A review of size-dependent continuum mechanics models for micro- and nano-structures

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Review

A review of size-dependent continuum mechanics models for micro- and nano-structures

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ABSTRACT

Recently, the mechanical behavior of micro-/nano-structures has sparked an ongoing debate, which leads to a fundamental question: what steps can be taken to investigate the mechanical characteristics of these structures, and characterize their performance? From the standpoint of the non-classical behavior of materials, size-dependent theories of micro-/nano-structures can be considered to analyze their mechanical behavior. The application of classical theories in the investigation of small-scale structures can lead to inaccurate results. Many studies have been published in the past few years, in which continuum mechanics models have been used to investigate micro-/nano-structures with different geometry such as rods, tubes, beams, plates, and shells. The mechanical behavior of these systems under different loading – resulting in vibration, wave propagation, bending, and buckling phenomena – is the focus of the review covered in this work. The present objective is to provide a detailed survey of the most significant literature on continuum mechanics models of micro-/nano-structures, and thus orient researchers in their future studies in this field of research.

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1. Introduction

Because of the growing applications of micro-/nano-electro-mechanical systems (M/NEMS) in various engineering disciplines including mechanics, civil, electrical, medical, as well as aerospace, modeling their behavior has become particularly important. Moreover, modeling of the mentioned structures requires an accurate understanding of their physical (mechanical, electrical, thermal, etc.) characteristics, which can be defined in rods, tubes, beams, plates, and shells. Table 1 gives information on the schematics representation of different types of micro-/nano-structures including micro/nanorods, micro/nanobeams, micro/nanoplates, and micro/nanoshells, are proposed in this research work [1–6]. It is interesting to note that after the discovery of technologies that allow to manufacture small-sized components, several practical applications regarding employing micro/nano-structures in different fields of engineering were raised [7–12]. For instance, the design and manufacturing of nano-fluid or nanoparticles in targeted drug delivery systems as well as micro/nano-robots in different categories of surgeries can be considered as major practical applications of these structures in medical engineering problems. Furthermore, numerous examples can be proposed in the design of components for airplanes and spacecrafts, submarines, smart electrical circuits, and so on. In order to model small-scale structures, several approaches can be utilized, such as molecular dynamic (MD), density functional theory (DFT), and tight-binding MD [13–17], which are all considered as atomistic simulation theories, continuum mechanics as well as the combination of them. On the other hand, classical and non-classical continuum mechanical models are simpler to analyze than atomistic models. Because of the possible lack of accuracy of classical continuum mechanics in modeling small-sized structures, various size-dependent effects were introduced in the literature. They give non-classical continuum mechanical models, as the nonlocal elasticity of Eringen, the modified couple stress theory, the strain gradient model, the Cosserat theory, the micromorphic model, and the nonlocal-stress gradient (as a hybrid model) theory [18–50]. Many researchers such as Reddy [35], Lim et al. [51], Gutierrez Rivera et al. [52], Fernández-Sáez et al. [53] and Khodabakhshi and Reddy [54], Challamel et al. [55–61], Ma et al. [62], Papargyri-Beskou et al. [63], Srinivasa and Reddy [64], Al-shujairi et al. [65], Wang and Wang [66], Wang et al. [67], Duan & Wang [68],

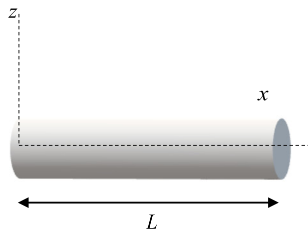
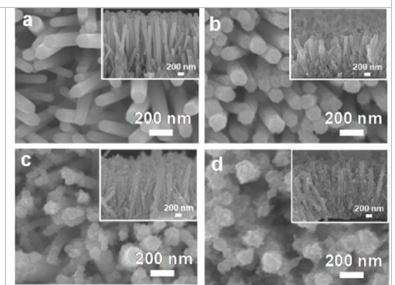
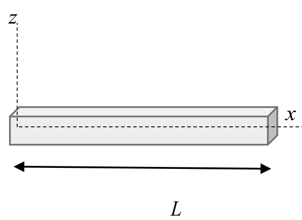
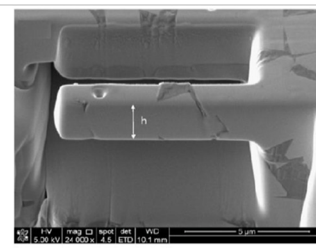
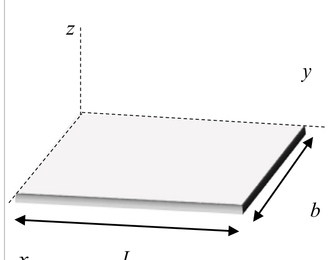
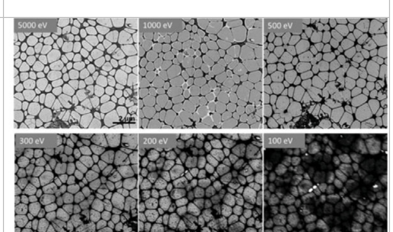
Lim and Wang [69], Challamel and Wang [70], Wang et al. [71], Wang [72], Arash and Wang [73], Wang and Liew [74], Hu, et al. [75], Wang and Varadan [76], Wanget al. [77], Wang et al. [78], Wang and Arash [79], Lin et al. [80], Lim and He [81], He et al. [82], Lim et al. [83], Lim [84], Zhou et al. [85], Scarpa et al. [86], Chowdhury et al. [87] , Murmu and Adhikari [88,89], Murmu et al. [90], Lei [91], Ranjbartoreh et al. [92,93], Ghorbanpour Arani et al. [94–96], Ghorbanpour Arani and Roudbari [97], Lü et al. [98,99] , Liu et al. [100], Liu and Chen [101], Zhang et al. [102], Ansari et al. [103–106], Elishakoff et al. [107–109], Hache et al. [110] studied the static and dynamic behavior of non-classical continuum mechanics models associated with vibration [104–146], wave propagation [147–165], and buckling [166–186] of micro-/nano-structures. Based on their kinematic relations and geometrical aspects, linear and nonlinear simulations were considered. In this regard, different analytical, semi-analytical, and numerical approaches were carried out to ascertain the fundamental variation of their models based on the related physical parameters.

It is significant to mention that there are many advantages, drawbacks, limitations, as well as key applications pertinent to using various types of size-dependent methods. Some researchers claimed that using the nonlocal elasticity theory, the modified couple stress theory, and the strain gradient model can change the stiffness estimation of the micro/nano-structures. For instance, by employing the nonlocal Eringen model, in particular at the higher values of the nonlocal parameters, the softening behavior can be observed in the small-scale structures, while using the modified couple stress or the strain gradient models yield hardening behavior of them. Therefore, the weaknesses pertinent to using the mentioned models, especially with their unconventional parameters, can restrict their application in the design of micro/nano-structures. Hence, having a hybrid size-dependent model can lead to having a more realistic model at micro/nano-scale. Table 2 provides comprehensive information related to the advantages, drawbacks, limitations, and key applications of using small-scale methods in practice [26,31–33,53,62,70,121,122,143,162,187–209].

On the other hand, the applications of the continuum-based models for the analysis of micro/nano sized-structures can be scrutinized based on the different loading conditions including thermal–electrical–magnetic–mechanical applied in the proposed models. Various mechanical behavior can be addressed under different external loadings, which can affect noticeably stiffness, stress, strain, and deformation of the micro/nano-structures. Therefore, the importance of investigating the effects of Fourier/non-Fourier heat conduction, coupled thermoelasticity analysis including the classical coupled and Lord-Shulman coupled models, uncoupled magneto-thermoelasticity analysis, as well as electro-thermoelasticity behavior is essential on the mechanical behavior of micro/nano sized-structures [210–229].

Moreover, various types of nano-material structures were investigated such as carbon (C), boron nitride (BN), and silicon carbide (SIC) with different shapes (tube, graphene, powder, and particle), sizes, number of layers (single-walled (SW), double-walled (DW), and multi-walled (MW)) and also crystal structures [230–244]. Figs. 1–3 demonstrate the different shapes of SWCNTs, DWCNTs, MWCNTs,

Table 1
Schematics representations of micro/nano-structures [1–3,5,6]. Reprinted with permission from Elsevier.

Structures	Schematic representation	
Micro/nano rods		 Scanning electron micrograph (SEM) images of (a) ZnO nanorod-P, (b) ZnO nanorod-T1, (c) ZnO nanorod-T2, and (d) ZnO nanorod-T5 [1]. Reprinted with permission from Elsevier.
Micro/nano beams		 Cantilever microbeam machining to calculate the microbeam thickness [2]. Reprinted with permission from Elsevier.
Micro/nano plates		 Micrographs of multi-layered graphene with 3-5 layers, 5000-100eV [3]. Reprinted with permission from Elsevier.

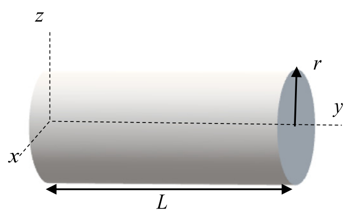
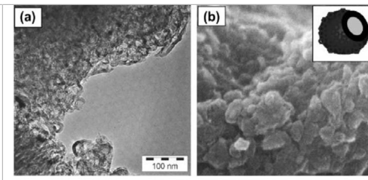
(*continued on next page*)

BNNTs, and SICNTs [245–251]. Also, Table 3. gives information on the crystallographic properties of h-BN, h-graphite, and h-SIC (h-means hexagonal) [247–279].

Some researchers studied the integral-based nonlocal theory instead of differential models. Because of some limitations peculiar to differential forms of the nonlocal theory in modeling structures under static and dynamic analysis, some new models were defined such as integral forms of the nonlocal theory and the combination of this theory with micro-structure models [53–57,252–271].

Recently, some investigations were carried out on the effect of surface stress on micro-/nano-structures [272–276], which can influence the stress and deformation characteristics. It is interesting to state that, since the surface-to-volume ratio is very large at the small-scale sizes, the effects of surface parameters could be important. Based on the assumption proposed by researchers, the surface is formed of a film with zero thickness value sticking to a substrate without slipping [272–276]. Generally, it is important to mention that, in practice, the presence of surface free energy effects in nanostructures has never been proven.

Table 1 (*continued*).

Micro/nano shells		 Images of nanoshell carbon by (a) transmission electron microscope (TEM) and (b) SEM [5]. Reprinted with permission from Elsevier.  The silicon cylindrical nanoshell simulated by LAMMPS [6]. Reprinted with permission from Elsevier.

Over the last decade, many interesting papers regarding the review of micro/nano-structures were proposed. Khaniki et al. [277] worked on the static/dynamic electrically actuated M/NEMS structures. Their review covered studies pertinent to the variation of constant/time-varying voltages as well as various theories or models and comparing their outputs together. Awrejcewicz et al. [278] examined size-dependent influences of various models including beams, plates as well as shells. They focused on some specific models such as the Euler–Bernoulli beam, Timoshenko beam, Kirchhoff plate, and Kirchhoff–Love shell. Likewise, various size-dependent effects including nonlocal elasticity, modified couple stress method as well as modified deformation gradient method using higher-order shear deformation theory were discussed in their review. Another review paper about the mechanical behavior of nanorods/beams/tubes/plates using nonlocal theories was given by Hosseini et al. [279]. They considered the literature on nonlocal as well as nonlocal strain-gradient models in their results. Thai et al. [280] overviewed size-dependent continuum models related to beams/plates/shells with attention to classical models as well as first-order/higher-order shear deformation models. They focused on the studies related to different size-dependent models such as the nonlocal Eringen method, modified couple stress method, and strain gradient method. It is worth mentioning that the major differences between the present review and the above mentioned ones are the assessment and overview of the most significant achievements related to the effects of the various size-dependent methods on different micro/nano-structure models proposed in the literature presenting suitable classifications. In this regard, the main motivation of this study is to thoroughly review the published literature and discuss future areas to be developed in the field of continuum mechanics at the micro/nano-scale applied to thin-walled structural elements.

The current study is organized in six sections, as follows: Section 2 focuses on the applications of couple stress and strain gradient models in statics and dynamics of micro-structures. Section 3 investigates the nonlocal elasticity properties of nanotubes, nanobeams, and nanoplates. Section 4 surveys the review of micromorphic models. The nonlocal strain-gradient hybrid models are discussed in Section 5.

2. Couple stress and strain gradient models

In micro-scale structures, size-dependent deformation characteristics were investigated in metal structures [281–286] and polymer structures [287,288], in which the size dependence can be captured using a classical couple stress theory while stretch and dilation gradients appear to be nonexistent. A non-classical continuum model can be introduced as the couple stress model in which higher-order stresses are considered as the couple stresses [289]. Yang et al. [290] introduced a modified couple stress theory with a novel higher-order equilibrium equation. They only employed conventional equilibrium equations of forces as well as moments of forces. In order to demonstrate the influences of modification, they considered a cylindrical bar and a flat plate and examined the torsion and pure bending of them, respectively. Different linear and nonlinear beam, plate, and shell theories can employ this theory to reveal the size-dependent effects [291–363]. Lam et al. [291] worked on the modified strain gradient theory, in which a set of higher-order metrics was expanded to describe the strain gradient model. Indeed, they added a novel additional equilibrium relation to the classical equilibrium relations of forces as well as moments to control the behavior of higher-order stresses. They proved that using this new model can reduce independent length scale parameter numbers from five to three including rotation, dilation, and stretch gradients, which is autonomous of the non-symmetric rotation gradient tensor. Their proposed total energy function can be written as follows:

$$U = U(\varepsilon_{ij}, \varepsilon_{mm}, i, \eta^{(1)}_{ijk}, \chi^{s}_{ij}), \tag{1}$$

Also, for elastic center-symmetric isotropic materials pertinent to the linear model, the constitutive relation is a quadratic equation of the constant strain metrics as follows:

$$U = \frac{1}{2}k\varepsilon_{ii}\varepsilon_{jj} + \mu\varepsilon'_{ij}\varepsilon'_{ij} + \mu l_0^2\varepsilon_{mm.i}\varepsilon_{nn,i} + \mu l_1^2\eta^{(1)}_{ijk}\eta^{(1)}_{ijk} + \mu l_2^2\chi^{s}_{ij}\chi^{s}_{ij}, \tag{2}$$

where ε_{ij}, $\varepsilon_{mm,i}$, $\eta^{(1)}_{ijk}$, χ^{s}_{ij}, and ε'_{ij} are, respectively, the strain tensor, the dilatation gradient vector, the deviatoric stretch gradient tensor, the symmetric rotation gradient tensor, as well as deviatoric strain. Likewise, l_0, l_1, l_2 present additional material length scale factors which indicate the size-dependent effects and are, respectively, dilatation, deviatoric stretch, and rotation gradients. Then, the constitutive equations

Table 2
The advantages, drawbacks, limitations, and key applications of using small-scale methods [26,31–33,53,62,70,121,122,143,162,187–209].

Size-dependent theories	Advantages-drawbacks-limitations	Key applications
Couple stress and strain gradient theories	-The formulation remains local in which stresses as well as higher-order stresses are pertinent point-wise to strains and higher-order strains [187]. - The governing relations and constitutive laws have relative complexity [188]. -Simple for the conventional FE codes [188]. -Involving asymmetric couple stress tensor as well as only one material length scale parameter in the modified model [189]. -The incapability to address some problems with more than one material length scale parameter [189]. -Using rotation–displacement as a dependent variable [195] -Better in numerical implementation and the representation of elastic characteristics [202]. -PDE indicating the continuum behavior has not been rigorously obtained and the thermodynamical stability of the model has not been cleared [202].	-Flow and creep of polycrystals [187–189]. -Two-phase alloys [187–189]. -Reinforced composites [187–189]. -Localization and hear bands [187–189]. -Crack propagation [187–189]. -Micro-indentation [187–189]. -Optical phononicrcrystals/meta-materials [193]. -Grain boundary influences in bi-crystals [195]. -Microscale safety devices [195]. -Micro-resonators/actuators [195]. -AFM [195].
Nonlocal elasticity of Eringen theory	-Acceptable results in various ranges on the nonlocal parameters [143]. -Having more flexible structures which are applicable for some specific design [190]. -Having more reasonable results peculiar to the integral form for the sake of the energy consistent quadratic relations [31–33] -Nonself-adjointness of energy performance [194–197]. - Using the differential model of Eringen theory in particular for the specific class of attenuation functions [26,53]. -Paradox outcomes pertinent to the obtained eigenfrequency values against the classical model (Softening response against hardening behavior of the model) [53,70] -Having contrary results for some special cases of boundary conditions and loading such as a concentrated load at free end, or a clamped–pinned as well as a clamped–clamped structure [53,70] - Having non-conservative problems for the differential form [53,70,196–198]	-Dislocations and disclinations (the elastic strain energy of defects) [53,196–198] -Wave dispersion especially for higher values and surface waves [31,32] -Stress field at the Griffith crack [31,32] -M/NEMS cantilever actuators [195] -Fracture and plastic yielding [31,32]
Cosserat theory	-Considered as a particular model of complex medium [200]. -Regularized descriptions of softening materials [200] -Show deformations localized onto bands of vanishing width [201]. -Numerical advantages associated with strain localization, reduced integration, FE formulation, as well as hourglass control [201]. -Improved kinematics and the non-symmetry of the stress tensor [202]. -Offering a more perceptive analysis of the nano-filler matrix mechanical interactions with preserving simplicity [202].	-Electrically polarized media [200]. -Granular materials [200]. -Rock mechanics [201]. -Soil mechanics for non-associative plastic flow behavior [201]. -Biological tissues [201]. -Energy harvesting [202]. -Localization in geomaterials [202].
Micromorphic theory	-Having the simplicity and the diversity of the numerical implementation by employing a scalar modification [203]. -Defect nucleation and evolution in solids such as dynamic crack propagation [204]. -Having the generality and can be utilized for a broad range of hierarchical material possessing characteristics across arbitrary numbers of scales [205]. -Capability of revealing the size-dependent influences arising from the micro-structure [206]. -Useful in investigating complex challenging problems in nano/micro-physics [207].	-Semiconductor physics [203–207]. -Liquid crystals, polymers, and suspensions [203–207] -Meta-materials, phononic, and photonic crystals [193] -High strength steels, anisotropicity, as well as the determination of non-associative flow [203–207].
Nonlocal strain-gradient theory	-Contains both nonlocal and material length scale parameters [62,208,209]. -Predict the stiffness-hardening influences using the length scale parameter [208,209]. -More accurate for modeling and analysis of micro/nano-structures using both stiffness reduction and enhancement influences [121].	-Wave dispersion analysis of FGM [62,208]. -Biological tissues [121]. -Energy harvesting [121,122]. -M/NEMS cantilever actuators [121].

Table 3
The crystallographic properties of h-BN, h-graphite, and h-SIC [247–276,278,279]. Reprinted with permission from Elsevier.

Properties	h-graphite	h-BN	4h-SIC
Lattice parameters (nm)	a : 0.246 nm, c : 0.670 nm	a: 0.250 nm, c: 0.666 nm	a = 0.30798 nm, c : 1.00820 nm
Bond length (nm)	0.142	0.145	0.189
Bond energy (eV)	3.7	4	3.3–4.9
Young's modulus (TPa)	1.1	0.81–1.3	0.35–0.65
Thermal conductivity (W/mK)	2600	400	330–490
Band gap (eV)	~0	5.5–6.0	3.26

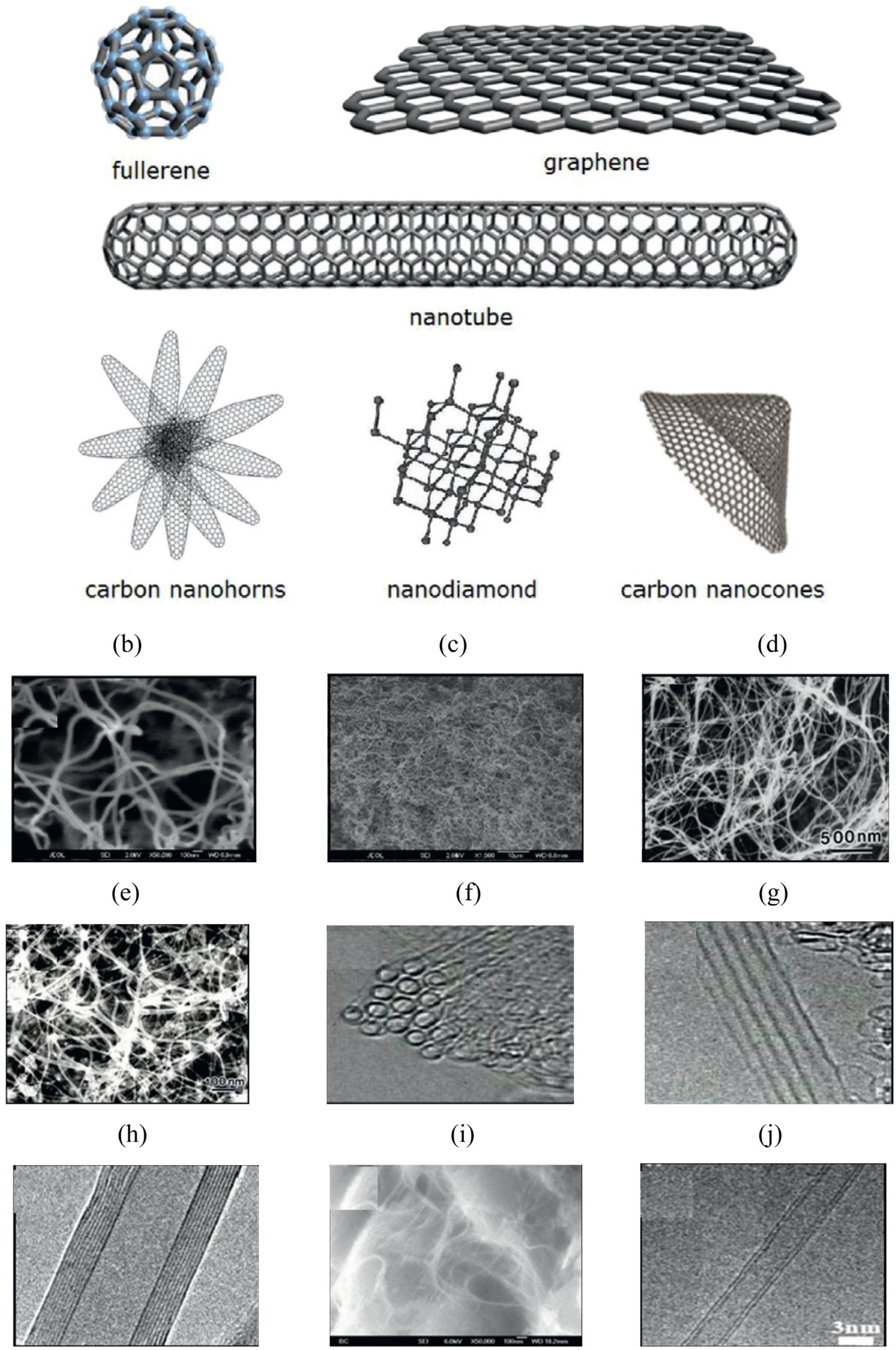


Fig. 1. (a)–(i). (a) Allotropes of carbon from synthetic process [245], (b) & (c) SEM images of MWCNT (with low and high magnification), (d) & (e) SEM images of SWCNT (with low and high magnification), (f) Cross-sectional view of a bundle of SWCNTs (TEM image), (g) transverse view of (f), (h) MWCNT (TEM image), (i) DWCNT (SEM image), (j) DWCNT (TEM image) [246]. Reprinted with permission from Elsevier.

