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The Effect of Social Background on the Development of Probabilistic Concepts

Robert Peard
B. Sc., M. Ed.

Submitted in fulfilment of the requirements for the degree of Doctor of Philosophy in the Faculty of Education,
Deakin University
March, 1994
Declaration

I hereby certify that this thesis entitled The Effect of Social Background on the Development of Probabilistic Concepts and submitted for the degree of Doctor of Philosophy is the result of my own research, except where otherwise acknowledged, and that this thesis (or any part of the same) has not been submitted for a higher degree to any other university or institution.

Signed

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Robert Peard

Date 23/3/94
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Summary

This research explores how the social backgrounds of a group of students contributed to their intuitive knowledge in probabilistic reasoning, and influenced their processing of the associated mathematics. A group of Year 11 students who came from families for whom the phenomenon of track gambling formed an important part of their cultural background was identified. Another group consisting of students in the same mathematics course (Year 11 Maths in Society) but from families for whom the phenomenon of gambling in any form was totally absent from their social backgrounds was identified. Twenty students were selected from each group.

The research employed a qualitative methodology in which a phenomenographic approach was used to investigate the qualitatively different ways in which individuals within the two groups thought about concepts involving probabilistic reasoning, and processed the related mathematical skills and concepts. The cognitive processes involved in the applications of probabilistic and related mathematical concepts in a variety of both gambling and non-gambling situations were studied in order to determine whether this culturally based knowledge could be viewed as a type of "ethnomathematics."

Data were obtained through individual structured interviews which enabled patterns of reasoning to be compared and contrasted. Analyses of these data enabled intuitive mathematical understandings possessed by the gamblers not only to be identified but also to be linked with their social backgrounds. Also differences between how individuals in the two groups processed probabilistic and associated mathematical knowledge were determined. This research complements and extends existing knowledge and theories related to culturally-based mathematical knowledge. Implications for further research, for classroom teaching, and for curriculum development in the study of probability in senior secondary mathematics classes are discussed.
Acknowledgements

First and foremost I would like to thank my supervisor, Professor Ken Clements. This project began from an idea I obtained from reading Peter Cary's novel *Oscar and Luccinda* while on my way to ICME VI in Budapest. Ken's enthusiasm to my suggestion to research the relationship between the social background of track gambling and the development of probabilistic concepts led to the instigation of the present study. His support, advice, patience, hospitality and friendship over the past five years have been greatly appreciated.

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CHAPTER 1

Overview of the Study

This study involves research in mathematics education within a general field which may be described as "ethnomathematics." In the reporting of his research, D'Ambrosio (1985a), defined ethnomathematics as:

The mathematics which is practised by identifiable cultural groups, such as national tribal societies, labour groups, children of a certain age bracket, professional classes and so on. (p. 45)

One of the cultural groups identified in this study is a group of Year 11 students who come from families that form a segment of Australian society for whom the phenomenon of track gambling is evident in their cultural background and forms a part of their everyday social experiences. Another cultural group identified is a group of students from families for whom the phenomenon of gambling in any form is totally absent from their social background. The present study involves the exploration of culturally-based mathematical concepts and skills that develop from gambling backgrounds. The mathematics is in the area of probabilistic reasoning, and includes related mathematical skills and concepts which are used by the students who are the subjects of this investigation. These skills and concepts will be compared with those held by the members of the sample of students whose school achievements are similar, but who come from family backgrounds in which gambling is completely absent, is foreign to their everyday social experiences, and is sometimes unacceptable as a form of social behaviour.

The present research explores how the social backgrounds described above contribute to the students' intuitive knowledge in the area of probabilistic reasoning, the processing of these concepts and whether this knowledge is sufficiently prevalent to be viewed as a type of "ethnomathematics" in the sense that D'Ambrosio has defined this term.

Since the research will examine how the students process these concepts in the construction of their own mathematical knowledge, attention will be given to the ideas of the field of constructivism in mathematics education. The research examines the cognitive processes involved in the applications of probabilistic and related mathematical concepts in a variety of both gambling and non-gambling probabilistic situations. As Lamon (1972) has stated, this kind of research is important in mathematics education.
to curtail some of our current problems in mathematics education the
teaching of mathematics should be directly influenced by the
psychological research (p. 8)

The research incorporates a "mapping study" similar to what Marton (1988, p. 141)
has described as "phenomenography," that is to say, a way of "mapping" the
qualitatively different ways in which students perceive, conceptualise and understand
their environment. Phenomenography investigates the qualitatively different ways in
which people experience or think about various phenomena, and this study will
examine the ways in which two groups of students think about concepts in
probabilistic reasoning and the related mathematical skills and concepts. Many of
these related concepts are in proportional reasoning and the use of
"phenomenography" has been shown to be an appropriate methodology to use when
examining the limited number of qualitatively different ways in which these concepts
are comprehended. For instance Lybec (1981, cited in Marton, 1988) studied
secondary students' understanding of tasks requiring proportional reasoning using a
phenomenographic methodology and found a number of distinctly different
perceptions.

In addition, in the present study, the relationship between students' perceptions
and the techniques of present classroom instruction in the area of proportional
reasoning will be examined. This will be done not only to determine how such skills
and concepts may be meaningfully incorporated into classroom practices, but also to
suggest how current practices can be improved by giving attention to the knowledge
that the students bring to school with them as a result of their social backgrounds,
thus making the teaching of probability more relevant. Consequently, it is expected
that the findings will have implications for curriculum development and teaching
methodologies.

**Reporting of Results**

It is anticipated that the results of this research will make a valuable contribution
to the body of literature relating to ethnomathematics, probabilistic reasoning and
social constructivism in mathematics education. As such, the results will be of interest
to researchers in the field, especially those involved in curriculum development in
upper secondary mathematics, teachers of mathematics, and possibly to social
scientists and sectors of the general public.
Background to the Study

A summary of the extent and effects of the phenomenon of social gambling in Australian society is presented in Chapter 3, "Gambling in the Australian Social Context", where it is established that there is an identifiable cultural group within the society for whom participation in track gambling in some form is very common. This common interest and its associated practices are the bases for identification of its members. It is the children of these families who are in their eleventh year of high school that form one of the samples of this study. This sample was selected from two state government high schools in mixed socio-economic regions in areas where access to track racing events (horse, harness, and dog racing) was identified. Some of the sample of non-gamblers were also selected from these two schools. However, for reasons given later in the methodology, it was decided to select some non-gamblers from a segment of society for whom the phenomenon of gambling would be totally foreign. A Seventh Day Adventist High School in Brisbane was chosen for this purpose.

Main Foci of the Study

The study focuses on the mathematical skills and concepts in ratio and proportion and probabilistic reasoning that individuals from the two groups possess. These will include probabilistic reasoning (including misconceptions), the concepts of simple and compound probability, fairness and expectation, and combinatorics.

The focus will be on the individuals within the two groups in order to gather data that will enable patterns of reasoning to be compared and contrasted. This in turn will generate hypotheses that may be subsequently tested in further research.

Summary of the Thesis

Issues Examined in the Study

The study will examine the cognitive processes involved in the development of probabilistic skills and concepts and related mathematics in order to determine what intuitive mathematical knowledge the gamblers possess as a result of their social background. It will also examine the ways in which this knowledge is processed by individuals within both groups in order to compare and contrast the skills and concepts held by the members of the groups. The prevalence of such intuitive knowledge will be assessed in order to determine whether such knowledge is a form of ethnomathematics as defined by D'Ambrosio.
Structure of the Thesis

Following this introductory chapter a more detailed rationale for the study is developed in Chapter 2, where it is established that there is a need to research the mathematical skills and concepts that a segment of the population might reasonably be expected to bring to the school environment with them as a result of factors within their social background. The importance of probabilistic ideas in school mathematics and the role of probability in modern school curriculums are also discussed in this chapter.

In Chapter 3, "Gambling in the Australian Social Context", it is established that the phenomenon of track gambling is a widespread within Australian culture and in consequence is a part of the social background of a large segment of the population.

In Chapter 4 historical considerations of the development of probability within the history of the development of mathematics and the historical relationship between gambling and probability are presented.

In Chapter 5 the literature relating to mathematics education in a number of areas is reviewed. These areas include: ethnomathematics, social and cultural factors in the learning of mathematics, constructivism in mathematics education, probabilistic reasoning and related mathematical skills and concepts.

The major research questions are formulated from the review of the literature and are presented in Chapter 6. The decisions for the methodology employed to answer these research question are discussed in Chapter 7, in which there is also a detailed account of this methodology.

A discussion of the results of data gathered by the study occurs in Chapter 8, followed by a more detailed analysis of selected data in relation to the first major research question in Chapter 9, and to the second and third major research questions in Chapter 10. Illustrations of specific student responses are presented in Chapter 11, in which samples of interview transcripts are given.

Conclusions, hypotheses generated by the results, implications of these results and suggestions for further research are presented in the final chapter, Chapter 12.

Concluding Comments

In this introductory chapter a broad overview of the study has been presented along with the background to the study and its main foci. The structure of the thesis indicates how the study will examine major issues raised. The first requirement is to establish clearly that there is a need for the study. This is done in the next chapter, The Rationale for the Research.
CHAPTER 2

Rationale for the Research

The topic of gambling has not been well researched in any academic area. Charlton (1987) points out that "until recently gambling has not been considered as a suitable topic for academic research" (p. 276), and the review of the literature which will be presented in Chapter 5, confirms that the topic has consequently received very little attention by mathematics education researchers. While probabilistic reasoning has recently become a subject of research in mathematics education, much of the earlier research of the understanding of the topic and the processing of the related concepts has been in the domain of psychology. Watson (1992) reported that historically, "psychologists have played a major role in the research into the understanding of probability concepts, most notably Piaget and Inhelder (1951), Fischbein (1975), and Kahneman and Tversky (1972)" (p. 9). Scholz (1991) provided an extensive historical overview of psychological research on the acquisition of probabilistic concepts, but this did not include research in mathematics education.

Perspectives

Given the fact that "Chance and Data", which incorporates the study of probability is now an important component of mathematics curriculums at all levels of instruction (see, for example, Australian Education Council, 1991), there is a clear need for research into probabilistic reasoning and related mathematical concepts. The need to undertake the present study within the social context of gambling will be developed, and the rationale for this selection will be supported from the following perspectives:

1. The activities of gambling are inherently mathematical in nature.
2. Gambling is widespread within Australian culture.
3. There is an established need for research into ethnomathematics and self-generated mathematics.
4. There is a paucity of research into the probabilistic reasoning of secondary school students.
5. The increased importance of probability in the curriculum is well established.
The Inherent Mathematical Nature of the Activities of Gambling

Many of the activities involved in track gambling are inherently mathematical in nature. These include the calculation of expected returns and winnings at various odds, comparing odds, relating odds and probabilities, and calculating numbers of combinations. Historical commentary on the development of a probability theory arising from a study of gambling outcomes in the seventeenth century, up to the inclusion of the study of probability in current mathematics curriculums are presented in Chapter 4. In addition, commentary related to epistemological considerations arising from the activities of track betting and other mathematical skills and concepts such as fraction concepts, proportional reasoning and concepts in combinatorics will be included in the study.

Bishop (1988a), in his study of what constitutes mathematical activities, supports the notion that probabilistic reasoning, including that involved in gambling and games, is part of general mathematical activity, and that investigations in our mathematical culture include experimental probabilities. According to Bishop (1988a), developing ideas about chance and prediction is "an important mathematical activity" (p. 106), and "gambling and games are part of modern western society" (p. 112).

Gambling is Widespread Within Australian Culture

The phenomenon of track gambling is widespread within Australian culture. Indeed, it can be argued that gambling is related to this culture in a unique way. This relationship is explained more fully in the next chapter. The need to research how individuals construct mathematical ideas from their cultural background is supported by Bishop (1998a) who argues that "it is the case that all cultures have necessarily developed their own symbolic technology of mathematics in response to the 'demands' of the environment as experienced through these activities" (p. 59), and that "the relationship which is of concern is ultimately that between learner and social environment not just between learner and teacher" (p. 131).

Such a view is further supported by the research of Carraher, Carraher and Schliemann (1985), who concluded:
There are reasons for thinking that there may be a difference between solving mathematical problems using algorithms learned in school and solving them in familiar contexts out of school. Reed and Lave (1981) have shown that people who have not been to school often solve such problems in different ways from people who have. This certainly suggests that there are informal ways of doing mathematical calculations which have little to do with the procedures taught in school. (p. 21)

This conclusion is also supported by the research results of Hope (1987), who found that in mental arithmetic calculations, people who used their own methods were more efficient than those who consciously attempted to use algorithms based on standard written computations. Thus this rationale clearly establishes the need for research in the field of "ethnomathematics."

**Research into Ethnomathematics and Self-Generated Mathematics**

As stated in Chapter 1, D'Ambrosio (1985a) defined the term "ethnomathematics" as meaning:

The mathematics which is practised by identifiable cultural groups, such as national tribal societies, labor groups, children of a certain age bracket, professional classes and so on. (p. 45)

The need for research in this field is well documented and will be presented in detail later. However, at this stage, there is a need to identify first the structural mathematics within the broad field of track gambling. This need was confirmed by D'Ambrosio (1985a) who maintained that:

Ethnomathematics is not recognised as a structured body of knowledge, but rather as a set of *ad hoc* practices. It is the purpose of research programs to identify within ethnomathematics a structured body of knowledge. (p. 47)

Thus this research will attempt to identify the structured mathematical knowledge within the *ad hoc* practices of the gamblers as they relate to the social context of participation in track events. In doing so, the research will draw attention to the learning of mathematics in circumstances unique to Australian culture. The need to do this is supported by Clements (1988), who claimed that it should be remembered that "often in Australia there are unique factors influencing how children learn mathematics" (p. 5).
It is expected that the research will have implications regarding how probabilistic concepts may be meaningfully incorporated into present classroom practices. This is important for, as D'Ambrosio (1985b) has stressed, there is a need to incorporate certain principles arising from the study of ethnomathematics into the curriculum in order to avoid the "psychological blockade" that is so common in mathematics. He has noted:

Only recently have cultural issues played a role in the discussion of mathematics curriculum in developed countries. With the increasing automation in industries and the spreading role of computers, the need to reshape the education of the lower strata of the population, is increasing. This carries naturally to cultural considerations. (p. 13)

Thus, the rationale relies, in part, on an acceptance that the learning of mathematics necessarily introduces culturally laden issues. This is supported by the extensive research in the field of culturally laden issues in mathematics education (see, for example Bishop, 1988a). These issues will be examined further in Chapter 5, the review of the literature. At this stage, it is sufficient to note that this premise is not at present universally accepted. Bishop (1988b) stated:

For some people this influence represents a thoroughly undesirable development. Mathematical knowledge has for them the attributes of clarity, universality and truth; values which imply certain educational goals, and therefore certain specific research tasks. The social perspective is for them at best an unnecessary diversion from the real tasks, and at worst an undesirable confounding of an already complicated field. (p. 117)

Bishop (1991) maintained that all mathematical learning takes place in a social setting and that we need to be able to theorise about "interpersonal" as well as "intrapersonal" mathematical learning. He stresses that learning mathematics in a social context, cannot be fully interpreted as an "intrapersonal" phenomenon because of the social context in which it occurs. Equally, "interpersonal" or sociological constructs will be inadequate alone since it is always the individual learner who must make sense and meaning in the mathematics. He concluded that therefore

it is vitally important to research the ways this intra-interpersonal complementarity influences the kind of mathematical knowledge acquired by pupils in the classroom. (p. xviii)

The need to identify probabilistic knowledge that both gamblers and non-gamblers acquire outside of school, as well as the intuitive concepts acquired by the gamblers from their social background is supported by Glaeser (1983) who made
the important point that within modern society the ideas of probability are very common:

When one starts to teach ... this subject ... [students] are certainly not without previous knowledge: ... everybody is familiarised with situations of betting, of drawing lots, or with decisions under uncertainty. The only basis for efficient teaching is the teachers' awareness of pupils' preconceived ideas which he (sic) will seek to discourage or promote. (p. 313)

This argument gains support from the evidence that informal procedures learned outside of school are often extremely effective. Gay and Cole (1967) for example showed that unschooled Kpelle traders estimated quantities of rice far better than educated Americans. They became convinced that it was necessary to investigate first the "indigenous mathematics," in order to be able to build effective bridges from this "indigenous mathematics" to the mathematics of the school.

The results of the present investigation should help construct "bridges" between the intuitive probabilistic skills and concepts of the gamblers and the probabilistic skills and concepts of the school curriculum. The need to do this is further supported by research showing that the teaching of much traditional school mathematics is ineffective. Eduardo Luna (in Gerdes, 1988) argued that the practical mathematical knowledge that children acquire outside the school is "repressed" and "confused" in the school. In a similar vein, Carraher and Schliemann (1985) have shown that children who have to make frequent and quite complex computations outside of school did so efficiently in out-of-school contexts, but were not successful with the same type of computation in a classroom context. Similar studies by Carraher and Schliemann (1988) have shown that fisherman with little formal schooling were able to calculate proportions and to transfer their techniques of solutions to unfamiliar situations.

Scholz (1983), noting that there is a poor level of performance by the general population in probabilistic tasks commented that

more knowledge is needed about how to put theory into practice, as well as about how to relate experimental findings to real life questions ... as a prerequisite for the development of suitable methods (e.g., curricula or decision aids) for improving behaviour. (p. 55)

This is particularly relevant in Australia today where recent initiatives in curriculum development have resulted in the inclusion of probability in new State syllabi and in

A Paucity of Research into Probabilistic Reasoning by Secondary School Students

Mathematics educators such as Watson (1992) have expressed concern that recent initiatives in curriculum development "have been taken without the benefit of previous educational research in Australia on the learning of probability" (p. 1). She stated that:

In Australian school systems teachers are currently implementing the Chance and Data curriculum using the best resources and advice they can get from educators and curriculum planners, all of whom are operating without the luxury of a local research base. (p. 5)

She then argued that since probability is such a relatively new area of the curriculum, research is needed to "provide a fundamental structure" (p. 11) for teaching and learning. She concluded that in the wake of the publication of *A National Statement on Mathematics for Australian Schools* there is "an urgent need for research into the understanding of concepts related to probability" (p. 13).

Watson (1992) also made the important point that the probability and statistics component of the mathematics curriculum is one part that is closely related to out of school experiences. The present research includes an examination of probabilistic concepts in real world, out-of-school contexts.

Shaughnessy (1992) confirmed that "there is very little large scale information about how secondary school students think about chance, random events and decisions under uncertainty" (p. 489). He attributed this paucity, at least in part, to the lack of instruction in the subject at that level, and makes the important observation that most of the recent research has been done with either elementary school students or college students, resulting in "a gap in our knowledge about students conceptions of probability at the secondary level" (p. 489). He supported the need for research in this field in order to answer the questions:

Do secondary students use heuristics similar to those exhibited by college students? Do they resort to nonstatistical or deterministic explanations of chance phenomenon? Do their conceptions change under the influence of instruction? (p. 489)

The present study will attempt to provide answers to some of these questions. The need for educational research in this field is supported further by Hawkins and Kapadia (1984) who, upon examining what conceptions of probability children have at various ages, concluded that "the question is not yet answered by the available research findings" (p. 374). Kapadia (1984) claimed that assessments of probabilistic understanding such as that carried out by the Assessment of Performance Unit, Her
Majesty's Scientific Office, (1981), and even by Piaget's research (from 1951 onwards) have yielded "little insight into children's conceptions of probability" (p. 47). On this matter, Green (1983a) noted that:

In our scientifically and technologically dominated society, with its deterministically oriented education system, it is perhaps inevitable that the concept of randomness should be neglected. Very little time or attention is given to this concept despite the fact that in our scientifically and technologically dominated society we are surrounded by random occurrences every day of our lives. (p. 782)

Garfield and Ahlgren (1986) claimed that research in probabilistic reasoning must expand in scope and become "more cross disciplinary and collaborative" (p. 273). Further evidence for this paucity of research in this field is given by Suydam and Brosman's (1993) report on research in Mathematics Education in 1992 in The Journal for Research In Mathematics Education, in which their index of research by topic did not include "probability" as a separate heading.

The Importance of Probability in the Curriculum

Finally, there can be no doubt that the topic of probability is an important one in the mathematics curriculum of today. The role of probabilistic reasoning in modern secondary mathematics curricula is discussed in detail later in this chapter. Further, it was noted by von Glasersfeld (1987) that the nature of probability is pedagogically suited to the contemporary mathematics educator's belief that children are active constructors of their own knowledge. This, he maintained, is due to the experimental nature of the topic and its emphasis on inquiry.

The importance of understanding probabilistic concepts in modern technological societies has been well established for some time now. Ashmore, Dodge and Kasch (1962), for example, argued that such understanding is essential since we require a "probabilistic rather than a deterministic view of nature and mathematics" (p. 530).

Writers, such as Jones (1979), have argued that it is essential that students be taught how to deal realistically with uncertainties otherwise they may respond to probabilistic situations with preconceived notions, emotive judgements and even a lack of awareness that chance effects are operating. Despite the recognition of the importance of probabilistic concepts by mathematics educators, the inclusion of probability into the mathematics curriculum is a relatively recent development, and the implication of this will be examined in detail later.
The Role of Probability in the Curriculum

In the 1981 Yearbook of the National Council of Teachers of Mathematics (NCTM) in the United States, Percira-Mendoza and Swift (1981) presented the following reasons for the inclusion of probability as part of every student's general mathematical education:

Individuals need a knowledge of statistics and probability to function in our society. Such things as consumer reports, cost of living indices, surveys and samples are part of our everyday life ... competence with the utilitarian aspects (of probability) will help them process the many data-oriented messages they receive. (p. 2)

They made the point that the number of disciplines that make use of probabilistic concepts is increasing and that this increased use brings with it the need to develop an "aesthetic" for the proper use of the concepts.

Kapadia (1984) reported in an address to the Royal Statistical Society Conference in London in 1984, that the eminent statistician, Lindley, appealed for mathematics education to "promote the probabilistic appreciation of the world" (p. 49). He then described this plea as "surely the fundamental rationale for teaching probability in schools" (p. 49). This need has been consistently reiterated by delegates at the 1986 Second International Conference on the Teaching of Statistics (ICOTS II) in Victoria (Canada) and the Third International Conference on the Teaching of Statistics (ICOTS III) held in Dunedin (New Zealand), in 1990.

Clearly, the study of probability and statistics is being more widely accepted as an integral component of the school mathematics curriculum. In 1984 the state of Wisconsin made the study of probability and statistics for at least one semester of secondary school mathematics mandatory for all students. Kepner, at ICOTS II (1986), described this mandate as an important development in the teaching of school mathematics. Ottaviani and Aureli (1986) examined the inclusion of probability and statistics for all junior secondary students in Italy since 1979, and Peard (1987) reported on the revised role of probability in the new Queensland secondary mathematics syllabus. In his opening address at ICOTS III, Jowett informed the audience that "data analysis and probability are becoming key components of educational programs that motivate and illustrate mathematical concepts by the use of applications to the real world of the students" (reported in Schaeffer, 1990, p. 1).

