Reconstruction of the refractive index profile of planar waveguides using ray tracing analysis

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Abstract

We propose a method for refractive index profiling based on measuring coordinates and angles of laser beams passing across the waveguide layer. Calculations are performed by solving an integral equation using new global optimization methods.

1. Introduction

The refractive index profile of a planar waveguide determines many of the transmission properties of the waveguide, such as bandwidth, mode profile and coupling efficiency, and also allows one to study waveguide fabrication processes. Various methods of nondestructive measurement of refractive index have been proposed [17, 14, 9, 10, 16, 7, 6], most of them are based on inverting WKB integral and nonlinear least squares fit. The goal of these methods is to compute, based on the performed measurements, the refractive index $n(z)$ of the waveguide layer as a function of depth. This paper presents an alternative method of computing the index profile, based on the geometrical optics model. The method uses positions and incidence and exit angles of thin laser beams propagating across the waveguide layer, which can be measured with relative ease. It then involves numerical inversion of a nonlinear integral equation relating the measured quantities with the index profile. This is a difficult ill-posed mathematical problem, but with recent advances in global optimization it can be efficiently solved even in the presence of substantial noise in the measurements. This paper examines the mathematical model behind the method, and analyses its performance on simulated experimental data.

2. Index profile reconstruction

Reconstruction of refractive index profiles falls into the category of inverse problems [15], and as such is ill-posed (with a range of far reaching negative implications). The interior of the waveguide layer is inaccessible for the measurements, and they can be taken only outside or on the boundary of the waveguide layer. It will be assumed throughout this paper that the index on the accessible boundary n_0 (at z=0 on Fig.1) is known (as it can be measured directly). It is also assumed that n is a monotone function of z only. The goal is to determine this function $n(z)$.

Consider the following experimental setup. A thin laser beam passes across the waveguide layer in the direction of increasing z, and exits at a certain distance h from the entry point, which is recorded. Such distance would depend on the incidence angle Θ, which is also measured and recorded. By changing the direction of the beam, we obtain the function $h(\Theta), -\Theta_{max} < \Theta < \Theta_{max}$. Because of the symmetry, we consider h only on $[0, \Theta_{max})$. Since we can perform only a limited number of measurements I, we consider a discretised representation of h as a set of pairs $\{\Theta_i, h(\Theta_i)\}_{i=1}^{I}$.

Fig. 1 illustrates the experimental setup. Because the index n varies with z, the paths of laser beams inside the waveguide layer are curved. However they cannot be observed. The question is to compute $n(z)$ from the observed pairs.

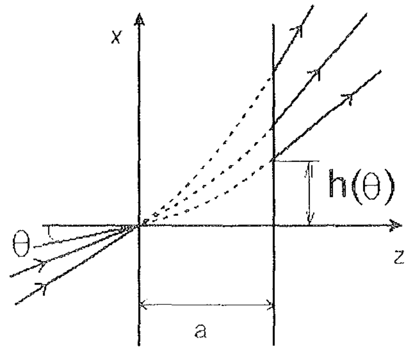

Fig.1. Propagation of rays across the waveguide layer.

Let us formulate the mathematical model of ray propagation in the waveguide layer, which will be subsequently used for index reconstruction. In geometrical optics approximation, the ray paths are related to n in the Eikonal equation, which is written as [5]

$$\frac{nx''}{1+x'^2} + x'n_z = 0, \qquad (1)$$

where $x=x(z)$ is the ray path parameterized by z. The solution to the Eikonal equation in the stratified media $n=n(z)$ can be found as [11]

$$p(z) = \frac{k}{\sqrt{n^2(z)-k^2}}, \qquad (2)$$

where $p(z) = x'(z)$ and $k = n_0 \sin\Theta$. Then

$$x(z) = k\int_0^z \frac{dz}{\sqrt{n^2(z)-k^2}}. \qquad (3)$$

Let the depth of the waveguide layer be a. Then we can relate the exit position h and k

$$h(k) = k\int_0^a \frac{dz}{\sqrt{n^2(z)-k^2}}. \qquad (4)$$

Eq.(4) is the main equation that will be used for index profile reconstruction.

Let us now make some observations about the proposed model. Without loss of generality, we can put a=1 and $n(1)$=1. $n(0)=n_0$ is known from the direct measurements. The condition that the ray eventually crosses z=1 and exits the waveguide layer is $n(z) > k$ and hence $\Theta_{max} = \sin^{-1}(1/n_0)$.

