory starts and remains forever in the positive orthant for all non-negative values [9, 10] because of the broad application in chemical industry [11], power electronics [12]. Biology and economy [13].
4. An Observer for a Positive System

A real application can be related to the variables in the control industry (see for instance, [14] and the references therein). The observer design comes in the input-output analysis.

\[ x_i = b_i + \sum_{j=1}^{n} z_{ij} (x_1, x_2, x_3, \ldots, x_n) i=1, \ldots, n \]  
(4.1)

Where the \( x_i \) is the amount of good produced in a given time period (e.g. 1 year) by the sector \( i \),

\( b_i = \) demand/consumption \( b_i \geq 0 \), \( z_{ij} \) = amount of good \( i \) purchased by sector \( j \).

From the above equation, the purchased quantities \( z_{ij} \) are given by

\[ z_{ij} = a_{ij} x_j \]  
(4.2)

where \( a_{ij} \) = technological coefficient i.e. amounts of good \( i \) needed to produce one unit of good \( j \). Obviously \( a_{ij} \geq 0 \) so that the matrix \( A \) is non-negative.

\[ i.e. \; x = Ax + b \]  
(4.3)

As clarified before in the presentation, an observer is utilized to evaluate the conditions of the genuine framework considering the information of the progression of the framework, the control input and the deliberate esteem. Basically, it is a virtual framework which recreates the conduct of the genuine framework. The level of control to be connected to any framework shifts relying upon the many-sided quality and use of the framework. Framework disappointments because of wasteful aspects in the control systems can prompt unfortunate results and tremendous misfortunes. The disappointments are intensified by the developing idea of digital assault and the soaring expenses of cutting-edge sensors.

A compelling control framework relies upon criticism from the deliberate factors, which are then compared with the reference (or set) values and the error sustained to the controller to make a remedial move. This may have a few difficulties when managing signals that are distant (or inaccessible). The inside and outside unsettling influences experienced in plant and unknown errors in the yield estimations can appear as unmeasurable inputs. This features the principal need to plan powerful control frameworks to improve the framework's dependability.

To control a linear positive system with time-delay all the state vectors should be available. Through the design of an observer, the system can provide all its states available for the controller through an observer estimation [15]. This is the main reason why an observer design in control theory is very popular amongst researchers. In control theory, a state observer is a system that provides an estimate of the internal state of a given real system, from measurements of the input and output. For instance, vehicles in a tunnel: the rates and velocities at which vehicles enter and leave the tunnel can be observed directly, but the exact state inside the
tunnel can only be estimated. The observer is used to estimating the state in a system. It is used to track the trajectory (how the state variable is responding to an input). In dynamical systems, a trajectory is the set of points in state space that are the future states resulting from a given initial state. Positive observer as given as constructing observer which ensure the non-negativity of the estimates of non-negative states.

A plant process diagram of an observer in a process is depicted in figure 1. An overview of the observer design is given in [16]. More recent works can be seen in [17] where state observer ensure the non-negativity of the state vector. (see for example [18] – [20]). To control a linear positive system with time-delay all the state vectors should be available. Through the design of an observer, the system can provide all its states available for the controller through an observer estimation.

![Observer schematic in a Control system](image)

**Fig.1.** Observer schematic in a Control system

### 4.1 Full Order State Observer

Most of the works are based on full order observer design which is done by estimating all the variables in a system. Since it is estimating all the variables, the cost and the computational complexity will be more. Therefore, the need for high-speed processors should be employed. To avoid this problem, the reduced order observer is introduced which estimates only the unavailable input variable [18].

There are some linear systems which have the effect of unknown inputs. This work also considers the balancing of unknown input excitation to the system. When we model a system, there are diverse types of system uncertainties such as parameter change, non-linearity, unknown external excitation, coupling change in large-scale systems and actuator faults can be represented as unknown inputs [19].

Numerous physical frameworks include amounts whose esteems are limited to a locale characterized by last and most extreme limits. This is to guarantee that the framework works in a protected or achievable area. A large portion of these amounts is intrinsically positive. For example, current moving through a circuit, weight or temperature in reactors, the volume of fluid in tanks, the populace of a city et cetera. They give more significant data when displayed by positive esteems. It is unreasonable to discuss the negative estimations of such parameters. The
controller intended for such frameworks must consider the energy imperatives to guarantee that the framework works inside the doable or safe area when the controller is actualized in the genuine plant.

4.2 Reduced Order State Observer

The unknown input observer (UIO) is also known as disturbance decoupled observers. The unknown input can relate to disturbance because there are conditions where some of the inputs cannot access so, therefore, the full order observer which estimate all variables use their knowledge to calculate. Early work such as [19,20], given some preliminary information about the unavailable inputs. In the earlier work [33] proposed a basic observer design for all the states along with unknown inputs. Since then several authors provided different techniques for the design of UIO observers. The reduced order UIO has been theoretically implemented for the system having no disturbance [20]. This makes the system smaller dimensions and equivalent system having no Unknown inputs.
5. Modelling of System

The system with unknown input and time delay can be represented as follows

\[ \dot{x}(t) = A \, x(t) + B \, u(t) + D \, d(t) \quad (5.1) \]
\[ y(t) = C \, x(t) \quad (5.2) \]
\[ x(\theta) = \varphi(\theta), \quad \theta \in [-\tau, 0] \quad (5.3) \]

where \( x(t) \) is the state vector

\( u(t) \) is the input vector

\( d(t) \) is the disturbance vector

\( A \in \mathbb{R}^{nxn} \), \( B \in \mathbb{R}^{nxm} \), \( C \in \mathbb{R}^{pxn} \) and \( D \in \mathbb{R}^{nxr} \) are constant matrices and \( \varphi(\theta) \) is the initial condition.

The time delay can sometimes occur either in control input/output or in the state. The time delayed state can be represented by the following set of equations

\[ \dot{x}(t) = A \, x(t) + A_d \, x(t - \tau) + B \, u(t) + D \, d(t) \quad (5.4) \]
\[ y(t) = C \, x(t) \quad (5.5) \]
\[ x(\theta) = \varphi(\theta), \quad \theta \in [-\tau, 0] \quad (5.6) \]

Whereas \( \tau \) is known as constant time delay [21]. The objective in designing the reduced order observer can be paraphrased as the reduced order state observer of order \( (n - p) \) to make sure that the states estimated \( x(t) \) converges asymptotically to the state which is true that is \( x(t) \). models in view of on (many) investigations and models considering physical first standards and (a couple) parameter identification tests. In modern control theory all the tasks are really complicated, that means the system comes with multiple inputs and multiple outputs. The need of complex design is coming priority. This new concept was the reason behind the term state. State can be defined as the smallest set of variables, where the information of this variables at the initial time completely determines the behaviour of the system [12]. The variables which defines the state is known as state variables. The collection of this state variables is collectively known as state variables. The coordinates formed by this state variables are known as state space. Set of equations defining this state space is called state space equations.

For example, consider the mechanical system below.

When considering the equilibrium position, the state equation comes as a second order equation as follows.

\[ m\ddot{y}(t) + b\dot{y}(t) + ky = u \quad (5.7) \]

The above system has two state variables and let it be

\[ x_1(t) = y(t) \quad (5.8) \]
\[ x_2(t) = \dot{y}(t) \]  

Then the state equation comes as \( x_1 = x_2 \)

\[ \dot{x}_2 = \frac{1}{m}(-ky - by) + \frac{1}{m}u \]  

(5.10)

\[ \dot{x}_2 = \frac{-k}{m}x_1 - \frac{b}{m}x_2 + \frac{1}{m}u \]  

(5.11)

Fig 2: Mechanical System

We can represent the above in a block diagram form as well as in vector matrix form as follows.