Nevertheless, until recently in Australia, probabilistic concepts have often been taught formally within the broader context of academic mathematics programs, designed to serve the needs of the more academically oriented students. This situation
still exists in many parts of the world. Travers (1989) reported in the Proceedings of the Sixth International Congress on Mathematics Education:

At the secondary level the pattern is likely to be that probability and statistics is taught only to pupils who are in specialised courses leading to university study — in no country is the topic taught universally across all grade levels. (pp. 348-349)

The Department of Education (1987a), in Queensland expects the topic of probability to be taught in years 1-10. Prior to 1987 probability concepts were not introduced until Year 9. The following diagram from the *Years 1 to 10 Mathematics Teaching, Curriculum and Assessment Guidelines* shows the broad nature of probability concepts as they are expected to be presented in grades 4 to 10 in Queensland schools.

<table>
<thead>
<tr>
<th>Probability</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3</td>
</tr>
<tr>
<td>Concept of probability and associated language</td>
<td></td>
</tr>
<tr>
<td>Qualitative descriptions of probability</td>
<td></td>
</tr>
<tr>
<td>Quantitative measures of probability</td>
<td></td>
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<tr>
<td>Sample spaces</td>
<td></td>
</tr>
<tr>
<td>Use of tables or tree diagrams</td>
<td></td>
</tr>
</tbody>
</table>

*Figure 1:* Probability in Grades 1 to 10.
(From Department of Education 1987a, p. 27)

According to the Queensland Department of Education, (1987b), probability is concerned with how often we may expect an event to occur, and it is the laws of probability which allow us to refine wild guesses in order to make predictions about the likelihood of an event occurring. The following learning experiences are to be found in The Queensland Department of Education’s (1987c) mathematical source books:
Year Four:
Analysing situations and explaining which events are possible, impossible or certain to occur.

Analysing situations and explaining the chances of an event's occurring.
Identifying all possible outcomes in different situations.

Predicting and validation the outcomes of:
- simple (that is, one-stage) probability experiments;
- two-stage experiments.

(Vol. 4, p. 250)

Year Seven:
Predicting the number of actual outcomes, given the total number of outcomes.

Comparing the probability of outcomes to determine events that are more/most, less/least, equally likely to occur.

Examining sample spaces to discover which one is more/most, less/least, equally likely to give a particular outcome.

(Vol. 7, p. 53).

Year Eight:
Exploring real-life situations that involve probability.

Analysing and interpreting outcomes of simple experiments.

Calculating and comparing probabilities to determine the likelihood of a particular event.

Representing, calculating and validating that the sum of the probabilities of any experiment is 1.

(Vol. 8, p. 82)

Year Ten:
Analysing and determining the complement of an event.

Conducting 1-, 2-, and 3-stage experiments to determine appropriate sample spaces.

Calculating the probability of an event using tree diagrams.

(Vol. 10, p. 107)
Since 1987, we have seen significant changes to curriculums in a number of countries in which probabilistic concepts have been included, in particular the United States of America, the United Kingdom, and in Australia.

In the United States of America, the publication of the *Standards for Schools* (National Council of Teachers of Mathematics, 1989) includes as Standard 11 the study of probability and statistical reasoning at all levels.

**Standard 11: Probability**

In grades 5-8, the mathematics curriculum should include explorations of probability in real-world situations so that students can -

model situations by devising and carrying out experiments or simulations to determine probabilities;

model situations by constructing a sample space to determine probabilities;

appreciate the power of using a probability model by comparing experimental results with mathematical expectations;

make predictions that are based on experimental or theoretical probabilities;

develop an appreciation for the pervasive use of probability in the real world.

(p. 109)

According to the *Standards for Schools* (National Council of Teachers of Mathematics, 1989), the study of probability in grades 5-8 should not focus on developing formulas or computing the likelihood of events pictured in texts. Students should actively explore situations by experimenting and simulating probability models. Such investigations should embody a variety of realistic problems, from questions about sports events to whether it will rain on the day of the school picnic. Students should talk about their ideas and use the results of their experiments to model situations or predict events.

Probability, the measure of the likelihood of an event, can be determined theoretically or experimentally. Students in the middle grades must actively participate in experiments with probability so that they develop an understanding of the relationship between the numerical expression of a probability and the events that give rise to these numbers (for example, two-fifths, as it relates to the probability of choosing a red marble from a hat). Students must not only understand the relationship between the numerical expression and the probability of the events
but realise that the measure of certainty or uncertainty varies as more data are collected.

At the secondary level, from Grades 9 to 12, the *Standards for Schools* (National Council of Teachers of Mathematics, 1989) suggests that the mathematics curriculum should include the continued study of probability so that all students can:

- use experimental or theoretical probability, as appropriate, to represent and solve problems involving uncertainty;
- use simulations to estimate probabilities;
- understand the concept of a random variable;
- create and interpret discrete probability distributions. (p. 110)

The document suggests that students in Grades 9-12 should understand the difference between experimental and theoretical probability. Concepts of probability, such as independent and dependent events, and their relationship to compound events and conditional probability should be taught *intuitively*. This statement is particularly relevant to the present study, since it gives support to the rationale to research students' intuitive probabilistic concepts.

According to the *Standards for Schools* (National Council of Teachers of Mathematics, 1989), formal definitions and properties should be developed only after a firm conceptual base is established so that students do not apply formulas indiscriminately when solving probability problems.

The Australian Education Council (1991) states that "making informed choices and decisions about social issues is fundamental to a democratic society" (p. 4) and that "sound concepts in the areas of chance, data handling and statistical inference are critical for the levels of numeracy appropriate for informed participation in society today" (p. 82). In fact, *A National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991) includes the following comments on "Chance and Data" that makes specific reference to gambling:

Many situations in the adult world involve chance. For example, gambling is a significant part of the live and customs of many Australian adults, as are decisions about the insurance of health and property. (p. 163)
The *National Statement* also refers to the language used in gambling, noting:

The language of chance is widely used in a colloquial way (tomorrow will probably be fine, it's an odds-on bet, it's a sure thing). Students should be helped to refine and extend their use of this language so that they are more able to make sense of their everyday experiences. (p. 163)

According to the *National Statement*, experiences in Band A (A1, the early primary years) with chance should be provided which enable children to:

Use, with clarity, everyday language associated with chance events.
Possible activities:

Clarify and use common expressions such as "being lucky", "that's not fair", "always", "it might happen", "tomorrow it will probably rain".

Use the vocabulary "certain", "uncertain", "possible" and "impossible" appropriately, recognising that, while there is an element of uncertainty about some events, others are either certain or impossible.

Use language such as "very likely", "unlikely", "more likely" and "equally likely" to describe events which relate to the experience of the child. (p. 166)

At the second level of Band A (A2), students should be able to describe possible outcomes for familiar random events and one-stage experiments. Possible activities include:

Identify all outcomes arising from one-stage chance experiments, such as tossing a coin, rolling a die, using a spinner, or selecting a marble from a container.

Record outcomes from simple experiments involving counters (for example, toss 8 counters and record the numbers falling each side of a piece of cotton, saying, for example, that "three fell one side and five the other side and none on the cotton, and three and five make eight").

List possible outcomes from one-stage experiments involving counters.

Use techniques for random assignment of roles. (p.166)
At the third level (A3), students should be able to place outcomes for familiar events and one-stage experiments in order from those least likely to happen to those most likely to happen.

Further experiences with chance during Band B included in the *National Statement* (The Australian Education Council, 1991) are:

For random events, systematically list possible outcomes, deduce the order of probability of outcomes and test predictions experimentally. Make and interpret empirically based predictions about simple situations. (p. 170)

Specifically, the *National Statement* (The Australian Education Council, 1991) includes in Band C suggested activities that relate to the probabilistic concepts associated with gambling. These include:

Study common games of chance by analysis and simulation to find the probability of winning a major prize, any prize, the expected return on the dollar, etc.

Describe probabilities on a scale between 0 and 1 and place informal expressions of chance on the scale (for example from impossible, through poor chance, even chance, good chance and certain).

Use language of chance to describe events which are mutually exclusive (for example, if one of these does happen then the other will not), including the special case of complementary events.

Discuss the term "odds-on" and note that statements of odds which appear in gambling contexts reflect statements of subjective probability as well as statements of return on money invested.

Investigate "odds" to determine how the bookmaker makes a profit (the related probabilities add up to more than one).

Examine various probabilistic misconceptions (for example, the gamblers' run).

Devise, play and analyse a variety of "fair" and "unfair" games. (p. 175)
In addition, the Community Guide to the National Statement, published by the Australian Education Council (1992) as a guide to parents and the wider community, includes the following information on the Chance and Data strand:

The world in which we live is filled with uncertainty, so it is critical for students to develop sound concepts in the areas of chance, data handling and statistical inference ... We need to understand the nature of chance processes and their effects on data collection, and the assumptions that underlie procedures and predictions. We need to understand about predictions and statistical inference. (p. 31)

This Guide suggests that students should:

develop an understanding of chance events and how they are described;
learn data-handling processes and make judgements on their appropriateness;
understand how people make predictions from data and be able to judge their reasonableness. (p. 31)

The Guide points out that uncertainty (or chance) is a familiar characteristic of our everyday experience. It underlies, for example, weather forecasts, forms of insurance, stock exchange transactions, opinion polls and gambling. Students need to clarify their understanding about chance events and the language used to describe them so that they are more able to make sense of their lives. Schools should provide practical activities involving chance, from the early years. In the secondary years, students should develop the concept of probability as a measure of chance and engage in a range of experimental and more formal activities.

In the United States, the National Research Council (1989, p. 8) recognised the importance of probability in the curriculum, and noted that legal cases often rely on probabilistic inferences as much as on direct evidence.

The senior secondary mathematics syllabus in Queensland includes objectives relating to probabilistic reasoning in both Mathematics A and Mathematics B as part of the "core" syllabus objectives. It is anticipated that a large majority of Queensland senior secondary students will take at least one of these subjects. The global objectives, as stated by the Board of Secondary School Studies, Queensland (1992a) include:

be aware of the uncertain nature of their world and be able to use mathematics to assist in making informed decisions in life-related situations. (p. 4)
The need to make mathematics more relevant has been of concern to mathematics educators for many years. One way of achieving this is to provide mathematical experiences with a social context. As Bishop (1988b) noted, "the influences of the social sciences is being increasingly felt" (p. 117). This need is reflected in the suggested learning experiences contained in the syllabus documents of the new Board of Secondary School Studies, Queensland (1992a, 1992b) Mathematics A and Mathematics B courses.

In Mathematics A, subject matter includes:

Probability as a measure of chance and likelihood; as a relative frequency of equally likely outcomes in an experiment; as a proportion between zero and unity.
Tree diagrams as a tool for defining sample spaces and estimating probabilities.
The use of probability distributions to compare two or more groups; overlapping areas and discrimination.
Discrete probability distributions, uniform (rectangular) and binomial, as data models, the table of binomial probabilities. (p. 23)

In Mathematics A, suggested learning experiences include:

Identify words used in English as expressions of probability.
Use a tree diagram to find the probabilities associated with successive births to a couple who are known to carry a specific genetic disorder (for example, the condition of haemophilia).
Use the tabulated information given in the newspaper about previous Gold Lotto draws to determine whether the numbers are drawn at random, that is, whether the numbers follow a uniform probability distribution model. (p. 24)

In Mathematics B subject matter includes:

Probability as a measure of chance and likelihood; as a relative frequency of equally likely outcomes in an experiment; as a proportion between zero and unity. (p. 33)

In Mathematics B, suggested learning experiences include:

Examine media reports and everyday expressions for the use of probability language and discuss the meaning and accuracy of terms used.
Use a tree diagram to find the probabilities associated with successive births to a couple who are known to carry a specific genetic disorder, for example, the condition of haemophilia. (p. 34)
An examination of selected textbooks of recent years shows that they contain specific reference to the mathematics of gambling. Such references are to be found in Interactive Mathematics A, Book 2 by Peard, Mowchanuk and Shield (1993a), and in Maths for Today's Society Statistics, Chapter 7: Probability in daily life - gambling, by Francis (1990).

Articles on the chance and data theme published by the Australian Association of Mathematics Teachers in their journal The Australian Mathematics Teacher include "Race Day? You Bet!" by Allan White (1991, p. 32), in which the author claimed that "the use of the theme of horse racing and gambling can provide an interesting and experiential setting for dealing with the topic of ratio." Mathematics Curriculum and Teaching Materials (MCTP) prepared by Lovitt and Clarke (1988) included a simulation of the Totalization Board Betting (TAB) system. Publications of the Australian Academy of Sciences include Modelling Your World by Ian Lowe (1988) which contains sections such as "Unit 15 - Taking your chances" and "Unit 17 - Punting on ponies," and Mathematics at Work- Taking your chances, by Treilbs (1980) which included the mathematics of lottery tickets and poker machines, as well as an analysis of the playing of "Crown and Anchor" and "Roulette."

In addition to changes in content, we have also witnessed changes approach to the teaching of mathematics. Most new curricula have considerable "flexibility" to incorporate the pupil's background into the mathematical context, as can be evidenced from statements of policy documents such as the following one taken from the Queensland Years 1-10 Mathematics Curriculum Framework (Curriculum Services Branch, 1986):

Intellectual growth of the individual is continuous yet marked by clearly discernible developmental characteristics reflecting the age, general competencies and language facility of the student. The syllabus is based upon the following beliefs about how students learn mathematics. (p. 2)

Furthermore, this document notes that:

The degree to which students want to learn mathematics varies considerably and depends upon:
the background of the student
personal interests
previous experience
success
the quality of the learning experiences provided
interaction between students and teacher
the value placed on learning
the value placed on mathematics

(p. 2)
The 1987 *Years 1 to 10 Mathematics Syllabus* developed by the Queensland Department of Education also advocated the introduction of probability concepts in the primary school. However, as Figure 1 shows, the concepts to be taught were very broadly stated.

Despite the recent inclusion of probability in current mathematics syllabi, and its recognised importance, there are still many problems with its implementation. Peard (1987), who researched teachers' knowledge of content and attitude to the teaching of probability and statistics in Queensland, Australia, reported extensive teacher unfamiliarity with the mathematical content, widespread unease with the teaching of topics, and a widespread belief that the content was not important. According to Peard (1987), comments such as "There's only so much time to teach what's in the syllabus," and "probability and statistics is generally left to the end, after the important mathematics, and there's generally no time left for it" (p. 17) were common.

Garfield and Ahlgren (1988) reported that "teaching a conceptual grasp of probability appears to be a very difficult task, fraught with ambiguity and illusion" (p. 57). Watson (1992) suggested that "it may be possible to use the results of research to make more precise and confident statements to help teachers and students understand and apply the concepts in the Chance and Data statement" (p. 5).

It is therefore an important objective of the present study to provide research results that will help Australian teachers with the implementation of the syllabus objectives as they relate to probabilistic concepts.

**Summary**

In this chapter it has been established that the topic of gambling has not been well researched in academic studies. This claim will be further supported in the literature review. Since the activities of gambling are inherently mathematical in nature, and since this mathematics is incorporated into the present school curriculum, research into the topic falls within the domain of mathematics education. While there is a large body of research into probabilistic reasoning by psychologists, little research by mathematics educators has been done. Furthermore, most of this has been with either primary students or college students, with little attention to secondary students. There is no longer any doubt about the importance of probability in the current school mathematics curriculum, especially as writers such as Watson (1992) have expressed much concern that all those involved in the implementation of the Chance and Data sections of new curricula throughout
Australia are operating without a local research base. The present research will add much needed information to the body of research knowledge required by those involved in the implementation of Chance and Data sections of the senior secondary school curricula.
CHAPTER 3

Gambling Within the Australian Social Context

Since this research involves an examination of the probabilistic reasoning of a group of Australian school students within a gambling context, it will be useful, at the outset, to examine the social context in which this gambling occurs. This is done from the perspectives of the expenditure on gambling, the extent of the social acceptance of gambling within the society, the history of gambling, and categories of gamblers within society.

Expenditure on Gambling

The total expenditure on gambling within any society is difficult to assess with accuracy. However the following account of monies spent on gambling in Australia takes into consideration data supplied from various sources in order to illustrate the magnitude of the expenditure, bearing in mind that all figures will be either estimates or approximations.

Legal and Illegal Betting

In the reports of expenditure on gambling by Caldwell (1985), Haig (1985), Haig and Reece (1985), and Quiggin (1985) two categories are considered: legal and illegal.

Legal Betting: The influence and extent of gambling within Australian society can be investigated by an examination of the relevant statistics relating to expenditure on various types of gambling. Although accurate data on the subject are not easily obtained, fairly reliable estimates are available from a variety of sources. Haig, (1985), has provided much data on the topic. He qualifies his estimates with the recognition that difficulties in obtaining accurate estimates relate to "the omission of illegal betting, understatement of bets by licensed bookmakers and expenditure on miscellaneous activities which are not taxed" (p. 77). He has also noted that attempts to make comparisons of expenditure between countries are difficult due to factors such as "inconsistent definitions and incomplete coverage" (p. 73).
Nevertheless, Haig (1985, p. 72) has presented an informative set of statistics which suggest that in 1982/1983 Australians spent $580 per head of population on legal betting. Even with the omission of estimates of illegal betting, Haig concluded that "the figures indicate that Australia has the highest level of gambling expenditure per head of population of any country" (p. 74).

Indeed, after calculating expenditure as a percentage of personal consumption, he concluded that "the level of expenditure relative to personal consumption is twice the average of all countries and 50% higher than that with the next highest expenditure" (p. 74).

It could be expected that the 1982/1983 figure of $580 per head of population would be much higher today. Even if expenditure had remained constant over this time, in terms of today's dollars, it would be over $1000. However there is considerable evidence that with the introduction of casinos in several states and an increase in the prevalence of poker machines throughout the country, overall expenditure on gambling has risen. Jones and Couchman (1981) estimated that each Australian family spent between $2000 and $3000 per year on gambling. Warneminde (1991) reported that a study by a private data firm, Australian Gambling Statistics, estimated that Australians lost a total of $4400 million in 1989-90 in legal gambling activities. This figure represents $2380 for every Australian over the age of 18. Warneminde (1991) commented:

There is statistical support for the reputation of Australians as a nation of people who like a bet. The evidence shows that we lead the world in our enthusiasm ... per capita spending here ... is 60% higher than in the U.S.A., ... 647% (sic) higher than in Britain and ... 716% (sic) higher than that of Canada. (p. 80)

Haig (1985) estimated that when the data were broken down into the various forms of gambling, 70% of all legal betting was on track events. However, more recent estimates reported by Warneminde (1991) suggested that this proportion is less than 50% (estimates varied between states and ranged from 28% to 48%). However, this decrease does not represent a dollar decrease in track betting but can be attributed to the increase in expenditure associated with the introduction of casinos and an increase in the number of poker machines following their recent introduction in many states.

An increase in casino betting throughout Australia was documented in a February 1991 issue of The Australian Magazine. In this report, it was stated by Mike Safe (1991) that casino profits had risen to $443 million in 1989-90. In another issue of the same magazine, in November 1992, it was reported by Steve Bunk (1992) that casino profit for the 1990-91 period was $525 million. Safe (1992) reported that up to 85%
of casino betting is "local trade" (p. 31), the remaining represented expenditure by tourists.

Safe (1992) reported that the expenditure on legal off-course track betting through the government appointed Totalization Board Betting institutions (the TABs) exceeded on-track betting through bookmakers and quoted a figure of $11 billion bet through the national TABs in 1991/1992. This has some implications for this study since the "odds" given when betting through the TAB are not determined before the race, but are calculated by "totalization" techniques of which the better may be unaware. This factor will be examined later in the methodology where it will be explained that in order to be selected as a "gambler" the student must follow the "odds" when betting or considering bets.

*Illegal Betting:* All figures relating to illegal betting will be estimates. Haig (1985) commented that "most illegal gambling is track betting, and it is likely that illegal gambling is higher in Australia than in other countries because of the relatively greater importance of track betting" (p. 74). Safe (1992) reported that the Queensland Criminal Justice Commission estimated the illegal off-course SP (starting price) market to be worth over $4 billion a year and that "Bookmakers across the country are crying out for phones, claiming they would bring back business as well as cornering a slice of the illegal market" (p. 28).

Further evidence showing the extent of illegal betting was furnished by the report of the Fitzgerald Inquiry into police corruption in Queensland in 1989. This inquiry resulted in a crackdown on illegal SP betting and in the following year the Queensland TABs reported a 20% increase in turnover.

**Social Acceptance of Gambling**

In addition to statistics relating to expenditure, we find evidence for the widespread occurrence and acceptance of gambling in the quantity of newspaper space concerned with track racing and the time given to the broadcasting of track events and their results on radio and television. Popular magazines, such as the "Australian Women's Weekly", which is a conservative, "family oriented" publication, have featured articles such as Lawnton and McNaughton's "The Great Aussie Gamble" (September, 1989) in which they claimed that "gambling is as much a part of Australian culture as the meat pie or barbie" (p. 115).

There is in Australia an acceptance of gambling as a "respectable" pastime that is not to be found in many other societies. Social scientist, Jan McMillen, of the Queensland University of Technology, cited by Warnemide (1991), observed:
There has not been a well organised moral opposition to gambling in this country ... we don't have the moral hang-ups of the USA where after the excess of the Wild West, it became viewed as a vice and subjected to prohibitions which still exist in all but a few states. (p. 81)

The phenomenon of the Melbourne Cup is unique to Australia. On this occasion every year, on the first Tuesday in November, the population of the city of metropolitan Melbourne (encompassing over two million people), take a public holiday for the running of a horse race. Throughout the country, office "sweeps" are conducted and virtually the whole country comes to a halt to listen to, or to watch the race. The TAB turnover nation wide on November 2, 1993 was $130 million, $67 million of which was bet on the one race alone (Courier Mail, November 3, 1993, p. 1).

Haig (1985) reported that interest in track events is common in the USA, Britain and Ireland, but expenditure per head of population is much lower than in Australia. Many Asian countries are also tolerant of gambling, but with the exception of Hong Kong, little gambling is related to track events and, unlike Australia, it does not generate large amounts of government revenue. Monies bet on track gambling in Hong Kong can be as much as $US100 million per meeting, though much of this is from international betting. In many Asian societies gambling is often accompanied with much superstition relating to "lucky" numbers and the like. Further, in all Moslem countries betting on any games of chance is illegal since it is specifically prohibited by the Koran.

The popularity of track betting in Australia can be observed in hotel bars throughout the country on Saturdays, on most week days and on some evenings when track events are being held throughout the country. On these occasions large numbers of track followers are to be seen.