Eq. (4) naturally leads to two distinct problems, the direct and the inverse. The *direct* problem consists of solving (4) for $h(k)$ with known $n(z)$, i.e., computing the exit position of a ray entering the media with known $n(z)$ at the origin at an angle Θ. It is a variant of the ray tracing problem. We will use it as a tool for simulating experimental data.

The *inverse* problem is the one we are interested in: to invert (4) and compute $n(z)$ given $h(k)$. We note that a similar problem arises in computing the index profile of a circular symmetric optical fibre (i.e., when $n=n(r)$). It was dealt with in detail in [8, 14], and it involves Abel type integral equations. In our case Eq. (5) does not allow explicit inversion [12], hence we need to solve (4) for n numerically.

3. Solution of integral equation

In the ideal case of exact measurements, the existence and uniqueness of solution for (4) follows from the fact that $h(k)$ is the solution of the direct problem (i.e., there is such $n(z)$ that generated $h(k)$ via (4)) and that Eq.(4) is not singular for $\Theta \in [0, \Theta_{max})$, and the integrand is bounded [12].

However, when we consider the case of noisy data $h(k)+\varepsilon$, the existence of the solution of the inverse problem (4) is not guaranteed. It is a common situation in the inverse problems [15], where neither existence nor continuous dependence on the data do not hold, and the problems are ill-posed.

Tikhonov regularization is a common approach to the solution of inverse problems. It consists in minimizing $\|An-h\| + \alpha\|Dn\|$, where A is the integral operator (4) acting on n and D is a differentiation operator. It is customary to use second derivatives, and thus we will minimize

$$\left\| h(k) - k\int_0^1 \frac{dz}{\sqrt{n^2(z)-k^2}} \right\| + \alpha\int_0^1 [n''(z)]^2 dz \qquad (5)$$

Let us now represent $n(z)$ through its values at certain points on [0,1] $x_j, j=1,...,J$. Under the earlier assumption of monotonicity of the index profile, $n_0 \le n_1 \le ... \le n_J \le 1$. Then we have the approximation of the integral

$$\int_0^1 \frac{dz}{\sqrt{n^2(z)-k^2}} \approx \sum_{j=1}^{J} w_j \frac{1}{\sqrt{n_j^2-k^2}}.$$

The nodes x_j are chosen as zeros of Legendre polynomials, and weights w_j are taken from the standard tables. For smooth profiles Gauss integration results in a very high precision, and only a few nodes are required (J=5 or 7). The second integral is approximated analogously, and the derivatives are approximated with second order divided differences. The regularization parameter α is adjusted dynamically depending on the noise in the data.

The norm in Eq. (5) is chosen to be Chebyshev (max) norm. It is stronger than the traditional least squares norm, but is not differentiable. However, the minimization algorithms we use do not require differentiability.

To minimize (5) we employ two new methods: the Cutting Angle method of global optimization (CAM) [1, 4, 13] and the Discrete Gradient method of local nonsmooth optimization (DG) [2, 3]. The reason for using CAM is that expression (5) may possess many locally optimal but globally suboptimal solutions (the well known problem of multiple local minima). The solution of the inverse problem corresponds to the global minimum of (5). CAM is well suited for locating (and confirming) global minima on the unit simplex (which is the domain of this optimization problem). However it requires a substantial number of iterations before it converges. If this algorithm is stopped because of time constraints, its solution can be improved using DG, taking the best solution by CAM as the starting point.

4. Test profiles

To test the feasibility and performance of the proposed method of index profile reconstruction, we need to simulate experimental data by solving the direct problem with chosen index profiles. Then a random noise is added to the simulated data, and the index profile is found by minimising (5). The reconstructed profile is then compared with the model used for simulating the experiment, and conclusions about the quality of reconstruction can be made. It is expected that the method will perform similarly using real experimental data with the same noise level.

To simulate the experimental data we need to solve (4) with respect to $h(k)$. We have used several formulae for index profiles (Table 1), which are flexible enough to model a variety of shapes using the appropriate values for the defining parameters (a,b,c) (Fig.2). We were able to take integral (4) analytically in these cases, thus avoiding losses of precision during the numerical integration. For each model index profile we generated 20 experimental points $(\Theta_i, h(\Theta_i))$ using analytical expressions for the integrals in Eq.(4). A normally distributed random noise ε with μ=0 and a fixed σ was then added to h. These 20 points were subsequently used in the numerical solution of the inverse problem.