The vector-matrix is derived as

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
\frac{-k}{m} & \frac{-b}{m} \\
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
\end{bmatrix} + \begin{bmatrix}
0 \\
\frac{1}{m} \\
\end{bmatrix} u
\]

(5.12)

\[ y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \]  

(5.13)
Where 

\[ A = \begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}, \quad C = [1 \ 0], \quad D = 0 \] (5.14)

The state space representation is defined as follows. The block diagram representation and the system space representation together form the state space representation.
6. Linear Programming

The computational approach to the positive observer can be done LP and Linear Matrix Inequality (LMI) ([21] - [22]). LP method allows finding maximum profit with least cost for a mathematical model. In comparing the previous work, it is concluded that the LP approach is simple and worth than LMI approach. Since the LP method has less computational complexity this is most preferable than LMI [22]. Because of the relation between constraints and the mathematical equation linear programming has a lot of similarity with control system modelling. That is the main reason why we involve the LP method to solve the constraints maximization subjecting to different variables. More constraint problems and preliminary results has been discussed in [23] – [26]. If under the proposed conditions, the system converges to zero exponentially, then the designed observer can estimate the states well accurate.

Linear programming is a method for determining the best type of activity needs to take place small amount of resources give rise constraints in Linear programming problem, to maximize or minimise the problems is the procedure to consider this problem. Optimal usage of the limited resources can be classified as the optimization problems which needed linear programming. One of the major requirement of a linear problem is that, the variable and the objective function should be linear. In different fields the linear programming techniques have been using. Formulation of a LP problems is necessary to specify three things they are,

- Objective Function
- Constraints
- Decision variables

For instance, if the profit from product A is 5 and profit from product B is 7 then the objective function comes as maximize \( z = 5x_1 + 7x_2 \). Where, \( x_1, x_2 \) are the decision variables. Also let the product A takes 3 hours and product B takes 8 hours total hours free are let be 40 then the constraints come as \( 3x_1 + 8x_2 \leq 40 \). This is how we formulate each Linear programming problems. Also, there are solution for this type of Linear Programming problems as graphical solutions also there are different types of linear programmes. Graphical method involves fining the feasible region for the problems using different coordinates.

Optimal values include the corner values for each variable and the intersection of all the variables forms a region. If it comes in a positive coordinate for maximization problem, then the corner point represents the value of the problems and hence the solution of the corresponding optimization problem is found using the graphical method. Because of the relation between constraints and the mathematical equation linear programming has a lot of similarity with control system modelling. Optimal usage of the limited resources can be classified as the optimization problems which needed linear programming. One of the major requirement of a linear problem is that, the variable and the objective function
should be linear. If it comes in a positive coordinate for maximization problem, then the corner point represents the value of the problems and hence the solution of the corresponding optimization problem is found using the graphical method.
7. Functional Observers

As an advance design procedure, the observer design has reached till reduced order observer functional observer for the system with time delay where it is found less complex and accurate than ordinary observers [37]. In the reduced order functional observer, the calculation derives only the linear function of the state vector. If the system has state vector in its hand then the stabilization, fault identification and monitoring will be easy in some practical processes [27] – [29]. Thus, the observer design has relevant and physical meaning in real-world applications.

The observation issue of the state vector of a direct time-invariant multivariable framework subjected to unknown inputs, has gotten extensive consideration over the most recent three decades [24]. Unknown inputs in a framework can be because of unavailability of the inputs or nearness of plant aggravations. This condition makes it difficult to utilize the regular observers which depend on the accessibility of the considerable number of inputs of a framework. These vulnerabilities influencing the framework can be demonstrated as inputs. Observers intended to assess the unknown inputs can be utilized to recognize shortcomings and furthermore for confinement plans. A portion of the prior work conveyed by the analysts depended on the presumption of the presence of data about the unknown inputs. For example, utilizing polynomial models to appraise these inputs was proposed and talked about carefully in [25], then again, construct their supposition with respect to a condition that the unknowns adjust to a given consistent coefficient differential condition [26].

Other writing expected the nonattendance of any learning on the unknown inputs and in this manner conceived different outline systems for settling this sort of observers. The idea of state arranges change was produced in [27] though in [28], the framework reversal calculation was connected. A summed-up grid approach was viewed as in ([21], [29] - [30]) and the solitary esteem deterioration procedure in [31]. The important and adequate condition for the presence of unknown input observers were accounted for in [30] and [32]. There has been a critical advancement in the hypothesis and outline methods for these sorts of observers with numerous present writing giving a clear, essential and adequate condition for their reality and in addition plan systems. Studies conveyed in [24], [33], [34] have broadly investigated different components for recreating inner states for multivariable direct frameworks driven by unknown inputs. A portion of the utilization of this kind of observers have been examined in [35] and [ 36]. The resolute mission for strategies to unravel the unknown input observers as prove by the mushrooming research regarding this matter provides a reason to feel ambiguous about no its incentive for applications to frameworks with unsettling influences (or non-quantifiable inputs) and in fault analysis.

Another important type of observer is linear functional observers [ 36]. Rank is determined by using observability index v. One of the major applications of an
observer is the disturbance estimation and fault detection [30] – [32]. Whether to minimise the disturbance in a system and to estimate the linear function of a state vector are crucial to any type of control processes. The proposed method designs a reduced observer which tracks disturbance as well as unknown inputs in a process so that the process can be controlled effectively than before.
8. Recent Research

The progression in the literature shows the relevance of this study in control system engineering. The research methodology includes how the numerical examples can prove the designed system and how the system performance is affected by the contemporary design in terms of linear programming.

No researches have been done recently on a combined study of disturbance estimation and basics in designing an observer on positive linear functional observer for positive systems having unknown inputs [37]. This project describes the state full order observer for a positive system with time delay and disturbance. The observer design is important when the linear variable estimation is needed for the control, stabilization and monitoring purposes must be done.

Even if this project describes mainly the system with disturbance it is also paid some attention to the systems like with delay and without delay. That means a general conclusion comes only after discussing different types of system along with system with disturbance. To reduce the complexity and order the project can be extended to functional observer easily by changing the error dynamics and do the same design again with a functional constant vector F. The only factor that needs to be considered in here is the surety of the positivity.

When the design progress consideration of positivity plays a major role. It leads to some necessary and sufficient conditions which is considered during the design and coding. In future, the software which can be used for this project is MATLAB as this helps to plot the graph of each variable, also helps to do all the calculation and matrix operations.
9. Positive Systems

Positive systems are the most important types of systems under consideration as they have a positive characteristic in all their aspects. Thus, it can be called as a most potent form of classification amongst other systems. These types of systems are positive by nature. The important characteristics of this types of systems are they are having non-negative states. Many physical systems have this property which has the property that whenever the initial conditions are non-negative they have non-negative states or variables. When the controlling of systems comes as first preference these positive constrains plays an important role. Otherwise, it will badly affect the stability of the entire system or it may affect its performance, thus the system will move to the infeasible region when controlling action takes place. As a result, the construction of observers and controllers for this type of systems should be done by preserving its positive constraints are the same on the estimated state. This is where the positive systems play a major role in defining control systems. It makes the theory of positive systems deep and abundant. As stated in [1], a positive system is a system in which the state variables are always positive or at least non-negative in value. As going deeper in to this definition some practical phenomenon can be quoted as examples of positive systems. They are as follows

- Time
- Concentrations
- Money
- Queues
- Data flow
- Temperature
- Volume
- Area
- Level of liquids in tank
- Light intensity levels

The above are a few examples of positive systems that are available in surrounding places. The main is to consider when dealing with positive systems is to maintain its positivity throughout the operation and to hold its positivity or to make the deviations small enough. Especially when the state variables are in equilibrium points the system should be stable as times goes.

There are some basic definitions for positive systems. And these are basic definitions and theorems that considered in this project, which is used to ensure that the system is positive even if the proper observer estimates its variables.