TAB offices are to be found in virtually every town of the country and, at these locations, one can place a bet on any major track event anywhere in the country.

*Figure 2: TAB Fortitude Valley.*
Following the increase in Casino betting and an increase in the number of poker machines in operation, the TAB have increased their advertising in an attempt to capture a greater proportion of the market. They have used advertisements to imply that betting on racing is more a "social activity", and have appealed to punters through advertisements such as those shown below, that track betting is "the thinking person's form of gambling" (see Figure 3).

Put your theories into practice.

Figure 3: TAB Advertisement "Put Your Theories Into Practice."
TAB offices and hotel bars broadcast live coverage of all major track events via satellite TV which results in an almost continuous broadcast of track races, and recently, we have seen the introduction of "PUB TABs" to 162 hotels throughout the state of Queensland. These consist of a TAB office attached to the hotel bar.

![Image: PUB TAB](image)

*Figure 4: PUB TAB.*

Newspapers print pages of information on the "form" of the horses for all major events in all capital cities. The "Race Guide" appears in all major daily papers generally three times a week. For example, the Saturday *Courier Mail* race guide is called the "Superform" and it is generally 16 pages of print. The results of major track races, generally accompanied with a replays of the finishes of the races, are broadcast on the evening television in the "sports" section. These broadcasts include a statement of the "odds" and the TAB payouts for the winning and placing (second and third) horses as well as any payouts for the "doubles" (first in two races), "extra doubles", "trebles", "quinellas" (first two in one race in any order), and "trifectas" (first three in one race in the correct order). Each capital city has a "race" radio station (in Brisbane, this is 4TAB), concerned solely with the broadcasting of race results. As a result of this exposure virtually all of the Australian population is to some extent familiar with track betting. Rosecrance (1988) claims that gambling is an integral part of Australia's self-image.

The general acceptance afforded the national image of Australians as heavy gamblers has given gambling a legitimacy rare in other countries. While Australian history is replete with abortive reform and prohibition movements, Australians' view of themselves as gamblers was sufficient to defeat these efforts. (p. 149)

More recently mathematics educators such as Lovitt and Clarke (1988) have recognised that "gambling is widespread in our community" (p. 75) and included a
simulation of the operation of the TAB betting system in their Mathematics Curriculum and Teaching Program materials package.

The Origins of Gambling in Australia

The origins of gambling in Australia go back to the very beginnings of European settlement. Booker Prize winning Australian author Peter Cary (1987), in referring to the social history of the country, commented that "it was as if the very colony was founded on gambling" (p. 167). Charlton (1987) reported that as early as 1905 the Melbourne newspaper, the Argus stated that government-sanctioned gambling was part of Australian social life. Historians such as Crowley (1974) and O'Hara (1988) have provided extensive accounts of the history of gambling in Australia.

Auchmuty (in Crowley, 1974) reported that "from the earliest days there had been organised horse racing" (p. 78). It is interesting to note that Charlton (1987, p. 276) observed that "until very recently, gambling was not considered a subject for academic research." In his introduction, Charlton quoted Frank Hardy, who attributed the phenomenon of gambling to the nature of the settlement:

From the very beginning, life was a gamble among the convicts and settlers. Our country was pioneered in the spirit of a gambler's throw. The settlers, the squatters and later the shearers and drovers who went into the hostile wilderness, gambled life itself on a bid to find success and security. Men were thrown together without women and without culture, sport or entertainment - so they gambled. (p. 1)

Many factors contributed to the popularity of horse racing. Charlton (1987) has argued that horse racing was "a method of achieving social mobility as well as being a sport; owning and racing horse conferred a kind of respectability on those who craved recognition"(p. 57). He maintained that by the 1870s "horse racing was, quite simply, our national sport" (p. 64) and that compared with other forms of gambling it was regarded as being both "eminently respectable and egalitarian" (p. 59). According to Charlton, in 1898 the Anglican Bishop of Newcastle observed:

Australia has caught the betting and gambling infection from two sources of infection. The early goldfields, with the recklessness and rowdysim characteristic of them, together with the commonness and cheapness of horses, has made horse racing the Monte Carlo of Australia. (p. 159)
In addition there are other historical records that have reported opposition to gambling at this time. Most of this opposition came from the Protestant churches and some opposition to gambling is still evident today from these groups. This opposition is largely based on their view of gambling as running counter to the Protestant work ethic and ethos of self-improvement based on hard work. Charlton (1987) maintains that the Protestant churches saw gambling as "a distinct and deliberate challenge to accepted Protestant values of thrift, hard work, and denial in pursuit of a Godly life" (p. 158). Although opposition to gambling is still in evidence today among these groups, their attempts to influence government policy have subsided. As Charlton has observed:

They (the churches) have lost. Australians now have more legal outlets for gambling than ever before and the churches' voices are somewhat muted. They have realised that for governments, gambling in all its forms is too lucrative a source of revenue. (p. 282)

Charlton also points out that this opposition was not common among the Catholic population many of whom had Irish backgrounds and "a long history of gambling and remarkable devotion to horse racing" (p. 158). On the other hand Baptist opposition was most forthright. They were "totally opposed to any form of gambling" (p. 186). While Catholics organised and ran fund-raising raffles, Baptists faced possible loss of church membership if caught buying a raffle ticket.

O'Hara (1988) reported that in their opposition to gambling, the churches appealed to the middle classes:

Successful gaming or betting by the working classes had the potential to undermine the middle class ethos, and unsuccessful gaming and betting would lead to greater impoverishment, thereby endangering not only the individual losers and their families, but also the social order. The role of the Protestant churches in defending and reinforcing the urban and middle class value system, and promoting a separate middle class consciousness, was most important. (p. 91)

Although such views were dominant in respectable sectors of society, it was unlikely that chance and data were considered as a part of the mathematics curriculum. Not surprisingly, with the reduced influence of such opponents, the study of gambling has come to be included in the study of chance events within school mathematics programs.
Categories of Gamblers

Gamblers within society have been categorised by psychologists according to such characteristics as frequency of gambling, motivation for gambling, and amount of money wagered. Dickenson (1985, p. 139) describes four distinct categories:

1. Professional gamblers.
2. Compulsive or pathological gamblers.
3. Regular gamblers.
4. Social gamblers.

Professional Gamblers

The identifying characteristic of the professional gambler is that of motivation. The professional gambler is motivated by an expectation of winning, and relies on gambling as a major source of income.

Peard (1990), in an examination of the operation of professional gamblers (including syndicates), observed that they operate from two mathematical axioms:

1. A positive mathematical expectation of return, that is to say, a belief that the odds are in their favour, and thus, that in the long run, they will come out ahead; and
2. The ability to sustain short term losses. (p. 15)

It is not expected that subjects in this study will come from family backgrounds belonging to the category of professional gambler. Numbers in this category are thought to be relatively low, though, for obvious reasons, little data on the number of professional gamblers in the population, or the incomes of these gamblers are available.

Compulsive or Pathological Gamblers

These also belong to a category beyond the scope of this research. This type of gambler has no long term expectation of winning, and invariably experiences long term losses which must be financed from other sources of income, often illegal. The study of the compulsive gambler is in the domain of psychology rather than education, and this type of gambler is not considered in the present study.

Regular Gamblers

Gamblers in this category are described by Dickenson (1985) as "those betting three or more times per week" (p. 140), although he does not qualify this with any reference to the amounts bet on these occasions.
Social Gamblers

Those in this category are described by Dickenson as those who gamble no more than two times per week. He has noted also that in terms of numbers the vast majority of gamblers fall into this last category. The motivation for gambling by the social gambler differs from the other categories in that an element of social enjoyment is often present. Since the majority of gamblers in Australian society belong to the category of social gambler, it would be reasonable to expect that the gamblers in this study will come from families who fall into this category.

The extent of gambling. Solonsch (1993) estimates that of all Australians over the age of 18, only 15% do not gamble at some time or in some form. He classifies gamblers as either "hard" or "soft", and estimates that there are more than two million "hard" gamblers in Australia. These are regular bettors on racing, poker machines or card games.

Motivation for gambling. Walker (1985) observed that the motivation and reasons for gambling by occasional gamblers are "not homogeneous across the different forms of gambling" (p. 149). He identified two distinct motivational patterns:

1. Those motivated by the hope of winning a large sum of money such as on Lotto and Pools; and
2. Those motivated by social reasons such as interest, excitement and enjoyment. These include track betting and betting on football and other events. (p. 149)

The present study will be confined to those motivated by the latter of these patterns. Motivation for gambling will be included in the data gathered and an attempt will be made to relate this to the cognitive processes used in processing of the probabilistic concepts involved.

Summary

Gambling is an important social activity for a large segment of Australian society. This has been clearly shown by the statistics relating to the expenditure on gambling throughout the country, and by the information presented illustrating the social acceptance of gambling within the society. Of the various forms of gambling, track betting is the dominant in consideration of monies expended, social acceptance, and familiarity. The activities of track gambling involve mathematical concepts and procedures, and this mathematics represents a culturally based form of knowledge that may be reasonably expected to be held by a segment of the population. This study will examine the nature of this knowledge.
CHAPTER 4

Historical Factors in the Development of Probabilistic Reasoning

The historical and philosophical setting for the development of probability provides an essential backdrop for an overview of research in stochastics learning (Shaughnessy, 1992, p. 467). An examination of aspects of the historical development of probability theory and probabilistic reasoning will also be useful in order to understand why it is commonly reported that the teaching and learning of the topic and its associated concepts is difficult. Epstein (1977) commented:

Throughout the entire history of man preceding the Renaissance, all efforts towards explaining the phenomena of chance were characterised by comprehensive ignorance of the nature of probability. Yet gambling flourished in various forms almost continuously from the time Palaeolithic man cast his polished knuckle bones and painted pebbles. Lack of knowledge has rarely inhibited anyone from taking a chance. (p. 92)

According to Epstein, the first reasoned considerations relating to chance came in the sixteenth century. Cardano (1501-1576) is credited with the first attempt to organise the concepts of chance into a cohesive discipline. Before Cardano, the connection between gambling and mathematics was not overtly realised, despite the prevalence of gambling in many societies for many centuries prior to this.

It is known that dice games were played for gaming purposes by the Egyptians as early as 2000 BC, and it is interesting to note that there is evidence that biased dice were in operation implying, at the very least, the existence of some intuitive notions of chance (Ashmore, Dodge & Kasch, 1962). Card games have been played by the Chinese from AD 1000 onwards, and possibly earlier.

Nevertheless a mathematical theory of probability did not emerge until the seventeenth century in Europe. This theory developed from the need of gamblers to quantify chance occurrences. Epstein (1977) noted:

Gamblers can rightfully claim to be the godfathers of probability theory, since they are responsible for provoking the stimulating interplay of gambling and mathematics that provided the impetus for the study of probability. For several centuries, games of chance constituted the only concrete field of applications of probabilistic methods and concepts. (p. 10)
Fermat and Pascal are together credited with being the first to place the theory of probability within a mathematical framework. Pascal's correspondence with Fermat in 1654 concerning a problem posed by the gamester, the Chevalier de Mere, is often cited as the first truly mathematical analysis of a game of chance. Epstein (1977) has described this event as "the most significant event in the history of the theory of gambling" (p. 3). Eves (1969) claimed:

It is generally agreed that the one problem to which can be credited the origin of the science of probability is the so-called problem of the points. This problem requires the determination of the division of the stakes of an interrupted game of chance between two supposedly equally-skilled players, knowing the scores of the players at the time of the interruption and the number of points required to win the game...the problem was discussed by Cardano and Tartaglia...but a real advance was not made until the problem was proposed, in 1654, to Pascal, by the Chevalier de Mere...in their correspondence Pascal and Fermat laid the foundations of the theory of probability. (pp. 262-263)

This relatively late start to the development of a calculus of probability has caused speculation by historians of mathematical development. David (1969) has noted that "the real problem which confronts the historian of the calculus of probability is its extremely tardy conceptual growth...its late birth as an offspring of the mathematical sciences" (p. 21). Shaughnessy (1992) claimed that "the field of probability and statistics is barely a mathematical adolescent when compared to geometry or algebra" (p. 468). Kendall (1970) suggested that there are four possible factors that have contributed to this slow growth:

(a) the absence of a combinatorial algebra (or at any rate combinatorial ideas)
(b) the superstition of gamblers
(c) the absence of a notion of chance events
(d) moral or religious barriers to the idea of randomness and chance. (p. 30)
Of these factors, Kendall (1970) claimed the last is the most significant:

If we discount such factors as ill-made dice, indifferent mathematical expertise, superstition and so forth ... then we seem driven to the conclusion that the late emergence of the probability calculus was due to some more fundamental factor. The very notion of chance itself, the idea of natural law, the possibility that a proposition may be true and false in fixed relative proportions, all such concepts are now so much part of our common routines of thought that perhaps we forget that they were not so to our ancestors. It is in basic attitudes towards the phenomenal world, in religious and moral teachings and barriers, that I incline to seek for an explanation of the delay. (p. 30)

David (1969) suggested that it is possible that early civilisations such as the Egyptians and Greeks did not develop the mathematics of dice throwing because of "their inability to link up theoretical concepts with empirical facts" (p. 36). Kendall (1970) gave support to this noting that the religious barriers were not as strong in these societies. Although the gods and goddesses of these societies had influence over the course of events, including dice throwing, they were only beings with superhuman powers, not omnipotent entities. However, the situation was radically changed by the advent of Christianity, and the belief that "nothing happens without cause .... nothing was random and there was no chance" (p. 31). This view prevailed and hindered the development of a theory of probability despite the noticeable advances in other branches of mathematics.

As Kendall (1970) noted "humanity as a whole has not yet accustomed itself to the idea [of randomness]" (p. 32). The difficulty in making the connection between mathematics and chance can be readily appreciated when one considers that mathematicians as eminent as Leibniz (1646-1716) incorrectly concluded that "the sums of 11 and 12 cast with two dice have equal probabilities" (Epstein, 1977, p. 4). Kapadia (1984) reported that:

Famous mathematicians, such as d' Alembert, erroneously assigned a probability of one-third (to the probability of getting a head and a tail on the toss of two coins), presumably using an incorrect equally-likely sample space of three possibilities instead of four. (p. 46)

Gardner (1989) reported that d' Alembert also mistakenly thought that the outcomes of tossing a coin three times were different to those obtained in a single toss of three coins simultaneously. Kapadia (1984) also commented on another famous mistake that occurred when
the Chavalier de Mere calculated that the chance of getting a six in four throws of a single die was the same as that of getting a double-six in 24 \((6 \times 4)\) throws of two dice. (p. 46)

In addition to the lack of a mathematical framework on which to base a theory of probability, other notions inhibited understanding in the field. For example, Epstein (1977) has cited notions of "hidden cause, determinism, miracles and the like that have inhibited the development of the connection between mathematics and chance" (p. 5).

Determinism

Of the factors that have inhibited the development of a mathematical theory of probability, the notion of determinism would appear to have been be a major obstacle. Indeed, to the present time it continues to inhibit the development of an understanding of randomness and chance events. Green (1983a) reported that the issue of determinism was raised when subjects in his study thought that an event was certain when, in fact, it was only highly probable. Shaughnessy (1983) reported that several studies have suggested that "people's misconceptions of probability may be due to ... an overemphasis upon deterministic reasoning in our schools and in society" (p. 332).

Shaughnessy also cited research by Falk (1981) which suggests that what people tend to view as "miracles" and "coincidences" are, in fact, much more likely to occur than people think. Paulos (1989) also made the same point when he noted that people tend to view relatively common coincidences as extraordinary.

The shift in ideology from determinism to probabilism has been forwarded in a recent report on mathematics curricula appropriate for the future (Australian Education Council, 1991). In *A National Statement for Mathematics in Australian Schools* (1991, p. 8) it is noted that there is an increasing emphasis on random models of the world rather than on an event's inevitable consequences that can be described by rules and equations. Steen (1990) noted that it is important to know that many events are the result of chance variation rather than deterministic causation so that it not always necessary to look for specific, often spurious reasons to explain an occurrence.

Despite this shift from determinism, notions such as astrological horoscopes and superstitions are common in everyday life and conflict with the ideas mathematical probability. Thus, it would appear that determinism has not only impeded the development of probability theory, but also continues to be an obstacle in its teaching and learning. This is in addition to other pedagogical impediments associated with related mathematical concepts such as those found in proportional reasoning and fraction concepts.
However, Hacking (1975, cited in Shaughnessy, 1992) rejected both the preoccupation with determinism and the influence of religious doctrine as being able to explain, in themselves, the historically slow emergence of mathematical probability. Hacking claimed that "this slow emergence was primarily due to the dual meaning that has been historically attached to the word probability ... to indicate 'degree of belief' and 'calculations of stable frequencies for random events" (p. 468).

Shaughnessy (1992) observed that "How we obtain stochastical knowledge, or even what it means to know something in a stochastical setting, are potential stumbling stones for researchers" (p. 467). He noted that the process of conducting research into what it means to know that something is true, has been influenced by the 17th-century epistemological schism between empiricism and rationalism. According to Shaughnessy (1992) there are still traces of this in the present and recent research on learning probability. He has argued:

In addition to the philosophical influences on the epistemology of stochastics, there are some historical developments in the probability concept itself, and in the notion of what constitutes "acceptable evidence for truth" that have influenced research in stochastics. (p. 468)

Hacking (1975, cited in Shaughnessy, 1992) noted that at the time when mathematical probability theory appeared in the mid-17th and early 18th centuries, the term probability still held both statistical (random events) and empirical (degree of belief) connotations. This led to a duality between scientia (knowledge) and opinio (belief). The "high sciences" such as mathematics and mechanics, sought absolute truths, while the "low sciences", such as medicine and astrology rendered opinions based on empirical evidence. It was held by such scientists as Galileo and Bacon that absolute truth could never be inferred from experimental evidence. Thus probability was considered as belonging to the low science of opinion. Prior to the 17th century, probability was considered a matter of "approval" rather than mathematics (p. 468). Shaughnessy (1992) wryly observed that "Unfortunately, the tradition of that notion of probability is still with us" (p. 469). In light of these historical considerations, it is not surprising that, as Garfield and Ahlgren (1988) have concluded, "the teaching of probability is fraught with difficulties" (p. 57). Konold, Pollatsek, Well, Lohmeir and Lipson (1993) have noted that "probability is notoriously difficult to teach" (p. 413). They recognise that it is important for teachers of probability to become familiar with the alternate conceptions of probability that students possess. The present study will examine such conceptions in the context of gambling.
Summary

In conclusion, we see that from a historical point of view, probability is a relatively recent development in mathematics, and the study of how people acquire probabilistic concepts is a very recent area of research in mathematics education. A number of factors have contributed to this tardy development. These include epistemological considerations which have resulted in difficulties in understanding concepts that persist to the present time. The acquisition of probabilistic concepts by gamblers led to the idea of experimental evidence gaining respectability through the work of respected mathematicians such as Pascall, Fermat and Huygens, and its final development as a branch of mathematics. The importance of probability in mathematics today has resulted in its inclusion in the modern school curriculum. This is examined further in the next chapter, the Review of the Literature.
CHAPTER 5

Review of the Literature

The research literature reviewed in this chapter is drawn from several different areas of mathematics education. Foremost among these are the reports on research concerned with the teaching and learning of probabilistic reasoning and related mathematical skills and concepts such as proportional reasoning and fraction concepts. However, this research is also concerned with ethnomathematics and cultural factors in mathematics education. In addition, literature relating to constructivism in the learning of mathematical concepts and the techniques and strategies of computation and estimation is reviewed.

Research Relating to Ethnomathematics

Definition

In Chapter 1 D'Ambrosio's (1985a) definition of the term "ethnomathematics" was given:

The mathematics which is practised by identifiable cultural groups, such as national tribal societies, labour groups, children of a certain age bracket, professional classes and so on. (p. 45)

In the present study the term will be taken to mean the inherent mathematical ideas that arise naturally out of cultural practices and norms and, in particular, the probabilistic skills and concepts that arise from the cultural practices of a segment of the school population whose social background includes gambling. These students might reasonably be expected to bring these skills and concepts with them to the school environment.

Australian society should be viewed as a complex mixture of various subcultures with vastly different social values and life styles. Any "ethnomathematical" concepts will, of necessity, be confined to the subgroup identified. Borba (1992) sees ethnomathematics as:

A field of knowledge intrinsically linked to a cultural group and its interest, being in this way tightly linked to its reality ... and being expressed by a language, usually different from the ones used by mathematics. (p. 134)
The members of one of the two cultural subgroups of this study are identified through
a common social interest in the field of track betting. This common interest includes
betting on horse racing, dog racing, and trotting events both on and off track. The term
"ethnomathematics" is used in the context of the definition given by D' Ambrosio
(1985a) who argued that its identity
depends largely on focuses of interest, on motivation, and on certain
codes and jargons which do not belong to the realm of academic
mathematics. (p. 45)

Nunes (1992, p. 557) observed that D'Ambrosio's use of the term ethnomathematics,
refers to forms of mathematics that vary as a consequence of their being embedded in
cultural activities whose purposes are other than the doing of the mathematics. This
working definition is in keeping with that employed by other researchers in the field.

Zepp (1989), in noting that the term is difficult to define precisely, suggested that
it is useful to consider what the term is not. He suggested that:

It is not a collection of interesting folk games, measuring techniques,
or counting systems used by various "primitive" cultures ... nor is it a
doctrine which states that differing races have differing mathematical
abilities. Rather, the term refers to ... the ideology that mathematics
education should be relevant to the goals and aspirations of students,
whatever they may be. (p. 211)

Graham (1988), in researching the ethnomathematics of some groups of Aboriginal
children in Australia, used the term to refer to "the mathematical understanding that
the Aboriginal children bring to the educational encounter ... the mathematical
relationships inherent in their own culture" (p. 121). Gerdes (1988) used the same
definition to describe the intuitive mathematics of a native culture in a post colonial
society. Borba (1992) identified examples of sociocultural groups including the
different people studied in Africa by Gerdes (1988), by Ferreira (1990) and Borba

Lampert (1986) and Leinhardt (1988), in separate studies, focused on what
students know before instruction as a result of interaction with their social background
in an endeavour to determine what "understanding mathematics" means to students
who are being taught new aspects of mathematics. Carraher, Carraher and Schliemann
(1985) in Brazil, used the term "ethnomathematics" to refer to "the everyday use of
mathematics by working youngsters in commercial transactions" (p. 21). Carraher
(1988a) subsequently reported:
Many invariants of mathematical concepts taught in school appear to be quite basic and necessary for solving problems in everyday life. These invariants can be understood outside school, without the benefit of teaching, through the understanding of problem situations. (p. 17)

It has been argued in Chapter 3 that track gambling is an identifiable cultural practice within the Australian social context. The pilot study, which will be described in Chapter 7, confirmed that at the two schools selected the interest in track gambling was widespread among the student population.