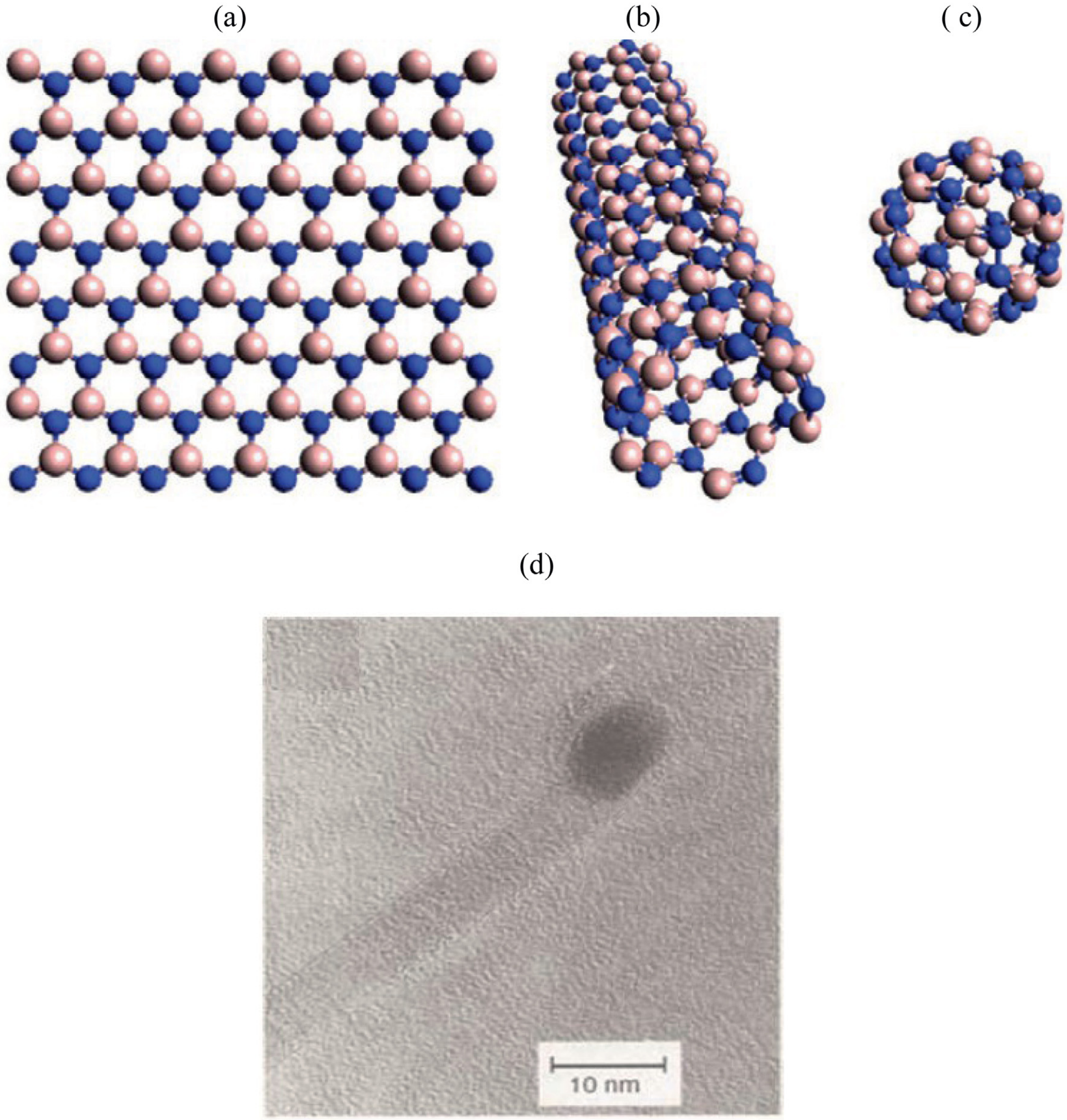


Fig. 2. (a)–(d). (a) single-layered low-dimensional BN nano-sheet (2D), (b) single-walled low-dimensional BN nanotube (1D), (c) single-shelled low-dimensional BN fullerene (0D) [247], (d) A BN nanotube with a metal cap (HRTEM view) [180]. Reprinted with permission from Elsevier.

can be proposed as [291]:

$$\begin{cases} \sigma_{ij} = k\delta_{ij}\varepsilon_{mm} + 2\mu\varepsilon'_{ij}, \\ p_i = 2\mu l_0^2 \varepsilon_{mm,i}, \\ \tau_{ijk}^{(1)} = 2\mu l_1^2 \eta_{ijk}^{(1)}, \\ m_{ij}^s = 2\mu l_2^2 \chi_{ij}^s, \end{cases} \tag{3}$$

in which, σ_{ij} is the classical stress tensor, and p_i, $\tau_{ijk}^{(1)}$, m_{ij}^s are higher-order stresses. Moreover, k, μ, δ_{ij} are, respectively, bulk modulus, shear modulus, and Kronecker delta function. Furthermore, the component of the strain tensor ($\hat{\varepsilon}$), the deviatoric stretch gradient tensor $\left(\hat{\eta}^{(1)}\right)$, as well as the rotation gradient tensor ($\hat{\chi}$) can be expressed as follows [291]:

$$\begin{cases} \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \\ \varepsilon_{mm,i} = \gamma_i, \\ \eta_{ijk}^{(1)} = \frac{1}{3}(\varepsilon_{jk,i} + \varepsilon_{ki,j} + \varepsilon_{ij,k}) - \frac{1}{15}\begin{Bmatrix} \delta_{ij}\left(\varepsilon_{mm,k} + 2\varepsilon_{mk,m}\right) \\ +\delta_{jk}\left(\varepsilon_{mm,i} + 2\varepsilon_{mi,m}\right) \\ +\delta_{ki}\left(\varepsilon_{mm,j} + 2\varepsilon_{mj,m}\right) \end{Bmatrix}, \\ \chi_{ij}^s = \frac{1}{2}(\theta_{i,j} + \theta_{j,i}), \\ \theta_i = \frac{1}{2}e_{ijk}u_{k,j}, \end{cases} \tag{4}$$

where u_i, θ_i, e_{ijk} are, respectively, the displacement vector, the symmetric rotation gradient tensor, the rotation vector. In general, based on their results, the strain energy in a linear elastic material occupying

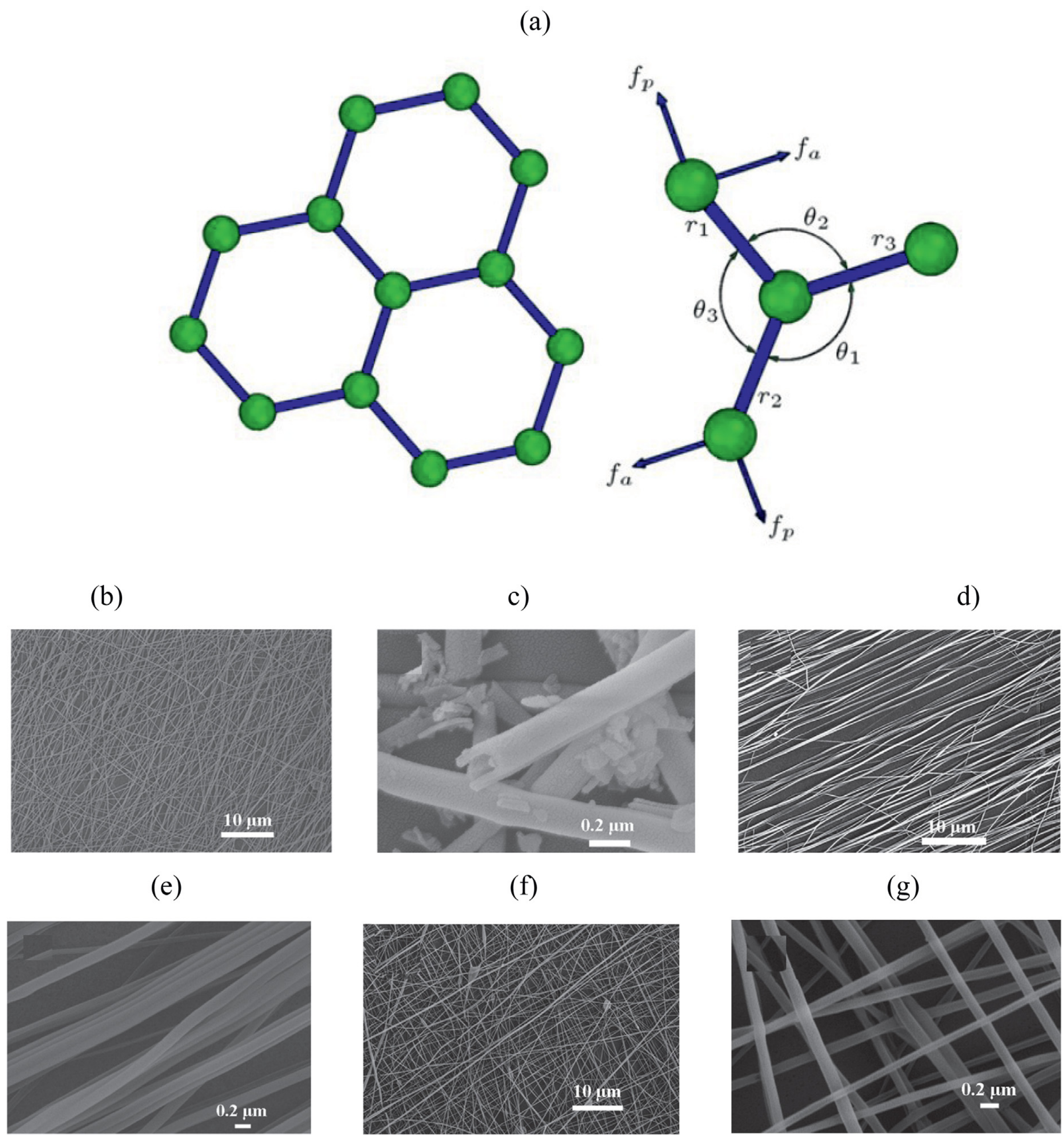


Fig. 3. (a)–(g). (a) A chiral single-walled SICNT with Hexagonal units as well as force distribution in bonds r_1 and r_2 [249], (b) Disordered SICNTs, (c) The cross-section view of a SICNT, (d) & (e) The parallel SICNTs, (f) & (g) the mat-like SICNTs [250]. Reprinted with permission from Elsevier.

volume V with infinitesimal deformation can be obtained as [291]:

$$U = \frac{1}{2}\int_V \left(\sigma_{ij}\varepsilon_{ij} + p_i\gamma_i + \tau^{(1)}_{ijk}\eta^{(1)}_{ijk} + m^s_{ij}\chi^s_{ij}\right) dV, \qquad \varepsilon_{mm,i} = \gamma_i, \tag{5}$$

Also, the constitutive equations associated with the stresses as well as the kinematic parameters effective on the strain energy related to the linear isotropic elastic model can be described as follows:

$$\begin{cases} \sigma_{ij} = \lambda tr(\hat{\varepsilon})\delta_{ij} + 2\mu\varepsilon_{ij}, \\ p_i = 2\mu l_0^2\gamma_i, \\ \tau^{(1)}_{ijk} = 2\mu l_1^2\eta^{(1)}_{ijk}, \\ m^s_{ij} = 2\mu l_2^2\chi^s_{ij}, \end{cases} \qquad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)};\ \mu = \frac{E}{2(1+\nu)} \tag{6}$$

where, the parameter λ is the Lame parameter. The advantages and limitations of the size-dependent models in micro-scale are different, in which each model should be employed appropriately to give better results in the design of these structures [329–333].

The calibration of the material length scale parameters is a very important part of the design of micro-structures. In general, the material length scale parameter is the square root of the curvature modulus ratio to the shear modulus (mathematically aspect), which is a measurement tool for the influence of couple stress. For instance, Yang et al. [290] showed that the rigidity of the micro-structures is around seven times the conventional outcome values when the material length scale parameter is equal to the radius of the micro-structures. Also, they revealed that in $\nu = 0.3$, the bending rigidity of this model is about five times its conventional rigidity when the material length scale parameter is close to the thickness of the micro-plate. Park and Gao [292] indicated that the material length scale parameter for the epoxy microbeam is $l =$ 17.6 µm when $b =$ 17.6 µm, $\nu = 0.38$, $l_0 = l_1 = 0$, and $l_2 = l$ (based on the proposed parameter in [293]). Fleck and Hutchinson [293] proposed the material length scale parameters as $l_1 = 0, l_2 = (1/3)\, l_{CS}, l_3 =$

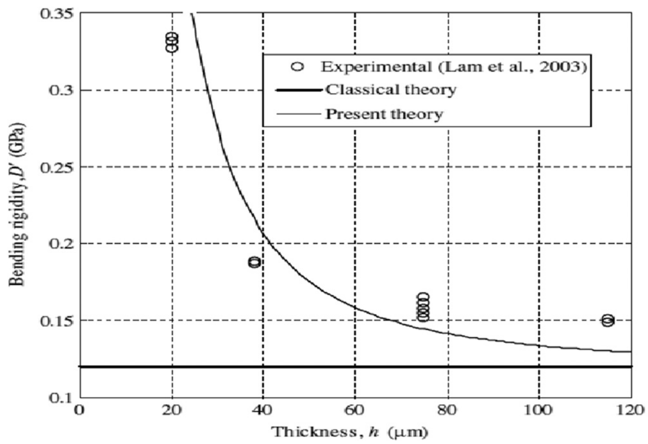


Fig. 4. Bending rigidity against thickness values [292].
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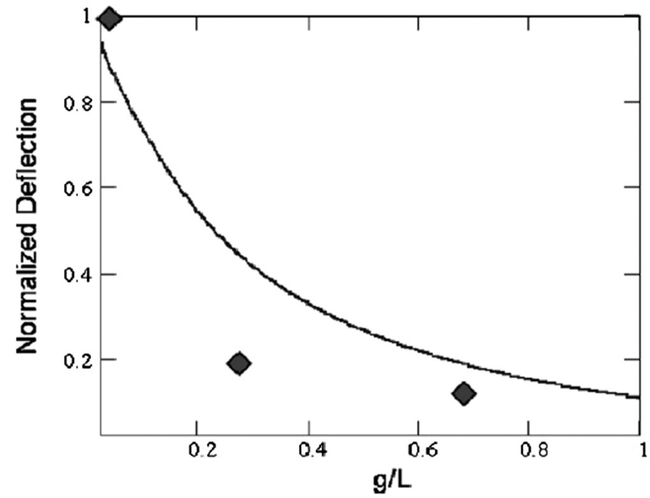

Fig. 5. Dependence of normalized deflection, at the free end of beam on against microstructural length over beam length [296]. Reprinted with permission from Elsevier.

$\sqrt{5/24}\, l_{CS}$ or $l_2' = l_{CS}, \nu' = 0$. They considered the description of effective strain and reexamined the parameter reduction using the compressibility consistent description of effective strain. In another interesting research, Gao et al. [294] obtained the relation between the material length scale parameters as $l_1 = l_2 = l_3 = (1/2)\, l_{MSG}, l_2' = \sqrt{11/10}\, l_{MSG}$, and $\nu' = -(1/11)$. In which, their obtained results were compatible with the Fleck and Hutchinson description of effective strain. Furthermore, Chong et al. [295] proposed that the material length scale parameter is $l = 3$ μm for torsion loading of the copper wires. They proved that in the micro-scale, elastically deformation of copper structures is strongly influenced by strain gradients.

2.1. Bending and buckling of small-scale structures

There are many published papers related to linear/nonlinear static and dynamic bending as well as linear/nonlinear buckling and post-buckling of micro/nano-structures associated with the couple stress and strain gradient models. Linear/nonlinear static bending as well as linear/nonlinear buckling and post-buckling of micro/nano-structures can be discussed in this section, and linear/nonlinear dynamic bending as flexural vibration of micro/nano-structures will be investigated in Section 2.2.

2.1.1. Linear static bending of micro/nano-structures

Having various types of solving procedures attracts researchers to select them as capable tools to obtain the mechanical responses of the model. Some researchers employed analytical methods to investigate the linear static bending responses of the micro/nano-structures. A novel theory for the static bending of an Euler Bernoulli beam with linear effects was proposed by Park & Gao [292] using a modified couple stress model. They used the variational method according to the minimum total potential energy. Based on their finding, the bending rigidity of a cantilever beam obtained by their theory was larger than that of the classical theory, which can be shown in Fig. 4. In another interesting work, structural analysis of gradient elastic components was done by Giannakopoulos & Stamoulis [296]. They examined the size-dependent effects on the linear static bending of a cantilever beam and cracked bar tension using the gradient elasticity theory. They demonstrated that the cantilever gradient elastic beam was remarkably stiffer with consideration of a more brittle response, while the gradient cracked bar showed acceptable toughening. They compared their results with those in their previously published work which was based on the two-dimensional finite element modeling, in which the variation of normalized deflection with microstructural length over beam length was investigated, see Fig. 5.

The influences of various parameters such as length to thickness ratio, internal length scale parameter, and aspect ratio on the numerical results were studied. The examination of Poisson effects on the linear static bending of isotropic microbeams was investigated by Dehrouyeh-Semnani & Nikkhah Bahrami [297]. Using Euler Bernoulli and Timoshenko beam models and under concentrated loads, the bending rigidity, as well as the deflection behavior of epoxy microbeam, were examined. They showed that the overestimation of bending rigidity can occur while the Poisson ratio parameter was considered in the epoxy microbeam.

Functionally graded (FG) materials have become a hot topic of investigation mainly to determine the deflection and stress distribution on these materials. It is significant to mention that FG nanobeams do not exist, in the sense that they have never been manufactured, and they are probably not feasible even in the future due to their small-scale.

The size-dependent effects on the linear static bending behavior of S-S FG microbeam using the Timoshenko beam model under uniform and concentrated loads were studied by Simsek et al. [298]. They assumed that the material characteristics of the FG micro-structure change in the direction of thickness, which was obtained using the classical method of the mixture as well as the Mori–Tanaka homogenization procedure. They proved that using the classical beam model leads to larger deflection values of the FG microbeam against the modified couple stress model.

Some researchers employed discretization approaches to acquire the mechanical responses of the micro/nano-structures. Alinaghizadeh et al. [299] published an article about the linear static bending behavior of size-dependent FG annular sector microplate using the first shear deformation model and modifies couple stress theory. A generalized differential quadrature method (GDQM) was used to discretize the governing relations. The influences of material length scale, power-law index, and geometrical parameters were investigated in their research. Kahrobaiyan et al. [300] worked on the static/dynamic bending of the FG Euler–Bernoulli beam model using the strain gradient theory. Using the variational principle, the classical, as well as non-classical boundary conditions, were acquired. For the investigation of the static bending, the generally valid closed-form method was proposed. They employed equivalent length scale parameters to scrutinize the bending behavior of FG microbeams with consideration of the functions of the length scale parameters.

2.1.2. Nonlinear static bending of micro/nano-structures

Because of the nature of the nonlinear geometric, usually only numerical approaches were utilized to scrutinize the mechanical behavior of micro/nano-structures. The nonlinear static/dynamic bending (related to the induced mid-plane stretching) of the Timoshenko beam theory using the modified couple stress model was investigated by Asghari et al. [289]. Using the Hamilton method, the PDE and boundary conditions for the hinged–hinged beam model were obtained. Also, using the numerical methods, the nonlinear static bending of the microbeam was analyzed and solved. Nonlinear static bending behavior of classical circular microplate according to the modified couple stress theory was investigated by Wang et al. [301]. They derived the governing relations using the minimum total potential energy method with consideration of von Kármán geometrically nonlinear as well as the modified couple stress models. In another research, Reddy and Kim et al. [302] studied the nonlinear bending of FG plates using the modified couple stress model and with consideration of the third-order shear deformation scheme. By employing the Hamilton principle, power-law through the thickness method, as well as the von Kármán strain–displacement with consideration of nonlinear effects the governing relations were derived. Their proposed methods could be utilized for developing finite element methods to investigate the geometric nonlinearity and size-dependent influences, as well as material grading effects on nonlinear static bending of microplates. Jafari-Talookolaei et al. [303] worked on the static bending behavior of Bernoulli–Euler microbeams using single delamination as well as the modified couple stress method. They used free as well as constrained mode models to investigate the interaction of delamination surfaces that occurs in the damaged area. Also, they considered the compatibility as well as continuity conditions for the neighboring sub-microbeams. The influences of finite strain and thermal environment on nonlinear static bending of the micro laminated composite Euler–Bernoulli beam model using modified couple stress theory were scrutinized by Ghasemi and Mohandes [304]. They considered the minimum potential energy principle was employed to drive governing relations with different boundary conditions as well as the Green–Lagrange strain tensor based on the finite strain procedure. Using the GDQ method, the obtained relations were solved. The utilized microbeam in their model was made of various materials including carbon-epoxy as well as glass-epoxy, in which a considerable difference between the von Kármán and finite strain assumptions was proved in some specific values of aspect ratio. Nonlinear static bending of a thin microbeam using the strain gradient model based on the Euler–Bernoulli beam model was investigated by Lazopoulos et al. [305]. They used the variational method to obtain the governing relations and relevant boundary conditions. Some new terms in their study increased the stiffness of the microbeam in particular for the thin microbeam. Dadgar-Rad and Beheshti [306] proposed a nonlinear static bending behavior of microbeams as well as microframes with consideration of the strain gradient finite element method. They employed the Newton–Raphson method to investigate and solve the nonlinear governing relations. Moreover, they examined three various case studies to scrutinize the performance as well as capability of their model at linear/nonlinear deformation regimes.