Let the state space representation of a system be

\[ \dot{x}(t) = A \ x(t) \]  \hspace{1cm} (9.1)
Definition 1. A vector $x$ or a matrix $A$ will be said to be positive and denoted by $x > 0$, $A > 0$, provided that its entries are non-negative but at least one of them is positive [1].

Definition 2. A linear system describes in the above representation is said to be a positive linear system iff for any positive initial state vector, the trajectory remains positive for all times [1].

Based on the above definitions the following theorems can be derived for a positive system of type 1.1 and 1.2

Theorem 1. A linear system described by the state representation is positive linear systems if $A > 0$, i.e. the dynamic matrix $A$ is positive [1].

In this project considering the above definitions and theorem the following conditions can be given for a positive system

Definition 3. The linear systems are said to be positive if, for any non-negative initial condition $\varphi(0) \in \mathbb{R}_+^n$, for every $t \in [-\tau, 0]$ and any input $u(t) \in \mathbb{R}_+^m$, for every $t \geq 0$, the corresponding trajectory $x(t) \in \mathbb{R}_+^n$ for all $t \geq 0$ [20].

Definition 4 [20]. A square real matrix $M$ is called Metzler matrix if its off-diagonal elements are non-negative i.e., $m_{ij} \geq 0$, $i \neq j$.

Therefore from [20], we can say the following lemma

Lemma 1. System 1.1 and 1.2 is positive if and only if $A$ is a Metzler matrix and $A$ and $B$ are non-negative matrices [20].

The non-negativity of the estimates must be protected when designing an observer for a positive system. The states should be non-negative at any instant of time. When the controlling of systems comes as first preference these positive constrains plays an important role. Otherwise, it will badly affect the stability of the entire system or it may affect its performance, thus the system will move to the infeasible region when controlling action takes place. The conditions must be considered while designing the observer also when numerically executing the Linear Programming problem. The stability as well as the positivity must be given same attention. These types of systems are positive by nature. The important characteristics of this types of systems are they are having non-negative states. The conditions must be considered while designing the observer also when numerically executing the Linear Programming problem.

The parameter uncertainty and time-delay make the positive system more challenging in control. The change in parameters affect the Properties of the positive systems in such a way that, first it reduces the computation of Eigenvector, secondly it provides direct correspondence between positive equilibrium point and finally stability and comparative statics which is applicable to stable systems. Every stable system has its own equilibrium point at rest. If parameter changes the
system will move to a new equilibrium point. Comparative statics is the question of how the change in equilibrium points is related to the parameter change that produced it.

Positive systems are dynamic systems whose initial conditions are non-negative also its trajectory remains always in the positive quadrant. If a positive system is unstable, then a proper controller being introduced which further makes the system robust and error free. The conditions such as Metzler and Hurwitz, come in design when the matrix selection starts. Also, the derived error dynamics is compared with necessary and sufficient conditions based or the positivity constraints.
10. State Observer for Different Types of Systems

There are always some states that are unavailable for measurements because of physical systems factors. In this situation, observer design is necessary to estimate the state details which are unavailable for the sensor to measure. Since this project deals with positive systems, the observer for this type of system can be called a positive observer. Observer design is useful when direct observation is not possible. In many physical situations, the external states are not possible to determine by using the sensors. Since this project deals with positive systems, the observer discussing in the below area is a positive state observer.

10.1 Introduction

Let’s begin with a basic state space representation of a system of type (1.1) and (1.2). State vector $x(t)$ has all the valuable information about a state variable. Therefore, state space representation has a lot of advantage over transfer function representation. Observer design has a good background on research and it is considered as an innovative topic in control system engineering. If a system thinks that it has some specified variable to measure or considered as its change might affect the system badly, instead of measuring all the variables or estimating all the variables, the observer can be designed specifically to design a variable. Since the characteristics of the systems are determined by the constant matrices in a state space representation, the observer estimation and design include the constant matrices as well. Which means they are equally important when the design of an observer takes place. The variable constraints contained in a system defines states of variables as well as the conditions that the system satisfy certain rules in control engineering. For instance, stability and error dynamics are related to observer design. When the state estimation is completed error, dynamics need to be checked to ensure that the variable that we estimated is of the right form. As error tends to zero as time goes that means there is not a gap between the estimated variable and the real variable, which gives us the information of how effective our design was. If the error converges asymptotically zero which gives the results that the system is stable and in equilibrium. The system can be controlled by a controller without worrying about the system performance and affecting unknown parameters.

The device which regenerates the state vector can be called an observer. The observer is given by inputs and outputs. The estimated or observed variable is given to the controller which uses a control law of the function of input and a constant vector. The output from the control law is given to the reference comparator which compares the reference signal with this control law. This all process is known as compensation. The all process is described in the below block diagram. This is how the feedback to the controller takes place and reconstructing the state variable is done. If the error between the real model and the observed model is zero, then the designed observer is perfect.
Figure explains the way how each block is connected according to the process description and working in a real plant. The control law is a function of input multiplied by a constant vector k. The operation of observer estimation and control law are together known as compensation.

10.1.1 Open Loop

In an open loop system, the input is applied to the process and the process output is compared with a reference set point and the comparator output goes to the controller and controller output is measured. There is no feedback action takes place. In this case, the error will be the value if the plant output subtracted from set point. This process will be ineffective since the controller output can only give us the value of error as digital values at each controller operation. The following block diagram explains the operation of an open loop state feedback controller.

10.1.2 Closed loop

For a system of type (1.1) and (1.2) the control law is

\[ u(t) = -k x(t) \]

this control law is applied to system model as feedback. This is how the closed loop controller works.

\[ \dot{x}(t) = (A - BK) x(t) \]  \hspace{1cm} (10.1)

the error can be reduced to asymptotically zero by defining the gain K matrix and the eigenvalues of the matrix (A-BK) lies on the left-hand side of the S plane. When
designing the observer, the \( u(t) \) input is found by assuming that the state vector information is completely available. That means the estimation of \( x(t) \) should be possible to all extend to define the input vector \( u(t) \). This is known as tracking. The control signal is calculated as the same value as reference and feed back to the controller. The error dynamics becomes zero as tracking comes accurate. To make this tracking good in a system the observer plays an important role. The observer stimulates the system precisely. If the observer is to me accurate and practical the error between the model and the real system should be zero.

### 10.2 Full Order Observer for Linear Systems

full order observer as it represents it estimates all the state variables. State variable order will be the same as the estimated observer. If an observer has \( n \) variables then the state will have \( n \) variables as its state variables. There are some conditions that an observer must satisfy. Amongst them, let \( \hat{x}(t) \) is the estimated variable of \( x(t) \) then a full order observer for the system (1a) and (1b) is of the form as follows

\[
\hat{x}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - c\hat{x}(t))
\]  

(10.2)

Where \( L \) is the gain matrix. The following diagram explains the structure of a full order observer. And the order of matrix \( L \) is \( nxp \). In the observer dynamics the term \( y(t) - C\hat{x}(t) \) is the difference between the real output of the system and the anticipated output. This difference is multiplied by the gain and it estimates the estimated variable using the above dynamics. Thus, given to the control law. This input is given as the real input to the actual plant.

![Diagram](image)

Fig 6: Closed-Loop Compensator (H. Trinh, T. Frernado,2012, Fig.2.3, p.08)

Let calculate the error equation of a full order observer. Estimated error be

\[
e(t) \triangleq x(t) - \hat{x}(t)
\]  

(10.3)

error dynamics can be obtained by taking the derivative on both the sides.
To make error $e(t)$ to zero $(A-LC)$ has negative real parted eigenvalues, so $(A-LC)$ should be Hurwitz. $L$ should be selected based on these conditions. Also, we must make sure the pair $(C, A)$ is observable.