The term "ethnomathematics" is used in this study with some flexibility. This is entirely appropriate since, as Bishop (1988c) has said "the term ethnomathematics itself is not well defined" (p. 180).

**Intuitive Knowledge of Mathematics Developed Outside Schools**

More generally, intuitive knowledge of mathematics developed outside schools has been well documented among children (Abraham & Bibby, 1988; Carraher, 1989; Carraher, 1986; Carraher et al., 1985, 1987; Carraher & Schliemann, 1985; Ginsburg, 1977), as well as among adults (Gay & Cole, 1967; Gerdes, 1988; Graham, 1988; Lave, 1988; Schliemann, 1984, 1988; Scribner, 1984).

Carraher et al. (1985) analysed the everyday use of mathematics by school children in commercial transactions out of school, and found computational strategies that were different, and more effective, than those taught in schools. They concluded that children who knew how to solve arithmetical problems that they encountered in daily life (at the market place, for example) before they went to school, could not solve the same problems later in school, using traditional class taught algorithms.

Schliemann and Carraher (1988) showed that fishermen who had never been taught about proportions at school but who dealt with proportions at work were able to invent appropriate procedures to solve proportional problems, thereby demonstrating an understanding of the logic mathematical relations involved in their everyday computations in the domain of multiplicative structures.

Acioly and Schliemann (1986) analysed the relative contributions of practical experience and school experience of 20 adult bookmakers on the development of mathematical knowledge in the process of determining the bet values in a popular Brazilian lottery game. They analysed the bookmakers' knowledge of mathematics in the work setting and on problems that differed from those encountered in the workplace.

In the present study, students will be interviewed for the purposes of gathering data pertaining to how algorithms are constructed and employed by Year 11 high school students in the context of track betting and in contexts outside of gambling. The
interview questions are also designed to examine the students' ability to employ traditional classroom algorithms in situations requiring the calculation of proportion in both gambling and traditional classroom contexts.

Schliemann and Magalhaes (1990) investigated the understanding of proportional relations by illiterate subjects in the context of commercial transactions. After investigating the type of mathematical thinking used by Brazilian domestic cooks, they concluded that their results supported the suggestion that "to ensure understanding and transfer of mathematical models, educators should provide students with experience to familiarise them with the problem context to which the models are relevant" (p. 71).

In the formulation of research questions for the present study, the conclusions from research investigations such as those cited above were borne in mind. In particular, Schliemann's (1984) conclusion that instruction which was not specifically related to practice was not used in solving problems was especially relevant as was the conclusion of Carraher (1988b) that learning mathematics outside of school does not always lead to correct responses to similar mathematics in schools, even when the content of the problems is familiar.

The present study will compare the ways in which probabilistic ideas are processed by individuals who are familiar with the out-of-school gambling contexts and will compare these with the responses of students who are unfamiliar with such contexts. As Acioly and Schliemann (1986) have concluded:

> Although mathematical knowledge may develop outside of school, training and use of this knowledge in new situations and an understanding of the relevant mathematical relations embedded in problem-solving rules seems to benefit from school experience ... Practice without schooling generates knowledge that is limited to the situations in which it originated. (p. 226)

With regard to intuitive mathematical knowledge developed outside of school, Carraher and Schliemann (1988) after analysing several sources of data, arrived at four main conclusions:
1. Reasoning principles underlying written and oral mathematics appear to be the same.
2. There are diverse ways of understanding and using mathematical concepts which depend upon the cultural conditions under which mathematics is practiced.
3. Schools transmit culturally perfectioned mathematical tools (such as numeration systems, algorithms and formulas) and tend to make students good model-users at the expense of meaning.
4. Mathematics learning in daily life produces meaningful procedures which may be of restricted applicability. (p. 11)

The present study examines the reasoning principles underlying both written and oral mathematics of probability, the cultural conditions in which this mathematics is practised and, in particular, whether these daily life procedures are restricted to application within that situation or whether they transfer to traditional classroom situations.

Borba (1990) claims that the "ethnomathematics developed by different groups are likely to be more efficient at solving problems related to their cultures than academic mathematics" (p. 40). After arguing that ethnomathematics and traditional education are really compatible, Borba went on to say:

Ethnomathematics is developed by the cultural groups' interest ... which is natural because it was generated by the members of the cultural group in response to their own situation. (p. 41)

Borba (1992) has also commented that the "very power of ethnomathematics and of the work done in the area (of ethnomathematics) challenges the notion that mathematics is only produced by mathematicians" (p. 135).

Social and Cultural Considerations

There is a need to distinguish between "social" and "cultural" factors as they relate to the present study. Gambling has been identified as a cultural phenomenon within the Australian society. Bishop (1990) says that until about ten years ago mathematics was considered to be culture free and that it "has somehow always been felt to be universal ... and for most people it continues to have today, the status of a culturally neutral phenomenon" (p. 51). In addition to the effect of gambling within Australian culture, there are other social factors in the learning of mathematics that will have bearing on this study. In a review of research of social dimensions of mathematical education Bishop and Nickson (1983) identified five levels. These were:
1. Cultural. In this study the influence of gambling will be such a factor.

2. Societal. There are many such factors identified in the learning of mathematics and in this study these will include the social interactions of the class.

3. Institutional. Two distinctly different institutions are involved in this study. The Seventh day Adventist School and the metropolitan high schools.

4. Pedagogical. Different schools and even different classes within one school may employ different pedagogies.

5. Individual. Although individuals in this study will be selected from the identifiable cultural group, they will display differences in the areas of achievement and other interests.

Since the present study will examine how individuals construct and process probabilistic ideas, the results will have pedagogical implications for the teaching of probability within the secondary school system.

The idea that mathematics is a culture-free body of knowledge somehow independent of human thought and physical existence, has been questioned for some time now. Writers such as Wittgenstein (1958) and Lakatos (1976) questioned the notion of universality, and over the last ten years there has been a growing acceptance that mathematics is a social construct. Bishop (1988b) is adamant and unequivocal that "mathematics is not a value free phenomenon" (p. 118). Bishop (1988c) further maintained that:

Mathematics must now be understood as a kind of cultural knowledge which all cultures generate but which need not necessarily look the same from one cultural group to another. (p. 180)

Bishop (1988c) has argued persuasively that mathematics is not universal and culture free, rather it is a pan-human phenomenon, and that as "each cultural group generates its own language, religious beliefs, etc., so it seems that each cultural group is capable of generating its own mathematics" (p.180). Nevertheless, Bishop (1990) commented that for most people it "continues to have today the status of a culturally neutral phenomenon" (p. 52).

Bishop's ideas however continue to generate debate. Many leading contemporary philosophers do not accept the social constructivist view of knowledge. Stigler (1989) in reviewing Bishop's book *Mathematical Enculturation*, claimed that Bishop's discussion of cultural values of mathematics is "weathered by its lack of empirical support" (p. 368).

Among the implications Stigler (1989) drew from Bishop's work regarding a cultural approach to mathematics education was the idea that the approaches to
teaching mathematics may result from a discovery learning strategy, a functional problem-solving strategy or "the definitital implication." This last term was explained:

Mathematics itself consists not only of symbolic technologies but also of other cultural products - such as values, beliefs and attitudes - that arose from the same universal activities as did mathematics itself. To truly teach mathematics means that these other cultural products must be taught as well ... Whereas the first two implications are often propounded, they are not often supported by the necessary empirical research, and thus they are easily dismissed by the sceptical critic. To argue, however, that mathematics consists of more than symbolic technology, and that we must therefore teach more than symbolic technology, is a judgement that can be made quite apart from theory and data related to children's learning processes - and one that makes a lot of sense. (p. 369)

Nevertheless, value-laden topics within mathematics have been researched in a number of contexts. Swadner and Soedjadi (1988) in Indonesia, for example, examined the social values inherent in a number of mathematical topics and discussed how the study of each of these may be used to promote social values that are consistent with national goals.

The importance of the cultural context in mathematics education. Abraham and Bibby (1988) noted that this has formed a central theme of their research. They claimed that a common element of projects in ethnomathematics and self-generated mathematics is that "the legitimation of the learners' experiences is recognised as being of fundamental pedagogical importance" (p. 3). D'Ambrosio (1986, cited in Nunes, 1992) researched the mathematical knowledge involved in boat construction by an Amazonian Indian community. He claimed that the activity was clearly mathematical in nature and involved the transmission of knowledge from one generation to another. Various other studies have researched mathematics used in the workplace (Acioly & Schliemann, 1989; Carraher, 1985; Cockcroft, 1982; Lave, 1988; Scribnor, 1984).

Nunes (1992) identified two distinct approaches to the study of cultural influences on mathematical knowledge. The first of these is the view held by Bishop (1988a, 1990), and Stigler and Baranes (1988), that mathematics is not a universal, formal domain of knowledge. According to Stigler and Baranes (1988):
As children develop, they incorporate representations and procedures into their cognitive systems, a process that occurs within the context of socially constructed activities. Mathematical skills that the child learns in school are not logically constructed on the basis of abstract cognitive structures, but rather are forged out of a combination of previously acquired (or inherited) knowledge and skills, and new cultural input. (p. 258)

Nunes (1992, p. 558) pointed out that this view stresses differences rather than similarities across cultures. She reported that the second perspective is that illustrated by D'Ambrosio (1986) which suggested that the analysis of cultural influences on mathematical knowledge can demonstrate both difference and invariance in mathematical knowledge across cultures. In this view mathematical reality is representing reality in such a way that more knowledge about the represented reality can be generated through inferences using mental representations. Invariant logical structures are embedded in mathematical knowledge, regardless of whether such knowledge is developed in or out of school.

Denys (1992) in reporting on the working group on Cultural Aspects in Learning Mathematics at the International Group for the Psychology of Mathematics Education's 16th annual conference in Durham, New Hampshire noted that the group had identified and were working on several types of studies related to the cultural field. These included:

1. Informal education and formal mathematical knowledge.
2. The effects of language and cultural environment on the mental representations of students.
3. Cognitive processes in learning mathematics. (p. 7)

Ernest (1989) after commenting that it has only been in recent years that it has been recognised that mathematical knowledge has been constructed as a result of social activity, went on to say:

Once it is admitted that mathematics is a living social construct, then the aims of teaching mathematics need to include the empowerment of the learners to create their own mathematical knowledge; mathematics can be reshaped, at least in schools, to give all groups more access to its concepts, and to the wealth and power its knowledge brings, the social contexts of the uses and practices of mathematics can no longer legitimately be pushed aside, the uses and implicit values need to be squarely faced. (p. 177).

The increasing acceptance of this viewpoint has both influenced and been influenced by curriculum reforms of the last decade. Driving these reforms has been the rapidly
changing technology which has influenced not only what should be taught but also how it can be taught. As Graham (1988) has observed "We are only now beginning to appreciate the significance of what children know when they come to school and how they learn and think about what they know" (p. 121).

Wilder (1981) proposed a philosophy that views all of mathematics as a cultural system. He asserted that to conceive of mathematics as a cultural system offers a way of explaining anomalies that have not previously been explained by philosophical or psychological means (p. vii). Wilder said that, according to this conception

> We consider mathematics as a subfield of our general culture, and instead of representing it as a tree, think of it as a system of vectors ... in which each vector is striving for growth and in which the different vectors impinge on one another, sometimes resulting in new consolidations that become vectors in their own right. (p. 16)

Interestingly, Smorynski (1988, p. 9) said that Wilder's view "is the first mature philosophy of mathematics."

**Intuitive mathematical knowledge.** Fischbein (1975, p. 5) defined this as "a form of immediate cognition in which the justifying elements, if any, are implicit." The acquisition of such knowledge as a result of interaction with social background has been researched by Cobb (1989); Fischbein (1975); Ginsburg (1977); Leinhardt (1988); Noddings (1985); and Resnick (1986). All of these researchers considered that the distinguishing feature of intuitive knowledge is that it is not derived from instruction but develops from interaction with real world situations. Bishop (1988a) argued that the this interaction leads to the legitimisation for the learners of their intuitive mathematical knowledge.

Barton (1992) asked "What is the process by which ethnomathematical knowledge becomes legitimised as mathematics?" (p. 2). He then attempted to establish a philosophical basis for ethnomathematics by explaining how it develops. In order to do this, he examined the history of navigation and the mathematics used in navigational systems. Barton described ethnomathematics as "the study of cultural relativity in mathematics" (p. 1). As such, it includes mathematical concepts, systems, modes of thinking, and metamathematics, e.g. what counts as proof, beliefs about how mathematics relates to the world, and values implicit in mathematics. (p. 1)
The present research will determine the intuitive knowledge of a segment of the population in order to determine whether it constitutes a form of ethnomathematics as described in other research reported in the literature.

**Research Relating to Constructivism in Mathematics Education**

According to constructivists such as Herscovics and Bergeron (1985), mathematical knowledge is the product of active, goal directed individuals consciously manipulating and reconstructing their own world. Romberg and Carpenter (1986) support this view and give a detailed description of the processes involved in this reconstruction (p. 853). von Glasersfeld (1991) commented that:

> The notion that knowledge is a result of the learner’s activity rather than of passive reception of information or instruction, goes back to Socrates and is today embraced by all who call themselves "constructivists". (p. xvi)

Constructivists believe that knowledge cannot be transferred ready made from one person to another, but must be built by the cognising subject from his or her own personal experiences. Blais (1988) maintained that all knowledge must be constructed and he made the distinction between knowledge which must be constructed and information which can be simply transmitted:

> Constructivism does not say that knowledge is something that learners *ought* to construct for and by themselves. Rather it says that knowledge is something that learners *must* construct for themselves (p. 627)

von Glasersfeld (1992, p. 444) claimed that the constructivists' view of knowing provides an explicit epistemological basis for the presupposition that students make sense of their experiences and that the sense they make must be understood and respected before any attempt to modify it will pay off. Ernest (1989) observed:

> In the last few years a new theoretical perspective has greatly illuminated the processes by which children learn mathematics. This is constructivism: the view that children construct their own knowledge of mathematics over a period of time in their own, unique ways, building on their pre-existing knowledge. (p. 151)

The research questions of this study will be concerned with the identification of this pre-existing knowledge in the field of probability and how this knowledge is processed.
processed and modified in the construction of new mathematical knowledge. Popkewitz (1988) maintained:

Mathematics cannot be treated solely as a logical construction or a matter of psychological interpretation. What is defined as school mathematics is shaped and fashioned by social and historical considerations that have little to do with the meaning of mathematics as a discipline of knowledge. (p. 221)

In a similar vein, Geiger (1990) said that in constructivism "every learner constructs for themselves a model of reality" (p. 8). Geiger added that this model of learning is based to some extent on external stimuli, such as what the pupils already know.

While the importance of constructivist ideas in mathematics has been recognised for some time, constructivism in mathematics education is a more recent development. Leder (1989), an Australian mathematics educator, referred to "the growing adoption by contemporary mathematics educators of constructivist perspectives" (p. 2), and Higginson (1989), a Canadian mathematics educator, referred to constructivism as "a conception of knowing and learning with its emphasis on the active involvement of the learner" (p. 11).

The notion that mathematicians create or construct mathematics is not new. Kroniker's statement of 1892 that "the integers were made by God, all else is the work of man" has been the source of much philosophical discussion in publications such as Kline's (1989) *Mathematics - The Loss of Certainty*. Snapper (1988), an eminent mathematician, maintains "mathematicians create mathematical structures in their minds and study those properties of these structures about which they can communicate" (p. 57). However, we need to distinguish between the construction of mathematics by mathematicians (constructivism in mathematics) and the construction of mathematics by learners (constructivism in mathematics education). The latter of these is not universally accepted. Klein (1992) observed:

Recently ... there has come to prominence a theory of teaching and learning which promises to shake the very foundations of teaching and forces a choice: "constructivism" or not ... it necessitates a reappraisal of what mathematical knowledge is, or "should" be, and what it means "to learn" (p. 35)

Noddings (1991) in discussing the strengths and weakness of a constructivist approach to mathematics education noted that "it is the centre of considerable controversy" (p. 7). Wheeler (1987) warned that "we must subject all references to constructivism (in mathematics education) to critical scrutiny" (p. 55). Fillerton and
Nevertheless, many mathematics educators today consider themselves constructivists. Ernest (1991) claims that constructivism has become one of the main philosophies of mathematics education research and concludes that, despite the problems, "constructivism remains one of the most fruitful philosophies of mathematics education research today" (p. 32).

Cobb (1988) explored constructivism, which he viewed as a philosophical and psychological position with regard to how learners process information. He addressed the contradiction between the assumptions that the goal of mathematics education is to transmit knowledge to students and the view that students construct mathematical knowledge by reorganising their cognitive structure. Although clearly supporting the constructivists' position he admitted that "further difficulties must be acknowledged before a serious attempt is made to implement constructivist mathematics instruction" (p. 100). These difficulties stem from the fact that this form of teaching requires far more of the teacher, who, according to Cobb (1988)

should have a deep relational understanding of the subject matter and be knowledgeable about possible courses of conceptual development in specific areas of mathematics. In addition the teacher should continually look for indications that students might have constructed unanticipated, alternative meanings. (pp. 99-100)

Ellerton and Clements (1992), summarised the strengths and weaknesses of constructivism in mathematics education. First, they saw its main strength is its emphasis on the need for learners to construct their own mathematical meanings and thereby develop a sense of ownership of mathematics by the learner. They noted that when someone actively links aspects of his or her physical and social environment with certain (mathematical) concepts "a feeling of 'ownership' is often generated" (p. 4).

A second strength that they described is its recognition and advocacy of quality social interaction as a basis for quality mathematics learning. Pateman and Johnson (1990, cited in Ellerton & Clements. 1992, p. 5) have claimed that the constructivist approach in mathematics education has largely been responsible for the recent move towards learning environments that nurture interest and understanding through cooperation and high quality social interaction.

A third strength is its identification, clarification and advocacy of principles for improving mathematics teaching and learning. Ellerton and Clements (1992) went on to outline what they saw as three weaknesses of constructivism. First, there was the "constructivist band wagon" or "the missionary zeal of some radical constructivists who tend to accuse mathematics educators outside their ranks of advocating and practising (only) transmission modes of education" (p. 1). Second, the downplaying of
the role of linguistic activity in the development of abstract thought is a weakness. Third, they noted as a weakness the tendency of radical constructivists to provide oversimplified answers to the question of what constitutes mathematical knowledge. Cobb, Yackel, and Wood (1992) claimed that all mathematical knowledge is constructed.

**Ethnomathematics, Constructivism, and Mathematics Education**

The decision to examine the relationship between ethnomathematics and constructivism in this study was motivated by the constructivist perspective that learners construct or invent knowledge on the basis of what they already know, and that much of what they already know has developed from cognitive interaction with factors in their social background. Other researchers, for example, Carpenter and Moser (1983) and Fuson (1988), have clearly shown that learners do invent useful strategies to solve novel problems. Constructivism acknowledges the relativistic nature of the constructions and recognises that constructed concepts are valued for how they can be used to deal with problems. Cobb (1988) noted that when some concepts become well accepted, it is necessary to act as if they exist in some ideal form because the alternative is too demanding.

The present research will examine how the gamblers construct concepts in probabilistic reasoning and how they apply these to solve problems in probability and related mathematical procedures.

It is recognised that such constructs will not necessarily be "correct" mathematics since research has shown that children can develop "buggy" algorithms. Brown and VanLehn (1980) defined bugs as "systematic calculational errors ... due to using incorrect or partially correct procedures" (p. 161). However even these bugs can be considered to be "invented or constructed from the students' existing knowledge" (Resnick, 1982, p. 143).

Studies researching mathematics used in the workplace (Carraher et al., 1985, 1987; Cockeroff, 1982; Lave, 1988; Schliemann, 1986; Scribner, 1984) have shown that such mathematics is often idiosyncratic. Cockeroff (1982) referred to "back of envelope" methods as opposed to formal algorithmic methods taught in school. Scribner (1984) discovered that dairy workers in filling orders were often required to perform lengthy computations. She found that they had invented context-specific methods that relied on contexts specific knowledge. For example, they knew that if one layer held 16 containers, to fill an order for 35 containers they filled two layers and placed three more on top. A similar procedure of decomposition (taking numbers apart) and recomposition (putting them back together again) was used to calculate costs.
The present research will examine context-specific procedures in probabilistic situations used by the gamblers and attempt to identify the context-specific intuitive knowledge required for such computations.

**Overview: Research Relating to Probabilistic Reasoning**

The review of the literature reveals that relatively little research into the learning of probability at the secondary school level has been done. Bell, Costello and Kucheman (1983) reported a number of research studies of primary pupils' knowledge of elementary probability, but cited no studies of secondary pupils. This may be due to the complexities of the concepts as previously noted, together with its relatively recent inclusion in the school curriculum.

Shaughnessy (1992) observed that it is not at all surprising that "there has not been much involvement by North American mathematics educators in research on the teaching and learning of probability" (p. 456). One other factor affecting this lack of research in this field of mathematics education is that as has been noted, until recently the topic has been largely considered the domain of psychology. Scholz (1991) provides an historical overview of the psychological research on the acquisition of probabilistic concepts by a wide segment of the population. However most of this research involves samples of adults and tertiary students. In particular, with reference to gambling, only studies of adult gamblers by psychologists are reported (Ceci & Leiker, 1986a, 1986b; Kahneman & Tversky, 1972, 1973, 1982a, 1982b; and Kahneman, Scholz, 1983; Slovic & Tversky, 1982). Watson (1992) in a review of research in probabilistic reasoning in Australia, reported that little research in stochastic reasoning has been carried out.

**Identification of Specific Questions from the Literature**

Garfield and Ahlgren (1988), after reviewing the literature on research on the learning of probability and statistics at the secondary level and above, stated:

The literature makes it clear that far more research has been done on the psychology of probability than on other statistics concepts. In spite of this research, however, teaching a conceptual grasp of probability still appears to be a very difficult task, fraught with ambiguity and illusion. (p. 57).

They then went on to make pragmatic recommendation for two research efforts that should proceed. One was to continue to explore the means to induce valid conceptions of probability, and the other to explore how useful ideas of statistical
inference can be taught independently of technically correct probability. They conclude that "what is needed, however, is not debate but research" (p. 57).

Thus research questions in the present study derive from the factors and recommendations presented by Garfield and Ahlgren (1988). The study will include interview questions that examine the prerequisite rational number concepts and add to the literature in the area of "valid conceptions of probability."

A more recent review by Shaughnessy (1992) provided a summary of research associated with judgments under uncertainty. He suggested directions for future research in a number of related areas including the cognition of secondary students. The research questions in the present study will examine such cognitive aspects of probabilistic reasoning of a group of Year 11 students.

Hawkins and Kapadia (1984) presented a review of the research done in probabilistic reasoning up to the first International Congress on the Teaching of Statistics. They discussed the debate between followers of Piaget and Inhelder (1951), who concluded that children could not handle probability concepts until the age of formal operations, and followers of Fischbein (1975) who used an intuitive definition of probability and came to a different conclusion. Hawkins and Kapadia (1984) pointed out that much confusion has arisen as a result of researchers using different definitions of probability itself. One of the difficulties associated with research in this area is that much of it has been done with convenience samples of tertiary students. The applicability of the results of this research to primary and secondary students is highly suspect given the expected different levels of development present.