2.1.3. Linear buckling/post-buckling of micro/nano-structures

Many researchers utilized analytical approaches to scrutinize the linear static buckling/post-buckling responses of the micro/nano-structures. A survey of the modified couple model in buckling investigation of micro composite laminated beam models was conducted by Mohammad-Abadi & Daneshmehr [311]. They used the minimum potential energy principle to drive the final equations of motion with consideration of different boundary conditions. Likewise, based on the new curvature tensor and modified couple stress theory, the influences of considering size-dependent effects against classical beam models were also examined. They investigated the variation of critical buckling load with length to thickness ratio which indicates that critical buckling load values decrease with an increase in the amounts of the length to thickness ratio which are the same for [0,90,0] and [90,0,90] laminations. The size-dependent biaxial buckling analysis of an orthotropic multi-microplate system with consideration of modified couple stress theory was developed by Hosseini et al. [312]. They utilized the total potential energy method and the Kirchhoff plate model to acquire the partial differential relations with relative boundary conditions. Furthermore, they revealed that the effects of the Pasternak foundation on the buckling load were decreased while the number of microplates was increased. Also, in another research, Ebrahimi & Mahmoodi [314] worked on the modified couple stress theory to characterize the buckling behavior of higher-order inhomogeneous microbeams with consideration of the exact position of the neutral axis and porosities. They considered material properties of the FG microbeam change in the thickness direction through a modified form of the power-law distribution. They worked on the geometry and position of the neutral axis for an FG microbeam using porosities. Furthermore, they proved that the critical buckling load decreases with an increase in the values of porosity coefficient with different magnitudes of thickness to length parameter. Moreover, Hadjesfandiari et al. [318] proposed a novel size-dependent Timoshenko beam model that was investigated using the consistent couple stress model. They scrutinized buckling, curvature, and pure static bending of the microbeam with consideration of the partially/fully clamped boundary conditions, and then solved the obtained equations analytically. Likewise, they compared their analytical method with the 2D finite element formulation procedure.

On the other hand, many types of research were carried out related to discretization approaches to obtain the mechanical responses of the micro/nano-structures. Size-dependent effects on dynamic stability of FG microbeams using the modified couple stress theory were investigated by Ke & Wang [310]. They utilized the DQ method to transform motion equations to linear Mathieu–Hill relations with consideration of the boundary points distributed on the stable regions using Bolotin's principle. Based on their outputs, dynamic stability behavior can be impressed remarkably by the size-dependent effects when the values of microbeam thickness and material length scale parameter are the same. Later, the size-dependent FG microplate model was reported by Ansari et al. [319] to investigate the buckling, bending, as well as free vibration behaviors using the classical as well as modified couple stress methods. According to the Mindlin plate theory and Hamilton's principle, the governing relations with their relevant boundary conditions were derived and then solved by the GDQ method. They showed that an increase in the values of the material gradient index as well as slenderness ratio, lead to a decrease in the magnitudes of the critical buckling load and fundamental natural frequency. Santos and Reddy [320] worked on the linear buckling behavior of a microbeam using the modified couple stress model based on the Timoshenko beam theory. They employed the Ritz method to calculate the buckling load and natural frequencies of the microbeam. They proved that there is a minimum in certain magnitudes of the Poisson's ratio when increasing the buckling load happens against an increase in the material length scale parameters. Khakalo et al. [321] studied the modeling of the buckling, bending, and vibration behavior of 2D triangular lattices using the strain gradient elasticity method. Based on the Euler–Bernoulli as well as Timoshenko beam models, the effective elastic moduli pertinent to the classical elasticity model were proposed with consideration of the computational homogenization procedure. They showed that there is no proof for depending on the higher-order material parameters as well as the problem type, the beam relations, or the relevant boundary conditions. They proved that, based on the Euler–Bernoulli beam model, the critical buckling load, bending rigidity, as well as eigenfrequencies merely relate to the lattice parameter associated with the micro-structure.

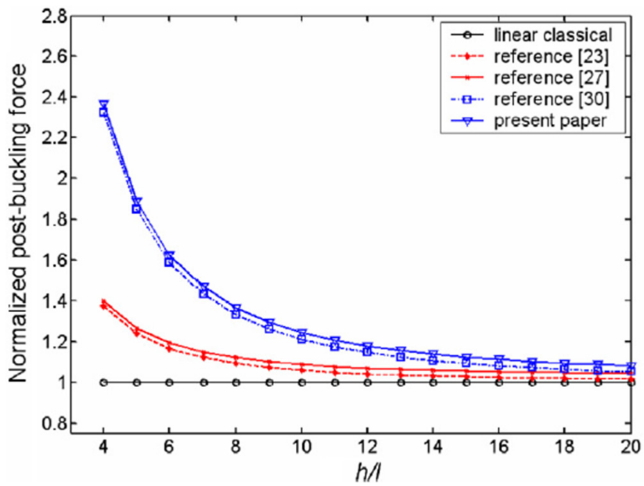

Fig. 6. Critical post-buckling forces for a slender microbeam with the ratio of thickness to material length scale parameter [307]. Reprinted with permission from Elsevier.

2.1.4. Nonlinear buckling/post-buckling of micro/nano-structures

According to the nonlinearity geometric, normally numerical procedures were employed to investigate the mechanical behavior of micro/nano-structures. In a comprehensive research work using strain gradient theory nonlinear post-buckling, static bending, and vibration of microbeam were reported by Zhao et al. [307]. The nonlinear static bending behavior of the clamped–clamped (C-C) boundary condition micro-structure acted upon by a transverse force, the nonlinear frequencies, and critical buckling load of the S-S micro-structure with consideration of initial lateral displacement were analyzed. Fig. 6 illustrates the trend of normalized post-buckling load versus the ratio of thickness to material length parameter compared with previously published studies in this field. The most notable feature was that the ratio of thickness to material length scale parameter is very significant, in which at higher values of the ratio, the influence of size-dependence was not remarkable (see Fig. 6). They obtained the critical post-buckling force for a slender microbeam as [307]:

$$N_{Cr} = \frac{\pi^2 EI}{L^2}\left\{\begin{matrix} 1+\frac{A}{I}\left(l_0^2+\frac{4}{15}l_1^2+\frac{1}{2}l_2^2\right)+\frac{\pi^2}{L^2}\left(l_0^2+\frac{2}{5}l_1^2\right) \\ +\frac{\eta}{\pi^2}\left(1+\frac{A}{I}\left(l_0^2+\frac{4}{15}l_1^2+\frac{1}{2}l_2^2\right)\right) \end{matrix}\right\}, \tag{7}$$

where E, I, L, and η are, respectively, the elastic modulus, the moment of inertia, the length of microbeam, and the symmetric rotation gradient tensor. Also, l_0, l_1, l_2 are the independent and additional material length scale parameters pertinent to the dilatation, deviatoric stretch, and symmetric rotation gradients, respectively.

Lou et al. [308] carried out pre-buckling and buckling scrutiny of FG microshells induced by axial and radial loads using the modified couple stress model. They employed the first-order shear deformation model in conjunction with von Kármán's geometric nonlinearity to capture the deformation characteristics. Moreover, they obtained the critical buckling load using the pre-buckling deformation effects. Based on their results, using the couple stress method leads to having a larger value of microshell stiffness against the classical model. The investigation of the thermal post-buckling behavior of the uniform FGM beams was carried out by Anandrao et al. [309]. They employed the classical Rayleigh–Ritz relations as well as the finite element method to analyze the micro-scale model. They proved that although, the FG microbeam responses were more accurate via FE relations separately, applying the classical Rayleigh–Ritz relations can be considered to obtain the approximate closed-form post-buckling solution associated with the FG microbeam. The buckling behavior of a micro composite plate with nano-coating using the modified couple stress model was done by Malikan [313]. Using the simplified first-order shear deformation

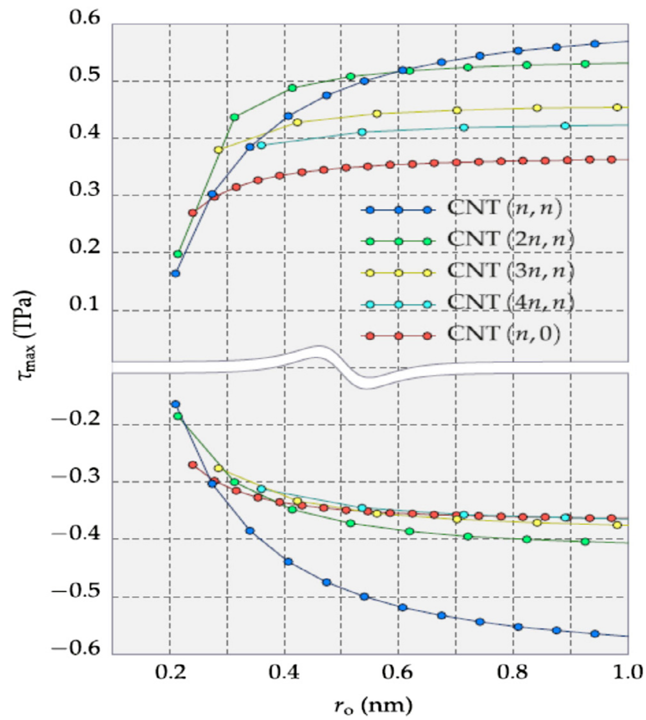

Fig. 7. Dual ideal shear strengths variation of SWCNTs against their radii [315]. Reprinted with permission from Elsevier.

model, the principle of Hamilton, as well as von Kármán's geometric nonlinearity the motion equations were derived. It was proved that the maximum and minimum critical load magnitudes are related to the clamped–free clamped–free (CF-CF) as well as simply supported–free simply (SS-FS) supported–free boundary conditions, respectively. Delfani & Shodja [315] worked on the dual ideal shear strengths for describing the elastic buckling behavior of chiral SWCNTs using material and geometrical nonlinearities. Dual ideal shear strengths variation of SWCNTs against their radii is shown in Fig. 7. It was clear that the torsional behavior of SWCNTs does not depend on the length of the nanotube. Also, using strain gradient theory can make some complexities, which was for the sake of the anisotropic characteristics of SWCNTs in this study. Wu et al. [316] investigated size-dependent buckling as well as post-buckling of an S-S FG microplate resting on a nonlinear elastic medium. By employing the modified couple stress and the Mindlin plate models with consideration of von Kármán's theory with geometric nonlinearity the deflection behavior of the microplate was examined. Also, the minimum potential energy principle was utilized to obtain the governing equations with the relevant boundary conditions. Furthermore, the Galerkin procedure was applied to obtain the closed-form solution method peculiar to the buckling load as well as the load–displacement equation in post-buckling. The effects of cross-section on the linear/nonlinear buckling behavior of an imperfect FG microtube were analyzed by He and Cai [317]. Using Euler–Bernoulli beam model, the size-dependent effects of the microtube were investigated by the modified couple stress model, and then by employing the conservation energy method, the governing relations with consideration of the nonlinear Von-Kármán method were derived. They also considered the linear, exponential, as well as convex in comparison with the uniform section pertinent to the radial distribution of ceramic or metal material using the power-law function. Likewise, they used the GDQ method to solve the obtained equations with consideration of the various boundary conditions including pinned, clamped, as well as the combination of them.

2.2. Vibration and wave propagation of small-scale structures

There are several published papers pertinent to linear/nonlinear free and forced vibration, band structure analysis of micro/nano-sized periodic structures, as well as elastic/thermoelastic wave propagation of micro/nano-structures associated with the couple stress and strain gradient models, which are discussed in the following parts.

2.2.1. Linear/nonlinear free vibration of micro/nano-structures

The number of studies pertinent to the analytical schemes for the investigation of the linear free vibration responses of the micro/nano-structures is considerable. Akgoz & Civalek [324] investigated longitudinal vibration analysis of microbars using the strain gradient model. Based on Hamilton's principle, motion equations pertinent to micro-scale of the elastic bar were obtained with consideration of different boundary conditions. They also showed that using the material length scale parameter is a key factor in modeling micro-scale structures. Likewise, as can be seen from Fig. 8, it was proved that the variation between natural frequencies obtained by micro-scaled and classical beam theories was more considerable for both higher vibrational modes and lower magnitudes of the aspect ratio of the microbar. They obtained the natural frequency for the C-F microbar as follows [324]:

$$\omega_n = \frac{(2n-1)\pi}{2L}\sqrt{\frac{1}{m}\left(EA + B\frac{(2n-1)^2\pi^2}{4L^2}\right)},$$
$$B = \mu A\left(2l_0^2 + \frac{4}{5}l_1^2\right), \quad n = 1, 2, \ldots . \tag{8}$$

where E, A, L, m, and μ are, respectively, the elastic modulus, the cross-sectional area, the length of microbar, the mass of microbar, and the shear modulus. Also, l_0 and l_1 are, respectively, the independent and additional material length scale parameters pertinent to the dilatation and deviatoric stretch gradients. In another interesting research, the second strain gradient model based on Timoshenko beam theory was carried out by Asghari et al. [325]. The static and vibration characteristics of a hinged–hinged microbeam were analyzed and then compared with the strain gradient, the modified couple stress, and the classical continuum theories. It is clear from Fig. 9 that at lower magnitudes of the thickness to lattice parameter ratio, the normalized dynamic deflection magnitudes of microbeam peculiar to the second strain gradient model are lower than other theories. A size-dependent Reddy–Levinson beam using a strain gradient elasticity theory was introduced by Wang et al. [326]. They employed three material length scale parameters related to the Euler–Bernoulli and Timoshenko beam models for capturing the size-dependent influences in micron or sub-micron.

They indicated that the differences pertinent to the mechanical characteristics obtained by their employed models were getting larger than the material length scale parameters. Moreover, the mentioned differences were decreasing against an increase in the values of the microbeam thickness.

In order to investigate the nonlinear behavior of free vibration micro/nano-structures, numerical methods were used by the researchers. An investigation into the nonlinear vibration of the curved microbeam with consideration of nonlinear foundation and the modified strain gradient model was reported by Allahkarami et al. [327]. Using the multiple time scales perturbation method, the frequency responses of the curved microbeam were examined. They concluded that the frequency response curves may use the optimum values of the design parameters. A size-dependent third-order shear deformable theory using strain gradient model of an FG circular–annular microplate was proposed by Zhang et al. [328]. They employed the refined third-order shear deformation model in which the in-plane and transverse displacements were divided into bending as well as shear parts related to the microplate. Using the DQM the governing motion equations with consideration of the different types of boundary conditions, the critical buckling load, as well as natural frequency were obtained. Awrejcewicz et al. [329] examined the mathematical modeling of

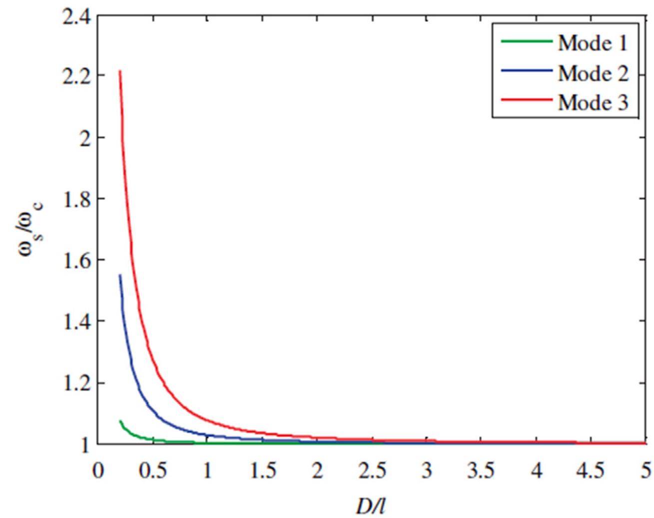

Fig. 8. Natural frequency for a clamped–free microbar via the slenderness ratio for different modes [324]. Reprinted with permission from Elsevier.

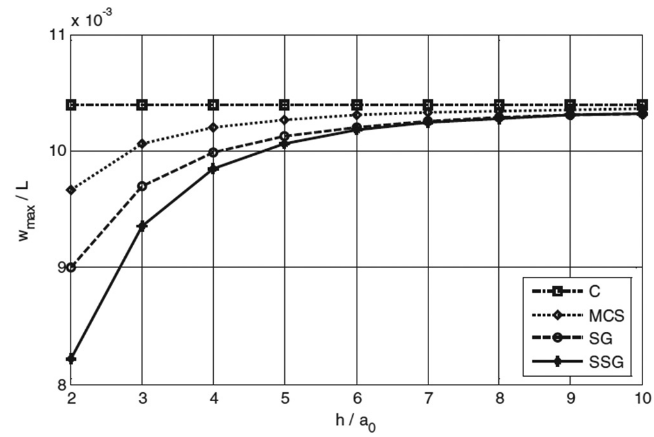

Fig. 9. Normalized dynamic deflection values of micro-beam with the thickness to lattice parameter ratio [325]. Reprinted with permission from Elsevier.

three-layer micro/nanobeams using Grigolyuk–Chulkov and modified couple stress models. They used Hamilton's principle which led to novel motion equations with consideration of the boundary/initial conditions pertinent to beams displacement. The obtained boundary model was of sixth-order and investigated analytically in the case of statics. They showed that the beam stress and deflections predicted by the couple stress theory are less than those obtained by the classical Grigolyuk–Chulkov model, in which the Eigen frequency values are higher, respectively. Recently, Nguyen et al. [330] worked on the vibration of cracked FG microplates using the strain gradient model with consideration of the extended isogeometric model (XIGA). They utilized one length scale factor and an additional micro inertia parameter to reveal the size-dependent effect. Using the refined plate theory, the displacement field of the microplate was demonstrated. In their research, the IGA approach with highly smooth basis functions of non-uniform rational B-spline made an efficient behavior of higher continuity necessities in the strain gradient model. They showed remarkable effects of microstructural features with consideration of size-effects on the dynamic responses of microplates rather than the classical structure. The Euler Bernoulli and Timoshenko beam models were also employed to model an FG microbeam and laminated composite materials. The investigation of FG microbeams using Euler Bernoulli and Timoshenko beam models with consideration of geometric nonlinearity was developed by Reddy [331]. He utilized the nonlinear von Kármán model, the modified couple stress model, as well as power-law variation. By employing the length scale parameter the differences between classic and size-dependent models were shown with consideration of the Euler–Bernoulli and Timoshenko beam models. Size-dependent effects on nonlinear vibration of FG microbeams using the Timoshenko beam model were investigated by Ke et al. [332]. Using the Mori–Tanaka homogenization technique, the material properties of FGM were considered and graded in the thickness direction based on the power-law function. They scrutinized the effects of the material property gradient factor, small-scale parameter, and slenderness ratio on the vibrational behavior of the FGM microbeams with nonlinearity influences. They investigated the influences of material property gradient parameter (n) on the nonlinear frequency ratio against vibrational amplitude curves. It is clear that the nonlinear frequency ratio of H–H FGM microbeams is related simultaneously to the sign and magnitude of the vibration amplitude. But, for the C–C and homogeneous case study, the results are completely independent of the sign of the vibration amplitude, in which symmetrical curves are obtained. Rajabi & Ramezani [333] worked on a nonlinear microbeam model using the strain gradient model. They indicated that the strain gradient effect on the increasing rate of the natural frequency was predominant when the thickness to length parameter ratio was close to unity. Moreover, they proved that increasing the thickness lead to a decrease in the strain gradient effects and thus the effects of geometrical nonlinearity on the increasing rate of natural frequency were remarkable. Also, they showed that the frequency ratio decreases with an increase in the magnitudes of thickness to length scale ratio which is similar to both linear and nonlinear strain gradient and the couple stress case studies. Georgiadis et al. [341] worked on the Rayleigh-wave propagation in microstructured solids using dipolar gradient elasticity. Based on the Mindlin–Green–Rivlin of dipolar gradient elasticity theory the Rayleigh type wave propagating along the surface of a half-space was investigated. They demonstrated that the obtained waves were dispersive at higher values of the frequencies. The shear wave propagation behavior of piezoelectric composite structures using the two-layer model was scrutinized by Gaur & Rana [342]. They revealed that the thickness and elastic constants have a significant effect on shear wave propagation behavior.

2.2.2. Linear/nonlinear forced vibration of micro/nano-structures

Several works were addressed related to the discretization procedures for the linear/nonlinear forced vibration characteristics of the micro/nano-structures. Simsek [323] worked on the dynamic analysis of an embedded micro-structure acted upon by a moving microparticle using the modified couple stress model. Based on the finite Fourier sine transformation, the closed-form solution of microbeam deflection was calculated. Also, based on Lagrange's equations in conjunction with the direct integration method of Newmark, the normalized dynamic deflection of microbeam was obtained. It was demonstrated that the effects of the material length scale, the Poisson's ratio, and the velocity of the microparticle parameters on the dynamic behavior of the microbeam are very remarkable. In another research, Vatankhah et al. [334] analyzed nonlinear forced vibration of the strain gradient theory of microbeam structures. They solved the nonlinear motion relations using the perturbation technique. Also, they considered primary, sub-, and super-harmonic resonances of the microbeam with consideration of small-scale effects pertinent to the Euler–Bernoulli beam model. It was shown that using size-dependent effects is very significant in the modeling of these structures. Ghayesh et al. [40] worked on the nonlinear forced vibration behavior of a microbeam with consideration of the strain gradient elasticity model. By employing Hamilton's principle and with consideration of the geometrically nonlinear von Kármán strain model, the nonlinear partial differential motion equations of the micro-structure were derived, and then a set of nonlinear second-order ordinary differential relations were obtained which is using the Galerkin procedure. Moreover, they employed the pseudo-arclength continuation method to obtain the frequency–response curves of the micro-structure. Later, Ghayesh and Farokhi [335] analyzed the small-scale effect of the global dynamics bending behavior of axially forced microbeams with imperfect influences, which was pertinent to improper microbeam manufacturing. Based on their research, the continuous expressions peculiar to the size-dependent potential and kinetic energy associated with the micro-structure were balanced dynamically with the energy procedure. Using the direct time-integrating reduced-order method, the bifurcation curves of Poincaré maps using were plotted for the imperfect micro-structure. The small-scale vibration behavior of a microplate acted upon a moving load using the modified couple stress and Kirchhoff–Love plate theories was investigated by Simsek et al. [336]. A trial function related to the dynamic deflection was proposed in a polynomial format to calculate the microplate dynamic responses. Likewise, they employed the time integration Newmark-β procedure to solve the motion relations with consideration of various boundary conditions. They proved that the dynamic deflections were remarkably varied with any changes in the values of micro-scale parameters as well as the load velocity. Mirjavadi et al. [337] scrutinized the dynamic responses of a metal foam FG porous cylindrical microshell induced by moving loads using the strain gradient method. They utilized the Laplace transform procedure, to derive the governing motion relations associated with the first-order microshell theory. Also, an inverse Laplace transform method was proposed to go back into the time domain equations. They showed that the forced vibration properties of the proposed model were related to the utilized type of porosity distribution. Likewise, lower stiffness values of the microshell, as well as larger dynamic deflection values, were obtained when higher porosity coefficients were employed. In another research, Dynamic responses of a size-dependent Timoshenko microbeam model under moving loads with consideration of the finite element method were examined by Esen et al. [338]. He used 2-node as well as 4 DOF microbeam finite element, in which moving mass influences and material length scale parameters were proposed simultaneously. Also, he represented that the frequency values did not change and tended to a certain magnitude against an increase in the amounts of the material length scale parameter. Nonlinear free/forced vibration behavior of an FG graphene nanoplatelet reinforced microbeams using geometrical imperfection and the nonlinear elastic foundation was reported by Mirjavadi et al. [339]. They utilized graphene Platelets which were uniformly/non-uniformly scattered in the proposed micro-structure, in which non-uniform distribution of graphene nanoplatelets was examined as linear/nonlinear models. Moreover, geometric imperfection was proposed associated with the first vibrational mode of the microbeam. Furthermore, they applied the uniform harmonic load to the top surface of the microbeam. In order to solve the nonlinear motion relations of the microbeam with quadratic as well as cubic nonlinearities, the Harmonic balance procedure was employed. The mechanical behavior of FG microcantilevers with viscoelasticity effects was investigated by Ghayesh [340]. He considered a nonlinear spring model to show the elastic support acted upon by neighboring devices. Also, size-dependent effects were employed with consideration of the modified couple stress theory, and Mori–Tanaka relation was used to represent FG property variations. Fig. 10(a)–(b) depict the frequency response of the elastically supported FG microcantilever with viscoelastic effects for classical and modified couple stress continuum mechanics models. It was clear that the peak amplitude decreases while using the modified couple stress theory.