### 10.3 Reduced Order Observer for Linear Systems

Reduced order observer represents the same concept as that of full order observer, the only difference is that the $(n-p)$ state variables are estimated by using an observer has to an order $(n-p)$. The design starting with forming a partitioned matrix from the original matrix as follows [37].

\[
\begin{bmatrix}
    x_p'(t) \\
    \hat{x}_u(t)
\end{bmatrix} =
\begin{bmatrix}
    A_{11} & A_{12} \\
    A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
    x_p(t) \\
    x_u(t)
\end{bmatrix} + \begin{bmatrix}
    B_1 \\
    B_2
\end{bmatrix} u(t) \tag{10.10}
\]

\[
y(t) = \begin{bmatrix} I_p & 0 \end{bmatrix} \begin{bmatrix}
    x_p(t) \\
    x_u(t)
\end{bmatrix} \tag{10.11}
\]

Where $A_{11} \in \mathbb{R}^{p \times p}, A_{12} \in \mathbb{R}^{p \times (n-p)}, A_{21} \in \mathbb{R}^{(n-p) \times p}, A_{22} \in \mathbb{R}^{(n-p) \times (n-p)}$ are constant matrices and $x_u(t)$ represents non measurable states that is $x_u(t) \in \mathbb{R}^{(n-p)}$ and $x_p(t)$ represents measurable states. Here $x_p(t)$ is directly available from y(t) as it has $I_p$ of the dimention p x p and gives the estimation straight from the output equation. Even if $I_p$ is not readily available in the given system then the following method can be used to identify the matrix $I_p$. Where $P = [C^+ C^\perp]$ is an invertible matix. Where $C^+$ denotes the Moore- Penrose inverse of C that means $CC^+ = I_p$ and $C^\perp$ is the orthogonal basis for the null space of C that means $CC^\perp = 0$. Up on finding the partitioning

\[
\hat{x}(t) = \begin{bmatrix}
    x_p(t) \\
    x_u(t)
\end{bmatrix} \tag{10.12}
\]

\[
\bar{A} = P^{-1}AP = \begin{bmatrix} A_{11} & A_{12} \\
A_{21} & A_{22} \end{bmatrix} \quad \text{and} \tag{10.13}
\]

\[
\bar{B} = P^{-1}B = \begin{bmatrix} B_1 \\
B_2 \end{bmatrix} \tag{10.14}
\]

then the system can be reduced to the following format[37].

\[
x_p'(t) = A_{11} x_p(t) + A_{12} x_u(t) + B_1 u(t) \tag{10.15}
\]
\[ \dot{x}_u(t) = A_{21}x_p(t) + A_{22}x_u(t) + B_2u(t) \]  \hspace{1cm} (10.16)

Rearranging the above equation [37]

\[ A_{12}x_u(t) = \bar{y}(t) = x_p'(t) - A_{11}x_p(t) - B_1u(t) \]  \hspace{1cm} (10.17)

\[ x_p'(t) = A_{11}x_p(t) + A_{12}x_u(t) + B_1u(t) \]  \hspace{1cm} (10.18)

\[ \dot{x}_u(t) = A_{21}x_p(t) + A_{22}x_u(t) + B_2u(t) \]  \hspace{1cm} (10.19)

\[ \bar{y}(t) = A_{12}x_u(t) = x_p'(t) - A_{11}x_p(t) - B_1u(t) \]  \hspace{1cm} (10.20)

\[ \hat{x}_u(t) = A_{22}\hat{x}_u(t) + A_{21}x_p(t) + B_2u(t) + L(\bar{y}(t) - A_{12}\hat{x}_u(t)) \]  \hspace{1cm} (10.21)

The graphical arrangement of reduced order observer is as follows

![Reduced Order Observer Schematic Diagram](image)

Fig 7: Reduced Order Observer Schematic Diagram (H. Trinh, T. Fernando, 2012, Fig. 2.5, p. 16)

Where

\[ \hat{x}_u(t) = z(t) + L_y(t) \]  \hspace{1cm} (10.22)

\[ \dot{z}(t) = Fz(t) + Gy(t) + Hu(t) \]  \hspace{1cm} (10.23)

L, F, G and H are constant matrices. The error dynamics can be calculated as

\[ \hat{x}_u(t) = x_u(t) - \hat{x}_u(t) \]  \hspace{1cm} (10.24)

\[ \hat{x}_u(t) = \hat{x}_u(t) - \hat{x}_u(t) = (A_{22} - LA_{12})x_u(t) - F\hat{x}_u(t) \]

\[ + (A_{21} + FL - G - LA_{11})y(t) + (B_2 - H - LB_1)u(t) \]  \hspace{1cm} (10.25)

Then
Provided the pair (C,A) is observable.

The method of converging the error asymptotically to zero, the method is to select the observer gain L whereas the eigenvalues of \((A_{22} - LA_{12})\) sits on the left half of the s-plane. For a single output system, the design of reduced order observer is not need. But when the system has multiple output the reduced order observer plays an important role to reduce the complexity.

All the other design steps are possible only after the partitioning of the augmented system. This can be called similarity transformation with an identity matrix \(P\). Also the obtaining the submatrices comes as the second stage of the design followed by calculation of matrix \(L\). The method pole-placing is used to perform this calculation. The following procedure is to obtain the matrices \(F\), \(G\), \(H\) and them estimated vector is obtained using equation (10.29).

As Reduced order observer represents the same concept as that of full order observer, the only difference is that the \((n-p)\) state variables are estimated by using an observer has to an order \((n-p)\) form the equation of \(\dot{x}(t)\) i.e. equation (10.29) we can see that the order of the estimated matrix vector is reduced.

10.3.1 Design of Reduced-Order State Observers for a Linear Time-Delay Positive System

Reduced order observer represents the same concept as that of full order observer, the only difference is that the \((n-p)\) state variables are estimated by using an observer has to an order \((n-p)\) A delayed system appears in many types of control system. That can be either in the internal system or in output, even it can be in the input. So, it is one of the factors that must be considered while designing an observer. Even if there is a time delay in the system, the complete control must ensure that all the state variables are available through estimation, means the observer estimation. The system equations can be written as

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + A_d x(t - \tau) + Bu(t) \\
y(t) &= Cx(t)
\end{align*}
\]

Where \(\tau > 0\) is constant time delay with an initial condition of

\[
x(\theta) = \phi(\theta).
\]
The reduced order observer (n-p) order estimates the state \( \hat{x}(t) \) which converges in to \( x(t) \) after the estimation.

\[
\dot{z}(t) = w(t) + Ey(t) \tag{10.33}
\]
\[
\dot{w}(t) = Nw(t) + N_a w(t - \tau) + Jy(t) + J_d y(t - \tau) + Hu(t) \tag{10.34}
\]

And let \( \hat{x}(t) \) be \( M_1 y(t) + M_2 \hat{x}(t) \). \( \tag{10.35} \)

By substituting the same equation to the above equation then,

\[
\hat{x}(t) = (M_1 + M_2 E) y(t) + M_2 w(t). \tag{10.36}
\]

The error vector can be calculated as follow

\[
\varepsilon(t) = w(t) - Fx(t) \tag{10.37}
\]
\[
e(t) = \hat{z}(t) - z(t). \tag{10.38}
\]

The conditions satisfy that the observer converges to real state variable must be derived using the error dynamics.