Hawkins and Kapadia (1984) stated that, because of differences in the terminology and methodologies employed, research has generated debate rather than provide answers to the questions posed. They identified and defined the following types of probability:

1. A priori (or theoretical) probability - obtained by making an assumption of equal likelihood in the same sample space.
2. Frequentist probability - calculated from observed relative frequencies of different outcomes in repeated trials.
3. Subjective and Intuitive probabilities - expressions of personal belief or perception.
4. Formal probability - that which is calculated precisely using the mathematical laws of probability. (p. 349)

The research questions in this study will address some of the issues raised by Hawkins and Kapadia. In particular, the intuitive conceptions held by Year 11 students and the relation between intuitive and subjective conceptions will be investigated.
Hawkins and Kapadia (1984) maintained that these different types of probability have important philosophical and psychological implications for research and, therefore, for the classroom. For example, they believed that subjective probability (which may rely merely on comparisons of perceived likelihoods) is an area which is often neglected in the classroom although it may be a fundamental precursor to formal probability (which requires some acquaintance with fractions).

As well as identifying the different types of probability that underlie much of the research, Hawkins and Kapadia (1984) isolated the major questions that have driven research in this area of mathematics and identified these as:

1. What conceptions of probability do children of various ages have?
2. How might these conceptions be changed?
3. What is the relationship between intuitive or subjective conceptions and those conceptions which are socially mediated in the classroom and which constitute formal probability knowledge?
4. Is there an optimum age at which to introduce the child to formal probability?
5. Are there optimum teaching and learning techniques which take account of the child's spontaneous conceptions of probabilistic notions while developing his or her understanding of the formal [computational] knowledge of probability? (pp. 373-374)

In their review of the current literature on research into the difficulties that students have in understanding probabilistic concepts, Garfield and Ahlgren (1988) note that "although many articles contain recommendations on how to teach probability and statistics, there is little published research into how the concepts are actually learned" (p. 47). They put forward three main reasons why students have difficulty in understanding probabilistic concepts:

1. Students have difficulty with the prerequisite rational number concepts and proportional reasoning.
2. Probability concepts often seemed to conflict with students' experience of the world.
3. Exposure to probability in a formal way discourages students from further interest in the subject. (p. 47)

The work of Piaget (1951) on children's probabilistic thinking was the first to appear in the literature. However, Kapadia (1984) has observed it would appear to be the case that "Piaget's work has been partly neglected since the English translation did not appear until 1975" (p. 46). Further research was conducted by Piaget and Inhelder (1975). Much of this was aimed at finding an answer to the question: "Could there be in normal man (sic) an intuition of probability just as fundamental and just as frequently
used as, say, the intuition of whole numbers?" (p. xv). Piaget and Inhelder conducted a series of experiments focusing on a priori probability with children of varying ages who conformed to their categories of preoperational (approximately 7 - 11 years) and formal operational (approximately 12 years on). From the results of these experiments, they hypothesized that, while the notion of chance appeared at the beginning of the second stage of cognitive development (that is to say from 7 years to 11 years approximately), true intuitions of probability emerged only when a system of distributions was established and this was not until the child had reached the formal logical stage of development (that is to say, after 11 years approximately). In consequence, Piaget's view was that probability concepts should not be introduced to very young children because they did not have the cognitive structures necessary for processing this information.

Piaget and Inhelder were of the opinion that it was quite natural for the young child not to have an idea of chance. This is because he or she "must first construct a system of consequences, such as position and displacement, before being able to grasp the possibility of the interference of causal series or of the mixture of moving objects" (1975, p. xvii). They stated that the "idea of chance and the intuitions of probability constitute almost without doubt secondary and derived realities, dependent precisely on the search for order and its causes" (1975, p. xvii). Therefore, it can be seen that Piaget and Inhelder distinguished between the concepts of chance and probability.

In other studies, however, Fischbein (1975) and Jones (1974) have revealed that children as young as 5 years of age have the cognitive structures required for processing elementary probability concepts such as sample space and the probability of an event's occurring.

Although Piaget's research involved children up to the age of only 14, a number of his conclusions are relevant to the present study. Most significantly, Piaget and Inhelder (1975) concluded that fundamental probabilistic notions develop only at the formal operational level (p. 159). They argue that the essential reasons for this are that the operations require previously learned concrete operations and that the concept is based on the notion of reversibility. For example, spreading objects apart can be reversed by putting them back together.

According to Piaget and Inhelder (1975), the prelogical child is characterised by irreversibility of thought. They conjecture that the discovery of the idea of chance comes after the understanding of reversible operations that will allow the child to grasp the opposite. "Chance, then, is a domain complementary to logic and understood only by comparison and contact with reversible operation" (p. xix).

Not all researchers agree with this. Kapadia (1984), for example, firmly rejects these conclusions while at the same time conceding "it may be so for formal ideas of
probability, but not for intuitive ideas of chance" (p. 46). By contrast, Fischbein (1975) suggested that effective instruction could set up "structures corresponding to formal operations already at the concrete operational stage with much greater ease and more stability than would be the case for the transition from the preoperational to the operational stage" (p. 187).

The research questions in the present study will relate to both formal ideas of probability and intuitive ideas of chance held by pupils who will most likely be at the formal operational level.

In the investigation by Piaget and Inhelder (1969) of children's understanding of probability, they noted that proportion is directly applied in the calculation of equal probabilities. According to Piaget (1969, p. 141), the concept of proportion develops at the end of the third (concrete operational) stage, but proportional ratios develop in the formal stage and are prerequisite to probability.

The present research will include the collection of data concerning the concepts of proportion, and related mathematical reasoning. The results of the exploration of intuitive ideas of chance held by the gamblers will form part of the research data of this study.

The main purpose of Fischbein's (1975) study was to explore systematically the evolution of probability notions across Piaget's three major stages of development (preoperational, concrete operational and formal operational), the effects of prior systematic instruction, sex differences and the influence of the total number of outcomes on the correctness of responses.

In his preliminary experiments, it was noticed that subjects at all year levels tended to estimate the probability of an event by using ratios (number of favourable outcomes as opposed to the number of unfavourable outcomes) rather than fractions (number of favourable outcomes out of the total number of outcomes in the entire sample space). Fischbein (1975) used the ratio and fraction procedures to differentiate between chance and probability, intimating that perhaps chance is a subjective informal estimate.

Between 1978 and 1981 the Social Science research Council in the United Kingdom, sponsored research in the East Midlands region investigating probability concepts of 11-16 year olds (Green, 1982). The three aims of this project were:

1. To survey the institutions of chance and the concept of probability possessed by the English school pupils aged 11 to 16 years.
2. To establish patterns of development of probability concepts and to relate these to other mathematical concepts.
3. To investigate pupil responses to GCE O-level and CSE probability questions, with particular reference to sex differences. (p. 2)
Green (1982) reviewed and analysed a number of tests and concluded:

Items requiring ratio concept are very poorly done.
Various strategies are employed by pupils when answering odds problems, and there is little consistency in choice of strategy either by individuals or by classes.
Items involving an appreciation of randomness, the stability of frequencies, and inferences were particularly poorly done, with little improvement with age.
Pupils' verbal ability is often inadequate for accurately describing probabilistic situations.
Very often, certainty and high probability are equated, as are impossibility and low probability.
Most pupils do not attain the level of formal operations in probabilistic thinking before they leave secondary school. (p. 167)

Interview questions in the present research will include items that assess knowledge of ratio concepts, strategies employed by pupils when answering odds problems and verbal ability in describing probabilistic situations. In addition the consistency of strategies used in these computations will be recorded.

**Difficulties with Probabilistic Reasoning**

The difficulties pupils encounter with probabilistic concepts have been confirmed in a number of recent studies. In the fourth National Assessment Education Program, (NAEP), for example, Brown, Carpenter, Kouba, Lindquist, Silver and Swafford (1988) reported widespread "difficulty with items involving probability, except those involving very simple concepts" (p. 242) and that "only 5% of (Year 11) students correctly answered questions involving compound probabilities" (p. 243). Interview questions that examine the understanding of the concepts of both simple and compound probabilities are included in the present study.

Fischbein and Gazit (1984), investigated how the teaching of probability can improve probabilistic intuition. They described the results of a twelve-lesson course given to middle school students, and concluded that many of the notions were too difficult for the pupils. Distinguishing between simple and compound events, and the concept of independence were among the concepts found to be too difficult. All pupils encountered difficulties with proportional reasoning in calculating equivalent probabilities. Although Fischbein and Gazit's (1984) study was confined to Grades 5-7, Green (1986), in reporting follow-up research based on his 1982 findings stated that "there were certain areas in which there was no improvement (in performance) from the age range 11 to 16" (p. 5).
In the present study, the cognition of students aged 15 to 17, will be investigated with data obtained on the students' concepts of simple probability and equal likelihood, independence of events, compound events, fairness, expected return and expectation.

In examining areas of difficulty in probabilistic reasoning, Garfield and Ahlgren (1986, p. 270) distinguished between concepts that are difficult because they are unlike anything the student has thought of before and concepts that run counter to intuitive ideas that the students already have.

In a study to determine the processes by which young children generate probability estimates, Lovett and Singer (1991) performed three experiments which involved 50 kindergarteners, 55 Grade 1s, 53 Grade 3s, 49 Grade 5s and 48 adults. From the results of their experiments, Lovett and Singer concluded that many children have a non-quantitative understanding of probability, preferring to use perceptual strategies when both perceptual and quantitative strategies are supported. The perceptual strategies they referred to were the use of the perception of qualities such as area or colour of a region of a spinner, in order to estimate probability of that region.

As a result of these findings, Lovett and Singer (1991) suggested that the relationship between perceptual and quantitative strategies needed to be explored to determine whether perceptual strategies eventually lead to quantitative strategies or whether the two strategies developed separately without ever interacting.

Konold (1989) researched the teaching of domains in which people are known to hold strong prior conceptions that are at odds with the concepts central to that domain. The major implication of his study was that the beliefs that students have prior to instruction in probability can interfere with their learning of the concepts introduced in the mathematics course:

It is unfortunate that these misconceptions do not prevent many students from learning a host of associated quantitative skills, because such skills can erroneously convince both teacher and student that the domain is being learned (p. 92).

Other researchers report that children encounter specific difficulties in probability other than fractional computations. Pegg (1988) reported that pupils experienced difficulties with concepts of independence of events, and the concept of "equally likely" events. Cognition and intuition are often intermingled.

Steinbring (1991) claimed that the basic concept of probability is "circular" in that one cannot define the probability of an event without either reference to the probability of another event or without some prior assumption of randomness. On the other hand, one cannot define randomness without reference to the notion of probability. He suggested that probability is best taught by starting with a problem situation and then using several representations to model the problem, thus developing an intuitive
understanding of the concept. This approach, however, does not overcome the problem of the circular nature of the definition.

Ahlgren and Garfield (1991) point to various difficulties with the teaching and learning of probability, including inadequate research regarding how to teach probability in the best ways and teacher unfamiliarity with the applications of the concept. In consequence, they suggest that the recommendations of the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards* (1988) for probability in the curriculum are overly ambitious.

Jones (1974) reported that although many educators suggested that probability concepts be introduced in primary school, there was considerable discrepancy among the researchers as to how the content of probabilistic reasoning should be prescribed in the primary syllabus. Teacher unfamiliarity with both content and pedagogy in the field of probability and statistics in Queensland schools was reported by Pear (1987), who found 60% of secondary teachers had no formal education in the field and 70% had taken no courses in the teaching of the topic. Many teachers of junior secondary classes were so unfamiliar with the subject that they totally omitted it from their lessons. Many others viewed the subject as "unimportant", commenting that it "gets in the way of the real mathematics" (p. 17).

**Cognitive Processes Involved in Probabilistic Reasoning**

It was noted in Chapter 2, The Rationale for the Research, that much of the research into probabilistic reasoning to date has been in the domain of psychology. In their review of the literature concerning theories of knowing, Putnam, Lampert and Peterson (1990, p. 65) commented that most current research into how students learn is conducted from a cognitive perspective and, therefore, focuses on what it means for the individual learner to know and understand mathematics. They claim that the change from a behaviourist perspective to a cognitive perspective has resulted in an increase in the attention given to the nature of understanding and to complex knowledge types. Schoenfield (1987) has noted that although the cognitive structures involved in the development of mathematical concepts are extremely rich and complex, attempts to understand these structures can yield significant insights into the ways that thinking and learning take place.

The present research will examine the cognitive structures of individuals who make up two well defined groups of the students. An attempt will be made to understand how probabilistic concepts are constructed by the students.

Shuell (1986) notes that most cognitive theorists share the fundamental assumption that an individual's knowledge structures and mental representations of the world play a central role in perceiving, comprehending and acting. Putnam et al. (1990,
p. 67) point out that this representation of the environment through the cognitive structures underlies the definition of knowledge in cognitive theories – knowledge is comprised of the cognitive structures of the individual knower, and that to know and understand mathematics from this perspective means to have acquired or constructed appropriate knowledge structures.

Garfield and delMas (1988) assert that "why people have difficulty understanding probability concepts has been a topic of interest to many researchers, particularly those with a background in educational and cognitive psychology" (p. 57). Watson (1992) suggests that the much needed research into chance and data should be located in developmental psychology. Such research should:

(i) build a cognitive model of student and teacher understanding of the topics,
(ii) make suggestions for the ordering of these new topics in the mathematics curriculum; and
(iii) provide assessment procedures for the evaluation of the implementation of the curriculum. (p. 13)

Scholz (1983), who has conducted extensive research into misconceptions in probability, is another to make the point that to date "research issues on probability are centred around the sub disciplines of psychology" (p. 2). Similarly, Shaughnessy (1977, 1981, 1992) confirms that research evidence suggests that many misconceptions in probability have psychological roots. Shaughnessy (1992) claims that the psychologists involved with research into stochastic reasoning are mainly observers and describers of what happens when subjects wrestle with cognitive judgemental tasks. Researchers in mathematics education, however have as their task the improvement of the students' knowledge by changing their perceptions and are therefore natural interveners.

It is expected that the present research will supplement the literature on the construction of such cognitive models of student understanding of the concepts of chance in gambling contexts and lead to the development of appropriate intervention strategies.
Misconceptions in Probabilistic Reasoning

Misconceptions in probabilistic reasoning are widely reported in the literature. Anderson and Pegg (1988), after noting that "much myth is often created in the young mind about probability," pointed out the common misconception that it is perceived to be more difficult to throw a "six" than to throw any other number on the die. Earlier research by Jones (1974) drew attention to this same misconception. Jones (1976), in reporting the results of subsequent research involving the comparison of probabilities noted that young children were not able to deal with probability comparisons.

Pedler (1977) carried out research into whether children believe that some numbers were harder to get than others with dice. He found that 80% of primary students surveyed believed that some numbers were harder to get than others and that 83% of those who believed this thought that a "six" was the hardest number to get.

Truran (1992) found similar evidence "to suggest that children will frequently nominate six as the most difficult number to throw possibly because of the requirement of many board games of a player to throw a six before the player can begin" (p. 1). This present study will gather interview data on this misconception, one aim being to determine whether it is still held by a significant proportion of older students.

Anderson and Pegg (1988) also reported that many children had difficulty with the concept of fairness and the use of sampling to answer probabilistic questions. Lovitt and Clarke (1988) questioned whether pupils about to leave school had realistic expectations about the outcomes of gambling. Activities to determine this were trialed in a number of schools. They reported that the trial schools observed a "huge gap between perception and reality" (p. 77).

The Use of Heuristics in the Subjective Assessment of Probability

Tversky and Kahneman (1982a), in researching the beliefs of members of the general public concerning uncertain events, paid special attention to what determines such beliefs. In researching how people quantify the probability of an uncertain event or the value of an uncertain quantity, they showed that people rely on a limited number of heuristic principles which reduce the complex tasks of assessing probabilities and predicting values to simpler judgmental operations. While generally speaking, these heuristics are quite useful, they sometimes generate severe and systematic errors. In referring to these heuristics, Tversky and Kahneman (1982a) explained the more general use of heuristics in making judgments, noting that the subjective assessment of probability resembles the subjective assessment of physical quantities such as distance or size. They explained that judgments are all based on data of limited
validity, which are processed according to heuristic rules, citing the example that the apparent distance of an object is determined in part by its clarity. The more sharply the object is seen, the closer it appears to be. This rule has some validity, because in any given scene the more distant objects are seen less sharply than nearer objects. However, the reliance on this rule leads to systematic errors in the estimation of distance. Specifically, distances are often overestimated when visibility is poor because the contours of objects are blurred. On the other hand, distances are often underestimated when visibility is good because the objects are seen sharply. Thus, the reliance on clarity as an indication of distance leads to common biases. They noted that "Such biases are also found in the intuitive judgment of probability" (p. 3).

Tversky and Kahneman (1982a) claimed that most misconceptions in probability concepts among adults could be attributed to one of two heuristics: representativeness and availability. Shaughnessy (1981) also reported that misconceptions used in estimating probabilities by college entry students could often be attributed to these heuristics.

Representativeness. Shaughnessy (1981) described representativeness as "a process by which those who estimate the likelihood of an event do so on the basis of how similar the event is to the population from which it is drawn" (p. 91). This results in a fallacy by which equally likely sequences are viewed as unequally likely. This can occur in two ways. For example, a subject observes a fair coin lands heads on eight out of ten tosses, and then incorrectly reasons that the likelihood of a head on the next toss is less than one-half. The subject is incorrectly reasoning that a balance is expected. This is commonly known as "belief in the gambler's fallacy of the law of averages" (del Mas & Bart, 1989). In the second instance, the subject erroneously reasons that a head is more likely using the preceding short run of outcomes as being a representative base rate. Barr-Hilliel (1982) also reported that this fallacy is widespread among the population.

Tversky and Kahneman (1982a) gave a number of other examples of the use of representativeness. For instance "the probability that Steve is a librarian ... is assessed by the degree to which he is representative of, or similar to, the stereotype of a librarian" (p. 4). They noted that the use of the representativeness heuristic
is not influenced by several factors that should affect judgments of probability. One of the factors that have no effect on representativeness but should have a major effect on probability is the prior probability, or base-rate frequency, of the outcomes. In the case of Steve, for example, the fact that there are many more farmers than librarians in the population should enter into any reasonable estimate of the probability that Steve is a librarian rather than a farmer. Considerations of base-rate frequency, however, do not affect the similarity of Steve to the stereotypes of librarians and farmers. (p. 4)

The questions asked in research reported by Kahneman and Tversky (1982a) included:

- All families of six children in a city were surveyed. In 72 families the exact order of births of boys and girls was G B G B B G.
- What is your estimate of the number of families surveyed in which the exact order of births was B G B B B B? (p. 34)

The two birth sequences are about equally likely, but Kahneman and Tversky noted that most people will agree that they are not equally representative, possibly because the sequence with five boys and one girl fails to reflect the proportion of boys and girls in the population.

The use of representativeness in gambling situations such as Roulette is also well established. Casinos provide patrons with forms enabling them to keep track of the frequencies of numbers. Tversky and Kahneman (1982b) noted:

- The law of large numbers applies to small numbers as well. This belief, we suggest, underlies the erroneous intuitions about randomness, which are manifest in a wide variety of contexts. The gambler's fallacy, or the negative-recency effect, is a manifestation of the belief in local representativeness. (p. 23)

Kahneman and Tversky (1982a) claimed that representativeness accounts for the counter intuitive nature of many conclusions of probability theory, such as the fact that most people are surprised to learn that in a group of as few as 23 people, the probability that at least two of them have the same birthday (i.e., same day and month) exceeds 0.5, and that in a group of 30 people this probability is more than 70%.

They conjectured that "the counter intuitive nature of many results in probability theory is attributable to violations of representativeness" and that "people view a sample randomly drawn from a population as highly representative, that is, similar to the population in all essential characteristics" (p. 37). Furthermore, they claimed that "the heart of the gambler's fallacy is the misconception of the fairness of the laws of chance" (p. 37). Gamblers therefore expect that any deviation in one direction will soon be cancelled by a corresponding deviation in the other.
It is interesting to note that the gambler's fallacy has been well known for many years. Dostoyevsky (1966) writing in 1866 described instances of the use of representativeness at the roulette table:

'Grandmamma, zero has only just turned up,' I said, 'so now it won't turn up again for a long time. You'll lose a lot of stakes; wait a little while.'
'Rubbish, put it down!'
'Very well, but it may not turn up again until evening. You may lose as much as a thousand, it has been known to happen!' (p. 90)

Availability. Shaughnessy (1981) referred to availability as a term used to explain the process that occurs "when people tend to make predictions about the likelihood of an event based on the ease with which instances of that event can be constructed or called to mind" (p. 93). This results in combinational naïveté. One event may be judged more likely than another if instances of it are easily recalled. Tversky and Kahneman (1982a) reported on research into the use of this heuristic, and gave a number examples of its use. "One may assess the risk of heart attack among middle-aged people by recalling such occurrences among one's acquaintances ... the probability that a given business venture will fail, by imagining various difficulties it could encounter" (p. 11). They noted that availability is a useful clue for assessing frequency or probability, because instances of large classes are usually reached better and faster than instances of less frequent classes.

However, availability is affected by factors other than frequency and probability, and the reliance on availability leads to predictable biases. Tversky and Kahneman (1982c) described several other instances where such biases occur and categorised these occurrences, such as "salience, which affects the retrievability of instances" (p. 164). They gave the example that the impact of seeing a house burning on the subjective probability of such accidents is probably greater than the impact of reading about a fire in the local paper. They noted also that recent occurrences are likely to be relatively more available than earlier occurrences, and that it is a common experience that the subjective probability of traffic accidents rises temporarily when one sees a car overturned by the side of the road. In these cases, the estimation of the frequency of a class of the probability of an event is mediated by an assessment of availability. They stated that:

A person is said to employ the availability heuristic whenever he (sic) estimates frequency or probability by the ease with which instances or associations could be brought to mind. (p. 164)
Thus, it can be seen that availability is a valid clue for the judgment of probability based on the notion of relative frequency, because frequent events are generally easier to recall or imagine than infrequent ones. However, availability is also affected by various factors which are unrelated to actual frequency. If the availability heuristic is applied, then such factors will affect the perceived frequency of classes and the subjective probability of events.

Common uses of the heuristics. There is ample evidence for use of the representativeness and availability heuristics in the selection of Lotto numbers. Newspapers in Australia publish a table each week showing the Lotto number frequencies for the last 200 draws, and the number of weeks since each number has shown. This information presumably enables prospective bettors to use representativeness to select numbers that have not shown for a long time. The availability heuristic may be also demonstrated in the selection of Lotto numbers by the reported avoidance of patterns (Gregory, 1993).