Table 1. Model index profiles for simulating experiment.

Model	$n(z)$
1	$az+b$
2	$b-ae^{cz}$
3	$a(1+cz)^{-1}+b$
4	$c-a\sqrt{1+bz}$
5	$b-c(1+e^{-a(z-0.5)})^{-1}$

5. Numerical results

Some of the models and reconstructed profiles are plotted in Fig. 2. Evidently, profiles of all tested shapes have been found correctly. Quantitative results are presented in Table 2. As expected, the quality of index reconstruction drops with increased noise in the data, however it is still acceptable even when the noise level is quite high (σ=0.001).

Table 2. Accuracy of index reconstruction from noisy simulated data (root mean square error)

Model	σ=0	$\sigma=10^{-4}$	$\sigma=10^{-3}$	$\sigma=10^{-2}$
1	$8.5\ 10^{-6}$	$8.4\ 10^{-4}$	$3.1\ 10^{-3}$	$7.9\ 10^{-3}$
2	$3.3\ 10^{-4}$	$1.2\ 10^{-3}$	$4.8\ 10^{-3}$	$7.0\ 10^{-3}$
3	$6.2\ 10^{-5}$	$1.0\ 10^{-3}$	$3.0\ 10^{-3}$	$5.5\ 10^{-3}$
4	$5.7\ 10^{-4}$	$1.3\ 10^{-3}$	$4.1\ 10^{-3}$	$6.0\ 10^{-3}$
5	$1.9\ 10^{-4}$	$9.1\ 10^{-4}$	$3.5\ 10^{-3}$	$7.1\ 10^{-3}$

6. Conclusion

This paper presents an alternative method of reconstruction of the refractive index profile of planar waveguides. It is based solely on the geometrical optics approximation, and requires measurements of the positions and incidence and exit angles of laser beams passing across the waveguide layer. The method involves numerical inversion of a nonlinear integral equation, which can be done using recently developed global and nonsmooth local optimization techniques. The index reconstruction problem is ill-posed, but adequate regularization methods can be applied. The quality of index reconstruction is acceptable even in the presence of significant noise in the data. The proposed method seems to be a feasible alternative to other index reconstruction methods.

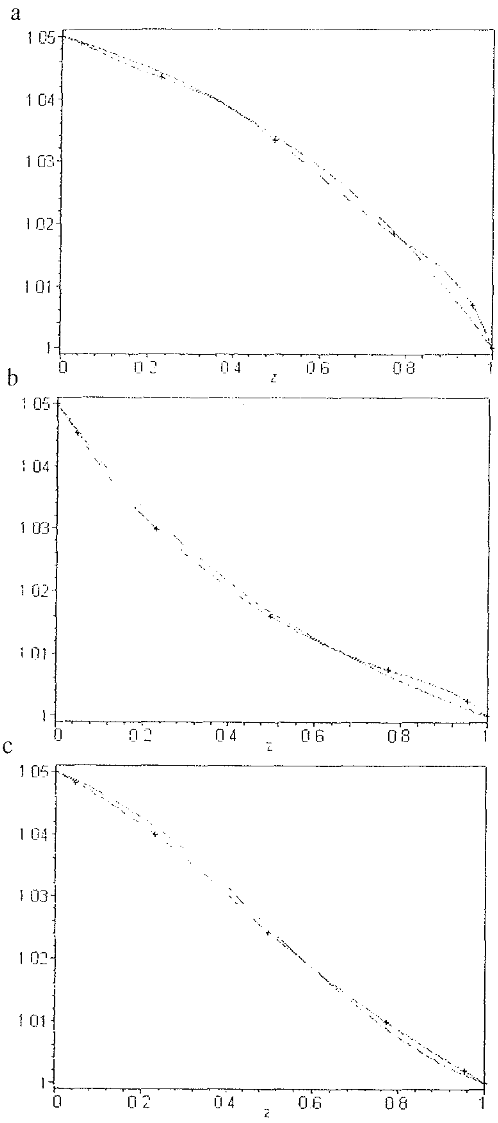

Fig.2 Comparison of model and reconstructed indices (noise in the data was σ=0.0001). a) model 2, b) model 3, c) model 5.

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