2.2.3. Elastic/thermoelastic wave propagation of micro/nano-structures

The analytical procedure as a capable solution method was utilized by many researchers to investigate the elastic/thermoelastic wave propagation of micro/nano-structures. Karami & Janghorban [343] investigated the effects of a magnetic field on the wave propagation properties of nanoplates using the strain gradient model with consideration of a length scale parameter as well as a two-variable refined plate model. It is obvious that increasing the magnitudes of wave

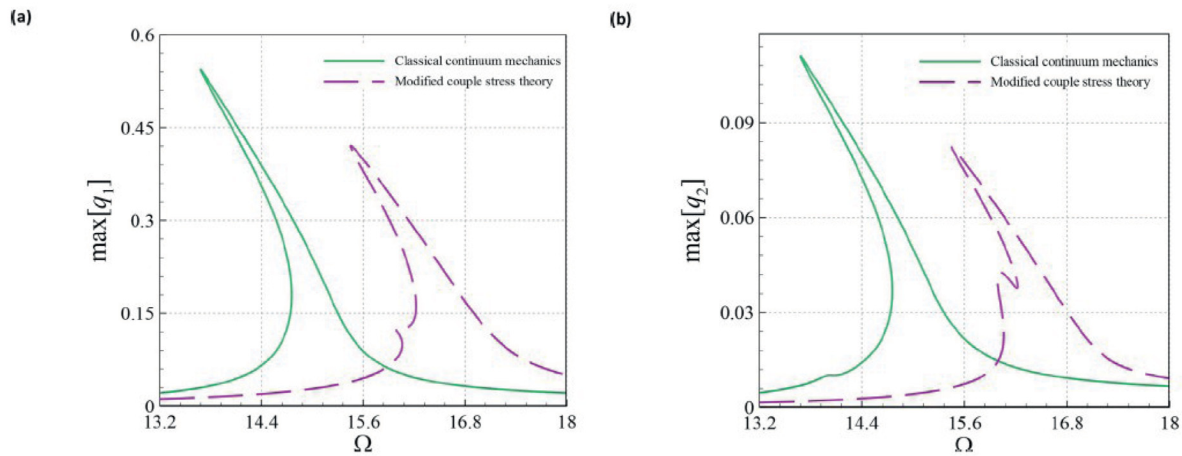

Fig. 10. (a)–(b). The frequency response of the elastically supported FG microcantilever with viscoelastic effects [340]. Reprinted with permission from Elsevier.

numbers leads to an increase in the values of nanoplate frequencies in all magnitudes of material length scale parameters ($l = 0 - 2$ nm). Selvamani et al. [344] studied the dynamic modeling of refined couple stress of thermoelastic wave propagation pertinent to linear elastic material with void as well as carbon fiber reinforced polymer composite cylinder induced by multi relaxation times. The partial differential relations peculiar to the modified couple stress as well as multiphase lag models were obtained for axisymmetric modes, and then using the linear model, the proposed equations were exactly solved. They scrutinized the effects of the changes in wave number as well as thickness on the field quantities including frequency, temperature, displacement values.

Furthermore, many interesting works were proposed related to the numerical approaches for the elastic/thermoelastic wave propagation of micro/nano-structures. The wave propagation behavior of a microbeam using the modified couple stress and in presence of an impact force method was analyzed by Kocatürk and Akbaş [345]. They used the triangular force in the transverse direction in conjunction with a harmonic motion to excite the microbeam. Using the Kelvin–Voigt model and Euler–Bernoulli beam theory, the micro-structure was modeled and then examined by the energy-based finite element method. By employing Lagrange's formulations, the motion equations were obtained as linear differential relations, which were transformed into linear algebraic relations, and finally solved via the average acceleration Newmark procedure. They proved that the modified couple stress model must be utilized instead of the classical model for small magnitudes of microbeam height. Liu et al. [346] examined theoretically investigation of elastic wave propagation behavior for an FG microplate using the modified couple-stress method. They employed global matrix as well as Legendre orthogonal polynomial methods, in conjunction with the couple stress model to scrutinize the transmission and reflection of primary waves in the FG microplate stabbed in liquid. Also, they indicated that the Legendre orthogonal polynomial method was more effective for the FG microplate model, where microplate delamination, as well as the displacement calculation pertinent to each partial wave, were not mandatory. Moreover, they showed that the influences of the couple stress model on the values of resonant frequencies of the FG microplate, and lead to a decrease in the number of resonant frequencies. In another interesting research proposed by Liu et al. [347], the investigation of Lamb wave propagation in an FG piezoelectric size-dependent plate with consideration of the modified couple stress model was carried out. The Legendre orthogonal polynomial model was proposed to implement the solving procedure. Also, they introduced the rectangular window function as well as extending the displacement vectors and electric potential to the Legendre orthogonal polynomial series to derive the governing relations associated with the electric-open as well as electric-short circuit boundary conditions. By employing the global matrix scheme as a solution method, the Legendre orthogonal polynomial model was verified. They demonstrated that the phase velocities of Lamb waves related to the first/second modes with various non-dimensional characteristic length scale parameters converged to two fixed magnitudes at various speeds.

The elastic characteristics at a metallic surface where crystalline lattice form restricts usually are not similar to the bulk counterparts. For size-dependent models where the surface to volume ratio is very large, the differences in elastic features of the model for the sake of the solid surface phenomena become increasingly considerable. Thus, defining a suitable theoretical approach that can be considered as a solid surface in the elastic behavior has attracted the attention of many researchers in this field [272,273]. In the Gurtin–Murdoch method, a zero-thickness layer was employed as a solid surface which was linked to the bulk, and also the surface stress can be represented by a second-order tensor. Zhou [348] studied the surface piezoelectricity effect on axisymmetric wave propagation in piezoelectric cylindrical shells. Using the first shear deformation and Gurtin–Murdoch theories, the equations of motion were derived. They indicated that, at higher modes, the surface layer has a remarkable influence on wave characteristics in small-scale structures. Fig. 11 gives the variation of wave frequency against the logarithmic ratio of thickness to the material intrinsic length for electric open circuit with and without consideration of surface piezoelectricity. It was obvious that at $\left(\left(h/h_0\right) > 100\right)$, the surface stress effect is not considerable and can be neglected, but when $\left(h/h_0\right)$ decreases, the influences of surface stress on the wave frequency are remarkable. A novel model for the investigation of band gaps for elastic wave propagation in a periodic composite Euler–Bernoulli beam model with consideration of the couple stress theory, rotational inertia, and surface energy was proposed by Zhang et al. [349]. They utilized the Bloch theorem as well as the transfer matrix method to obtain the equations of the periodic micro-structures. They showed that the influences of the rotational inertia were larger once the exciting frequency values were higher as well as the unit cell length magnitudes were smaller.

3. Nonlocal elasticity of Eringen

Based on the nonlocal elasticity Eringen theory [18,20,22–26,31,32], the stress components at any known reference point (x) are pertinent not only on the related strain components at the same point but also on the different strain components with various directions at every position of the body. The basic functions for linear isotropic and homogeneous elastic solids without considering body forces are as

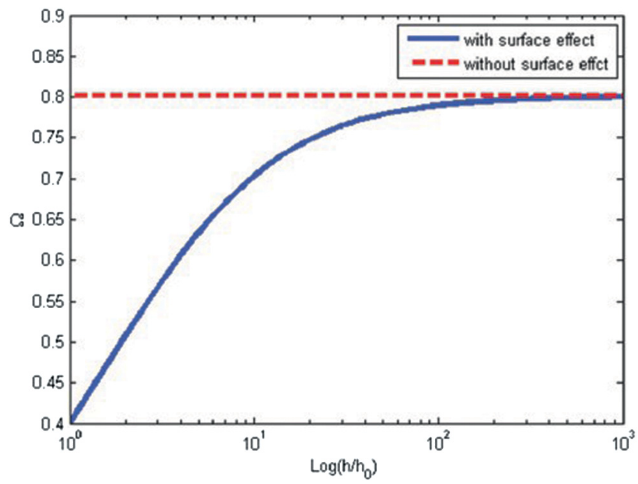

Fig. 11. The variation of wave frequency against the logarithmic ratio of thickness to the material intrinsic length for electric open circuit with and without consideration of surface piezoelectricity [348].
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follows [18,20,22–26,31,32,364–438]:

$$\begin{cases} \sigma_{ij,j} = 0, \\ \sigma^{nl}_{ij}(x) = \int \lambda\left(\left|x - x'\right|, \alpha\right) C_{ijkl}\varepsilon_{kl}(x')dV(x'), \\ \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \end{cases} \tag{9}$$

in which, σ^{nl}_{ij}, ε_{ij}, C_{ijkl}, u_i, $\lambda(\left|x - x'\right|, \alpha)$ are, respectively, nonlocal stress tensor, strain tensor, elasticity tensor, displacement vector, and attenuation function, also α is the material constant that pertains to the internal (such as the granular distance and the lattice parameter) and external (such as crack length) characteristics length. The characteristics of kernel function can be proposed as follows:

I. The maximum of this function happens at $x = x'$ and then attenuates with $\left|x - x'\right|$.
II. Having classical elasticity theory instead of the nonlocal elasticity model, $\alpha \to 0, \lambda$ becomes the function of the Dirac delta.
III. λ can be acquired using the matching of plane wave dispersion curves with atomic lattice dynamics. The kernel function can be defined as:

$$\lambda(|x|, \alpha) = (2\pi\, l^2\alpha^2)^{-1} B_0(\frac{\sqrt{x.x}}{l\alpha}), \tag{10}$$

where B_0 and l are, respectively, the Bessel function and the external characteristic length. Moreover, λ is considered a Green's function of the linear differential operator $\mathfrak{F}$:

$$\mathfrak{F}\lambda(\left|x - x'\right|, \alpha) = \delta(\left|x - x'\right|), \tag{11}$$

Combination of Eqs. (9) and (11) gives:

$$\mathfrak{F}\sigma^{nl}_{ij} = \sigma^{l}_{ij}, \tag{12}$$

By employing $\mathfrak{F}$ operation we have $\left(\mathfrak{F}\sigma^{nl}_{ij}\right)_{,k} = \mathfrak{F}\sigma^{nl}_{ij,k}$. Therefore, the equilibrium relation for the 2D linear elastic isotropic body can be proposed as follows:

$$\sigma_{kl,k} + \mathfrak{F}(\rho f_l - \rho \ddot{u}_l) = 0. \tag{13}$$

in which, instead of the integro-PDE, Eq. (13) should be examined and solved. Based on the mentioned equations, with consideration of the components of Hooke's law for σ^{l}_{ij} (the local stress tensor), Eq. (9) can be rewritten in another format as follows:

$$\begin{cases} (1 - \mu\nabla^2)\sigma^{nl}_{ij} = C_{ijkl}\varepsilon_{kl}, \\ \mu = \left(e_0 a\right)^2. \end{cases}, \tag{14}$$

where e_0 and ∇^2 are the material constant as well as is the 3D Laplace operator, respectively. There are some significant limitations pertinent to employing the nonlocal model including paradox outcomes for some special case models of boundary conditions and loading such as a concentrated load at free end, or a clamped–pinned (C-P) as well as C–C boundary condition models. Furthermore, the differential model, higher-order model, and integral form give various results in different ranges of loading or boundary conditions with various applications [252–255,369–378].

Obtaining suitable values related to the material constant is very significant in the design of nano-structures. Based on Eq. (14), using a suitable small-scale parameter leads to having a more reasonable design that seeks a correct calibration of the material constant. This can be applicable by fitting the outputs with those obtained by multi-scale or experimental methods [55,378]. One applicable method for calibration of the material constant is employing lattice or discrete structural schemes. For example, the Hencky bar-chain model can be utilized to obtain the material constant appropriately [55,378]. There are some similarities between the Hencky bar-chain and nonlocal Eringen models such as the similarities of mechanical behavior of each adjacent point in a model as well as an increase in the values of the external versus internal characteristic length ratio associated with the nonlocal Eringen models or segmental number pertinent to the Hencky bar-chain model leads to an increase in the magnitudes of vibration frequencies and buckling load of beams and plates [378]. Wang et al. [379] revealed that the material constant shows various magnitudes for various beam models. For instance, for buckling and vibration of Euler–Bernoulli beam models, these values were, respectively, $e_0 = \frac{1}{2\sqrt{3}} \approx 0.289$ and $e_0 = \frac{1}{\sqrt{6}} \approx 0.408$, in which the mentioned values were calculated using pseudo-differential operator as well as Pade approximation.

3.1. Bending and buckling of small-scale structures

Over the last three decades, the investigation of linear/nonlinear static and dynamic bending, as well as linear/nonlinear buckling and post-buckling of micro/nano-structures associated with the nonlocal Eringen model, was carried out by many researchers. The following parts are indicating a survey of the assumptions, formulations, advantages, and drawbacks of using the nonlocal elasticity of Eringen to capture the size-dependency effects associated with bending/buckling of the nano-structures.

3.1.1. Linear static bending of micro/nano-structures

Different types of solving procedures have been proposed by researchers as capable tools to obtain the mechanical responses of the model. Some of them utilized analytical methods to investigate the linear static bending behavior of the micro/nano-structures. Nguyen et al. [380] worked on the analytical solutions for bending behavior of axially or transversely FG beams using nonlocal elasticity theory. They assumed that the elastic modulus of the beam changes through the thickness or longitudinal directions based on the power law. The bending investigation of agglomerated CNT-Reinforced composite nanoplates with consideration of nonlocal effects and Pasternak substrate was investigated by Daghigh et al. [381]. They used the two-parameter micromechanics model using agglomeration to investigate the effective mechanical characteristics of the nanoplate model. Wang and Li [382] examined the static bending properties of the size-dependent nanoplate resting on the elastic foundation associated with the Mindlin as well as Kirchhoff plate theories. They proposed the difference between employing the mentioned nanoplate theories under bending loading. Based on their obtained results, the displacement ratio was increased with an increase in the values of the shearing layer stiffness as well as Winkler modulus. The static bending of Euler–Bernoulli nanobeams using nonlocal composite materials was evaluated by Feo and Penna [383]. They used a second-order differential relation related to the nonlocal elastic equation was proposed as equilibrated bending moments as well as transverse displacements. Furthermore, several benchmark case models were employed to scrutinize the small-scale influences in C-S supported as well as C-F nanobeams associated with the various magnitudes of the nonlocal parameter. The static/dynamic bending behavior of nanotube-reinforced composite plates resting on the elastic medium was investigated by Bakhadda et al. [384]. They assumed four types of distributions related to uni-axially aligned SWCNT for reinforcing the proposed plates. Using the hyperbolic shear deformation plate model, the governing equations were derived, and then solved analytically. They showed that the deflection of the plates diminished with an increase of the elastic modulus. Static/dynamic bending and buckling behavior of nonlocal FG-SWCNT-reinforced composite nanobeams with consideration of the Timoshenko beam model was investigated by Borjalilou et al. [385]. The governing relations were decoupled to an ordinary differential relation with the fifth-order pertinent to the cross-section rotation associated with the static bending as well as buckling. Decoupling of their proposed equations led to extend the exact solutions related to transverse defection as well as buckling load. Their results showed that the small-scale influences on the mechanical characteristics of proposed nanobeams were remarkably related to boundary conditions as well as loadings. Likewise, they proved that the hardening effects of the cantilever nanobeam by employing the small-scale influences were obvious, where the obtained results for C–C and S–S nanobeams were different. Novel analytical bending/buckling/free vibration behavior of rectangular nanoplates using the symplectic superposition scheme with consideration of the clamped as well as S–S edges was proposed by Zheng et al. [386]. By employing Hamilton's principle as well as the symplectic space the governing equations were derived based on the mathematical solution comprised of two significant eigenvalue problems which were the construction and the expansion of them. For various basic problems, the analytic symplectic methods were utilized, where the final method was obtained based on their superposition. Moreover, Lévy-type/non-Lévy-type schemes were obtained, whereas they could not be acquired by conventional analytical procedures.

Some researchers utilized discretization approaches to acquire the mechanical responses of the micro/nano-structures. Nejad et al. [387] worked on the bending behaviors of two-directional FG Euler Bernoulli nanobeams. Material characteristics of the nanobeam were considered to vary along with the thickness as well as length direction based on the arbitrary function. Using the minimum potential energy principle, the governing relations were derived and then with GDQM, the Euler Bernoulli nanobeam with arbitrary boundary conditions was analyzed to investigate the deflection of the nanobeam. The static bending behavior of microtubules with consideration of the nonlocal Euler–Bernoulli beam model was investigated by Civalek and Demir [388]. According to the shear as well as moment resultants of the nanobeam and stress/strain formulation associated with the 1D nonlocal elasticity model, the governing relations were derived and then solved with the DQ method. They indicated that the nonlocal elasticity model was superior to the local elasticity model, in particular for some specific boundary conditions.

Differential forms of nonlocal elasticity Eringen's model are widely used to reveal the size-dependent influences on the behavior of nanostructures. However, while using differential forms of nonlocal constitutive models such as cantilever beam models with consideration of different loading analysis, paradoxical results are taken, and therefore a reference must be made to the original form of the nonlocal Eringen's theory called the integral form of the Eringen's model. Saez et al. [53] worked on some inconsistencies pertinent to using the nonlocal elasticity of Eringen that were not understandable. They formulated the

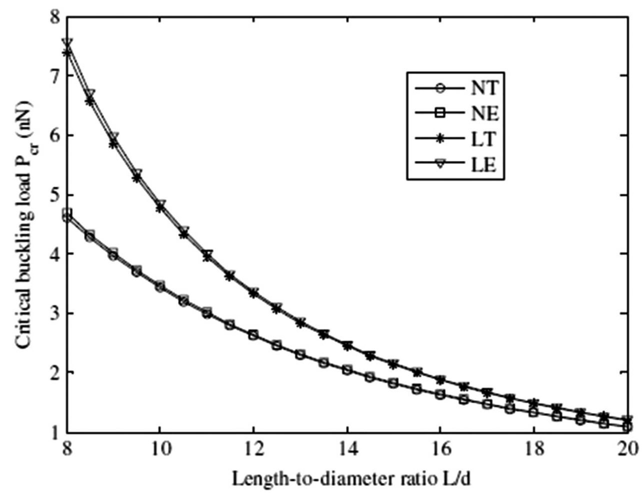

Fig. 12. Critical buckling load for both local and nonlocal Euler Bernoulli and Timoshenko beam models against the value of length to diameter ratio [395].
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problem related to the linear static bending of the Euler–Bernoulli beam model associated with the Eringen integral constitutive relation. Also, they proved that the differential form of the nonlocal Eringen theory was dissimilar to the integral form of the Eringen model, and then proposed a general scheme to analyze and solve the problem rigorously with consideration of the integral form. Using the integral form, a paradox appeared pertinent to solving a cantilever nanobeam with consideration of the differential form pertinent to increasing in stiffness values with the nonlocal parameter was solved. Ansari et al. [265] studied the static bending behavior of nanoplates using the integral formulation of the nonlocal Eringen model with consideration of the finite element procedure. They obtained the formulation in a general form, and then the arbitrary kernel functions were proposed. By employing the first-order shear deformation plate model, the governing relations associated with the integral as well as differential forms were derived. It was proved that using the integral method of the nonlocal Eringen theory, the mentioned paradox observed peculiar to the cantilever nanoplate was solved.

3.1.2. Nonlinear static bending of micro/nano-structures

Numerical approaches can be used to examine the nonlinear mechanical behavior of micro/nano-structures. Nonlinear static scrutiny of single layer annular/circular graphene sheets using nonlocal elasticity theory with consideration of Pasternak medium was studied by Dastjerdi et al. [389]. By employing the first-order shear deformation method with consideration of the von Kármán strain field geometry nonlinearity the equations of motion were derived. Moreover, using the DQM as well as the semi-analytical polynomial scheme, the obtained equations were transformed to nonlinear algebraic relations. Then, the final equations were solved using the Newton–Raphson iterative procedure. In another research, Dastjerdi & Jabbarzadeh [390] investigated the bending behavior of multi-layer orthotropic annular/circular graphene sheets with nonlinear effects with consideration of the elastic foundation. They applied the DQM and semi-analytical polynomial scheme to solve the obtained motion equations. Also, it was proved that the difference between local and nonlocal models increases with an increase in the magnitudes of van der Waals (vdW) interactions. Nonlinear static bending, as well as stretching of a circular graphene sheet model induced by the central point load, were examined by Duan and Wang [391]. Using the von Kármán nanoplate model with nonlinearity effect, the concentration of stress close to the loaded zone as well as the boundary for the sake of the graphene sheet bending rigidity were scrutinized. They proved that using proper parameters, the von

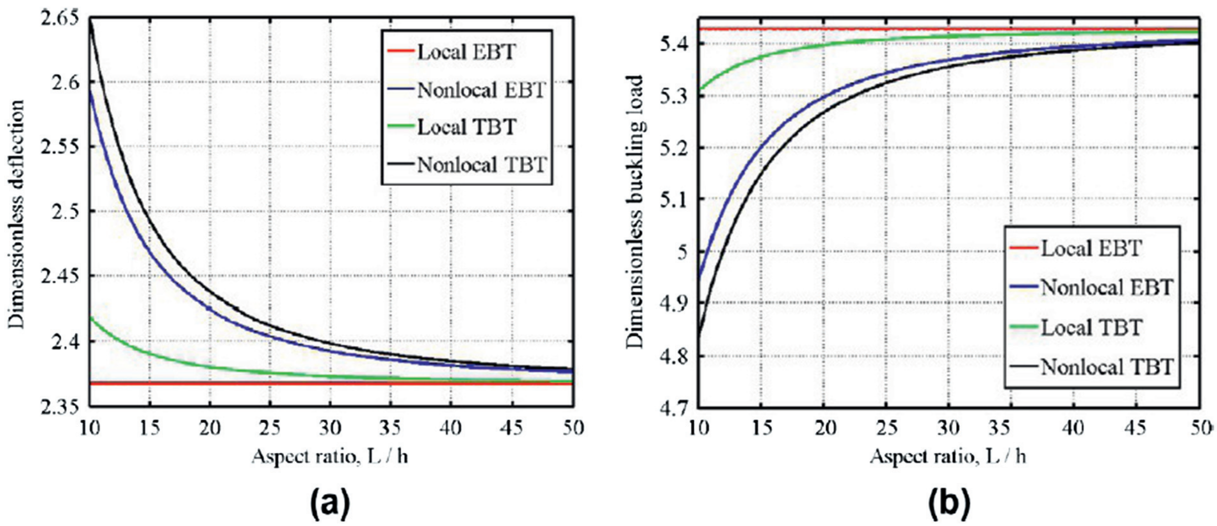

Fig. 13. (a)–(b). The influences of the aspect ratio on a non-dimensional deflection for uniform load and non-dimensional buckling load for local and nonlocal Euler Bernoulli and Timoshenko beam theories [396]. Reprinted with permission from Elsevier.