The perceived difficulty of throwing a six with the throw of a single die reported by Green (1983b), Jones (1974), Pedler (1977), and Trurin (1992), results from the ease of recall by the child of instances in which a six was wanted and not obtained. This misconception is therefore an example of the misuse of the availability heuristic.

Shaughnessy (1977, 1981) cited the example of a person who is told that (a) the odds of winning a particular game are 50:50, and (b) that out of the last 15 people to play, only 6 of the people won. The subject was then asked if she thought that the next player was more likely to win or to lose, or if the chances were equal. A subject using an availability heuristic might reply that she expects the next person to lose, because she knows that people tend to lose at this type of game. Other questions asked by Shaughnessy (1981) included:

The probability of a baby being a boy is about one half. Which of the following sequences is more likely to occur in having six children: (a) BGGGBG, (b) BBBBGB, (c) about the same chance for each of these sequences.(sic) (p. 91)

Subjects using an availability heuristic will choose the first alternative because they believe there are more instances of these available to recall. Research into other misuses of representativeness and availability among the general public, including educated professional people, by Tversky and Kahneman (1982a) suggested that the use of the heuristics are widespread and are "not limited to naive subjects," and that "people view chance as unpredictable but essentially fair" without the realisation that "deviations are not cancelled as sampling proceeds, they are merely diluted" (p. 5).
Criticisms. It has been noted that there have been some criticisms of the research into misconceptions in probability. Shaughnessy's (1981) main study involved 80 students and Kapadia (1984) has suggested that: some of the "misconceptions" he cites may actually refer to misinterpretations of the question rather than genuine misconceptions. Kapadia maintains that without detailed clinical data it is difficulty to know. Scholz (1991) has also questioned the validity of the use of multiple choice tests in determining misconceptions in probabilistic reasoning.

The ways in which the clinical interviews will be used in the present study will be discussed in the next section. This, along with the restriction in the study to a sample size of forty, will prevent the occurrence of misinterpretations of the questions.

The role of misconceptions in the school curricula. It was noted in Chapter 2 that misconceptions in probabilistic reasoning are specifically referred to in many new school curricula. Specifically, A National Statement for Mathematics in Australian Schools (1990) includes:

Misconceptions about chance processes are widespread. Many become established while children are still quite young and are then difficult to overcome. Therefore chance activities should be provided in schools from the earliest stages in order to help students develop more inclusive conceptions. (p. 165)

It is considered particularly relevant in the present research to determine whether the students demonstrate these same misconceptions that are reported in the literature, and to compare and contrast these occurrences between the two groups of students in situations where the misconceptions arise.

Independence of Events

The importance of the concept of independence of events in the study of probability is confirmed by educators and researchers in the field. Mathematics educators such as Reys, Suydam and Lindquist (1989, p. 242) noted that "independence of events is an important concept in probability." After claiming that independence is a concept that does not develop intuitively, they went on to define it "If two events are independent, one in no way affects the outcome of the other." Yet, as Peard et al. (1993a) noted, this is an intuitive and somewhat circular definition since there is no way of determining the effect of one event upon another without reference to the concept itself.
Sometimes we cannot be sure whether or not events are truly independent. The probability of a first born child being female is approximately 0.5. If the event of the birth of the next child is truly independent then this probability remains constant ... However we cannot be sure that such events are truly independent. (p. 25)

The formal mathematical definition of independence must wait until after the development of the notions of compound probability and conditional probability. Thus introductory ideas of independence must be intuitive. Peard et al. (1993a) state:

If we find that $P(A \mid B) = P(A \mid \bar{B})$, it is clear that the probability of A does not depend on the probability of B. In this case events A and B are said to be independent. This can be considered as a more formal definition of independence. (p. 34)

The need to research the understanding of the concept of independence is supported by Steinbring (1986) who claimed that "the concept of stochastical independence is an important example for analysing the transition from an empirical to a theoretical understanding" (p. 5).

Misconceptions of independence can result from the misuse of either availability or representativeness. For example, the misconception of the perceived difficulty of throwing a six with a single die reported earlier results from the ease with which other numbers are recalled. It has been reported earlier in this chapter that people tend to avoid sequences and patterns in the selection of Lotto numbers and that this avoidance results from the difficulty in recalling sequences such as 1,2,3,4,5,6. The fact that Lotto numbers are independent and therefore all equally likely means that this sequence is just as likely as any random sequence. However "random" sequences in Lotto are easily recalled while patterns and sequences are not.

Representativeness may also be used to falsely conclude that the outcomes of an event are no longer independent. The publication of lotto number frequencies, and the weeks since the last drawing of each number in the newspaper implies that the understanding of independence is not well understood by a segment of the population who presumably pay attention to such frequencies and dates.

The "gamblers' fallacy" mentioned earlier in this section is another example of the use of "local representativeness" and illustrates the lack of understanding of the concept of independence. Brown et al. (1988) reporting the results of the fourth National Assessment Education Program noted that "Students of all ages often fail to recognise the independence of certain events" (p. 243). To the question:
If a fair coin is tossed, the probability that it will land tails up is 1/2. In four successive tosses, the coin lands tail up each time.
- It will most likely land heads up.
- It is more likely to land heads up than tails up.
- It is more likely to land tails up than heads up.
- It is equally likely to land heads up as tails up. (p. 243)

They reported that only 56% of Grade 11 and 47% of Grade 7 students answered correctly. The present research will examine students' intuitive understanding of independence in contexts similar to those reported above.

**Fairness and Expectation**

Bright, Harvey and Wheeler (1981), in a study of fair and unfair games claim that "fairness" is best described by calling attention to an intuitive understanding of "unfairness." In referring to students in years 4-8 they claim that "Helping students recognise when a situation is fair or unfair is a reasonable expectation of the school curriculum" (p. 50). Research by Anderson and Pegg (1988) also reported that primary school pupils encountered difficulties with the determination of fairness.

**The mathematical definition of expectation.** In an experiment in which one of "n" outcomes must occur, the mathematical expectation is defined as:

\[ X - p_1 r_1 + p_2 r_2 + ... + p_n r_n \]

where \( p_i \) is the probability of each outcome for which the return is \( r_i \) (Packel, 1981, p. 22). That is to say, the expected return of the experiment equals the sum of the products of the probability and the return from each possible outcome.

The expectation of an experiment involving a return on a betting situation with two possible outcomes, may be defined as the product of the probability of a favourable outcome (a win) and the return of that outcome, less the corresponding product for an unfavourable outcome (a loss). Packel (1981) defines mathematical fairness as follows:

The expectation need not be equal to any of the possible payoffs, but represents the weighted sum of the payoffs with the probabilities providing the weights. The sign of the given payoff represents whether it is a win or a loss ... A game is defined to be fair if its overall [algebraic] expectation is zero. (p. 22)
The relationship between fairness and expectation. In a mathematically fair game each participant has an equal (zero) expectation. In "simple" games of chance involving only two players, the game is fair if the expectation of both players is zero. The product of the probability of a win and the return from that win equals the product of the probability of a loss and the amount that would be lost. Since, for either player the probability of a win for one is the same as the probability of a loss for the other, there exists an inverse relationship - the product is constant.

This mathematical concept of fairness is not necessarily the connotation that "fairness" implies in ordinary colloquial usage. Fairness may be taken to mean equal likelihood, such as with a "fair" coin, or it may mean absence of interference, such as in a "fair" football game.

Expected return. The expected return on a bet is defined as the product of the probability of winning and the amount returned if a win occurs. For example, if the probability of winning is 0.5 and the odds are "even money," for each dollar bet two dollars is the return (one dollar won plus the original outlay) and the expected return is $0.5 \times 2 = 1$ or 100%. The mathematical expectation is defined as the average long term win or loss and as such is the arithmetic sum of the probability of winning times the amount won, and the probability of losing and the amount lost. In the previous example this is $1 \times 0.5 + (-1) \times 0.5 = 0$. In a perfectly fair game each player has an expected return of 100% and an expectation of zero.

These concepts are clearly beyond the elementary level but require the application of only basic probabilistic reasoning. For simple games of chance involving only two players, one need determine only the probabilities for each to win and then calculate the required amounts for each to be a constant product (or inverse proportion). This constitutes an effective concept of equal mathematical expectation for both players. As Bright et al. (1981) have noted, in complex situations it can be difficult to determine mathematically whether a situation is fair.

Lovitt and Clarke (1988) concluded that there is a huge gap between perception and reality regarding students' conception of expectation resulting in widely over-optimistic estimations of expected return. Although they did not refer to any heuristic involved in arriving at these estimations, it would appear that an "availability" heuristic was involved in which ease of recall was an operative factor.

Data gathering questions in the present study will include those requiring the computation of expected mathematical return in gambling contexts in which a range of probabilities will be involved, and the determination of the fairness of a number of games of chance.
Simple and Compound Probability

The concept of simple probability has been already discussed in the overview. Specific research relating to this has been widely reported. Jones (1976), for example, reported that primary students were unable to deal with probability comparisons. Shaughnessy (1983) cited evidence showing that secondary pupils and college entrance students were poorly prepared in understanding basic probability concepts. They "may believe that all outcomes of an event are equally likely" (when, in fact, they are not), and "possess a myriad of mathematical misconceptions" (p. 325).

Glaeser (1983) researched the perceptions of the concept of simple probability in pupils aged 12 to 14 using questions that required them to draw marbles from bags. Brown et al. (1988), in reported the results of the Fourth NAEP Mathematics Assessment. Questions asked in this study included the drawing of coloured objects from jars, the tossing of coins and the rolling of dice. They reported that:

Students had difficulty with items concerning probability, except those items involving very simple concepts or skills. In one multiple choice item, students were asked to select the probability of selecting a red object from a jar containing 2 red and 3 blue objects. About half the seventh graders and two-thirds the eleventh graders answered correctly. (p. 242)

In the present research interview questions rather than multiple choice items will be used to determine the knowledge of simple probability of eleventh grade students in similar contexts.

Compound Probability. Brown et al. (1988) state that in the Fourth NAEP study only eleventh grade students were asked to respond to items involving compound probability. They reported:

One item asked the students to determine the chances that at least one tail would appear if two fair coins were tossed. Only 5 percent of the students answering the item selected the correct response of "3 in 4", whereas 70 percent of the students responding selected "1 in 2". (p. 242)

The present study will examine the concept of compound probability in the same context as was done in the NAEP study instead using an interview format.

Shaughnessy (1983) reported on secondary and college entry students' knowledge of compound probability and noted that many believe that the chance of getting a sum of four on a roll of two dice is the same as getting a sum of seven.
Truran (1992) asked primary students:

If you roll two dice which is more likely: that you will get the same number with both dice or that you will get two different numbers? (p. 3)

She reported that almost all answered by saying "different" and that most were resistant to change this view if the dice were different colours. Further, the pattern of response was the same across all the year levels.

Fischbein, Nello, and Marino (1991) researched factors affecting probabilistic judgement in children and adolescents among 618 pupils in Italy. Questions relating to compound probability involved throwing coins and dice together, and the rolling of two dice. They reported that earlier research results suggested that the computation of the probabilities of compound events raised many difficulties in students of all ages, citing studies by Fischbein, Barbat and Minzat (1991), Fischbein and Gazit (1984), and Lecoute and Durrand (1988). In order to explore these difficulties further Fischbein et al. (1991) asked a number of questions such as:

Let us consider the rolling of two dice. Is it more likely to obtain 5 with one die and 6 with the other, or 6 with both dice? Or is the probability the same in both cases?
When tossing two coins which result is more likely: to get 'head' with one coin and 'tail' with the other, or to get 'head' with each of the two coins. Or is the probability the same for both results? (p. 532)

They reported that most subjects answered these questions incorrectly stating that the probabilities in each case were equally likely. They noted that "this bias is very resistant. Numerous attempts to change it did not have a significant effect (for instance, colouring the two dice differently)" (p. 533). They further reported that the results were the same irrespective of age, grade or previous instruction in probability and concluded that "there is no natural intuition for evaluating the probability of a compound event" (p. 534).

Since the present research involves the examination of the intuitive understanding of a group of students it is important to include questions in compound probability.

Fischbein et al. (1991) also included the general case: "One rolls two dice. Which is more probable: to obtain the same number with both dice, or different numbers?" (p. 536). They reported that at all age levels the proportion of correct responses were now "visibly higher" (p. 537) and concluded:
The notions of simple and compound events raise interesting psychological and didactical problems. It is a topic which deserves more attention, considering its intuitive complexity and the variety of situations it may generate. (p. 547)

Green (1983a) explored the concept of compound probability held by children aged 11 to 16 by asking them:

A robot is put into a maze, which it begins to explore. At each junction the robot is as likely to go down any one path as any other (except that it does not go back the way it came). There are eight traps at the ends of the eight paths (see picture). In which trap or traps is the robot most likely to finish up, or are all traps equally likely? (p. 776)

He reported that the item proved to be extremely difficult for all ages.

Related Mathematical Concepts and Skills

Fractional Knowledge

It has been argued that basic fraction concepts are needed before probabilistic reasoning can be performed. Fischbein (1975), for example, stated that the mathematical measurement of probability requires an understanding of basic fraction concepts, including the ability to compare fractions. Hawkins and Kapadia (1984) noted that formal probability requires a sound knowledge of fraction concepts, yet difficulties with fraction concepts are documented as being extremely common. Davis (1989) for example, maintains that "many people in the community are totally unaware how one can compare fractions like 3/5 and 5/8 let alone fractions involving larger numbers" and that "problems involving fractions seem artificial and unnecessary ... quite different from natural numbers and decimals which are useful in daily life" (p. 40).
The use of fractions in probability, especially in the context of gambling would appear to be one of the few instances of the use of fractions in a real-life context. Thus interview questions in the present research will include those relating to the comparison of simple fractions and the use of fraction concepts in comparing probabilities and odds.

The examination of the intuitive knowledge of the gamblers will include fraction concepts since, as Clements and Del Campo (1990) point out "fraction knowledge is not natural in that it is not acquired intuitively" (p. 181). In examining the question "what fraction knowledge do young children bring to school?" they concluded:

For the large majority of school children fraction knowledge is something which is crammed into their heads in school mathematics lessons. Outside the classroom it has little relevance. It is not natural, and is only needed for the purpose of studying more mathematics. (p. 190)

They also noted that fraction knowledge developed by cultures outside the dominant European Western tradition "is not naturally acquired by children of all cultures." (p. 5)

**Proportional Reasoning and Equivalence**

Children's understanding of ratio and proportion concepts has been well researched. (See Hart, 1980, 1981, 1984, 1988; Karplus, Karplus & Woolman, 1972; Karplus, Pulos & Stage, 1983; and Nik Pa, 1989). An extensive study by Hart (1984), for example, found that common school algorithms for processing ratio and proportion problems are not well understood. Significantly, Hart (1984) found that understanding did not greatly improve over the two years of schooling from ages 13-15 years (p. 2).

Difficulties with the cognitive processing involved in proportion tasks has been well documented (Hart, 1984, 1988; Inhelder & Piaget, 1958; Karplus, Karplus & Wollman, 1972; Luzner & Pumfrey, 1966; Streefland, 1985). Subsequently, Hart (1988) summarised the findings of several other research studies relating to the ways students obtain ratios, the importance of the numbers used in ratio calculations, the avoidance of the manipulation of fractions and the prevalence of incorrect strategies.

Grugnetti (1991) researched the concept of ratio working with 288 12 year-old and 140 14 to 15 year-old pupils in Mexico. Grugnetti examined the relationship between the "power" of the additive structure and difficulties in recognising ratio questions and reported persistence with the use of additive procedures among pupils aged from 11 to 14 years. Questions asked by Grugnetti included those requiring the use of ratio in enlargement calculations.

Since the odds stated in gambling situations are ratios, the interview questions in the present study will include traditional ratio and proportion questions such as
finding enlargements. In addition, the research will examine the techniques and strategies employed by both groups of students in traditional classroom contexts and in gambling contexts.

**Techniques and Strategies**

Hart (1984) analysed the responses of over 2000 students between ages 11 and 16 years, her aim being to identify the strategies used in solving ratio and proportion problems. She included both correct "school taught" strategies and incorrect strategies. The three "traditional" school-taught strategies for solving problems in ratio and proportion she identified were:

1. The unitary method (find how much for one)
2. The formula $\frac{x}{a} = \frac{y}{b}$
3. An enlargement technique using a scale factor. (p. 4)

She identified a range of incorrect strategies which were employed. These included: The incorrect addition strategy in which the reasoning pattern interprets enlargement as the addition of a fixed amount rather than the multiplication by a scale factor. According to Hart (1984), "the child who employs this method often explains that fractions are to be avoided and that questions for which the requirement is doubling are much preferred" (p. 8). Hart (1984) also researched the strategies used by the "adders" on the ratio questions and found that for many pupils these were specific to the contexts and that a change in configuration or complexity would lead them to employ different strategies. She reported that the "adders" were likely to change from the incorrect addition strategy on items where a conversion rate could be obtained, and that "they were unlikely to seek a fractional multiplier ... and could not produce a standard method" (p. 32). She also stated that:
The method of adding an amount to produce an enlargement was common to a number of children in "average" classes. It was persistent over time, in that children seldom changed their approach between the two sets of interviews. This consistency had been apparent in the CSMS Longitudinal Study when 50% of those children identified as adders at the end of their second year in secondary school were still giving the same type of answer at the end of their fourth year. The method appears to be favoured on enlargement questions and is sufficiently tenacious that children who are aware that it produces an incorrect answer in one situation still use it on another question. (p. 75)

A basic premise of Hart's research was that "the errors made by children are indicative of their cognitive functioning and not mere carelessness" (p. 75). Hart also reported the use of correct "additive" procedures.

From numerous studies of mathematical knowledge developed at work, Acioly and Schliemann (1989) identified a number of strategies employed in proportional reasoning noting that "very often school procedures were used in interaction with procedures developed at work, yielding the most rapid and least cumbersome way to solve a problem" (p. 218). They identified a correct additive technique and a functional, or unitary approach to proportion problems used by both schooled and unschooled adults in the work force. The term "scalar additive" was used to describe the correct additive technique. The following is an illustration of the difference between the use of an incorrect additive and a correct additive procedure to the proportional problem 4:10 = 5:?

The incorrect additive reasoning would be:

5 is one more than 4
So the answer is one more than 10,[additive]
5:11 [incorrect]
The correct "scalar additive" reasoning would be:

\[ 4:10 \]
\[ 5 \text{ is one more than } 4 \text{ [additive reasoning]} \]

\[ 1: \frac{1}{4} \text{ of } 10, \ 1: 2 \frac{1}{2} \text{ [scalar]} \]

so \[ 5: 10 + 2 \frac{1}{2} \text{ [additive scalar]} \]

\[ 5:12 \frac{1}{2} \text{ [correct]} \]

In the present research, the term "scalar additive" is used throughout the report to refer to this type of reasoning. Hart (1982), in referring to the use of correct additive procedures, commented: "What is astonishing is how many children continue to use additive methods long after they have outgrown their usefulness and are very cumbersome" (p. 544).

In their review of the literature on proportional reasoning, Tournaire and Pulos (1985) described the type of tasks used in research to investigate the concept of proportionality and the variables found in this research to affect children's performance on proportion problems. They classified tasks and distinguished between types of proportion problems such as missing-value problems and comparison problems.

For example, "Find \( x \) in \( \frac{3}{x} = \frac{2}{7} \), and, "Which is greater, \( \frac{3}{7} \) or \( \frac{4}{9} \)" (p. 182).

They found variables affecting students' performance included numerical complexity, context and familiarity with the situation.

Harel, Behr, Post and Lesh (1991) argued that "an intuitive understanding of the physical principles underlying the situation of the task is necessary for solving the task correctly" (p. 128). They described functional and relational invariance in missing value proportion problems:

*Functional invariance* occurs "Find \( x \) in \( 3/x = 7/9 \). One can view this problem as asking: if 3 undergoes a natural change which results in the number 9, what change must 7 undergo so that the value of 3/7 is left unchanged ... We call this functional invariance because this change and its corresponding compensation each have only one outcome value ... that constitutes the idea of a function."
Relational invariance is when changes are interpreted in terms of the directionality of the order of relation between corresponding quantities ... unlike functional invariance ... changes are interpreted in terms of "increase" or "decrease" relations. (p. 130)

Studies by Carragher (1989) compared the methods of doing problems in proportion by seventh graders using school algorithms with those used by foremen on construction sites. She concluded that while both used the same type of strategy, neither group used the proportional algorithm taught in schools and reported that:

School algorithms are set up to achieve generality and set aside specific meanings; students seem to learn to set aside meaning also. In contrast, when mathematical knowledge is acquired outside the school, it is related to mastering situations, the meaning of which gives basic sense to the mathematics that is carried out. (p. 645)

Research by Grando (cited in Carragher, 1989) also confirmed that many school practices are performed without meaning. His study of area calculations by seventh graders and uneducated farmers provided further evidence for loss of meaning among the students. Acioly and Schliemann (1989, p. 218) also noted that the use of school procedures seems to be more determined by the type of numbers involved in the computations, and that in particular, problems with "nonround" numbers such as those in $3/8 = x/10$, leads to the use of written procedures more often than do problems with "round" numbers such as those in $3/5 = x/10$.

Estimation Strategies

The importance of estimation and the development of estimation strategies in mathematics education is confirmed by educators such as Reys, Suydam and Lindquist (1989), who state that research has confirmed that "good mental computation skills and number sense provide the foundation for the successful development of computational estimation techniques" (p. 214).

Reys (1992) commented that estimation is an "important skill that has not received a great deal of emphasis in mathematics education" (p. 279).

The National Council of Teachers of Mathematics (1989) has called for more attention to be given to computational estimation. Research into adult learning has shown that much of the mathematics used in everyday living relies far more on computational estimation than traditional methods of computation (Fitzgerald, 1985). Reys (1992) defines "computational estimation" as "the process of obtaining an approximate answer to a (mathematical) problem without using traditional pencil-and-paper algorithms or recording devices" (p. 279). Sowder (1992) has noted that:
The existence or development of approximation skills has not been a focus of research. Yet it is an extremely important skill to have, particularly when mathematics is applied to real world situations. (p. 373)

Furthermore, she has observed that:

Computational estimation is a complex process that develops slowly.... development of estimation strategies for rational numbers is not universal, and many individuals never develop this ability on their own. Instruction should build on mathematical intuitions as they develop (p. 387)

Since gamblers at the track generally perform calculations using mental computation (see Chapter 4), the present research will gather interview data relating to the techniques and strategies of mental computational estimation that some of the students use in track betting contexts.

**Combinatorics**

Combinatoric operations are those that require the calculation of the total number of possibilities in order to determine simple combinations and permutations. Piaget and Inhelder (1975) claimed “the formation of the ideas of chance and probability depend on a very strict manner on the evolution of a combinatoric operation” (p. 161).