10.3.2 Design of Reduced-Order State Observers for a Linear Positive System without Internal Delay

Reduced order observer represents the same concept as that of full order observer, the only difference is that the (n-p) state variables are estimated by using an observer has to an order (n-p). The system with no internal delay can be written as follows.

\[
\dot{z}(t) = w(t) + Ey(t) \tag{10.39}
\]
\[
\dot{w}(t) = Nw(t) + Jy(t) + J_d y(t - \tau) + Hu(t) \tag{10.40}
\]

with the same initial condition as the above. The term \( w(t - \tau) \) is omitted in case of internal delay whereas when designing a system without delay, considering the above equation changes made to the observer equation as follows.

\[
\dot{w}(t) = Nw(t) + Jy(t) + Hu(t). \tag{10.41}
\]

Where the conditions are as given as

\[
N \text{ is Metzler} \tag{10.42}
\]
\[
JC \geq 0 \tag{10.43}
\]
\[
NF + JC - FA = 0 \tag{10.44}
\]

and the realization of the linear programming (LP) is grouped as follows with a condition that the variables \( n_k \) and \( f_k \) is feasible.
\[ n_{kj} \geq 0, n_{kk} > \sum_{j=1, j \neq k}^{r} n_{kj}, k, j = 1, 2, \ldots, r \quad (10.45) \]

\[ c_i^T j_k^T \geq 0, i = 1, \ldots, n, k = 1, \ldots, r \quad (10.46) \]

\[ f_i^T n_k^T + c_i^T j_k^T = g_{ik}, i = 1, \ldots, n, k = 1, \ldots, r \quad (10.47) \]

These are the LP problem of the reduced observer for system without internal delay. The same approach has been used for this project when it comes to LP section. This will give the designer a clear view of how someone can achieve the sufficient conditions for a system.
11. Functional Observers

The functional observer is a recent term while we speak about the observers. Instead of knowing all the variables and its information a functional observer doesn’t include all the state variable. It gives a functional estimate of a variable. Instead of estimating a whole state vector, the functional observer identifies or estimates only the function of a variable. A functional observer a linear estimation is possible which further makes the system less complex and stable.

The following part describes how the reduced order is defined in terms of rank. Let the observability index be \( \nu \). Where \( \nu \) is the observability index.

The system of the type

\[
\begin{bmatrix}
    x_p(t) \\
    \dot{x}_u(t)
\end{bmatrix} = \begin{bmatrix}
    A_{11} & A_{12} \\
    A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
    x_p(t) \\
    x_u(t)
\end{bmatrix} + \begin{bmatrix}
    B_1 \\
    B_2
\end{bmatrix} u(t)
\]  

(11.1)

\[
y(t) = \begin{bmatrix}
    I_p & 0
\end{bmatrix} \begin{bmatrix}
    x_p(t) \\
    x_u(t)
\end{bmatrix}
\]  

(11.2)

Where \( x_u(t) \) represents non measurable states that is

\( x_u(t) \in \mathbb{R}^{(n-p)} \) and \( x_p(t) \) represents measurable states.

(11.3)

Also it agrees that \( \text{rank} \begin{bmatrix}
    C \\
    CA \\
    CA^2 \\
    \vdots \\
    CA^{\nu-1}
\end{bmatrix} = n. \)  

(11.4)

If \( \nu = 2 \) then,

\[
\text{rank} \begin{bmatrix}
    C \\
    CA
\end{bmatrix} = \text{rank} \begin{bmatrix}
    I_p & 0 \\
    A_{11} & A_{12}
\end{bmatrix}.
\]  

(11.5)

For a perfect observer \((\nu-1) \leq (n-p)\).

when it comes to the realization of a functional linear observer then \( z(t) \) be the state vector needs to be reconstructed and

\[ z(t) \in \mathbb{R}^r \] and

\[ z(t) = F x(t), \]  

(11.6)

where \( F \) is designed by the observer depends on which variable needs to be reconstructed.

If \( F = \begin{bmatrix}
    0 & 0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 0 & 1
\end{bmatrix} \)

then

the \( z(t) \) will be \[ \begin{bmatrix}
    x_4(t) \\
    x_5(t)
\end{bmatrix} \]  

(11.7)
The following diagram explains the functional observer

\[
\begin{align*}
\hat{z}(t) &= w(t) + E y(t) \\
\hat{w}(t) &= Nw(t) + Jy(t) + Hu(t)
\end{align*}
\]

The error can be calculated by using \( e(t) \triangleq w(t) - Fx(t) \)

And the error dynamics gives a set of conditions that should be satisfied in order for \( \hat{z}(t) \) to converge asymptotically to \( Fx(t) \).
12. State Order Observer for a Linear Positive System with Time Delay and Disturbance

Full order state order observer with disturbance and time delay can be constructed by the following architecture. This type of architecture describes a full order observer to estimate the value of \( x(t) \) as it estimates \( \hat{x}(t) \). The designed observer in this project is a full order observer for systems with time delay and disturbance. The observer is as follows.

Let the system under consideration is

\[
\dot{x}(t) = A x(t) + A_d x(t - \tau) + B u(t) + D d(t) \quad (12.1)
\]

\[
y(t) = C x(t) \quad (12.2)
\]

then the designed observer is of the following form

\[
\dot{\hat{x}}(t) = \dot{w}(t) + E y(t) \quad (12.3)
\]

Where

\[
\dot{w}(t) = N w + J y + H u \quad (12.4)
\]

By substitution

The final observer structure is of the form

\[
\dot{\hat{x}}(t) = N \hat{x}(t) + N_1 \hat{x}(t - \tau) + (JC + ECA - NEC)x(t) + (J_1 C + ECA_d - N_1 EC)x(t - \tau) + (H + ECB) u(t) + ECD d(t) \quad (12.5)
\]

Where \( w(t) \) is the observer state vector and \( \hat{x}(t) \) is the estimate of \( x(t) \). The following are the dimensions of the matrices. \( A \in \mathbb{R}^{nxn} \), \( B \in \mathbb{R}^{nxm} \), \( C \in \mathbb{R}^{pxn} \) and \( D \in \mathbb{R}^{rxm} \). And \( N \in \mathbb{R}^{nxn} \), \( J \in \mathbb{R}^{rxp} \), \( H \in \mathbb{R}^{rxn} \) and \( E \in \mathbb{R}^{rxp} \) whereas the dimensions of the state vectors are \( x(t) \in \mathbb{R}^{n} \), \( u(t) \in \mathbb{R}^{m} \), \( y(t) \in \mathbb{R}^{p} \) and \( d(t) \in \mathbb{R}^{m} \).

Fig 9: Observer Architecture for Disturbance Estimation (H. Trinh, T. Frernado, 2012, Fig. 2.6, p. 20)
When it comes to error dynamics, the error vector $e(t)$ can be written as follows

$$e(t) = \dot{x}(t) - x(t)$$  \hspace{1cm} \text{(12.6)}

substituting the value of $\dot{x}(t)$ and $x(t)$ the error dynamics reduced to

$$\dot{e}(t) = Ne(t) + N_1e(t - \tau) + (JC + EAC - NEC + N - A) x(t) + (J_1C + EA_dC - N_1EC + N_1 - A_d)x(t - \tau) + (H + ECB - B)u(t) + (ECD - D)d(t)$$  \hspace{1cm} \text{(12.5)}

From the above equation, $\dot{e}(t) \rightarrow 0$ as $t \rightarrow \infty$ if the following conditions are satisfied.

- $N$ is a Metzler matrix, $N_1 \geq 0$  \hspace{1cm} \text{(12.6)}
- $\dot{e}(t) = Ne(t) + N_1e(t - \tau)$ is asymptotically stable  \hspace{1cm} \text{(12.7)}
- $JC + ECA - NEC \geq 0, J_1C + ECA_d - N_1EC \geq 0$  \hspace{1cm} \text{(12.8)}
- $JC + EAC - NEC + N - A = 0$  \hspace{1cm} \text{(12.9)}
- $J_1C + EA_dC - N_1EC + N_1 - A_d = 0$  \hspace{1cm} \text{(12.10)}
- $H + ECB - B = 0$  \hspace{1cm} \text{(12.11)}
- $ECD - D = 0$  \hspace{1cm} \text{(12.12)}

The proof for the above conditions can be given as two parts, the first part proves that $\dot{x}(t) \in R^n_+$ for all $t \geq 0$ if and only if

- $N$ is Metzler, $N_1 \geq 0$, $JC + EAC - NEC + N - A = 0$ and
- $J_1C + EA_dC - N_1EC + N_1 - A_d = 0$. For that lets form an augmented system of the above observer $\hat{x}(t)$ which estimates the value of $x(t)$.