Kármán nanoplate model gave an accurate prediction of the mechanical behavior of the graphene sheet with consideration of linear/nonlinear bending as well as stretching. Xu et al. [392] worked on the nonlinear static bending of the size-dependent bilayer graphene sheets induced by transverse loads immersed in thermal environments. The vdW interactional forces were defined between the nonlocal double-layered nanoplate atoms with consideration of the von Kármán geometrical nonlinearity model. The thermal influences were considered in which the material characteristics were size- and temperature-dependent, and were acquired from MD simulations. They showed that the moderate influences of the stacking sequence were obvious, whereas the significant influences of the temperature change and the slenderness ratio on the nonlinear bending properties of the bilayer graphene sheet were observable. Also, it was indicated that the size-dependent effects diminish the static large deflections of the bilayer graphene sheet. Nonlinear bending/buckling/post-buckling behavior of the nanobeams using nonlocal elasticity as well as Gurtin–Murdoch surface elasticity energy influences was examined by Nguyen et al. [393]. They adopted the pre-residual stress produced in the bulk material, enforced by the residual surface tension that happened in the material layers. Based on the Euler–Bernoulli beam model as well as the elliptic integral scheme, the exact algebraic governing relations were established. They employed the discretization-free solution method using the Newton iterative procedure as well as the selected numerical quadrature scheme for solving nonlinear relations. They indicated that the proposed method leads to accurate outputs against analytical solutions. Small-scale effects of the nonlinear static bending behavior of a flexo-electric FG nanoplate using electro–thermo–mechanical loads based on the Kirchhoff classic model were analyzed by Ghobadi et al. [394]. By employing the variational scheme as well as the minimum potential energy principle, the nonlinear coupled governing differential relations of the nanoplate with their associated boundary conditions were derived. Furthermore, the FG nanoplate was proposed with consideration of a power-law formulation along the direction of nanoplate thickness. The direct/reverse flexo-electric influences were utilized to investigate the mechanical behavior of the FG nanoplate. They proved that using flexo-electricity yielded an increase in the values of the rigidity of the nanoplate. Likewise, they showed that the deflection, as well as the produced electric potential along nanoplate thickness, descended. Moreover, in the presence of linear temperature the induced polarization values were reduced.

3.1.3. Linear buckling/post-buckling of micro/nano-structures

Many researchers employed analytical methods to analyze the linear static buckling/post-buckling responses of the micro/nano-structures. Linear buckling analysis of micro- and nanorods/tubes using nonlocal Timoshenko beam theory was proposed by Wang et al. [395]. Based on the virtual work theory, they obtained the final governing equations. As it is shown in Fig. 12, increasing the values of length to diameter ratio leads to a decrease in the magnitudes of critical buckling load for both local and nonlocal Euler Bernoulli and Timoshenko beam models. Simsek & Yurtcu [396] studied the analytical investigation of bending/buckling of FG nanobeams using the Timoshenko beam model. The Navier-type method was employed for S-S nanobeam and then exact formulas were obtained for the buckling load and deflection. The influences of the aspect ratio on the non-dimensional deflection for uniform load and non-dimensional buckling load for local and nonlocal Euler Bernoulli and Timoshenko beam theories are given in Fig. 13 (a)–(b). It is quite clear that increasing the magnitudes of slenderness ratio leads to a decrease in the amounts of non-dimensional deflection and buckling load. Later, Rahmani & Jandaghian [397] investigated the buckling behavior of FG nanobeams using the nonlocal third-order shear deformation theory. They assumed that the material characteristics of FG nanobeams using the power law. Likewise, based on the nonlocal elasticity of the Eringen model and Hamilton's principle, the motion equations were derived with consideration of various boundary conditions. At the same time, Chaht et al. [398] examined the thickness stretching effects on the bending and buckling behavior of FG nanobeams. They used the nonlocal elasticity model, shear deformation, and thickness stretching influence with consideration of the sinusoidal model for all proposed displacements. By employing the Navier-type method the critical buckling load and the deflection of nanobeam were obtained. Thermal buckling of MWCNTs was proposed by Li & Kardomateas [399]. They utilized MWCNTs as thin shells which were coupled with vdW forces between nanotube layers. Also, they employed the closed-form method for radial/axial thermal buckling of the nanotube. They indicated that when the radial buckling occurs during thermal loading, the axial buckling has not happened. Evaluation of continuum mechanics to obtain buckling characteristics of SWCNTs was reported by Zhang et al. [400]. The authors examined two distinct buckling modes including the shell-type and the beam-type buckling modes pertinent to small and large slenderness ratios, respectively. It was proved that for the moderate slenderness ratios, using the more refined nonlocal Eringen beam model or the Timoshenko beam model is mandatory. Elmerabet et al. [401] worked buckling temperature of single-walled boron nitride nanotubes (SWBNNTs) with consideration of a novel nonlocal beam theory. The obtained results showed the significance of using small-scale effects in the thermal buckling scrutiny of BNNTs.

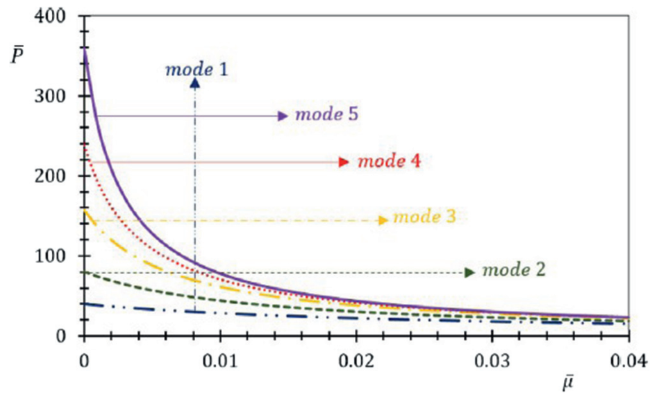

Fig. 14. The variation of critical buckling load against nonlocal parameters with different modes [402]. Reprinted with permission from Elsevier.

There are some researchers who employed discretization methods to examine the mechanical responses of the micro/nano-structures. Nejad et al. [402] worked on the buckling behaviors of two-directional FG Euler Bernoulli nanobeams. Using the minimum potential energy principle, the governing relations were derived and then with the GDQM, the Euler Bernoulli nanobeam with arbitrary boundary conditions was analyzed to obtain the critical buckling load. Fig. 14 demonstrates the variation of critical buckling load against nonlocal parameters with different modes.

It is clear that with an increase in nonlocal parameters, the dimensionless critical buckling load decreases for all different modes. The buckling load of rhombic, skew, rectangular, and trapezoidal nanoplates using different geometrical parameters were predicted in detail. Pradhan & Murmu [404] examined nonlocal effects on the buckling behavior of mono-layer graphene sheets using biaxial compression. It was proved that the small-scale effects had a significant influence on graphene sheets and a decreasing effect on the buckling loads. They compared the changes in the values of load ratio with the mode numbers for different magnitudes of aspect ratio, in which increasing the amounts of mode numbers leads to a decrease in the values of load ratio.

3.1.4. Nonlinear buckling/post-buckling of micro/nano-structures

Based on the nonlinearity geometric, numerical procedures can be employed to investigate the mechanical behavior of micro/nano-structures. The nonlinear investigation of nonlocal buckling of nanobeams was proposed by Emam [405]. It was revealed that the critical buckling load and the amplitude of buckling reduce and increase, respectively, with an increase in the values of nonlocal parameters which is correct for different beam theories such as Euler Bernoulli, first-order Timoshenko, and higher-order shear deformation models. In another research work, Ruocco & Reddy [406] studied the elastic–plastic buckling investigation of thick, rectangular nanoplates resting on a Winkler–Pasternak medium using the Reddy third-order plate model and nonlocal elasticity of Eringen. They employed the J_2 flow incremental and deformation models with consideration of the Ramberg–Osgood model to simulate the Elasto-plasticity model. Navier flowchart was used for solving the eigenvalue problems. Elastic and plastic buckling stress parameters versus the ratio of length for SS-SS nanoplates and classical plate models induced by a uniaxial load can be seen in Fig. 15.

It is obvious that the variation between the DT and FT models decreases with an increase in the magnitudes of nonlocal parameters. Norouzzadeh et al. [192] studied pre-buckling analysis of Timoshenko nanobeam models using the integral and differential approaches of nonlocal elasticity theories. The governing equations were obtained using the Timoshenko beam model and were utilized in a vector–matrix form which was applicable to the finite-element model. Moreover, an IG analysis was proposed to investigate the buckling behavior of the problem. Ansari et al. [408] worked on the analytical scheme for the nonlinear behavior of buckling/post-buckling of the cylindrical nanoshells with consideration of the surface elasticity method. Based on the Gurtin and Murdoch model, the surface energy influences were proposed. With consideration of geometrical nonlinearity associated with the nanoshell model made of Al and Si, the classical Donnell shell model as well as the von Kármán formulation the governing equations were derived. They proved the significance of using the surface stress on pre and post-buckling behavior of nanoshells with size-dependent effects. The zeroth-order shear deformation model for nonlinear post-buckling behavior of nanobeams with consideration of the nonlocal elasticity effects was proposed by Bellifa et al. [409]. Instead of rotational displacement employed in the shear deformation models, they utilized the shear deformation model according to the axial displacement with consideration of the shear forces. In order to derive the governing equations, the virtual work principle in conjunction with the nonlocal Eringen differential equations were adopted. Also, they employed the closed-form method to calculate the amplitude of the static nonlinear response as well as the critical buckling load in the post-buckling stage pertinent to S-S as well as C–C nanobeams. Thai et al. [410] studied the post-buckling behavior of FG nanoplates using the nonlocal model as well as IG analysis with consideration of Reddy's third-order shear deformation model and geometrical nonlinearity of the von Kármán formulation. The Mori–Tanaka procedure was used to obtain the effective material characteristics. Also, they used the virtual work principle to derive the governing relations. Moreover, in order to satisfy the C^1-continuity demand efficiently, the IG analysis was utilized for the discretization procedure. Likewise, they used the Newton–Raphson iterative method with consideration of the imperfection to investigate the post-buckling behavior of the nanoplate. They demonstrated that having larger gradient indices led to higher values of post-buckling deformations. Sidhardh et al. [411] scrutinized the post-buckling behavior of nanoplates with consideration of the nonlocal method and fractional-order continuum mechanics model based on the energy-based procedure. The application of Koiter's asymptotic scheme was proposed to scrutinize post-bifurcation branches associated with nanostructures. They utilized a 2D fractional-order finite element model to propose the Newton–Raphson as well as a path-following arc-length iterative scheme to analyze and solve the governing relations.

3.2. Vibration and wave propagation of small-scale structures

Recently, employing nonlocal elasticity of Eringen model for the investigation of vibration and wave propagation of micro/nano-structures have been increased by researchers. The following parts show the investigation of the works done by researchers pertinent to linear/nonlinear free and forced vibration, band structure analysis of micro/nano-sized periodic structures, as well as Elastic/thermoelastic wave propagation of micro/nano-structures related to the nonlocal elasticity of Eringen model.

3.2.1. Linear/nonlinear free vibration of micro/nano-structures

Analytical approaches were considered by many researchers to analyze the linear free vibration behavior of the micro/nano-structures. Reddy and Pang [412] analyzed nonlocal continuum theories of CNTs as beam structures. Both Euler Bernoulli and Timoshenko beam models were utilized in their research work. Also, the bending, vibration, and buckling behavior of beam structures with different boundary conditions were examined in detail. Based on their results, natural frequencies of both beam theories using S-S boundary conditions at

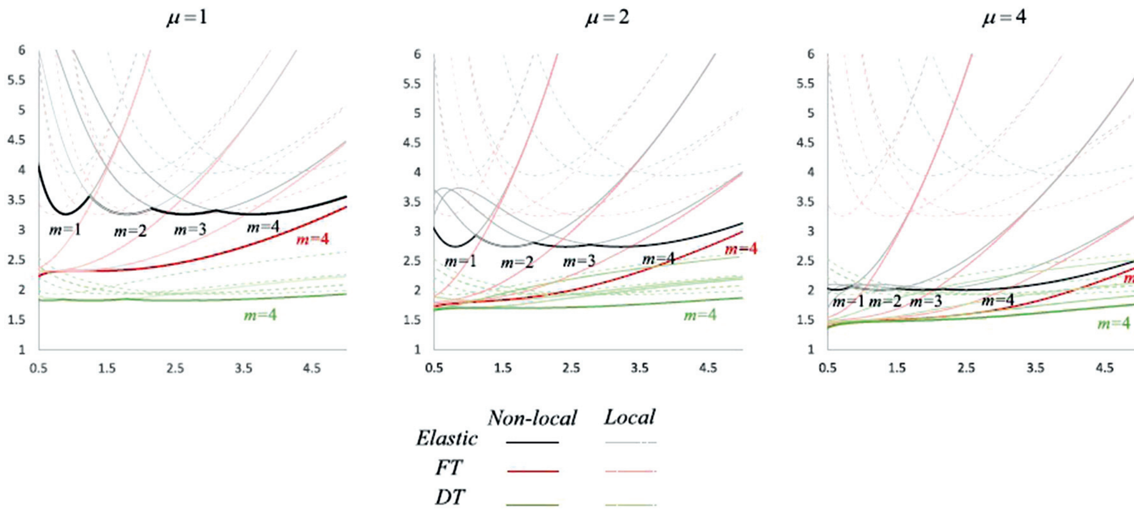

Fig. 15. Elastic and plastic buckling stress parameter versus the ratio of length for SS-SS nanoplates and classical plate models induced by a uniaxial load [406]. Reprinted with permission from Elsevier.

both ends with the consideration of the nonlocal differential theory can be obtained as [412]:

$$\begin{cases} \omega^E = \left(\frac{n\pi}{a}\right)^2 \sqrt{\dfrac{EI}{\left(m_0 + m_2\left(\frac{n\pi}{a}\right)^2\right)\left(1+\mu\left(\frac{n\pi}{a}\right)^2\right)}} \\ \omega^T = \left(\frac{n\pi}{a}\right)^2 \sqrt{\dfrac{EI}{m_0}}\left(\dfrac{1}{(1+n^2\pi^2\Omega)\left(1+n^2\pi^2\overline{\mu}\right)}\right) \end{cases}, \tag{15}$$

$$m_0 = \rho A,\ m_2 = \rho I,\ \mu = \left(e_0 l\right)^2,\ \overline{\mu} = \left(e_0 l/a\right)^2, n = 1, 2, ..$$

where Ω, a, and l are, respectively, the shear deformation parameter, the internal characteristic length, and the length of the nanobeam. In another important research work, dynamic properties of flexural beams based on a nonlocal elasticity model were studied by Lu et al. [413]. The frequency response and modal shape functions of a nanobeam with various boundary conditions were derived. The first four eigenvalues of S–S, C–C, and C–F nanobeams with consideration of the nonlocal parameter were calculated [413] (see Table 4).

Murmu and Adhikari [414] worked on the nonlocal frequency scrutiny of nanoscale biosensors. Using the frequency shift of the fundamental vibration mode, a novel nonlocal frequency sensor relation with consideration of energy principles was developed. They defined the stiffness mass and nonlocal calibration constants in their research. Two different points and distributed masses were utilized to simulate deoxythymidine molecules. Higher-order mathematical modeling for the vibration/wave propagation behaviors of protein microtubule nanoshell structures using shear deformation and nonlocal effects was examined by Daneshmand et al. [415]. They considered microtubules in mammalian cells which are cylindrical protein polymers and dynamically form functional activities in living cells. They also proved that microtubules were sensible to mechanical loading and the physical environment. The vibration and wave propagation features of the microtubule were scrutinized with and without consideration of the cytosol. They considered a microtubule and its modeling via an orthotropic elastic nanoshell. Likewise, the influences of microtubule shear to Young's modulus ratio (β) on the lowest wave velocity of the microtubule against wave vector using third shear deformation shell model for different values of circumferential modes ($n = 1, 2$) were investigated. In another research work, Kamil Zur et al. [417] studied the free vibration and buckling behavior of FGM nanoplates using the nonlocal modified higher-order sinusoidal shear deformation model with consideration of electro–magneto–elastic effects. They derived the motion equations of the rectangular FGM nanoplate based on Hamilton's principle, and then solved them using the closed-form analytical scheme as well as the Navier method. They also obtained the critical electrical, mechanical, and magnetic loads against various parameters related to the FGM nanoplate.

In order to analyze the nonlinear behavior of free vibration micro/nano-structures numerical methods were utilized by some researchers. Thermo-mechanical vibration of SWCNT resting on an elastic medium using nonlocal elasticity theory was done by Murmu and Pradhan [418]. The authors employed nonlocal elasticity theory to capture the size-dependency effects of the SWCNT. Also, the SWCNT was embedded in a Winkler-type elastic medium. They indicated that the differences between nonlocal and local frequency values were high for the lower magnitudes of temperature changes. Transverse vibration examination of an axially-loaded non-prismatic SWCNT using the two-parameter elastic medium was developed by Mustapha and Zhong [419]. They employed a polynomial power law to define the range of tapers along the longitudinal axis of the continuum. Using Bubnov–Galerkin method, the variable coefficient governing equations of the tapered SWCNT was obtained and then solved. Li et al. [375] published an article about nonlocal influences in nanobeams. They examined the existence of nonlocal effect in the bending of a nano-cantilever beam under a concentrated force at the free end and then studied the variation of equivalent stiffness of nanobeam rather than the classical beam model. Malekzadeh and Shojaee [420] investigated the nonlocal effects on the nonlinear vibration of non-uniform nanobeams using surface energy. They showed that the normalized natural frequency decreases with an increase in the values of nanobeam's length, which is correct for both the first and second vibrational modes and different small-scale parameters (see Fig. 16(a)–(b)).

Arefi et al. [421] investigated the free vibration behavior of FG polymer composite curved nanobeams reinforced with graphene nanoplatelets using the nonlocal elasticity theory and Pasternak substrate. By employing the principle of Hamilton and the nonlocal integral elasticity method in conjunction with the special bi-exponential averaging kernel the governing motion equations with consideration of size-dependent effects were derived. They described the displacement field in polar coordinates using the first-order shear deformation scheme. They proved that a gradual increase in the values of geometrical opening angle leads to a decrease in the magnitudes of the natural frequencies.

Nonlocal-integro-differential vibration of elastically supported nanorods was established by Kiani [422]. Using a meshless methodology, for fixed–fixed and fixed–free nanorods the obtained relations

Table 4
The first four eigenvalues of simply supported, clamped, and cantilevered nano-beams [413].
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e_0a/L	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Simply-supported beam									
q_1L	3.1416	3.0685	2.8908	2.6800	2.4790	2.3022	2.1507	2.0212	1.9102
q_2L	6.2832	5.7817	4.9581	4.6013	3.8204	3.4604	3.1815	2.9585	2.7754
q_3L	9.4248	8.0400	6.4520	5.4422	4.7722	4.2941	3.9329	3.6485	3.4174
q_4L	12.5664	9.9161	7.6407	6.3630	5.5509	4.9820	4.5565	4.2234	3.9536
Clamped beam									
q_1L	4.7300	4.5945	4.2766	3.9184	3.5923	3.3153	3.0837	2.8893	2.7246
q_2L	7.8532	7.1402	6.0352	5.1963	4.5978	4.1561	3.8156	3.5462	3.3251
q_3L	10.9956	9.2583	7.3840	6.2317	5.4738	4.9328	4.5231	4.1996	3.9360
q_4L	14.1372	11.0158	8.4624	7.0482	6.1504	5.5213	5.0505	4.6816	4.3828
Cantilever beam									
q_1L	1.8751	1.8792	1.8919	1.9154	1.9543	2.0219	2.1989	–	–
q_2L	4.6941	4.5475	4.1924	3.7665	3.3456	2.9433	2.4809	–	–
q_3L	7.8548	7.1459	6.0674	5.2988	4.8370	–	–	–	–
q_4L	10.9955	9.2569	7.3617	6.1385	5.2399	–	–	–	–

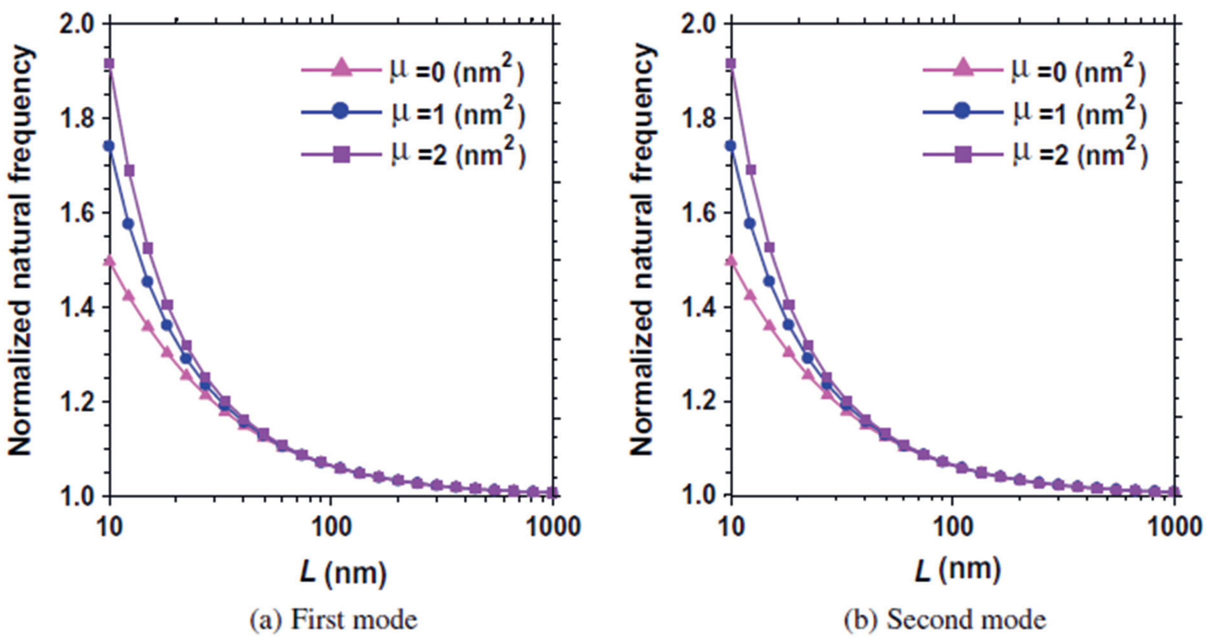

Fig. 16. (a)–(b). Normalized natural frequencies with the length of nanobeams for both first and second vibrational modes and different small-scale parameters [420]. Reprinted with permission from Elsevier.

were solved. The effects of the nonlocal parameter, elastic supports, surface energy, and kernel function on natural frequencies of the nanorod were examined. Visco-nonlocal–nonlinear piezoelasticity influences on the dynamic stability behavior of graphene sheets with ZnO sensors and actuators with consideration of surface elasticity based on a refined zigzag theory were published by Ghorbanpour Arani et al. [423]. They employed nonlocal piezoelectricity, Kelvin–Voigt, and differential cubature methods in their research. Likewise, a proportional-derivative controller was utilized as active control of the system's dynamic stability. They proved that magnetic fields and external voltages were considered effective control parameters for the dynamic instability region of the system. Norouzzadeh and Ansari [424] studied the finite element model of Timoshenko nanobeams based on the nonlocal integral elasticity model. They compared the maximum deflection of a nanobeam against the thickness to the nonlocal ratio based on the differential and integral models. The obtained results depicted in Fig. 17 were related to a cantilever nanobeam. In a different study, Arefi et al. [425] carried out nonlocal elasticity bending examination of curved nanobeams fortified with graphene nanoplatelets. Based on the first shear deformation theory and principle of virtual work, they derived the motion equations. Moreover, based on the Halpin–Tsai model and the rule of mixture, the effective Poisson's ratio and Young's modulus were examined. In another study, Alavinasab et al. [426] performed modeling of SWCNT composites using the nonlocal elasticity method. A representative volume element of the CNT composite was employed to obtain unknown parameters in the nonlocal elasticity theory. Moreover, stress distributions in representative volume elements with consideration of the nonlocal, finite element, and classical theories were examined.