Brown et al. (1988), in reporting on the fourth NAEP results, noted that a number of items dealt with combinations and that students of all ages experienced difficulties. However, English (1992), in a study of the strategies that 7 to 12 year-old children spontaneously apply to the solution of novel combinatorial problems reported that:

The study highlights the importance of discrete mathematics as a source of problem-solving activities in which children are motivated to create, modify and extend their own theories. (p. 225)

English (1991), in a study of even younger children (4 years 6 months to 9 years 10 months) attempting a series of novel tasks requiring the formation of different combinations of two items selected from a discrete set of items claimed that “children as young as 7 years can discover a procedure for forming \( n \times n \) combinations prior to the stage of formal operations postulated by Piaget and Inhelder” (p. 452). She commented that the domain of combinatorics is a particularly fertile field for research in mathematics education.

Thus the present study will include questions in the field of combinatorics in both gambling and non-gambling contexts.
Use of Language in Mathematics

The importance of the use of language in the learning of mathematics has been well documented (see, for example, Bickmore-Brand, 1990; Ellerton, 1989; Ellerton & Clements, 1991; Gawne, 1990; Irons & Irons 1992). Language learning is considered to have basic and vital links to the development of mathematics learning. Ellerton and Clements (1991) formulated a theoretical model which examines the interface between mathematics and language (See Figure 5).

![Figure 5: The interface between mathematics and language.](From Ellerton & Clements, 1991, p. 19)

Gawne (1990) applied the socio-psycho-linguistic model of language learning to the construction of a model of the language of mathematics which examines "the major language components contributing to mathematics instruction and the student's construction in the meaning of mathematics" (p. 41).

![Figure 6: A socio-psycho-linguistic model of language in mathematics.](From Gawne, 1990, p. 30)
Irons and Irons (1992) claimed that language helps build understanding of mathematical concepts and that this understanding requires more than the notation that is used to record them. They identified four stages of language usage in the development and processing of mathematical concepts. These categories are not discrete but may overlap. They described these as:

Stage 1: Child's Language. The natural language a child uses to describe the concept in a familiar situation, often a real-world story.

Stage 2: Material Language. The new language that might be used with concrete or pictorial materials as a child acts out or represents the story.

Stage 3: Mathematical Language. The use of a few words to record the language that describes the action of the materials. This stage leads to using more specific mathematical language.

Stage 4: Symbolic Language. The use of mathematical symbols as an even shorter way of recording action. (p. 79)

Watson (1993) discussed the use of probability language and everyday language and noted that, in this field, there is much common language. In the present research, data gathered will include observation of the use of language in the processing of probabilistic concepts by both groups of students. This will be done in order to gather data on the understanding of the concepts involved and the relationship between the language and the notation used to record the concepts.

**Gender Issues**

The importance of gender factors in the learning of mathematics has been a subject of a great deal of recent research (see, for example, Barnes, 1989; Brush 1980; Burton, 1986; Clements, 1989; Fennema, 1985, 1990; Leder, 1982, 1984, 1986, 1992; Willis, 1989). However very little of this research has been in the field of probabilistic reasoning. Research in probabilistic reasoning that has included assessment of sex-related achievement includes that of Fischbein (1975) and Green (1983a). Both of these studies reported small but significant sex-related differences. The males in Green's study achieved 0.5% above females. However Izard (1992, p. 9) analysed research into the development of probabilistic concepts in Quebec, Brazil and Hungary and reported that females in Quebec and Brazil scored slightly, but statistically significantly, higher than males.
It has been noted that Dickenson (1985) reported that both males and females are represented in the category of social gambler. Data from the Queensland Department of Education and an inspection of school records at each of the three schools involved in the study confirmed that for Year 11 students of "Maths in Society" there were no strongly noticeable differences in achievement according to gender. Vale (1993), referring to this issue of gender related achievement in Australian schools in general, reported:

A recent study into sex differences in mathematical achievement found that the differences in favour of males is very small and reducing over time. (p. 563)

Earlier studies, such as those reviewed by Leder (1986) have reported that sex differences in favour of males are more likely to occur in the senior secondary school than in the lower grades. Furthermore, results from the Australian Mathematics Competition would indicate that at the more advanced levels males achieve noticeably higher than females on questions relating to probability.

In the present study attention will be paid to gender issues. Approximately equal numbers of male and female students will be selected and any obvious differences by gender in patterns of response will be recorded.

Comparable Studies

The review of the literature reveals that, to the best of the researcher's knowledge, no comparable studies to that proposed, have been carried out. Although studies of adult gamblers in other countries (U.S.A. and Brazil) can be cited, they are not strictly comparable to this study. This may be due to the uniqueness of the social context as described in Chapter 4, together with the relatively recent inclusion of the subject in the school curriculum.

Acioly and Schliemann (1986) in their study on mathematical knowledge among lottery bookies in Brazil, described the combinatorial system as part of the everyday experiences of people who deal with a special kind of game, the Animal Lottery. Acioly and Schliemann (1986) compared the procedures bookmakers employed according to their level of schooling and found that although no differences could be attributed to degrees of schooling "schooled subjects provided more elaborate explanations" (p. 224). They concluded that since most of these had not learned probability in school, the contribution of formal schooling did not appear to be restricted to topics taught in the classroom. In reporting later research Schliemann (1988) commented:
It appears that everyday experience of the sort provided by the lottery game improves skills at working out permutations. On the other hand, general school experience also seems to promote better understanding of how the permutations can be systematically generated. However, specific school instruction on the combinatorial system does not appear to be crucial. The overall superior better performance of the students seems to result more from their length of schooling than from specific instruction on the combinatorial system. (p. 6)

In a subsequent report of the same study, Acioly and Schliemann (1989) made the observation:

It is interesting to note that most of the schooled subjects had not learned about probabilities at school, because this topic is usually only taught at the university level. However, some of them were able to analyze the game in probabilistic terms. This suggests that the influence of schooling is not limited to topics explicitly taught in classrooms but that school experience provides a different way of analyzing and understanding everyday activities. (p. 216)

All of the students in the present study were enrolled in Year 11 and would therefore have completed Year 10 mathematics. It is expected that most of these had some formal schooling and possibly some informal schooling in elementary probability, but nothing beyond this.

As we have noted, much of the research into probabilistic reasoning has been in the domain of psychology. Billet (1986, p. 5) in his doctoral dissertation in psychology, reported his research in the decision making behaviour of (tertiary) "student game players, student non-game players and non-student backgammon players" in games of chance. His study examined subjective representations of probability and subjective evaluations of outcome values. In these games the amount to be won was the dominant dimension. Decision making required the computation of the mathematical "expectation" of the situation under varying probabilities of winning. He noted that "the majority of work in decision theory has centred on a class of models in which the mathematical expectation is the key component" (p. 5). He concluded that, in such situations, "the generalizability of laboratory decision making studies to real life situations might be suspect" (p. xii).

The interview questions in the present study were designed to include several questions relating to the concept of mathematical expectation and expected return under varying probabilities of winning.
In a different type of research context, psychologists Ceci and Liker (1986a) conducted an extensive study of the mental structures and processes of "expert handicappers" at the race track. Their sample of "expert handicappers" consisted of 50 regular race goes experienced at predicting the odds of horse races. Although the subjects of the present research do not possess the same degree of expertise as Ceci and Liker's subjects, it is interesting to note the conclusions of Ceci and Liker particularly in light of the controversy their report generated.

They reported that it would be virtually impossible to model the reasoning process used, and that the professional track gamblers' ability to analyse and predict odds showed no correlation with measures of I.Q. (p. 130). Ceci and Liker's (1986b) claim that the complex activity of predicting odds showed no correlation with IQ, generated considerable controversy. Regan (1987) called into question the claim of independence between IQ and this ability, noting that a reported negative correlation between IQ and years of following horse racing should not have been dismissed. Detrimer and Spry (1988) claimed that the conclusions of Ceci and Leiker were not supported by the data they presented, and claimed that there were experimental errors in the conduct of the study. Ceci and Liker (1987, 1988) argued that such errors did not exist and refuted the arguments of Detrimer and Spry, maintaining the validity of the original conclusions.

The interview questions in the present study are designed to gather data relating to the structures and processes of non-experts in performing calculations involving odds and other probabilistic concepts.

Summary

The review of the literature has shown that there is a large body of mathematical skills and concepts that can be described as ethnomathematics. The importance of intuitive mathematical knowledge acquired outside of school as a result of cultural activities has been well documented. It has been argued that mathematical knowledge may be viewed as a kind of cultural knowledge generated by identifiable social subgroups within cultures.

Thus the research questions in this study that arise from the review of the literature will include the determination of intuitive concepts in probabilistic reasoning held by an identifiable cultural group within Australian society for whom gambling is a common interest.

The literature has shown that people do invent their own useful strategies for solving mathematical problems in out-of-school situations. The literature further shows that difficulties with probabilistic reasoning and misconceptions in probability are
reported as being widespread among all segments of the population and at various ages. Topics in probabilistic reasoning that have been researched include: simple and compound probability, related mathematical concepts in ratio and proportion, expectation and fairness, the use of heuristics resulting in misconceptions, independence of events, and computations in combinatoric situations. The present study will gather data in each of these fields in order to answer the research questions which follow from the synthesis of this review. In addition, data gathered will relate to techniques and strategies of computation including approximation and mental computation. The literature further identifies the use of language as an important issue in the learning of mathematics and data relating to this will be included in the study.

The extensive review of the literature provided in this chapter suggests that no comparable studies have been done in the specific field of gambling and probabilistic reasoning of Year 11 students.
CHAPTER 6

The Research Questions

The review of the literature has shown that there is a documented need to research the mathematical knowledge that students bring to the classroom as a result of their social backgrounds. One way of conducting this research would be to contrast the knowledge of two groups of students with markedly different social background so far as a particular class of mathematical concepts is concerned. For example, the probabilistic concepts of two disjoint groups of students, from what might be described as gambling and non-gambling families could be investigated and contrasted. This, in fact, will be done in the investigation which will be described. In order to provide a focus for such an investigation it is necessary to define characteristics of the groups to be investigated.

For the purposes of the study which will be described, three major research questions were formulated. These are now stated, and associated comments are provided.

Major Research Question 1

The first major research question in this study is concerned with the intuitive probabilistic concepts and understandings of two groups of Year 11 students. Specifically:

Do Year 11 students of "Mathematics in Society" (a lower stream course in Queensland) whose social background includes extensive familiarity with track gambling (the "gamblers") have different intuitive probabilistic concepts and understandings from students for whom track betting is absent from their family and social background (the "non-gamblers")?

Some of the expressions in this question will need to be defined. Definitions of expressions such as "extensive familiarity with track gambling," and "intuitive probabilistic concepts and understandings," will be provided later in this chapter. Details of how the samples were selected and other methodological issues will be discussed in the next chapter.
**Major Research Question 2**

The second major research question is concerned with the examination of how these concepts and understandings are processed in a variety of situations. Specifically:

What are the cognitive processes employed in the application of probabilistic and related mathematical concepts in traditional classroom situations and in out-of-school contexts involving track gambling? Do these processes differ between the two groups specified in the first major research question, and, if so, what are the differences in the ways individuals in the groups tend to process these concepts?

From the literature review six categories or aspects of mathematical knowledge have been identified as being relevant to the type of probabilistic reasoning employed in this study. The contexts in which these will be studied are categorised as follows:

- **Category 1.** Proportional reasoning, fraction knowledge and construction of rational number algorithms.
- **Category 2.** Simple probability; comparison of probabilities; comparison of odds.
- **Category 3.** Misconceptions in probabilistic reasoning; the use of the heuristics of representativeness and availability.
- **Category 4.** The calculation of compound probability in a variety of contexts.
- **Category 5.** The concept of mathematical expectation: its determination and relationship with mathematical fairness.
- **Category 6.** Combinatorial situations: the determination of simple combinations and permutations in a variety of contexts.

The meaning of the term "cognitive processes" in the context of the present study and the range of concepts covered by the expression "probabilistic and related concepts" will be discussed later in this chapter.

**Major Research Question 3**

The third major research question is concerned with the determination of the extent to which this knowledge pervades the gambling group. Specifically:

Is the knowledge acquired by the "gamblers" as a result of socially induced cognitive interactions in gambling contexts sufficiently pervasive to be regarded as a form of "ethnomathematics" in the sense that this term has been described in the review of the literature?
This last question is the broadest of all three and its answer will be developed from a synthesis of the responses to the other two questions.

**Additional Comments on the Major Research Questions**

**Major Research Question 1**

The "gamblers" would be identified as having a social background that includes either the attendance and betting at the track, or the following of track betting situations at home, or both of these characteristics. The methodology described in Stage 2 of the next chapter ensures that this familiarity with track gambling in their background will be extensive. The "non-gamblers" are a group of Year 11 students from a social background in which there is no interest whatsoever in track betting or any other form of gambling, and for some of whom such interest is contrary to their social values.

The "intuitive probabilistic concepts and understandings" of this question refer to those mathematical concepts and procedures that might be associated with or applied to track betting. These include: the comparison of "odds," the calculation of expected winnings at various odds, techniques of computation and estimation, the relationship between odds and probability, and the language used in describing these processes.

**Major Research Question 2**

The cognitive processes employed in the applications of these probabilistic concepts and associated mathematical skills are concerned with the construction of algorithms and mathematical procedures that are used in situations that involve probability, ratio and proportion. The traditional classroom situations have been selected with references to those reported in the literature, as have a number of out-of-school situations. However in some situations, particularly those relating to track gambling, no comparable studies have been identified in the review of the literature. Therefore the present writer could not rely on the literature to provide the appropriate questions in these circumstances and the questions used in the research instrument described in the next chapter are those which have been constructed by the present author.

*Categories selected for major research Question 2.* In Category 1 questions that were chosen for the present study are similar to those used by Hart (1981, 1982, 1984, 1988) in her extensive research into proportional reasoning by secondary pupils. Strategies examined include those identified by Hart (1984, p. 8). These are repeated from Chapter 5:
the unitary method (find how much for one);  
the use of the formula  
an enlargement/scale factor technique;  
double/halve strategy;  
and an incorrect addition strategy.

Hart (1988) also identified as relevant factors "the importance of the numbers used, the  
avoidance of the manipulation of fractions, and the existence of incorrect methods of  
reasoning" (p. 204).  

Acioly and Schliemann (1989) identified strategies employed in a number of out-of-school contexts. These include the "additive scalar" and "functional" techniques  
described earlier in the review of the literature. Other researchers who have asked  
similar questions in out-of-school contexts include Carraher (1989), Carraher et al.  
(1985), Harel et al. (1991), Lave and Wenger (1991), Schliemann and Magalhaes  

The importance of fraction knowledge in both proportional and probabilistic  
reasoning has been well documented. Fischbein (1975), for example, stated that the  
mathematical measurement of probability requires an understanding of basic fraction  
concepts, including the ability to compare fractions.

Interview questions in Category 1 will therefore include the comparison of  
fractio...
The understanding of the concept of compound probability, Category 4. This has been researched in a number of situations. Green (1983a) asked a large sample of school pupils aged from 11 to 16 years questions involving a robot put into a maze which it begins to explore. Fischbein et al. (1991) asked questions involving the throwing of a pair of die to students of the same age group. The fourth NAEP included questions asked to Year 11 students on compound probabilities obtained from the tossing of coins. The interview questions in Category 4 of the present study are drawn from those identified in this literature.

Questions in Category 5. These examine the understanding of the concept of mathematical fairness. This concept has not been widely researched. Bright, Harvey, and Wheeler (1981), in a study of fair and unfair games claim that fairness is best described by calling attention to an intuitive understanding of "unfairness." This is done in the research instrument developed for the present study. The mathematical concept of fairness as opposed to a merely intuitive understanding requires some understanding of "mathematical expectation" or "expected return". These concepts have been defined in Chapter 5, and they are examined by interview questions from this category. An extensive review of the literature revealed that little research on the intuitive ideas and beliefs associated with the topic of expectation has been done. This is unfortunate but not unexpected since the topic, until very recently, has not been included in secondary school syllabi.

Category 6, questions in combinatorical situations. These were included as a part of the research instrument developed for the present study as a result of widespread reference in the literature to relationship between combinatorics and probability. Piaget and Inhelder (1975) concluded that "the formation of the ideas of chance and probability depend in a very strict manner on the evolution of a combinatoric operation" (p. 161). English (1991) claimed "The domain of combinatorics is a particularly fertile field for research in mathematics education" (p. 451).

Major Research Question 3

The literature reveals that there are many mathematical concepts and skills that can be considered as being within the domain of "ethnomathematics." Although, as Bishop (1988c, p. 180) has noted, "the term is not well defined," there are in the research literature many well documented instances of situations in which children bring to school specific mathematical skills and concepts that have been acquired outside the classroom as a result of factors within their social and cultural background.
Bishop (1988a) further reports that concepts of this type include those associated with the playing of games, including games of chance.

Thus this research question is concerned with the determination of the prevalence of mathematical concepts associated with probabilistic reasoning among a large cultural segment of the society. The extent to which these important concepts are understood by individuals within this group will be determined in order to provide an answer to this research question.

Concluding Comments

The answer to the third question, which will be synthesized from the responses to the other two, will add valuable information to the body of knowledge within the domain of what can be described as "ethnomathematics" as identified in the literature. The review of the literature has shown that numerous researchers have examined situations in which people acquire mathematical skills and concepts as a result of interaction with their social background. (Among children by Abraham & Bibby, 1988; Carraher, 1986, 1989; Carraher, Carraher & Schliemann, 1985; Carraher & Schliemann, 1985; Ginsburg, 1977; among adults by Gay & Cole, 1967; Gerdes, 1988; Graham, 1988; Lave 1988; Schliemann, 1984; Scribner, 1984). In addition Lave and Wenger (1991) have looked at the mathematics of a variety of situations in which people acquire specific mathematical skills and concepts from factors within their social and cultural background. Their extensive work includes the study of concepts and skills associated with measurement in several apprenticeships, navigation, and machine repair work, but does not include studies of probabilistic concepts. Thus the answers to the research questions of this study will add valuable information to the understanding of an important field of mathematics that has not been greatly researched to date.
CHAPTER 7

The Research Methodology

The research methodology was designed to enable the major research questions formulated in the previous chapter to be answered authoritatively. These questions were:

**Major Research Question 1**
Do Year 11 students of "Mathematics in Society" (a lower stream course in Queensland) whose social background includes extensive familiarity with track gambling (the "gamblers") have different intuitive probabilistic concepts and understandings from students for whom track betting is absent from their family and social background (the "non-gamblers")?

**Major Research Question 2**
What are the cognitive processes employed in the application of probabilistic and related mathematical concepts in traditional classroom situations and in out-of-school contexts involving track gambling? Do these processes differ between the two groups specified in the first major research question, and, if so, what are the differences in the ways individuals in the groups tend to process these concepts?

**Major Research Question 3**
Is the knowledge acquired by the gamblers as a result of socially induced cognitive interactions in gambling contexts sufficiently pervasive to be regarded as a form of "ethnomathematics" in the sense that this term has been described in the review of the literature?

Since these questions called for research into (a) intuitive concepts and understandings, (b) the ways in which these concepts are applied and processed, and (c) the pervasiveness of such knowledge, it was recognised that the data needed for this study should be mostly qualitative in nature, and therefore it was decided that qualitative research methods would be employed.

The aim of qualitative research in education according to Sherman and Webb (1988) is to understand "experience as nearly as possible as its participants feel it or live it" (p. 7). They note that human behaviour is shaped in context and that events cannot be understood adequately if isolated from their context. They claim that the function of qualitative research is "to interpret, or appraise, behaviour in relation to
contextual circumstances" (p. 10). The present research will examine the mathematical behaviour associated with the processing of probabilistic ideas in the social context of track gambling, and is therefore well suited to the use of qualitative methods. It was recognised from the outset that the chosen qualitative research methodology should permit every effort to be made to preserve advantages arising from high "standards of design, analysis and statistical reliability" (Lamon, 1972, p. 8).

Generally speaking, qualitative research data are not suited to statistical analysis and it is not the intent of the present investigation to make inferences regarding the populations from which the samples were selected. Nevertheless, care in the identification of the population and the selection of the samples was taken since the study is expected to generate hypotheses which, in later studies might be tested for their validity and degree of possible generalisation to the population from which the samples were selected.

Preamble: The Four Stages

The methodology comprises four distinct stages.

Stage 1. This was a brief pilot study. This involved the confirmation of the prevalence of track gambling in the social background of students at one of the schools selected, the trialing of an identification questionnaire (Appendix 1) and the development and trialing of the interview questions. These interview questions were either drawn from the literature or constructed by the present author. Before finalising, it was decided that the set of these questions should be initially trialed in the pilot study at School 1, a government secondary school in metropolitan Brisbane.

Stage 2. This was concerned with the selection of the two groups of Year 11 students, the "gamblers" and "non-gamblers." It was decided that this would be best achieved by:

1. The administration of a short identification questionnaire to several classes of Year 11 "Mathematics in Society" classes.

2. A subsequent brief one-to-one interview administered to some respondents whom the questionnaire identified as possible subjects in either category. It was expected that this would validate the responses to the questionnaire and confirm or disconfirm whether the respondents did, in fact, meet the criteria for selection for either group.
Clearly the criteria for the allocation to either group would need to be specified, and it was decided that two criteria be taken into account: (a) responses to the items on the questionnaire, and (b) responses to questions posed to students who were identified by the questionnaire and selected for the subsequent interview.

The criteria are elaborated in further detail later in this chapter. Following the validation of responses by the interviews, students not clearly representative of one the background populations would be omitted from the final research samples. It was intended that twenty students from each group who were willing to participate further in the study, would be identified by this procedure.

Stage 3. In this stage of the investigation the probabilistic skills and concepts identified and described in the previous chapter would be compared and contrasted between the two groups of students. In Stage 3 it was decided that each of the subjects identified in Stage 1 would be interviewed using a structured clinical interview schedule in which a predetermined set of questions would be asked. The written and verbal responses to these questions would be recorded and written and verbal protocols which were obtained would then be analysed.

Romberg and Uprichard (1977) identified five distinct types of clinical interviews. These were:

1. The structured individual interview.
2. Piaget's "method clinique".
3. The teaching experiment.
4. The ethnographic case study.
5. Process development evaluation.

They made the point that the structured individual interview has been used in studying mathematical knowledge for some time, citing a study by Betchel as an example of this kind of investigation. In this study, the students were presented with a structured set of questions. Betchel limited his questions in a systematic way and taped the interviews. The behaviours of all the subjects were then coded into categories and the relationships examined.