The augmented system is as follows

$$
\begin{bmatrix}
\dot{x}(t) \\
\dot{\hat{x}}(t)
\end{bmatrix} =
\begin{bmatrix}
A & 0 \\
JC + ECA - NEC & N
\end{bmatrix}
\begin{bmatrix}
x(t) \\
\hat{x}(t)
\end{bmatrix} +
\begin{bmatrix}
A_d & 0 \\
J_1C + ECA_d - N_1EC & N_1
\end{bmatrix}
\begin{bmatrix}
x(t - \tau) \\
\hat{x}(t - \tau)
\end{bmatrix} +
\begin{bmatrix}
B & 0 \\
H + ECB & D & ECD
\end{bmatrix}
\begin{bmatrix}
u(t) \\
d(t)
\end{bmatrix}, \quad \forall \theta \in (-\tau, 0)
$$  \hspace{1cm} \text{(12.13)}

Where $\begin{bmatrix} x(\theta) \\ \hat{x}(\theta) \end{bmatrix} \in R_+^{n+r}, \forall \theta \in (-\tau, 0)$  \hspace{1cm} \text{(12.14)}

Also as $H + ECB \geq 0$ by lemma 1 and $\dot{x}(t)$ is a positive system $\dot{x}(t) \in R^n_+$ for all $t \geq 0$ if and only if $\begin{bmatrix} A & 0 \\
JC + ECA - NEC & N
\end{bmatrix}$ is Metzler and $\begin{bmatrix} A_d & 0 \\
J_1C + ECA_d - N_1EC & N_1
\end{bmatrix} \geq 0$ which is the same as the above. The All the other conditions are sufficient conditions that ensure $\dot{x}(t) \rightarrow x(t)$ as $t \rightarrow \infty$.

The below is the proof for $N$ is Metzler and $N_1 \geq 0, e(t) \rightarrow \infty$ as $t \rightarrow 0$ if and only if $N_1 + N_1$ is Hurwitz.
Let \( N = \begin{bmatrix} -n_{11} & \cdots & n_{1r} \\ \vdots & \ddots & \vdots \\ n_{n1} & \cdots & -n_{rr} \end{bmatrix} \) \( n_{kk} > 0, \; n_{kj} > 0, k \neq j \) \( (12.15) \)

By Gersgorian theorem, \( N_C \) is Hurwitz if \( N_C \) is strictly row/column diagonally dominant.

\( n_{kk} > n_{kk}^1 + \sum_{j=1,j \neq k}^r (n_{kj} + n_{jk}^1), k = 1, 2, \ldots, r \) \( (12.16) \)

Hence \( N_C \) is strictly Metzler & Hurwitz therefore the above condition is satisfied. That means \( \dot{e}(t) = N e(t) + N_1 e(t - \tau) \) must be asymptotically stable.

As an easy representation let express \( N, J, C, N_1, J_1 \) as follows:

\[
N = \begin{bmatrix} n_{11} \\ \vdots \\ n_{r1} \\ \vdots \\ n_{rr} \end{bmatrix}, J = \begin{bmatrix} j_1 \\ \vdots \\ j_r \end{bmatrix}, C = [c_1 \ldots c_n], N_1 = \begin{bmatrix} n_{11} \\ \vdots \\ n_{r1} \end{bmatrix}, J_1 = \begin{bmatrix} j_1 \\ \vdots \\ j_r \end{bmatrix}, j_k = \\
[j_{k1} \ldots j_{kp}], = j_k^1 = [j_{k1}^1 \ldots j_{k1}^1], n_k = [n_{k1} \ldots n_{k1}], n_k^1 = [n_{k1}^1 \ldots n_{k1}^1], i = 1, 2, \ldots, n, k = 1, 2, \ldots, r
\]

The existence of the observer is calculated based on the feasibility of the formulated linear program. The observer can be possible only if the variables in the following LP program \( n_k, n_k^1, j_k, j_k^1 \) is feasible.

\[
n_{kj} \geq 0, n_{kj}^1 \geq 0, k, j = 1, 2, \ldots, r
\]

\[
c_i^T j_k^T - c_i^T E^T n_k^T \geq A^T C^T E^T, i = 1, \ldots, n, k = 1, \ldots, r
\]

\[
c_i^T (j_k^1)^T - c_i^T E^T (n_k^1)^T \geq A_d^T C^T E^T, i = 1, \ldots, n, k = 1, \ldots, r
\]

\[
n_{kk} > n_{kk}^1 + \sum_{j=1,j \neq k}^r (n_{kj} + n_{kj}^1), k = 1, 2, \ldots, r
\]

\[
c_i^T j_k^T - w_i^T n_k^T + n_k^T = g_{ik}, i = 1, \ldots, n, k = 1, \ldots, r
\]

\[
c_i^T (j_k^1)^T - w_i^T (n_k^1)^T + (n_k^1)^T = h_{ik}, i = 1, \ldots, n, k = 1, \ldots, r
\]

Where \( g_{ik}, h_{ik}, w_i \) are the components of the matrices \( (A^T - A^T C^T E^T) \), \( (A_d^T - A_d^T C^T E^T) \) and \( C^T E^T \)

And the matrices \( E \) and \( H \) can be found out using the rest of the sufficient conditions

\( ECD = D \) \( (12.23) \)

\( D(\varepsilon C I - I) = 0 \) \( (12.24) \)

\( D \neq 0, \) \( (12.25) \)

\[ E = D(CD)^+ + Z(I_p - (CD)(CD)^+) \] \( (12.26) \)
where \((CD)^+\) is the Moore-Penrose inverse of \((CW)\) and \(Z\) is an arbitrary matrix of dimension \(Z \in \mathbb{R}^{nxp}\).

The solution for \(E\) exist only if \(\text{rank}\left[\begin{bmatrix} CD \\ D \end{bmatrix}\right] = \text{rank}\ [CD]\). (12.27)

As the rank of a full column matrix \(D\) is \(m\) i.e. \(\text{rank} (D) = m = \text{rank} (CD)\) is a sufficient condition for the existence of disturbance observer.

Substituting the value of \(E\) in \(H + ECB - B = 0\), (12.28)

\[ \rightarrow H = (I_n - EC)B \] (12.29)

Hence the value of \(E\) and \(H\) can be find out.
13. Design Algorithm

The design algorithms describe how the derivation and mathematical calculations done based on the architecture chosen. As there are no literature in detail about how these conditions are achieved even if the system under consideration is different, these page gives the reader the basic approach that must be done before simulating the real examples or numerical examples as instance. Also, dimensions discussed here will give a clear information to the reader about the steps must be followed to reach a conclusion about a positive observer with time delay and disturbance estimation.

As this project needs more time frame, the design approach and linear programming problem formulation is discussed in very detail. This work will give a next researcher a detail literature review at a glance with all the very basics. This work opens a wide research in future as it gives all the aspects about designing an observer for a positive system with time delay and unknown inputs and converting the sufficient and necessary conditions to linear programming problem.

Observer dynamics is discussed as below

\[ \dot{x}(t) = \dot{w}(t) + Ey(t) \]  
\[ \dot{x}(t) = \dot{w}(t) + EC\dot{x}(t) \]

The reason why this architecture is used is because as we can see the output dynamics is feedbacked through a constant matrix E which makes the structure more consistent and less distracted to output disturbance. If there is any sort of disturbance happens on the output for instance because of the sensors which measure output or any sort of by-product which from at the output: this architecture solves those disturbance up to some extent.