3.2.2. Linear/nonlinear forced vibration of micro/nano-structures

Many papers were addressed related to the discretization procedures associated with the linear/nonlinear forced vibration characteristics of the micro/nano-structures. The dynamic response of the SWCNT acted upon by a moving harmonic load using modified nonlocal elasticity theory was examined by Rahmani et al. [416]. They used nonlocal relation for a homogeneous isotropic Euler Bernoulli beam as follows:

$$\sigma_{xx} - (e_0a)^2 \frac{\partial^2\sigma_{xx}}{\partial x^2} - (e_0a)^4 \frac{\partial^4\sigma_{xx}}{\partial x^4} - (e_0a)^6 \frac{\partial^6\sigma_{xx}}{\partial x^6} - \cdots - (e_0a)^{2N} \frac{\partial^{2N}\sigma_{xx}}{\partial x^{2N}} = E\varepsilon_{xx}. \quad (16)$$

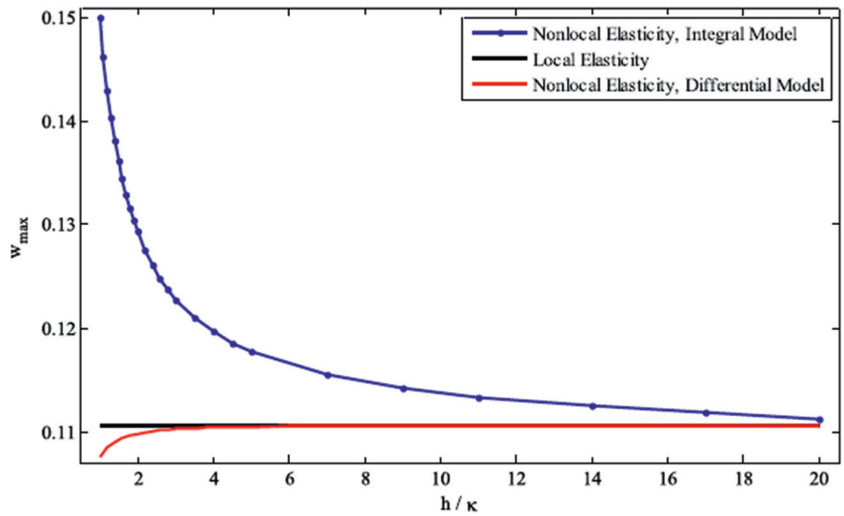

Fig. 17. The maximum deflection of nano-beam against the thickness to nonlocal ratio based on the differential and integral models [424]. Reprinted with permission from Elsevier.

where E, σ_{xx}, and ε_{xx} are, respectively, the elasticity modulus, the stress, and strain counterparts. The authors showed that increasing the frequency ratio yields an increase in the values of the maximum deflection as well as decreasing in the magnitudes of the frequency ratio leads to a decrease in the amounts of the maximum deflection which was correct for higher values of load velocity than a specified magnitude. Roudbari et al. [427] studied transient responses of similar interacting SWBNNTs acted upon by the movement of a nanoparticle. Electro-mechanical coupling behavior and nonlocal elasticity effect proposed by using respectively piezoelectricity as well as nonlocal theories. The interactional behavior between the two adjacent BN atoms was mooted in the calculation of vdW interactional forces among various atoms on the respective SWBNNTs. Using the transformation method, the vdW parameter was analytically obtained via the mentioned fourfold integral. They studied the influence of the non-dimensional velocity factor on the normalized axial displacement, and deflection of both mutual SWBNNTs for different magnitudes of aspect ratios shown in Fig. 18(a)–(f). For large magnitudes of the non-dimensional velocity factor, the normalized deflection will be negligible which the same for all case studies.

Also, the normalized axial displacement and deflection magnitudes for both BNNTs in $\Gamma = 60$ are lower than $\Gamma = 20$. Likewise, the normalized deflection magnitudes of the Timoshenko beam model overestimate the obtained results by the Rayleigh beam theory, for both without and with electric field cases. Civalek et al. [428] investigated the forced vibration behavior of the composite beam model reinforced by CNTs. They used a polymeric matrix in the composite beam and reinforced the SWCNTs with their different distributions. Also, the first-order shear deformation beam model was employed in their research. Using the Lagrange method, the governing motion equations were obtained, and then the Ritz method in conjunction with the algebraic polynomials were utilized to solve the equations with consideration of the trivial functions. Likewise, the Newmark average acceleration scheme was used to examine the forced vibration behavior of the nano-structure. Nonlinear forced vibration scrutiny of the nanocomposite-reinforced thick annular system with viscoelastic effects induced by the hygrothermal environment as well as mechanical loadings was analyzed by Al-Furjan et al. [429]. The matrix was reinforced via CNTS or the fibers or Carbon with consideration of the third-order shear deformation as well as von Kármán nonlinear shell models. In order to obtain the relations of motion, Hamilton's principle was utilized, and then solved using the GDQ method as well as the perturbation technique. They proved that the dynamic behavior of the proposed system becomes harden and soften while the magnitudes of the nonlinearity factor were positive as well as negative, respectively.

Studies dealing with nanotubes with flow inside, where the flow is described by potential flow, or other Newtonian flow models, can be incorrect. At this scale, the fluid does not behave like a Newtonian fluid. Therefore researchers should pay heed when they are modeling these types of structures. Nonlinear free/forced vibration characteristics of CNTs conveying magnetic nanoflow induced by a longitudinal magnetic field with consideration of the stress-driven nonlocal integral scheme and various boundary conditions were investigated by Mahmoudpour and Esmaeli [430]. They derived the partial differential relations based on the Euler–Bernoulli beam model with consideration of the von Kármán kinematic relations. Also, the governing motion equations were established by the Galerkin procedure, in which a set of nonlinear ordinary differential relations were obtained. Moreover, using the perturbation technique, the nonlinear primary, subharmonic, and superharmonic frequency responses were calculated and the divergence/flutter instability of the nano-structure associated with the increase in the velocity of nanofluid was scrutinized. They revealed that an increase in the values of the magnetic field leads to an increase in the magnitudes of the critical flow velocity, which yields a delay in the divergence/flutter phenomenon.

3.2.3. Elastic/thermoelastic wave propagation of micro/nano-structures

Analytical method as a capable solution methodology were employed by many researchers to investigate the elastic/thermoelastic wave propagation of micro/nano-structures [431–435]. Transverse wave propagation in SWCNTs under longitudinal magnetic fields based on nonlocal elasticity models was proposed by Kiani [432]. Using nonlocal Rayleigh, Timoshenko, and higher-order beam theories, and also with consideration of a surrounding elastic medium, the dimensionless governing motion equations were obtained. He proved that increasing the magnitudes of nonlocal parameters and axial magnetic field effects, respectively, leads to a decrease and an increase in the values of dimensionless frequencies. Wave propagation investigation of fluid-conveying SWCNTs with consideration of magneto–hygro–mechanical loads using a two-phase local/nonlocal mixture theory was conducted by Farajpour et al. [433]. They used the Beskok–Karniadakis model to examine slip effects between the fluid and SWCNT as a correction factor. The authors indicated that using the two-phase integral scheme leads to reducing the restrictions of the differential nonlocal elasticity peculiar to higher values of the wave numbers. The nonlocal electro-magnetic wave propagation behavior of DWCNTs was investigated by Basmaci [434]. The influences of material characteristics parameters associated with the inner and outer layers of CNTs on the electro-magnetic wave propagation were scrutinized. Moreover, the electro-magnetic wave

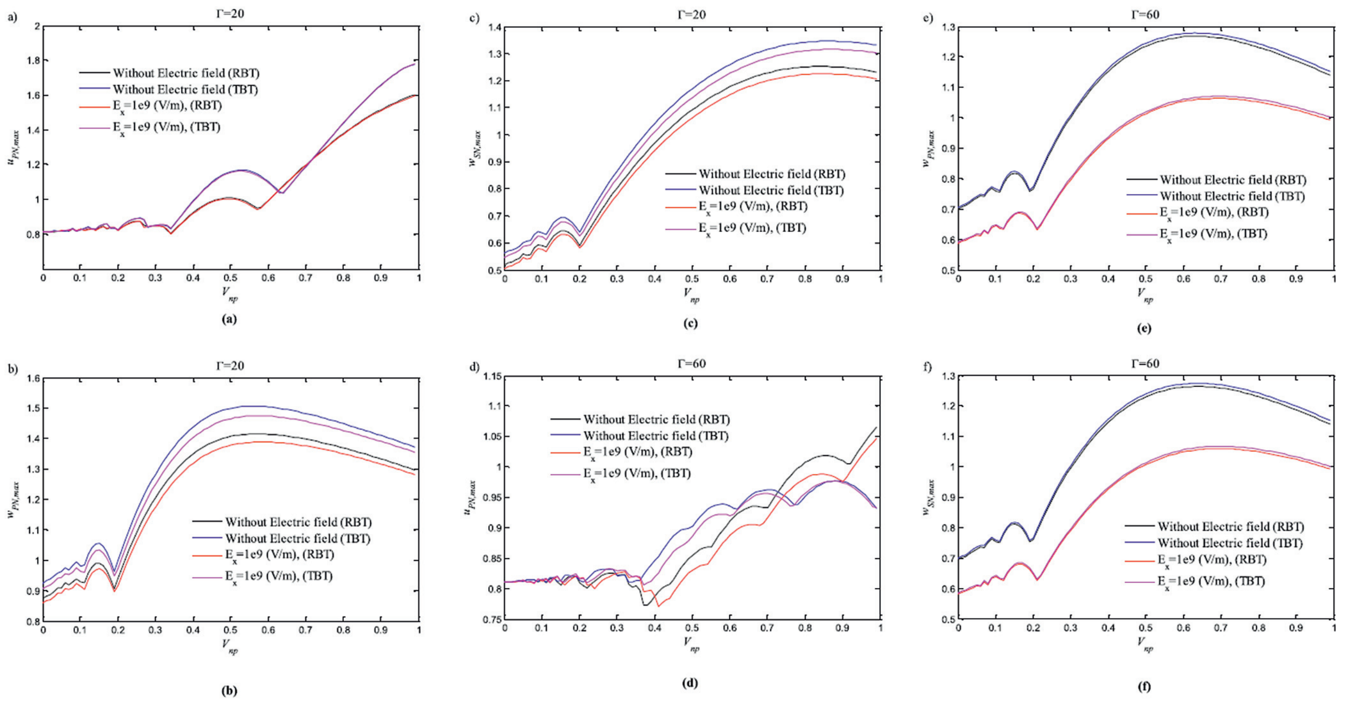

Fig. 18. (a)–(f). The effect of the non-dimensional velocity parameter on the normalized dynamic axial displacement and deflection of the primary and secondary SWBNNTs [427]. Reprinted with permission from Elsevier.

frequencies were remarkably examined. They demonstrated that when the wave number was larger than 1.8×10^{10}, the wave frequencies of the fundamental and second modes converged to the value of 3.554×10^{8} Hz. Likewise, the nonlocal electro-magnetic wave frequencies descended with an increase in the magnitudes of the nonlocal parameters and a decrease in the values of material parameters. Das et al. [435] worked on the generalized model of the Green/Naghdi model III thermo-elasticity theory using the nonlocal elasticity of Eringen theory to investigate the wave propagation of the harmonic plane waves with consideration of the nonlocal thermo-elastic medium. They employed a vertically shear wave with dispersive effects to encounter the critical wave frequency. The influences of the elastic behavior on the reflection parameters as well as the ratios of energy were investigated with consideration of the nonlocality of the model.

Wave propagation and free vibration properties of FG cylindrical nanoshell with nonlocal influences with consideration of wave-based procedure were scrutinized by He et al. [436]. The nonlocal elasticity of Eringen as well as the first-order shear deformation shell models were utilized. Also, the wave propagation properties associated with the longitudinal/circumferential wave number were examined. Moreover, with consideration of the effects of power-law exponent, small-scale parameters, thickness to radius ratios, as well as wave number, the wave dispersion behavior of wave frequency/phase velocity were scrutinized. Nonlocal piezoelectric wave propagation behavior of double-nanobeams associated with the Euler–Bernoulli beam theory was given by Ghorbanpour Arani et al. [437]. They utilized the proposed piezoelectric nanobeams were had been coupled using an elastic substrate (Pasternak medium). Also, according to Hamilton's principle, the general differential relations were established based on the nonlocal piezoelectricity model. By employing the analytical procedure the phase velocity, cut-off as well as escape frequency values were calculated. They showed that the applied external voltage was a capable controlling parameter for the proposed system. Moreover, the obtained phase velocity of in-phase wave propagation was not related to the stiffness of the elastic foundation. Ebrahimi et al. [438] studied wave propagation properties of the elastic waves applied in heterogeneous nanobeams using a novel two-step porosity-dependent homogenization method. Using the principle of Hamilton, the Navier formulations were established associated with the Euler–Bernoulli beam theory. Moreover, the motion equations were acquired based on the nonlocal elasticity model. Then, the proposed equations were analyzed and solved to investigate the wave frequency of the nano-structure. Likewise, they used some parametric studies to demonstrate the importance of using size-dependent effects on the wave dispersion properties of the proposed porous FG nanobeams.

3.3. Specific classes of the nonlocal models

There are several novel specific classes of the nonlocal models including peridynamic (PD), dual-horizon peridynamics (DH-PD), as well as the nonlocal operator methods (NOMs) [439–445]. Recently, researchers carried out several outstanding studies on the mentioned capable classes of the nonlocal models associated with some specific applications. For modeling of the complex fractures such as crack branching as well as the combination of multiple cracks, the PD model as a capable method in computational mechanics is employed, in which this model is not considered as a part of the problem, but proposed via the solution. The proposed method can be considered as a premium technique and applicable for all dimensions than other traditional schemes including the partition of the unity model and meshless procedure, etc. [446,447]. Silling [448] proposed a traditional type of PD method which was utilized for some specific applications such as composite delamination [449], fragmentation [450], and beam/plate models [451]. Bond-based peridynamics (BB-PD), as well as state-based peridynamics (SB-PD), are considered two major types of PD models. Likewise, ordinary state-based peridynamics (OSB-PD) and non-ordinary state-based peridynamics (NOSB-PD) can be proposed as two types of the SB-PD model. For modeling the materials with specific values of the Poisson's ratio ($\nu = 1/3$ and $\nu = 1/4$ for 2D and 3D materials, respectively), the BB-PD models can be employed in which bonds act similar to independent springs [448]. In contrast, there are no restrictions for using specific magnitudes of the Poisson's ratio in modeling materials once the SB-PD is utilized. Indeed, this model can be considered as the extension of the BB-PD model and can be used for all materials with complicated models [448]. PD models are based on the interacting horizontally between points of the materials where the points' distances should be considered within a threshold magnitude. For removing the mentioned restrictions pertinent to the PD model, the DH-PD model can be employed to boost the computational efficiency using less modification regarding the PD model. In the DH-PD model, the points can have their horizon size within the computational domain [448]. In general, both brittle and ductile fractures can be modeled by the PD model. Furthermore, two mentioned types of PD models (BB-PD and OSB-PD) is capable to model the brittle fracture in the linear type of solids. On the other hand, the NOSB-PD model in conjunction with other techniques such as Gurson–Tvergaard–Needlemam (GTN) model can be employed for modeling ductile fracture with consideration of large deformation as well as the nonlinear characteristics of materials [452].

Ren et al. [439] worked on the investigation of the NOSB-PD model using the GTN procedure associated with ductile fracture. Using large deformation as well as the variational calculus scheme, the NOSB-PD model was obtained. In order to model the bulk properties, the GTN model was utilized. They used the penalty method to decrease the mode of zero-energy. In another interesting research, Ren et al. [440] scrutinized the DH-PD procedure with consideration of the varying horizon distances which solved the "ghost force" problem. They considered the interactions with unbalanced behavior between various horizon sizes particles to propose the importance of dual-horizon. Balances of linear and angular momentum were considered completely in their research. They did not consider the "partial stress tensor" or the "slice" methods to improve the ghost force problem. They showed that the DH-PD method was employed in PD codes with consideration of minimal variations. Zhao et al. [441] examined a novel fracture criterion for the PD and DH-PD models peculiar to a crack tip. They considered the bond forces were disappeared when the relative deformation pertinent to particles of the PD bonds approaches the acute value of the opening displacement associated with the crack tip. Their novel relations were applicable for the elastic solid with linear effects which were extended to the materials with nonlinear influences. Likewise, they validated their results to the experimental data as well as other computational procedures. A new NOM with consideration of the variational method was investigated by Rabczuk et al. [442], which was applicable for the partial differential equations (PDEs) related to electromagnetic waveguide problems. Based on their formulations, they could assemble the matrix of tangent stiffness readily due to the analysis of eigenvalues related to the waveguide problems. Using electric fields and their assumed nonlocal formulations (nonlocal integral method), their obtained equations were analyzed and solved. Furthermore, to eliminate the modes of zero-energy, the hourglass energy function was proposed. In another study, Ren et al. [443–445] investigated the NOM to analyze PDEs. They used higher-order nonlocal operator method (HONOM) as well as higher-order hourglass energy functional in their study. According to the HONOM model, multiplications matrices from the residual as well as stiffness matrices were obtained using the introduced functional.

4. Micromorphic theory

The micromorphic theory (MMT), defined by Eringen and Suhubi [453,454] is a top-down micro-scale procedure with distinguishing features, is a suitable method for considering the microstructure effects of materials. It can be considered as a general form of micropolar theory (MPT), in which the major difference between the classical model and micromorphic continuum mechanics is peculiar to the additional degree of freedom (DOF) of material particles and balance equations. Based on this model, each material particle experiences an additional micro-motion that can be described by three deformable vectors as "director".

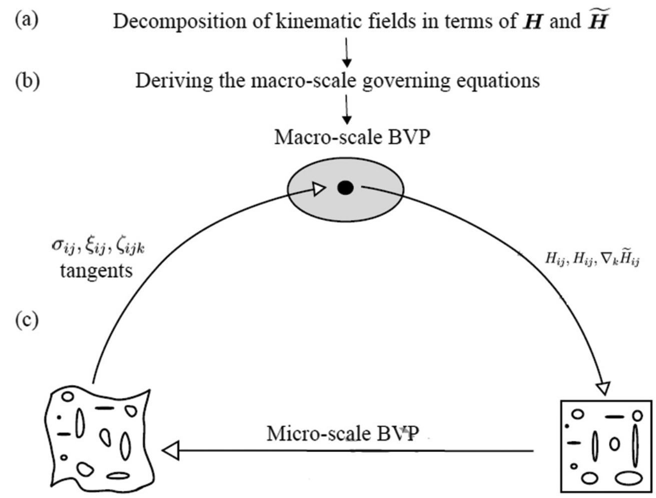

Fig. 19. MMT computational homogenization strategy [455]. Reprinted with permission from Elsevier.