Substituting the value of \( \hat{x}(t) \) from (14.1) and (14.2) yields the following equation

\[ \dot{x}(t) = Nw(t) + N_1w(t - \tau) + Jy(t) + J_1y(t - \tau) + Hu(t) + EC \left[ A x(t) + A_d x(t - \tau) + B u(t) + D d(t) \right] \]  
(13.3)

Rearranging the above equation

\[ \dot{x}(t) = Nw(t) + N_1w(t - \tau) + JCx(t) + J_1Cx(t - \tau) + Hu(t) + ECA x(t) + EC A_d x(t - \tau) + ECB u(t) + ECD d(t) \]  
(13.4)

Adding some terms to get the equation in appropriate form for error dynamics

\[ \dot{x}(t) = Nw(t) + NECx(t) - NECx(t) + N_1w(t - \tau) + N_1ECx(t - \tau) - N_1ECx(t - \tau) + JC + ECA)x(t) + (J_1C + ECA_d)x(t - \tau) + (H + ECB) u(t) + ECD d(t) \]  
(13.5)
\begin{align*}
\dot{x}(t) &= N((w(t) + ECx(t)) - NECx(t) + N_1((w(t - \tau) + ECx(t - \tau)) - \bar{N}_1 ECx(t - \tau) + (JC + ECA)x(t) + (J_1 C + ECA_d)x(t - \tau) + (H + ECB) u(t) + ECD d(t)) \\
\dot{\hat{x}}(t) &= N\hat{x}(t) - NECx(t) + N_1\hat{x}(t - \tau) - N_1 ECx(t - \tau) + (JC + ECA)x(t) + (J_1 C + ECA_d - N_1 EC)x(t - \tau) + (H + ECB) u(t) + ECD d(t) \\
\dot{\tilde{x}}(t) &= N\tilde{x}(t) + N_1\tilde{x}(t - \tau) + (JC + ECA - NEC)x(t) + (J_1 C + ECA_d - N_1 EC)x(t - \tau) + (H + ECB) u(t) + ECD d(t) - A x(t) - A_d x(t - \tau) - B u(t) - D d(t) \\
\dot{\bar{e}}(t) &= N\bar{e}(t) + N_1 \bar{e}(t - \tau) + (JC + ECA - NEC - A)x(t) + (J_1 C + ECA_d - N_1 EC - Ad)x(t - \tau) + (H + ECB - B) u(t) + (ECD - D) d(t)
\end{align*}

Where once again confirming the dimensions of the above matrices are defining as

\begin{itemize}
  \item \(A \in R^{nxn}, A_d \in R^{nxn}, B \in R^{nxm}, C \in R^{pxn}, D \in R^{rxm}, N \in R^{nxn}, N_1 \in R^{nxn}, J \in R^{nxp}, J_1 \in R^{nxp}, H \in R^{nxn}, E \in R^{nxp}\)
\end{itemize}

And the dimensions of vectors come as follows

\begin{itemize}
  \item \(x(t) \in R^n, u(t) \in R^m, y(t) \in R^p, d(t) \in R^m\)
\end{itemize}

Error Dynamics is explained as shown below

Error dynamics gives rise to necessary conditions that must be formulated as linear programming late in discussion. The real aim of designing an observer is to get the error dynamics converges to zero as times goes. Which denotes as \(\dot{e}(t) = 0 \ as \ t \to \infty\).

\begin{align*}
   e(t) &= \hat{x}(t) - x(t) \\
\dot{e}(t) &= \dot{x}(t) - \dot{x}(t) \\
\dot{\bar{e}}(t) &= N\bar{e}(t) + N_1 \bar{e}(t - \tau) + (JC + ECA - NEC - A)x(t) + (J_1 C + ECA_d - N_1 EC - Ad)x(t - \tau) + (H + ECB - B) u(t) + (ECD - D) d(t)
\end{align*}

Adding some terms to get the equation in appropriate form for error dynamics

\begin{align*}
\dot{\bar{e}}(t) &= N\bar{e}(t) + N x(t) - N x(t) + N_1 \tilde{x}(t - \tau) + N_1 x(t - \tau) - N_1 x(t - \tau) + (JC + ECA - NEC - A)x(t) + (J_1 C + ECA_d - N_1 EC - Ad)x(t - \tau) + (H + ECB - B) u(t) + (ECD - D) d(t) \\
\dot{\bar{e}}(t) &= N(\hat{x}(t) - x(t)) + N_1(\hat{x}(t - \tau) - x(t - \tau)) + (JC + ECA - NEC - A + N)x(t) + (J_1 C + ECA_d - N_1 EC - Ad + N_1)x(t - \tau) + (H + ECB - B) u(t) + (ECD - D) d(t)
\end{align*}

Substituting the value of \(\hat{x}(t) - x(t)\) as \(e(t)\)
\[ \dot{e}(t) = Ne(t) + N_1 e(t - \tau) + (JC + ECA - NEC - A + N)x(t) + (J_1 C + ECA_d - N_1 EC - Ad + N_1)x(t - \tau) + (H + ECB - B) u(t) + (ECD - D) d(t) \]

(13.16)

Now, the error dynamics and observer equations are derived, so the necessary and sufficient condition for the above architecture to be a successful positive observer is as follows.

To get error dynamics as zero \( \dot{e}(t) = 0 \) as \( t \to \infty \) and based on equation (13.13) discussed in the previous chapter

\[ N \text{ is Metzler, } N_1 \geq 0 \]  
(13.17)

\[ \dot{e}(t) = Ne(t) + N_1 e(t - \tau) \] is asymptotically stable  
(13.18)

\[ JC + ECA - NEC \geq 0 \]  
(13.19)

\[ J_1 C + ECA_d - N_1 EC \geq 0 \]  
(13.20)

\[ JC + ECA - NEC - A + N = 0 \]  
(13.21)

\[ J_1 C + ECA_d - N_1 EC - Ad + N_1 = 0 \]  
(13.22)

\[ H + ECB - B = 0 \]  
(13.23)

Proof has explained in the previous chapter

The corresponding Linear programming problem for the above conditions are found as follows

\[ n_{kj} \geq 0, n_{kj}^1 \geq 0, k, j = 1,2, ..., r \]  
(13.24)

\[ c_i^T j_k^T - c_i^T E^T n_k^T \geq A^T C^T E^T, i = 1, ..., n, k = 1, ..., r \]  
(13.25)

\[ c_i^T (j_k^1)^T - c_i^T E^T (n_k^1)^T \geq A_d^T C^T E^T, i = 1, ..., n, k = 1, ..., r \]  
(13.26)

\[ n_{kk} > n_{kk}^1 + \sum_{j=1,j\neq k}^r (n_{kj} + n_{kj}^1), k = 1,2, ..., r \]  
(13.27)

\[ c_i^T j_k^1 - w_i^T n_k^T + n_k^T = g_{ik}, i = 1, ..., n, k = 1, ..., r \]  
(13.28)

\[ c_i^T (j_k^1)^T - w_i^T (n_k^1)^T + (n_k^1)^T = h_{ik}, i = 1, ..., n, k = 1, ..., r \]  
(13.29)

Where \( g_{ik}, h_{ik}, w_i \) are the components of the matrices \( (A^T - A^T C^T E^T), (A_d^T - A_d^T C^T E^T) \) and \( C^T E^T \)

Feasibility of the Formulated LP Problem is discussed below

By considering the numerical example as follows as a fifth order time delay system with a disturbance matrix.
\[
A = \begin{bmatrix}
-15 & 2 & 3 & 1 & 2 \\
2 & -9 & 1 & 3 & 2 \\
1 & 2 & -14 & 1 & 0 \\
3 & 1 & 1 & -10 & 0 \\
1 & 2 & 3 & 3 & -16 \\
\end{bmatrix},
B = \begin{bmatrix}
1 \\
2 \\
3 \\
4 \\
5 \\
\end{bmatrix}
\]