4.1. Linear/nonlinear static bending and buckling of small-scale structures

The investigation of micromorphic effects of the first-order shear deformable plate was reported by Ansari et al. [456]. The influences of small-scale and other parameters on the bending behavior of micromorphic plates were proposed. They also indicated the significance of present finite element formulation on the analysis of small-scale structures which is for the sake of using micro-deformation and rotation DOF of material particles. In another study, Ansari et al. [457] worked on the micromorphic prism elements. They claimed that the matrix representation of their relations was given from a prism micromorphic element using the influences of micro-DOF of material particles. The obtained element was performed to the bending behavior of micromorphic circular and rectangular plates induced by different boundary conditions. Forest and Sab [458] analyzed finite-deformation second-order micromorphic procedure and its relations to strain and stress gradient models. Using the finite deformation framework, dynamical balance laws and hyperelastic constitutive relations were obtained. They showed an extension to finite deformation of stress gradient continuum model with consideration of second-order micromorphic model. The non-classical finite element model for the nonlinear mechanical behavior of micropolar plates was studied by Ansari et al. [459]. A new 3D formulation for the micropolar model in conjunction with the finite element theory was proposed. Furthermore, the final general formulation was reduced to the Mindlin plate model due to investigate the micropolar plates. Different types of boundary conditions were utilized to demonstrate the capability of the developed element. In another study, Hassani et al. [460] proposed an efficient numerical approach to the micromorphic hyperelasticity which was the variational DQ-finite element method. In their novel method, the domain was transformed into some finite element methods, and then a variational discretization technique was utilized within each element. Micromorphic approach to gradient plasticity and damage was reported by Forest [461]. Eringen and Mindlin micromorphic models were proposed and then extended to finite elastic–plastic deformations. A systematic approach to develop size-dependent plasticity and damage models was presented relating to phase-field models, and can be used for determining the effects of the hardening or softening material characteristics. The author classified the mechanics of generalized continua in local and nonlocal actions, in which the local model can be classified into two simple and non-simple material models. The simple material was ended in the Cauchy continuum classical model, whereas the non-simple model based on the order or grade medium models can be categorized into cosserat, micromorphic, second gradient, as well as the gradient of the

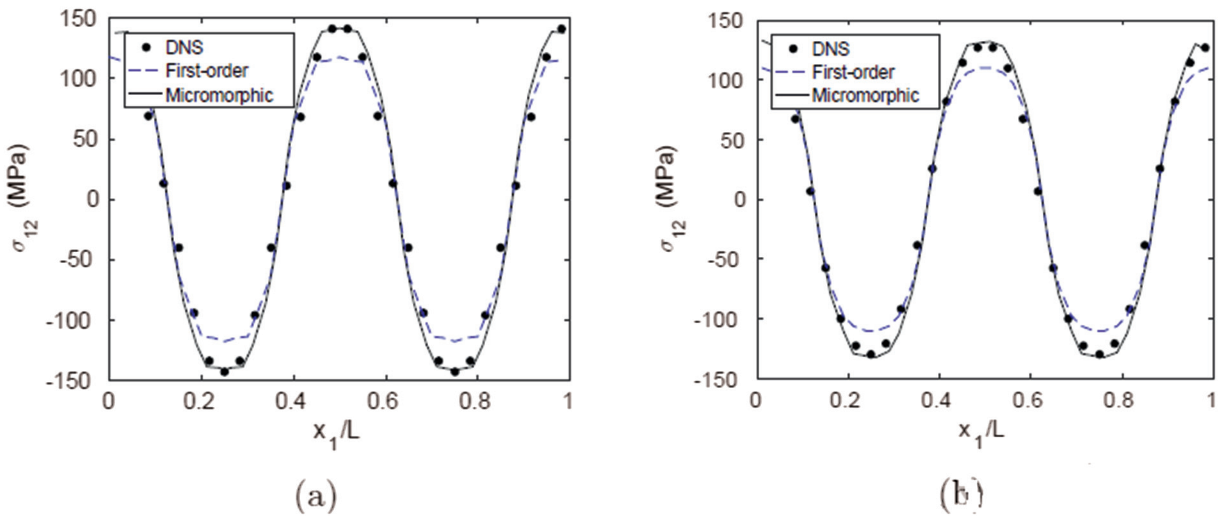

Fig. 20. Macro shear stress (σ_{12}) variation against the reference DNS and different representative volume elements [455]. Reprinted with permission from Elsevier.

internal variable. Micromorphic homogenization of a porous medium with consideration of elastic features and quasi-brittle damage was carried out by Hutter et al. [462]. They mentioned that MMT needs the formulation of related constitutive rules, which include the microstructural interactions. They showed that the application of MMT to investigate size-dependent effects of porous media like foams was very remarkable. In another research, Hutter [463] worked on the micromechanical gradient extension of Gurson's model of ductile damage via microdilatational media model. A sound micromechanical basis was combined with computational efficiency. Based on homogenization with unconstrained microdilatational media theory, a limit-load scrutiny was accomplished for a unit cell using void leading to Gurson-type with a closed-form yield function with consideration of additional terms of the microdilatational media model. Likewise, in another study in that year, homogenization of a Cauchy continuum using an MMT was proposed [464]. Using static or kinematic boundary conditions, the boundary value problem (BVP) was solved based on the microscale formulation. The author derived a closed scheme to homogenize the microscale of the Cauchy continuum to a micromorphic continuum model with consideration of the explicit definitions of all proposed generalized macroscopic stress as well as the deformation values. The MMT computational homogenization framework for heterogeneous materials was developed by Biswas and Poh [455]. Using matrix–inclusion composites, a novel computational homogenization framework was introduced in which standard continuum models at the micro-scale can be mapped into the macro-scale models to regain the MMT. Moreover, with consideration of representative volume elements, their relations proposed an additional macro kinematic field to demonstrate the average strain in the inclusions. They proved that without consideration of a clear separation of length scales between macro and micro, the homogenized MMT could show the material responses. Fig. 19 depicts MMT computational homogenization strategy. Fig. 20 demonstrates macro shear stress (σ_{12}) variation, which was obtained from the first-order shear deformation model and MMT, against the reference direct numerical simulations (DNS) and different representative volume elements. For the two presented morphologies, the MMT predicts an accurate structural response. In two recent studies, Norouzzadeh et al. [465,466] worked on the large elastic deformation of micromorphic shells in two different parts including variational formulation (Part I) and an IG method (Part II). In Part I, by the novel matrix–vector format utilized for the kinematic model, energy functions, and constitutive relations, an IG method-based solution strategy was proposed, which can readily assess the macro/micro-deformation field components. Also, using size-independent elasticity tensors, high-order stress–strain equations were derived for the isotropic micromorphic solid model. And then, to simplify the solution method of the obtained tensor-based relation, an equipollent matrix–vector form was introduced. On the other hand, in Part II, various types of locking behavior were accomplished in the finite element model (conventional form) of thin shells using low-order elements. Furthermore, a non-standard method of finite element model including additional DOFs and mixed interpolation was the available scheme to tackle locking problems.

4.2. Linear/nonlinear vibration and wave propagation of small-scale structures

Madeo et al. [193] presented the first evidence of nonlocal behavior in real band-gap metamaterial structures using the relaxed MMT, with real experiments of longitudinal plane wave transmission across an interface with a classical Cauchy material as well as a phononic crystal structure. The relaxed MMT in an isotropic model in addition to the investigation of micro-inertia parameters was utilized to characterize the constitutive parameters. A comparison transmission coefficient against that of another work for a real phononic crystal was done. Good fitting was reported up to frequencies around the order of 1 MHz. Also, Bragg scattering phenomena in their model was clear. Nonlinear regularization scrutiny of micromorphic approach to gradient elasticity, viscoplasticity, and damage was carried out by Forest [467]. A novel balance relation for generalized stresses and MMT constitutive relations were combined to generate the regularization operator. The approach was capable of a various range of elastoviscoplastic and damage models with multiphysics and anisothermal coupling. A finite element model of vibrating microbeams and micro-plates with consideration of a 3D Micropolar Element was developed by Ansari et al. [468]. They employed a non-classical 3D element to examine the free vibration behavior of micropolar beams and plates. Also, the micro-structure effect on the frequencies of microbeams and micro-plates under various boundary conditions was studied. They considered a 3D micropolar element. A 27-node cubic element was utilized in which each node has 6 DOFs with 3 displacement and 3 micro-rotations. They investigated the mode shapes of CC microbeams and CCCC micro-plates for different values of frequencies. Later, Eremeyev et al. [469] reported acceleration waves in the nonlinear elasticity micromorphic continuum. An acoustic tensor for the micromorphic medium was derived and then formulated the conditions for the existence of acceleration waves. And finally, through an example, their assumed conditions were examined for the linear micromorphic medium, and the relaxed micromorphic model. Faraji-Oskouie et al. [470] examined finite element modeling of micromorphic continua using three-dimensional elasticity. They formulated a linear MMT with consideration of three-dimensional elasticity, and then a three-dimensional size-dependent element was introduced using micro-deformation and rotation DOF of material particles. As it is shown in Fig. 21, they compared their results with the classical theory and MPT. Good agreement between their results was obtained for a fully clamped plate model.

5. Nonlocal strain-gradient theory

The use of an appropriate size-dependent method is important to investigate the mechanical behavior of micro-/nano-structures at micro-/nano-sized scales. The nonlocal strain-gradient model has been employed to discern both nonlocal and strain gradient methods, simultaneously. Using the higher-order theory by Lim et al. [51], the internal energy density potential $U_{ED}(\varepsilon_{ij},\varepsilon'_{ij},e_0a)$ based on the nonlocal strain-gradient model can be expressed as:

$$U_{ED}(\varepsilon_{ij},\varepsilon'_{ij},e_0a)=\frac{1}{2}\varepsilon_{ij}C_{ijkl}\int_V \alpha_0(\left|x-x'\right|,e_0a)\varepsilon'_{kl}dV + \frac{l^2}{2}\varepsilon_{ij,m}C_{ijkl}\int_V \alpha_1(\left|x-x'\right|,e_1a)\varepsilon'_{kl,m}dV, \tag{17}$$

where e_0a, e_1a, C_{ijkl} and l are, respectively, the small-scale parameters, elastic modulus tensor, and material length scale parameter. Moreover, ε_{ij} and ε'_{ij} are the Cartesian terms of the strain tensors. Likewise, $\alpha_0(\left|x-x'\right|,e_0a)$ and $\alpha_1(\left|x-x'\right|,e_1a)$ are the nonlocal attenuation kernel functions that were employed to define nonlocal effects for the classical stress as well as strain gradient tensors, respectively. For a two-dimensional problem, the nonlocal attenuation functions are proposed as $\left(\mu_0=(e_0a)^2,\mu_1=(e_1a)^2\right)$:

$$\begin{cases}\alpha_0(\left|x-x'\right|,e_0a)=(2\pi\mu_0)^{-1}B_0(\dfrac{\left|x-x'\right|}{\sqrt{\mu_0}})\\ \alpha_1(\left|x-x'\right|,e_1a)=(2\pi\mu_1)^{-1}B_0(\dfrac{\left|x-x'\right|}{\sqrt{\mu_1}})\end{cases}, \tag{18}$$

where B_0 is the Bessel function. Therefore, the total stress tensor will be obtained as follows:

$$\tau_{xx}=\sigma_{xx}-\frac{d\sigma^1_{xx}}{dx}, \tag{19}$$

in which σ_{xx} and σ^1_{xx} are the classical and higher-order stresses, respectively. Then, according to Lim's theory and using $e=e_0=e_1$, the final constitutive relation of the nonlocal strain-gradient procedure can be expressed as [51,471–492]:

$$\left(1-(ea)^2\nabla^2\right)\tau_{xx}=E\left(1-(l)^2\nabla^2\right)\varepsilon_{xx}. \tag{20}$$

where E, τ_{xx}, and ε_{xx} are, respectively, the elasticity modulus, the stress, and strain counterparts. Based on the mentioned assumptions pertinent to the nonlocal strain-gradient model, there are some limitations related to using this model in the design of micro/nano-devices. Designers should pay attention to increasing or decreasing the values of nonlocal and material length scale parameters simultaneously, in which having higher magnitudes of nonlocal parameters lead to more softening models than the hardening effect and vice versa [476–496].

5.1. Bending and buckling of small-scale structures

In recent years, the scrutiny of linear/nonlinear static and dynamic bending, as well as linear/nonlinear buckling and post-buckling of micro/nano-structures associated with the nonlocal strain-gradient model, was proposed by several authors in this field. The following parts present a survey of the assumptions, formulations, advantages, and drawbacks of using the nonlocal strain-gradient model to demonstrate the size-dependency influences associated with bending/buckling of the nano-structures.

5.1.1. Linear static bending of micro/nano-structures

Many studies reported analytical approaches to investigate the linear static bending behavior of the micro/nano-structures. The investigation of a unified nonlocal strain-gradient model pertinent to the nanobeams based on the significance of higher-order terms associated with Euler–Bernoulli and Timoshenko beam theories was proposed by Lu et al. [471]. The obtained equations were established using Hamilton's principle based on the relevant boundary conditions. Likewise, Navier's procedure was employed to examine the bending/buckling of an S-S nanobeam analytically. It was proved that the proposed nanobeam showed stiffness-softening/hardening influences according to the selected values of small-scale parameters. Arefi et al. [472] investigated the effects of the nonlocal strain-gradient model on the bending behavior of the sandwich porous nanoplate with consideration of piezomagnetic face-sheets using the first-order shear deformation theory. They utilized the power-law function to investigate changing the porosity along the direction of thickness of the nano-structure surrounded by the Pasternak substrate. Also, virtual work was employed to establish the governing relations based on the primary functions. Moreover, the nonlocal, as well as material length scale parameters, were used to indicate the stiffness reduction and enhancement associated with the nanoplate, respectively. By employing Navier's method, the governing relations were solved analytically. Allam and Radwan [473] worked on the nonlocal strain-gradient behavior of static/dynamic-bending and buckling of the viscoelastic FG curved nanobeam surrounded by an elastic substrate with various boundary conditions. The higher-order refined curved beam model was employed to indicate the shear deformation effects which did not need any shear correction parameters. Also, they utilized the two power-law models to demonstrate changes in material characteristics continuously for viscoelastic FG curved nanobeams. It was found that the bending, as well as shear components associated with the radial displacement, were considered as the increasing functions of the angle of the nanobeam. Nonlocal strain-gradient model behavior of nano-structures with consideration of non-standard as well as constitutive boundary conditions was scrutinized by Zaera et al. [474]. They revealed that the obtained solution did not comply with energy conservation. Moreover, they confirmed the inconsistency of the nano-structure model and proved that it must be prevented to use in the mechanical characteristics of nano-structures.

5.1.2. Nonlinear static bending of micro/nano-structures

Numerical approaches were utilized by researchers to investigate the mechanical characteristics of micro/nano-structures. Reddy [497] investigated nonlocal and strain gradient theories in structural mechanic problems. The author employed three various size-dependent models including modified couple stress, Srinivasa-Reddy gradient elasticity, as well as unified integral schemes for microstructure-dependent size effects. He proposed the discrete peridynamics method which was considered as a new scheme against the conventional peridynamics [448]. Zhu and Li [498] investigated the closed-form solution method for static bending of a nonlocal strain-gradient rod model in tension. They used an integral rod model which was both more self-consistent and well-posed than other proposed models. In a different study, Xu et al. [499] examined the bending/buckling behavior of nonlocal strain-gradient elastic nanobeams. They used weighted residual approaches and the Euler Bernoulli beam model using the nonlinear geometric von Kármán equation. They indicated that the higher-order boundary conditions have no effect on the static bending deflection of beams, but the mentioned condition and material length parameters have a remarkable influence on the buckling loads. Fig. 22(a)–(c) depict the effects of the higher-order boundary conditions on the first three buckling loads of nonlocal strain-gradient model for (a) C–F, (b) S–S, and (c) C–C boundary conditions.

Critical buckling loads values decrease with an increase in the dimensionless magnitudes of nonlocal terms which are the same for different modes. Barretta et al. [500] worked on the boundary conditions

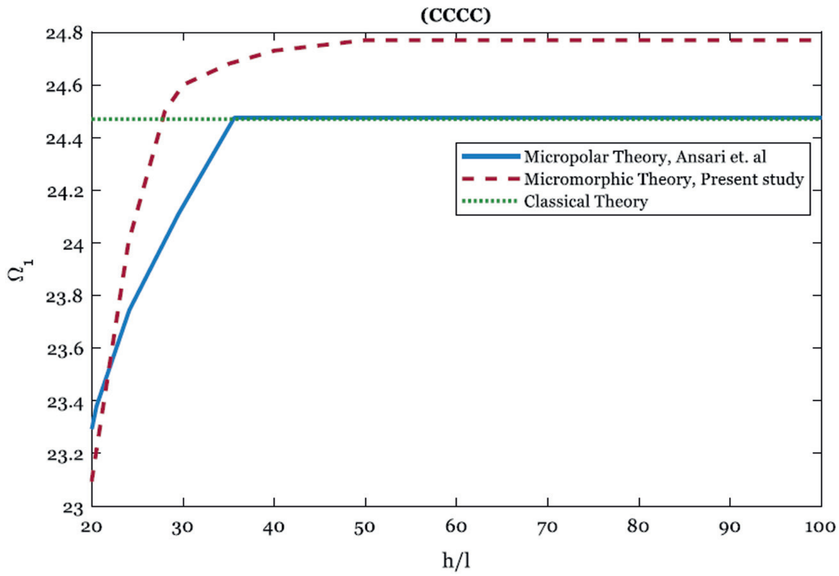

Fig. 21. Variation of dimensionless fundamental frequencies of a fully clamped plate versus thickness-to-length scale parameter ratio [470].
Source: Reprinted with permission from R. Ansari (Co-author).

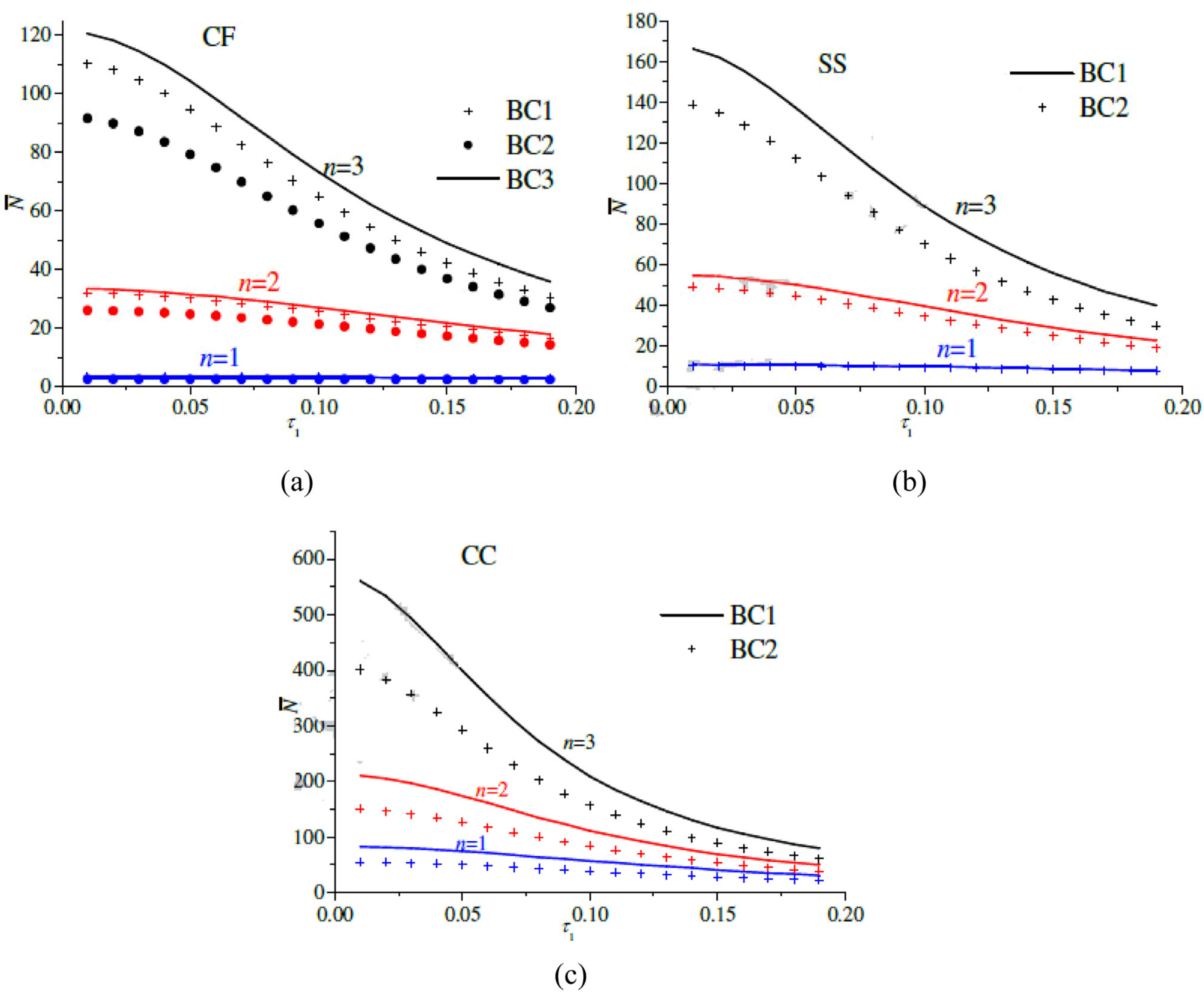

Fig. 22. (a)–(c). The effects of the higher-order boundary conditions on the first three buckling loads of nonlocal-strain gradient model for clamped–free (CF) (a), simply support-simply support (SS) (b) and clamped–clamped (CC) (c) boundary conditions [499]. Reprinted with permission from Elsevier.

for nonlocal strain-gradient nanobeam models. They demonstrated that the constitutive boundary conditions should be added to close the constitutive model. Also, they turn out the equivalence between the nonlocal strain-gradient integral procedure and the differential model with consideration of constitutive boundary conditions. Nonlinear bending scrutiny of nanobeams via the nonlocal strain gradient model with an IG finite element method was investigated by Norouzzadeh et al. [501]. The authors employed the strain gradient method which was simpler than other strain gradient theories including the modified strain gradient and couple stress schemes. Furthermore, they utilized the integral form of nonlocal elasticity. Using the minimum total potential energy procedure, the motion equation was acquired, and then based on the non-classical IG scheme, the matrix–vector form of the relations was analyzed and solved. Nonlinear static bending properties of an FG porous nanobeam induced by multiple physical fields using the nonlocal strain-gradient model were examined by Gao et al. [502]. FG thermo–magneto–electro–elastic materials with consideration of both constituent materials as well as the porosity gradient distribution were proposed. Also, the displacement function pertinent to the physical neutral surface was considered to scrutinize the mechanical characteristics of the proposed FG beams. They also acquired the nonlinear equations based on Hamilton's principle. By employing the two-step perturbation scheme in non-uniform electric as well as magnetic fields, the governing relations were solved analytically, and then they employ it to solve the nonlinear relations. Moreover, they investigated the influences of different physical parameters on the deformation behavior of nanostructures in detail. Static bending behavior of the laminated thin/thick nanoplates using Reddy's third-order shear deformation model based on the nonlocal strain-gradient model was investigated by Bacciocchi and Tarantino [503]. They proposed their model to generalize both the classical laminated plate and the first-order shear deformation models. The proposed constitutive laws were altered based on the nonlocal strain-gradient procedure. Then, they solved analytically the fundamental relations using the Navier method and cross-ply/angle-ply lamination procedure. They proved that for higher magnitudes of thickness, the stress components were remarkably different by changing the theory of the nanoplate.