This system is positive as \( A \) is Metzler and \( A^1, B \geq 0 \). The calculation of matrices \( g_{ik}, h_{ik}, w_i \) can be done and rewrite the linear programming problem for \( i = j = k = 1 \) as follows.

\[
n_{11} \geq 0, n_{11}^1 \geq 0,
\]

\[
c_1^T j_1^T - c_1^T E^T n_1^T \geq 0
\]

\[
c_1^T (j_1^1)^T - c_1^T E^T (n_1^1)^T \geq 0
\]

\[
n_{11} > n_{11}^1 + (n_{21} + n_{11}^2)
\]

\[
c_1^T j_1^T - c_1^T E^T n_1^T + n_1^T = 0
\]

\[
c_1^T (j_1^1)^T - c_1^T E^T n_1^T + (n_1^1)^T = 0
\]

The solution is feasible by graphical method as we are getting the constraints all along the positive quadrant of the real axis.
14. Result Analysis, Discussion and Future Scope

This project gives an overall basic idea behind where and how to start the design of an observer for any type of system. As the literature focused on the positive systems, the design is done based on Positive observer. Since there is no one has done a design for a positive observer with time delay and unknown input, this project will open an innovation to the research fellows to start with. The major steps of designing an observer can be concluded as follows.

Step 0: Start

Step 1: Propose an architecture for a system under consideration

Step 2: Derive the observer equation

Step 3: Derive error dynamics

Step 4: Collect the conditions which satisfy the error dynamics is become zero as times tends to infinity

Step 5: Formulate a Linear Programming (LP) based on results in Step 3 and Step 4

Step 6: Validate the existence of the proposed observer by checking its feasibility

Step 7: Execute a numerical example and find the values of unknown variables

Step 8: Find the estimation of the variable using a simulation platform as MATLAB

Step 9: Stop

Step 7 to 8 has not done as part of this work as the time frame was less. Need more future work and time frame to do the simulation and plotting the estimation. But this can be done as a perfect future work as an extension of this project as all the other information is collectively discussed here. Also, by extending the work in to discrete systems, systems with multiple delay, systems with internal input and output delay further gives the researcher an exact future scope of study of the control engineering.

The same approach can be extended to functional observer which reduces the complexity and gives the linear estimation of the unmeasurable state variable. It is less time consuming for a future student who will be doing a work related to positive observer, to read and understand the detailed work explained throughout this project. As there is no other literature on this topic yet, this will improve the effectiveness and the milestone in Observer design.

This work can be further extended to real time systems by choosing the variables and do a mathematical modelling first before starting the design procedure. The will improve the practical significance and very much effective from application point of view.
15. Conclusion

In this work, I have explained results of designing Positive observer for a linear time delayed-unknown input system. All the necessary and sufficient conditions are derived, and Linear Programming L P has been done to find whether the proposed architecture exist or not. The thorough explanation of the mathematical derivation behind an observer design is very effective for a researcher to understand and implement in future. The different cased of observer has discussed which improve the effectiveness of the project. The clear-cut explanation of the design improves the accuracy and functionality of this work and make it as a log book from a designer point of view. The MATLAB coding of the given example is given in the appendix section for future reference.
References


Appendix

The MATLAB coding for the given example is quoting below for the future work.

clc

clear

A=[-15 2 3 1 2;
    2 -9 1 3 2;
    1 2 -14 1 0;
    3 1 1 -10 0;
    1 2 3 3 -16];

A1=10^(-1)*[10 2 3 4 50;
    2 10 40 30;
    10 2 30 5 0;
    2 10 4 20 0;
    1 3 2 10 60];

eig(A+A1);

C=[1 0 0 0 0;
    0 1 0 0 0];

B=[1;2;3;4;5];

D=[-1;0;0;0;0];

M=C*D;
m=pinv(M);
E=D*m;

%MOORE-PEMROSE INVERSE TO CALLCULATE E
% E=D*MOORE-PEMROSE INVERSE(C*D)

In=[1 0 0 0 0;
    0 1 0 0 0;
    0 0 1 0 0;
    0 0 0 1 0;
    0 0 0 0 1];

%Y=(In-E*C);
%N=Y*A-K*C;
\%N1=Y*A1-K*C;
\%p=[-1 -1.1 -1.2 1.3 -1.4];
\%K=place(A',C',p.');
\%J=K+N*E;
\%J1=K+N1*E;
\%Simulation

t=0:0.001:40;
tau=1000;
\%Initail condition for x(t)
for i=1:tau+1
    x1(i)=1;
    x2(i)=2;
    x3(i)=3;
    x4(i)=4;
    x5(i)=5;
end
y1(i)=x1(i);
y2(i)=x2(i);

for i=1:40000;
    \%    u(i)=0.1*exp(0.0001*i);
    u(i)=1+sin(-0.001*i);
    d(i)=1+sin(-0.001*i);
end
for i=tau+1:40000;
x1dot(i)=A(1,1)*x1(i)+A(1,2)*x2(i)+A(1,3)*x3(i)+A(1,4)*x4(i)+A(1,5)*x5(i)+A1(1,1)*x1(i-tau)+A1(1,2)*x2(i-tau)+A1(1,3)*x3(i-tau)+A1(1,4)*x4(i-tau)+A1(1,5)*x5(i-tau)+B(1,1)*u(i)+D(1,1)*d(i);
x2dot(i)=A(2,1)*x1(i)+A(2,2)*x2(i)+A(2,3)*x3(i)+A(2,4)*x4(i)+A(2,5)*x5(i)+A1(2,1)*x1(i-tau)+A1(2,2)*x2(i-tau)+A1(2,3)*x3(i-tau)+A1(2,4)*x4(i-tau)+A1(2,5)*x5(i-tau)+B(2,1)*u(i)+D(2,1)*d(i);
x3dot(i)=A(3,1)*x1(i)+A(3,2)*x2(i)+A(3,3)*x3(i)+A(3,4)*x4(i)+A(3,5)*x5(i)+A1(3,1)*x1(i-tau)+A1(3,2)*x2(i-tau)+A1(3,3)*x3(i-tau)+A1(3,4)*x4(i-tau)+A1(3,5)*x5(i-tau)+B(3,1)*u(i)+D(3,1)*d(i);
x4dot(i)=A(4,1)*x1(i)+A(4,2)*x2(i)+A(4,3)*x3(i)+A(4,4)*x4(i)+A(4,5)*x5(i)+A1(4,1)*x1(i-tau)+A1(4,2)*x2(i-tau)+A1(4,3)*x3(i-tau)+A1(4,4)*x4(i-tau)+A1(4,5)*x5(i-tau)+B(4,1)*u(i)+D(4,1)*d(i);
x5dot(i)=A(5,1)*x1(i)+A(5,2)*x2(i)+A(5,3)*x3(i)+A(5,4)*x4(i)+A(5,5)*x5(i)+A1(5,1)*x1(i-tau)+A1(5,2)*x2(i-tau)+A1(5,3)*x3(i-tau)+A1(5,4)*x4(i-tau)+A1(5,5)*x5(i-tau)+B(5,1)*u(i)+D(5,1)*d(i);
\begin{verbatim}
x1(i+1)=x1(i)+0.001*x1dot(i);
x2(i+1)=x2(i)+0.001*x2dot(i);
x3(i+1)=x3(i)+0.001*x3dot(i);
x4(i+1)=x4(i)+0.001*x4dot(i);
x5(i+1)=x5(i)+0.001*x5dot(i);

y1(i+1)=x1(i+1);
y2(i+1)=x2(i+1);
end

plot(t,y1,'r',t,y2,'b');
plot(t,x3);
\%plot(t,x4);
\%plot(t,x5);
\end{verbatim}

Fig: response of variable x3 vs time

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