5.1.3. Linear buckling/post-buckling of micro/nano-structures

In recent years, researchers utilized analytical methods to scrutinize the linear static buckling/post-buckling responses of the micro/nanostructures. Beddia et al. [504] worked on a novel hyperbolic associated with two-unknown micro/nanobeam model pertinent to buckling/bending properties based on the nonlocal strain-gradient model. The significance of using their model was based on the two-unknown displacement field associate with the Euler–Bernoulli beam model, and also an acceptable accuracy for capturing the influences of shear deformation models without considering the shear correction parameter. Based on Navier's method, the governing relations were solved analytically with consideration of the S-S boundary condition.

On the other hand, many researchers utilized discretization approaches to examine the buckling/post-buckling behavior of micro/nano-structures of the model. A higher-order nonlocal strain-gradient model for buckling of an orthotropic nanoplate immersed in the thermal environment was proposed by Farajpour et al. [505]. The authors used the DQM method to solve the equation of motions which were derived by the virtual work method. Acceptable results between the exact and numerical outputs were obtained. Buckling investigation of the nonuniform nonlocal strain-gradient model of nanobeams using GDQM was examined by Bakhshi Khaniki et al. [506]. It was proved that nonuniform effects play a significant role in critical buckling loads pertain to the ratio between small-scale parameters. Table 5 gives critical buckling values in uniform nonlocal-strain gradient nanobeams compared to the predicted analytical solutions by other researchers, reporting the average relative error.

Dynamic instability scrutiny of sandwich piezoelectric nanobeam with FG carbon nanotube-reinforced composite (FG-CNTRC) face-sheets using different high-order shear deformation and nonlocal strain-gradient theory was studied by Arefi et al. [507]. They used three patterns of CNTs to reinforce the top and bottom face-sheets of the nanobeam. Furthermore, various higher-order shear deformation beam models including trigonometric shear deformation beam model, exponential shear deformation beam model, Aydogdu shear deformation beam model, and hyperbolic shear deformation beam model were examined to ascertain the governing equations with consideration of different boundary conditions. Buckling/frequency behavior of the micro/nanoshell reinforced with graphene nanoplatelets based on the nonlocal strain–stress gradient model was proposed by Mohammadgholiha et al. [508]. The material characteristics of the proposed piece-wise based graphene-reinforced composites were considered to be graded in the direction of thickness associated with the nanoshell, which were examined with consideration of a nano-mechanical model. They established the governing relations as well as boundary conditions based on Hamilton's principle and then solved using the GDQ procedure. They indicated the importance of using nonlocal and material length scale parameters, the pattern of graphene platelet distribution and its weight function, as well as GPL number of layers on the buckling/frequency of the micro/nanoshell reinforced with graphene nanoplatelets. Buckling/bending/vibration behavior of axially FG micro-nanobeams using the nonlocal strain-gradient and Euler–Bernoulli beam models was investigated by Li et al. [509]. The governing relations, as well as the boundary conditions, were established by Hamilton's principle, and then solved by the DQ method. Using the through-length grading of the proposed FG material, the mechanical characteristics were changed, and by selecting appropriate magnitudes of the power-law index the buckling load/deflection/frequency magnitudes were controlled. Also, for the concentrated as well as uniformly distributed loads, the maximum deflection increased with a decrease in the values of the material length scale parameter. Moreover, based on the magnitudes of the proposed size-dependent parameters, the stiffness-softening/ hardening effects of the axially FG micro/nanobeam applied on the critical buckling load/natural frequencies were changed.

5.1.4. Nonlinear buckling/post-buckling of micro/nano-structures

Numerical methods were used by researchers to analyze the nonlinear scrutiny of buckling/post-buckling of micro/nano-structures. Mehralian et al. [510] worked on the calibration of a nonlocal strain-gradient shell model for the axial buckling of nano-structures. They employed the first-order shear deformation model, von Kármán method by considering geometrical nonlinearity, and the minimum potential energy principle to establish final equations. Small length scale parameters were calibrated for the axial buckling model of CNTs based on the MD simulation. They indicated that the influence of small length scale parameters on the critical axial buckling load becomes more prominent with an increase and a decrease, respectively, in values of thickness and length ratio. Also, they showed that the calibrated small length scale parameters were useful while using nonlocal strain gradient model for the analysis of SWCNT. Based on Fig. 23, the effects of small length scale parameters on the critical buckling load were considerable. Zhong et al. [511] carried out deep post-buckling and bending investigation of nanobeams with nonlocal strain-gradient theory with consideration of nonlinear effects. They considered a large deflection model and scrutinized the multi-scale analysis of the Euler Bernoulli nanobeam. They employed a two-step perturbation procedure to analytically solve the deep post-buckling and nonlinear bending equations of nanobeams. They revealed that, as the maximum deflection increases, the high-order nonlinear terms and in-plane conditions in the bending curvature expression can be considered as a significant effect on the nonlinear behaviors of nanobeams. Torsional stability of an SW-composite nanoshell model using the nonlocal strain-gradient model and the 3D magnetic field was carried out by Malikan et al. [512]. They used first the shear deformation model to analyze the torsional stability of their model. According to the hybrid size-dependent model, the softening and hardening of the nanoshell stiffness

Table 5
Critical buckling values in uniform nonlocal strain-gradient nanobeams [46,506]. Reprinted with permission from Elsevier.

α_2	α_1	[46]	6 Nodes	7 Nodes	8 Nodes	9 Nodes	10 Nodes
0	0	9.86960	9.87040	9.86961	9.86961	9.86961	9.86961
	0.25	15.95767	15.95748	15.95772	15.95770	15.95768	15.95768
	0.5	34.22187	34.21878	34.22205	34.22195	34.22189	34.22189
	0.75	64.66221	64.65429	64.66259	64.66238	64.66223	64.66223
	1	107.27869	107.26401	107.27938	107.27897	107.27870	107.27870
0.5	0	2.84639	2.84647	2.84641	2.84641	2.84641	2.84641
	0.25	4.60219	4.60190	4.60222	4.60221	4.60221	4.60221
	0.5	9.86960	9.86817	9.86966	9.86963	9.86961	9.86961
	0.75	18.64861	18.64532	18.64874	18.64867	18.64862	18.64862
	1	30.93922	30.93331	30.93945	30.93931	30.93923	30.93923
1	0	0.90800	0.90802	0.90801	0.90801	0.90801	0.90801
	0.25	1.46810	1.46799	1.46811	1.46811	1.46811	1.46811
	0.5	3.14840	3.14791	3.14843	3.14842	3.14841	3.14841
	0.75	5.94890	5.94777	5.94895	5.94893	5.94891	5.94891
	1	9.86960	9.86758	9.86969	9.86964	9.86961	9.86961
Average relative error (%)		0	9.58234E−5	6.26586E−6	3.76102E−6	2.13373E−6	2.13373E−6

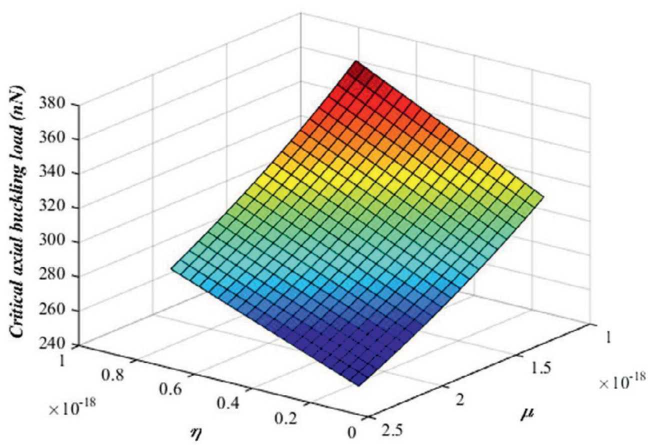

Fig. 23. The variation of critical buckling loads with small length scale parameters [510]. Reprinted with permission from Elsevier.

were examined. Likewise, with consideration of the 3D magnetic field, the transverse influence of the mentioned field was the most significant effect on the torsional stability of the nano-structure. The post-buckling behavior of FG nanobeams with consideration of nonlocal stress as well as strain gradient influences using the nonlinear geometric of von Kármán theory was investigated by Li and Hu [513]. They employed the physical neutral surface to remove the coupling between bending-stretching which was for the sake of geometric nonlinearity of the FG micro/nano-structure and also the coupling rigidity between the bending as well as extensional rigidities of the proposed model. Likewise, they utilized the closed-form method to investigate the post-buckling behavior as well as the critical buckling load based on the hinged-hinged boundary condition model. Mao and Zhang [514] scrutinized the buckling/post-buckling behavior of FG graphene reinforced piezoelectric micro/nanoplate induced by electric potential as well as axial forces. They employed the Halpin–Tsai parallel procedure to calculate the effective Young's modulus associated with each layer pertinent to the proposed model. Using the first-order shear deformation model, von Kármán geometric nonlinearity formulation as well as the virtual displacement method the governing equations were derived, and then solved by the DQ method in conjunction with the iterative technique. They proved that graphene platelet has a remarkable effect on the buckling/post-buckling strength of the FG piezoelectric micro/nanoplate.

5.2. Vibration and wave propagation of small-scale structures

The following parts demonstrate the published studies pertinent to the linear/nonlinear free and forced vibration as well as elastic/thermal wave propagation of the micro/nano-structures, which can be discussed as follows:

5.2.1. Linear/nonlinear free vibration of micro/nano-structures

Based on the analytical method, the solution procedure was proposed related to the linear free vibration of micro/nano-structures. The physical investigation of SWBNNT as a bio/nano-sensor for sensing attached nano-scale objects was reported by Roudbari and Ansari [515]. Various boundary conditions such as S–S, C–C, and C–F were used to illustrate the vibrational behavior of the model. Rayleigh and Timoshenko beam theories were employed in their research. Moreover, the nonlocal strain-gradient procedure was used to demonstrate the size-dependent effects. They indicated that for the cantilever case study of SWBNNT the dimensionless boundary conditions are as follows:

$$\begin{cases} \left(\overline{w}\right)^{R,T}=0, \left(\dfrac{\partial\overline{w}}{\partial\xi}\right)^{R}=0, \left(\overline{\theta}\right)^{T}=0, \quad \xi=0, \\ \left(\left(-1+\dfrac{\mu^2\omega^2}{\Gamma^2}-\mu^2\overline{N}_x\right)\dfrac{\partial^2\overline{w}}{\partial\xi^2}-\mu^2\overline{\omega}^2\overline{w}+\mu_l^2\dfrac{\partial^4\overline{w}}{\partial\xi^4}\right)^{R}=0, \quad \xi=1, \\ \left(\left(-1+\dfrac{\mu^2\omega^2}{\Gamma^2}-\mu^2\overline{N}_x\right)\dfrac{\partial^3\overline{w}}{\partial\xi^3}+\mu_l^2\dfrac{\partial^5\overline{w}}{\partial\xi^5}-\left(\mu^2\overline{\omega}^2+\overline{N}_x+\dfrac{\omega^2}{\Gamma^2}\right)\dfrac{\partial\overline{w}}{\partial\xi}\right)^{R}=0, \quad \xi=1, \\ \left(\omega_G\left(1-\dfrac{\mu^2\omega^2}{\Gamma^2}\right)\dfrac{\partial\overline{\theta}}{\partial\xi}-\mu_l^2\omega_G\dfrac{\partial^3\overline{\theta}}{\partial\xi^3}-\mu^2\omega^2\omega_G\overline{w}-\mu^2\overline{N}_x\dfrac{\partial^2\overline{w}}{\partial\xi^2}\right)^{T}=0, \quad \xi=1, \\ \left(\overline{\theta}-\mu_l^2\dfrac{\partial^2\overline{\theta}}{\partial\xi^2}+\left(1-\mu^2\omega_G\omega^2+\overline{N}_x\right)\dfrac{\partial\overline{w}}{\partial\xi}-\left(\mu_l^2+\mu^2\overline{N}_x\right)\dfrac{\partial^3\overline{w}}{\partial\xi^3}\right)^{T}=0, \quad \xi=1, \end{cases} \tag{21}$$

where $\overline{w}$, ξ, $\overline{\theta}$, μ, ω, Γ, $\overline{N}_x$, μ_l and ω_G are, respectively, the non-dimensional deflection, the non-dimensional length, the non-dimensional rotation, the non-dimensional nonlocal parameter, the non-dimensional frequency, the non-dimensional aspect ratio, the non-dimensional axial load, the non-dimensional length scale parameter, and the non-dimensional shear modulus. Also, the superscripts R and T are related to the Rayleigh and Timoshenko beam models. In another research, a modified nonlocal strain-gradient model for nanorods and its application to CNTs was developed by Barretta et al. [516]. They considered an integral elasticity model, with axial force and strain fields and then compared it to a differential elasticity model. Also, they examined the variation of Young's modulus against different parameters and then compared the obtained results with an MD simulation.

Numerical methods were employed to investigate the mechanical properties of size-dependent structures. Norouzzadeh et al. [407] investigated the IG vibration behavior of size-dependent Timoshenko beams using the nonlocal strain-gradient model. The authors derived the motion equations using an IGA scheme or finite element method. They also revealed that the maximum deflection values increase with a decrease in the magnitudes of the nonlocal parameters. Size-dependent nonlinear secondary resonance of micro-/nano-porous beams including truncated cube cells was analyzed by Sahmani et al. [517]. They investigated the mechanical properties of nano-porous biomaterials based on a truncated cube cell model using a refined hyperbolic shear deformation. And then, using the nonlocal strain-gradient model they analyzed the nonlinear secondary resonance of the porous model with consideration of subharmonic and superharmonic excitations using different boundary conditions. Sahmani and Safaei [518] studied nonlinear vibrations of bi-directional FG micro/nanobeams using nonlocal stress and microstructural strain gradient size-dependent influences. They employed GDQM in conjunction with Galerkin and pseudo-arc-length continuation methods to solve the obtained equation of motions. They showed that, for lower values of maximum deflection, the increment induced by the strain gradient method in the magnitudes of the frequency was more remarkable than the reduction made by the nonlocal effect. They worked on the variation of nonlocal strain-gradient nonlinear dynamic response of 2D FGM against different values of the axial material property. Changing the material of 2D FGM from ceramic to metal will diminish the nonlinear frequency values of the micro/nanobeam. Nonlinear vibration scrutiny of sandwich nanobeam with FG-CNTRCs face-sheets in electro-thermal medium with consideration of the nonlocal strain-gradient model and various shear deformation theories was analyzed by Arefi et al. [519]. They used nonlinear von Kármán equations and then with the Hamilton principle the governing equations of motion were obtained. Ghayesh et al. [475] worked on the coupled dynamics of nanofluid-conveying nanotubes. They used the Beskok–Karniadakis method to capture the size-dependent effects of the nanofluid. Also, Coriolis acceleration influences in conjunction with the effects of the centrifugal acceleration were considered in their research work. The time integration approach was employed to drive the global dynamic features of the nanofluid-conveying nanotubes. They investigated the coupled bifurcation variation of the nanofluid-conveying nanotubes at a specific speed of nanofluid. It is obvious that the chaos in the coupled motion dramatically spreads over a wider range of the forcing amplitude when the nanofluid speed increases. Nonlinear vibration of fluid conveying cantilever SWCNT embedded in a visco-Pasternak medium based on the nonlocal strain-gradient theory was carried out by Roodgar Saffari et al. [139]. The material characteristics of the cantilever SWCNT were modeled by Kelvin–Voigt viscoelastic constitutive equation and then slip boundary conditions of fluid conveying cantilever SWCNT were considered. They proved that the mentioned parameters have remarkable influences on the dynamic response of the SWCNT.

5.2.2. Linear/nonlinear forced vibration of micro/nano-structures

Using numerical approaches can be considered as a capable method to examine the linear/nonlinear forced vibration of the model. Forced vibration behavior of the heterogeneous porous FG, as well as metal foam micro/nanoplates based on the general nonlocal stress–strain gradient model, was examined by Barati [209]. They proposed the stiffness-softening/hardening influences to analyze the mechanical behavior of the model. According to the Galerkin procedure, the dynamic deflections, as well as resonance frequencies, were obtained. Also, they showed that the forced vibration behavior of the heterogeneous porous FG micro/nanoplates was remarkably affected by the porosities, excitation frequency, small-scale parameters, elastic medium as well as dynamic load properties. Roudbari and Doroudgar Jorshari [268] investigated the vibrational control behavior of thermally-magnetically SWCNT induced by a moving nanoparticle based on the nonlocal strain-gradient and the Rayleigh beam theories. There was a gain matrix and displacement–velocity feedback in the linear classical optimal control method to diminish the vibrational behavior of the nanotube. Based on their model, the PZT patches as nano-sensors were connected to charge amplifiers for actuating the nanotube. Forced vibration properties of various size-dependent beam models with consideration of surface influences and the nonlocal strain-gradient model acted upon by moving harmonic loads based on the Euler–Bernoulli, Timoshenko, as well as modified Timoshenko beam theories, were proposed by Rajabi et al. [520]. Analytical closed-form methods associated with the various nonlocal strain-gradient beam theories using S-S boundary conditions were utilized with the combination of the Galerkin method and the Laplace transform procedure. They showed that the dynamic deflection decreases with an increase in the values of surface elastic modulus. Moreover, the dynamic deflection of the proposed nanobeam increased with an increase in the values of the moving harmonic load velocity to a certain magnitude. Wu et al. [521] reported forced vibration analysis of FG graphene platelet-reinforced nanocomposite microbeams using the nonlocal strain-gradient method and the refined hyperbolic shear deformation beam model. By employing the Halpin–Tsai procedure, the effective material characteristics of laminated FG graphene platelet-reinforced nanocomposite microbeams were acquired. Also, based on Hamilton's principle, the size-dependent governing motion equations were established. Using the GDQ method in conjunction with the Galerkin scheme, the nonlocal strain-gradient frequency as well as the amplitude responses were obtained. They proved that the nonlocal parameter yields an increase in the peak of the associated excitation frequency as well as the jump phenomenon, but the material length scale parameter leads to a decrease in them. Nonlinear resonant characteristics of a thick multi-layered nanoplate based on the nonlocal strain-gradient model as well as the first-order shear deformation of the Mindlin plate model were examined by Mahmoudpour [522]. He considered the interactional vdW forces between the proposed adjacent layers of the multi-layered nanoplate. According to the harmonic balance procedure, the nonlinear resonance properties of the thick nanoplate were analyzed. Likewise, the primary as well as secondary resonances including superharmonic as well as subharmonic resonances were proposed. He indicated that the subharmonic resonance disappears with an increase in the values of the thickness of the nanoplate and then the curves of the frequency response show the non-resonant properties. Vahidi-Moghaddam et al. [523] studied the nonlinear behavior of forced vibration properties pertinent to the nonlocal strain-gradient beam model based on the homogeneous Euler–Bernoulli beam theory as well as C–C boundary conditions. The governing motion relations were established based on the mid-plane stretching as well as damping influences. They proposed a reduced motion relation under a central harmonic load based on the Galerkin procedure, and the perturbation technique was employed to solve the obtained equations.

5.2.3. Elastic/thermoelastic wave propagation of micro/nano-structures

The number of studies pertinent to the analytical approaches pertinent to the investigation of the elastic/thermoelastic wave propagation responses of the micro/nano-structures is noticeable. Wave propagation analysis of viscoelastic SWCNT based on nonlocal strain-gradient theory was conducted by Tang et al. [524]. They proved that the obtained blocking diameter was completely dependent on the damping coefficient, Winkler modulus of the surrounding elastic medium, as well as the nonlocal and the strain gradient length scale parameters. Ma et al. [62] studied wave propagation analysis of electro–magneto–elastic nanoshells with consideration of the nonlocal strain-gradient model using the Kirchhoff–Love shell as well as the first-order shear deformation shell models. They investigated the external electrical/magnetic potential, temperature change, external force, and small-scale parameters' effects on the wave propagation properties of the proposed model. It was proved that having external loadings including the electrical, magnetic, and mechanical yield to the cut-off wave number, where the frequency values tend to zero. Norouzzadeh et al. [163] worked on the wave propagation behavior of micro/nanobeams associated with the Timoshenko beam theory using nonlocal strain-gradient influences. They utilized the integral nonlocal model with the general form in combination with the modified strain gradient model. Likewise, Hamilton's principle was used to obtain the governing partial differential relations. According to the numerical integration method as well as periodic grid scheme, they employed a semi-analytical procedure to solve the proposed equations. It was found that at a certain wave number, the classical beam model underestimates and overestimates, respectively, the wave frequencies pertinent to the micro/nanobeam for without and with consideration of the nonlocal parameter.

In contrast, discretization schemes were utilized to analyze the mechanical behavior of nonlinear models associated with the elastic/thermoelastic wave propagation responses of the micro/nano-structures. Wave propagation scrutiny of the FG inhomogeneous nanoplates with consideration of the nonlinear thermal effect and based on the nonlocal strain-gradient theory was carried out by Ebrahimi et al. [208]. They employed shear deformation influences associated based on the refined shear deformation plate model with four-variable. Also, with consideration of the heat conduction model through the nanoplate thickness, the nonlinear thermal loading formulations were acquired. They considered that the material characteristics are temperature-dependent, which change gradually through the thickness based on the Mori–Tanaka theory. Moreover, based on Hamilton's principle the governing relations were established. They showed that in the absence of the material length scale parameter and at larger wave numbers, the gradient index, as well as the nonlocal model, had no significant effect on phase velocity. Huang et al. [525] analyzed the modeling of the flexural wave propagation behavior based on the nonlocal strain-gradient model using associated with Euler–Bernoulli and Timoshenko beam models. They utilized the Caputo fractional derivatives model instead of the Laplacian operator in the constitutive relations. They compared their results with the integer-order nonlocal strain-gradient theory as well as the MD simulation technique to verify the correctness of their model. In another interesting research, Huang and Wei [526] investigated the modeling of the flexural wave propagation in the infinite homogeneous nanoplate with consideration of the fractional-order nonlocal strain-gradient theory as well as thermoelastic behavior of the mode. They utilized the spatial as well as temporal fractional-order differential to show the spatial nonlocal as well as the history-dependent properties related to the thermoelastic characteristics of the size-dependent micro/nano-structures. They revealed that their proposed solution related to the deflection-dependent approximation associated with temperature distribution was effective while the thickness of the nanoplate was much lower than the wavelength values.

6. Conclusion

In this review, a comprehensive survey of continuum mechanics models of micro-/nano-structures was presented. Different geometries, such as rods, beams, plates, and shells, were considered. The sections of the review are organized according to the small-scale theories used to capture size-dependent effects associated with various external loading conditions and physical parameters. The main motivation of this study was to thoroughly review the published literature and discuss future areas to be developed in the field of continuum mechanics at the micro/nano-scale. The recent literature on the subject is particularly vast; therefore, this review is aimed to give various classifications of the size-dependent effects on the mentioned small-scale structures. Finally, directions for future studies are given; future research should focus more on the physics of the problem pertinent to the practical applications in this field, and use meaningful material data and geometries. Too many computations address problems that are fully unrealistic for incompatibility of dimensions in the framework of the physics involved.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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