The present study will proceed precisely in this manner, using structured clinical interviews, in which all interviews proceed in a predetermined structured manner, but digressions are permitted. It was decided that during the structured clinical interviews, the behaviours and responses of students would be coded and categorised in order to facilitate the examination of possible relationships between the groups. The major research instrument therefore would be the set of interview questions and no standardised tests or other such measurement techniques would be employed.
The interview questions trialed in Stage 1 would if necessary, be modified and a final version would be used in Stage 3. This final version, which would be the major data gathering instrument, will be described in detail later in this chapter.

Stage 4. This stage would be concerned with the analysis of the data which were gathered. Observation and interview protocols would be grouped into the six response categories described in the previous chapter. The forty students, twenty in each of the two categories would, in fact, define forty case studies, and for each the data set would contain a large number of responses to a similar set of questions. All responses would be recorded. Since it was likely that there would be a number of different responses to each item within each question of the six categories, it was recognised from the outset that the data set would be voluminous.

To assist with the analysis of such a large body of data, it was decided that a spreadsheet should be employed. This would permit easy identification of the responses by nature of response, characteristic of subject, and category of question. From both the spreadsheet and the raw data an attempt would be made to identify patterns of response which would assist in the comparing and contrasting of the two groups.

The emphasis of the analysis was to be on hypothesis generation. As was recognised by Hutchinson (1988), qualitative research is often better suited to this "theory generation, rather than verification" (p. 113). Lamon (1972) identified two kinds of researchers in mathematics education: the hypothesis builder, whose methods, as in the present research, are more clinical in nature, and the hypothesis tester who takes over after the clinical researcher. Lamon maintained that the two types of researchers "differ in their methods and objectives of research" (p. 9), and further observed that "analysis and assessment of findings produced by both will augment an understanding of mathematics learning and contribute to mathematics teaching methodology" (p. 9).

The present study was therefore designed to generate hypotheses which in turn could be tested for their validity and degree of possible generalisation to the population from which the samples were selected. However, it was recognised from the outset that the generalisability of the findings could be affected by the limitations imposed on the sampling method. This is discussed in detail later in this chapter.

A diagramatic representation of the four stages of the research methodology is shown in figure 7.
Stage 1 (Pilot Study)
(a) The development of a short identification questionnaire and the trialing of this on a small group of students
(b) The development of a set of interview questions and their trialing with three "gamblers" and three "non-gamblers"

Stage 2
(a) Short identification questionnaire given to large group (approximately 140) of students
(b) Brief interviews conducted with possible "gamblers" and "non-gamblers"

Stage 3
(a) Administration of the structured clinical interviews
(b) Recording and coding of responses

Stage 4
(a) Transfer of coded data to spreadsheet
(b) Qualitative analysis of raw data
(c) Conclusions and hypotheses generation

Figure 7: Details of the four stages.
Stage 1: The Pilot Study

Three gamblers and three non-gamblers were identified by the Special-Needs teacher at School 1 who was familiar with the student's social backgrounds. Interviews with these students were conducted for the purpose of validating the research questionnaire (Appendix 1). The six students interviewed in Stage 1 were not included in the samples described in Stage 2.

Each of the gamblers was asked to explain the meanings of terms such as "regular occurrence" and "greatly interested in" in order to validate the meanings of such terms.

The set of interview questions that was developed from the review of the literature was also trialed with the same six students. Questions that failed to yield adequate information were deleted, some questions were reworded to remove any ambiguity, and other minor changes were made. The final set appears in Appendix 2.

Stage 2: Identification and Selection of Groups

The research populations. The prevalence of track gambling within the Australian social context has been established in Chapter 3, where it was shown that there is a large segment of Australian society for whom the phenomenon is part of their social background. It was the Year 11 students from families with this kind of background and who were attending schools in the Brisbane metropolitan region in 1991, who were regarded as forming one of the populations for the research which was carried out. The other population consisted of Year 11 students attending schools in the Brisbane metropolitan region in 1991 from families for whom such gambling was completely absent from their social backgrounds.

The research samples. Two schools were identified in regions in which it could be reasonably expected that there would be an interest in social gambling within many families. These were School 1 and School 2. Both schools are in the vicinity of racing tracks (horse, dog, and trotting), and student familiarity with track betting was confirmed by consultations with special-needs support teachers in both schools.

With the assistance of the special-needs support teachers, and the classroom teachers of "Maths in Society" classes, the questionnaire developed in Stage 1 was administered to six classes each of approximately 25 students.

In the selection of both samples, an attempt was made to achieve a stratification by sex and achievement level of the students.


*Gender balance within the samples.* The literature suggests that there are many gender-related differences in achievement in mathematical topics (see for example, Brush 1980; Burton, 1986; Clements, 1989; Fennema, 1985, 1990; Leder, 1982, 1984, 1986, 1992; Willis 1989).

In all of the schools used in this study, and in Queensland in general, "Maths in Society" courses contain approximately the same numbers of males as females (Department of Education data for 1991 revealed that 55% of students enrolled in this course were female). Consequently, it was decided not to incorporate any special techniques in the methodology to effect gender balance of the samples.

*Balancing achievement levels.* It was decided that in the sampling procedure attention would be paid to pupil achievement level, even though it was recognised that this could be a difficult area of classification due to its subjective nature and possible variations between schools. Nevertheless, it was deemed essential that it be included in the sampling procedure since otherwise comparisons between gamblers and non-gamblers might be obscured by differing achievement levels. It would be inappropriate, for example, to compare only low achieving gamblers with high achieving non-gamblers.

It should be noted that in the Queensland School system, the "Maths in Society" course is the "lower" level course of the available senior mathematics courses. Nevertheless, in recent years, data from the Queensland Department of Education has shown that over 50% of secondary mathematics students have selected this course and, in consequence, many very able students are included in the population resulting in a wide range of achievement among "Maths in Society" students. In all schools used in this study, and generally in most Queensland schools, students taking this course are heterogeneous in composition.

Every effort was made therefore to ensure that both groups contained approximately equal numbers of high and low achieving students. A deliberate decision was made to select approximately equal numbers of high and low achievers and to omit the middle level. In this way, differences between the levels would be more easily identified.

In Queensland schools, all student assessment is school based and teachers are required to keep extensive and up to date records of student achievement. The reliability of informal teacher assessment of student achievement is supported in the literature. Clarke (1992), for example, points out that "teachers formulate and define quite accurate opinions concerning the competence of their pupils" (p. 24). The National Council of Teachers of Mathematics (1989) recognise that reliable pupil assessment is part of the normal teaching procedures. Thus, for the purposes of this
research, it was decided that the classroom teacher would be asked to classify the
student informally from personal knowledge and from teacher records as belonging to
one of the five levels of achievement currently in use in Queensland schools, namely:

1. Very High Achievement (VHA)
2. High Achievement (HA)
3. Sound Achievement (SA)
4. Low Achievement (LA)
5. Very Low Achievement (VLA)

It was decided that subjects classified as either VHA or HA should be recorded as
"High Achievers", and those classified as VLA or LA should be recorded as "Low
Achievers." Those classified as SA would be omitted from the samples in order that
the sample would represent two distinct achievement groups.

It was nevertheless recognised that even with the above provision on
achievement, the groups selected (HA and LA) could differ in terms of "achievement"
but not necessarily "ability." While the "achievement" of each student is measurable
from teacher assessment (even though this assessment may be imperfect), it was felt
that any attempt to measure "ability" with respect to probabilistic concepts would
have produced results that would be too subjective for practical use.

In addition, in order to give greater credence to the distinction between high and
low achievement, for each student identified in Stage 3(b), the assessment of
achievement was checked in the interview by asking the student for a measure of self-
assessment. In none of the forty cases was there a marked discrepancy between the
student's self assessment and the teacher's assessment of the level of achievement.

This was not totally unexpected since the importance of self-assessment is
becoming recognised. Ahmed (1987), for example, states that "If the assessment
process does not allow for pupils to leave school thoroughly versed in self evaluation,
then the assessment process must be considered incomplete" (p. 66).

The Criteria for the Selection of "Gamblers"

To be eligible for selection as "gamblers," the students had to indicate that they
either attended track events themselves or had parents who were "very interested in"
track events.

A subsequent interview was used to establish the meaning of the terms used. For
example, the term "very interested in" was taken to mean that they followed track
events from newspaper forms, race guides, or the television, or from a combination of
these, and that they discussed these events in their family or social settings. In these
settings they would discuss such factors as "odds" and "returns." The actual frequency
of attendance at track events was established during this interview. The terms "very regularly," for example, had to reflect an average of at least once a week. During the interview, questions relating to a subject's background both in and out of school would also be asked. Subjects who were selected for inclusion in the research sample according to the questionnaire criteria (Stage 2(a)), but who did not in fact satisfy the interview criteria (Stage 2(b)), could be rejected.

As a result of the implementation of these selection procedures 20 gamblers were selected; 13 from School 1 and seven from School 2. It was found that the proportion of male/female positive responses to the criteria for gambling was roughly equal, and the sample was stratified with respect to the proportion of male/female in the student population without intervention on the part of the author. This resulted in a sample of 11 male and nine female. Of the 20 gamblers, nine were classified as high achievers, and 11 as low achievers.

Criteria for the Selection of "Non-Gamblers"

To be eligible for selection as "non-gamblers" students had to indicate not only that their parents had "no interest at all" in track gambling but also that they themselves had no interest in any form of gambling, including games of chance involving the playing of cards or the rolling of dice. Since these activities are common among the student population of most government schools, and participation in track gambling is widespread, it was decided to use a third school in which students would be less likely to have these characteristics in their background. A Seventh Day Adventist High School (School 3) was selected in order to obtain this part of the sample of non-gamblers. Although it was not a requirement for selection as a non-gambler to hold negative or hostile opinions towards gambling, there were nevertheless some students from this school from social backgrounds in which this was the case. After students were selected as a result of the implementation of the above procedures, 20 non-gamblers were selected; 10 from School 1, three from School 2 and seven from School 3. Of these eight were male and 12 female, and 10 were high achievers and 10 were low achievers.

Composition of the samples: A summary. Ideally, as a result of the above procedures, we would expect two samples of 20 students and five students in each of the eight cells of Table 1. However, it was decided that since the criterion for gamblers as opposed to non-gamblers was the most important criterion in the selection, no prior rigid decision on the numbers in each cell of Table 1 would be made. Deviations from the desired rectangular distribution came largely from the fact
that the gamblers were, in general, in greater proportion among the low achievers, and that females were more represented among the high achievers.

**Table 1**

**Composition of Samples**

<table>
<thead>
<tr>
<th>Code</th>
<th>Group of Respondents</th>
<th>Low Achievement</th>
<th>Total</th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High Achievement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Male</td>
<td>Female</td>
<td>Male</td>
<td>Female</td>
<td>Male</td>
<td>Female</td>
</tr>
<tr>
<td>Gamblers</td>
<td>5</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>Non-Gamblers</td>
<td>3</td>
<td>7</td>
<td>5</td>
<td>5</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>Totals</td>
<td>8</td>
<td>11</td>
<td>11</td>
<td>10</td>
<td>19</td>
<td>21</td>
</tr>
</tbody>
</table>

**A Note on the Generalisability of Results**

It was recognised, at the outset, that the method for selecting the samples of gamblers and non-gamblers would not result in the generation of strict random samples from the populations. In-built constraints such as the limited amount of time available for interviewing and the need to obtain extensive background information on the students in the samples, prevented randomization. Furthermore, these constraints resulted in the selection of the samples from only three schools.

It was recognised that this may have resulted in sample bias. For example, it was possible that instruction in probability could have occurred in earlier years in these and other schools within the population region. All students would have completed Year 10 mathematics and as a consequence could have been expected to have had some formal instruction in the related topics of "ratio" and "proportion." Some students would have had instruction in introductory probability in Year 10, although the extent of this instruction could have varied from school to school and even between classes within the same school. To attempt to control such factors would have been impractical. However, one common factor was that none of the students could have been expected to have had formal instruction in probability in Year 11, as this occurs in Year 12 at all the schools selected.

Nevertheless, it was intended that the study would generate hypotheses that apply to the populations, especially if particularly marked patterns and differences were observed. The nature of the qualitative research methodology to be employed is such that these generalisations would arise naturally from the analysis of the categories and would lead to the generation of hypotheses for further testing.

**Other factors in the selection of the samples.** Each student included in the final sample indicated his or her willingness to participate. This was in accordance with one of the criteria of the "Ethical Clearance" form that the Bio-medical Research Ethics
Committee at Queensland University of Technology required the selected students to sign (This is explained in Appendix 4). It was found that the vast majority of those identified in Stage 2(a) were not only willing, but also generally enthusiastic, to participate. Only two declined, both citing "lack of time" as the reason.

Other features in the Ethical Clearance form included a guarantee of student anonymity, the disclosure of the purpose and nature of the research as well as the role of the researcher and participants, and the confidentiality of the results. Students who were selected for the study and who were prepared to sign this form (Stage 2(b)), became participants in Stage 3 of the research.

Stage 3: Data Gathering

The major research instrument used for data gathering was the structured clinical interview as previously described by Romberg and Uprichard (1977). The flexibility of this was a factor in its selection, since as Ginsburg, Kossan, Schwartz, and Swanson (1983) have stated, "it allows the interviewer to present problems and questions in a flexible manner and that this in turn allows for contingencies that may arise" (p. 11).

Ginsburg (1981) presented an analysis of the clinical interview procedure as used in research on mathematical thinking. He maintained that this technique has legitimate uses for both the discovery of cognitive processes and their identification or specification.

The appropriateness of the clinical interview for the proposed study is supported by the statement of Ginsburg et al. (1983):

In the study of mathematical cognition, initial work focuses on the identification of interesting phenomena, the identification of the task environment to be explored and the development of useful categories of mathematical knowledge and cognitive processes. (pp. 35-36)

This research proposes to employ procedures such as these with the aim of gathering "a rich corpus of data ultimately represented in written protocols, for the purpose of making inferences about underlying cognitive processes" (Ginsburg et al., 1983, p. 7).

Support for the use of the structured clinical interview for research of the kind that was being conducted is also given by Lamon (1972) who stated:

Work with small groups of subjects ... using qualitative methods ... should be conducted for the purpose of penetrating the mental activity of the subjects ... We should operate in this manner for the purpose of arriving at the principles of the mathematical learning process itself. (pp. 8-9)
The use of the interview as an instrument for gathering data in probabilistic reasoning was specifically supported by Scholz (1991), who noted that much of the recent research by cognitive psychologists on the acquisition of probabilistic concepts has methodological flaws, because tasks are posed in a multiple-choice, forced-answer format, where subjects' understanding of the tasks and their reasoning processes are not evident.

In support of the clinical interview, Romberg and Uprichard (1977, p. 1) maintain that the traditional experimental methodology, involving hypothesis testing and using complex statistics, is often unable to capture the dynamic characteristics associated with the learning of mathematics. Ellerton (1989b) is another to lend support to qualitative methodologies in her comment that "in recent years interviews with children as they work through mathematics questions have been shown to be powerful diagnostic tools" (p. 1).

Ginsburg (1992) elaborated on the areas of activity within the clinical interview and noted that "they include: preparation; development of rapport; monitoring and control of motivation; exploration of cognitive processes; specification of cognitive processes and establishing competence" (p. 2). He maintains that the successful interviewer takes great care in developing potentially fruitful tasks before the interview begins. The present research incorporates this feature in that the interview questions have been drawn from both the literature and the extensive teaching experience of the present author. Ginsburg notes that in order to develop potentially fruitful tasks the interviewer must have "some expectation about the kind of thought processes which will be discovered or investigated" (p. 2). As the tasks become more difficult the interviewer discovers the limits of the subject's understanding, and as they become more specific, the interviewer is able to pinpoint particular strengths and difficulties. The extensive classroom teaching experience of the present author enabled this to be taken into account in the present study.

The establishment of rapport with the student is another feature of the interview that Ginsburg considers to be important. This is necessary in order to create an interpersonal relationship of trust in which the interviewer presents a non-judgemental and supportive attitude. This would be accomplished in the present study by explaining the purpose of the interview in a direct manner in order to motivate the student to participate fully. Ginsburg notes that such motivation is necessary to ensure "adequate and reliable testing" and to encourage the student to "perform as well as possible on the tasks, and to verbalise as fully as possible in response to questions" (p. 2). It will be seen later that, in the present research the students' verbalisations of responses and reasoning form an integral part of the data to be gathered. Therefore, this type of interview is particularly suited to the present research.
Although the present study was not an ethnographic study, the methodology to be employed in data gathering stage of this study involves strategies that are similar to those described by Wolcott (1988, p. 192) in his account of research techniques used in ethnographic studies. These are participant observation, interviewing, use of written sources, and analysis of nonwritten sources. Wolcott's description of ethnographic research as the study of the "social behaviour of particular culture bearing groups of people" (p. 196), an expression which would seem to have particular relevance to the proposed study.

Wolcott (1988, p. 196) contrasts the structured formal interview with the informal interview citing both as valid research techniques. The interview format to be used in the present study is that of the structured formal interview. Additional support for the use of interview techniques in qualitative research can be found in Hutchinson (1988), Marton (1988), and Woods (1988).

It has been stated earlier that the qualitative research methodology of the present study incorporates some of the techniques that Marton (1988, p. 154) described as "phenomenography." Marton noted that "interviewing has been the primary method of phenomenographic data collection" and that open ended interview questions are particularly suited to situations in which the subjects need to be able to "choose the dimensions of the questions they want to answer." The present research is representative of phenomenographic research in that after the interviews have been completed, they are transcribed and the transcripts become the data to be analysed. In the present research, as with the phenomenographic research described by Marton (1988), this is done in order to determine "the qualitatively different ways in which people experience or conceptualise specific phenomenon" (p. 154).

**Construction of the Major Research Instrument**

**The Interview Questions**

In order to study the broad questions formulated in the previous chapter, a large set of specific interview questions was constructed. Many of the questions were drawn from the literature, and others were composed by the present writer, based on experience as a secondary school teacher, lecturer in mathematics education curriculum studies, and writer of secondary mathematics textbooks. See, for example, Peard et al. (1992). Other questions relating to such factors as a student's understanding of the meaning of the written questions, the reasoning processes used when answering the questions, and the reasons for decisions and choices of strategies were asked throughout the interview.
Swanson, Schwartz, Ginsburg, and Kossan, (1981) noted that "a defining characteristic of clinical interview methodology is its contingent structure. The specific direction the interview takes ... varies as a function of the subject and the subject's answers to earlier questions" (p. 34). Examples of how this applied in the present study appear in Chapter 11. Results IV - Case Studies.

Each of the written questions was administered to interviewees one at a time, with discussion and probing of each. The students were encouraged to respond using whatever technique they chose; written pencil-paper, using a calculator, mental computation with verbal response, or a combination of techniques.

Marton (1988) notes that qualitative research techniques are suited to situations which require the organisation of observations into categories for analysis to assess the appropriateness of such techniques. This was clearly the case with data generated in the present study.

Additional data obtained during the interview. These related to:

1. The various methods of computation: pencil/paper, calculator, mental reasoning, or a combination of methods were observed and recorded at the time. Whether the method varied according to the nature of the numbers or the context of the computation for each interviewee was also recorded.

2. The language used in describing methods, techniques and computations was noted. Students were encouraged to verbalise their reasoning throughout the interview and they were often asked to explain verbally what they had done. Whenever possible, the verbal protocols were recorded, as they were expected to form an important tool for the triangulation of responses. Furthermore, in some instances, data were gathered regarding the consistency of the language used. Irons and Irons (1992) stressed the need to check for the consistency of the students' informal language, the mathematical language, and the symbolic language, when students solve mathematical problems. It was planned that the interview data would enable the extent to which such consistency was demonstrated by members of each sample group.

3. The students' attitudes to school and mathematics (positive, negative, neutral) were noted.

4. The techniques of estimation which the students used and the mental computations they employed were observed. Several educators have stressed the need to research the techniques students use when estimating and making mental computations.

Reys (1992), for example, commented that "estimation is an important skill that has not received a great deal of emphasis in mathematics education" (p. 279).
Fitzgerald (1985), who conducted research into adult learning, has shown that much of the mathematics used in everyday living relies far more on computational estimation than traditional methods of computation. Since most of the calculations made by gamblers at the track are performed mentally, data will be gathered on how the students in the gambling group perform such calculations. It was planned that informal interview questions would be used in order to address this matter (see examples in Chapter 11, Results IV).

The final set of interview questions. This was a set of predetermined written items that were used as a basis for the interview (see Appendix 2). Not all questions would be given to all students. Those probing and digressing questions that would be asked only of certain students have been indicated with an asterisk. All other questions were administered in a written format, with verbal commentary by the present author. This set of questions forms the major research instrument which took into account the six categories of probabilistic and related concepts identified in the literature and discussed in the previous chapter. These categories, and the corresponding questions that would be used in the data gathering, are listed below together with an account of how they were incorporated into the major research instrument, the structured clinical interview.

**Category 1: Proportional Reasoning, Fraction Knowledge and Construction of Algorithms**

In Category 1 the questions used were similar to those used by Hart (1980, 1981, 1984, 1988) in her extensive research into proportional reasoning by secondary pupils. Question 1(a), (i) is similar to that asked by Grugnetti (1991, p.97) who worked with over 400 pupils aged from 12 to 15.

![Figure 8: Which number do you replace ? with to obtain an enlargement of the first figure? (From Grugnetti, 1991, p. 97).](image-url)
Lybeck (1981, cited in Marton, 1988) studied secondary school students' understandings of tasks requiring proportional reasoning using the methodology of phenomenography. Problems of the type "If 4 cm$^3$ weighs 6 grams, how much does 6 cm$^3$ weigh?" (p. 150) were presented to the students. The qualitative research methodology to be employed in the present study is similar to the phenomenography described by Marton (1988), in that it seeks to identify "the limited number of qualitatively different ways that each phenomenon, concept or principle can be understood" (p. 143).

Question 1(a), (i)
A photographic negative 2 cm wide and 3 cm long is enlarged to a photograph 9 cm wide.
How long is the photograph?

Hart (1984) identified a number of strategies likely to be used for this task. These were presented in Chapter 5, and included a reasoning pattern in which the pupil incorrectly interprets enlargement as the addition of a fixed amount rather than the multiplication by a scale factor.

Hart found that the choice of various strategies was both number and context dependent. Number dependence occurred when the selection of a strategy varied with the type of numbers involved (for example, whether one was a multiple of another) and context dependence occurred when the selection of strategies depended on the context (for example, a familiar context as opposed to an unfamiliar one). Hart noted for example that the "adders" used correct (but limited) strategies on easier items but used the incorrect addition of differences on harder items.

Lybeck (1981, cited in Marton, 1988) found that secondary students who answered this type of question correctly used one of only two techniques. These corresponded to the unitary technique and the scale factor or enlargement technique (p. 150).

It was decided that in the interview incorrect response to Question 1(a), (i) would be probed in order to explore such number dependence. Students would be asked, if necessary, in Question 1(a) (ii), the same question but with the width of 9 cm replaced with 8 cm (a multiple of 2, which Hart (1984) describes as "round" as opposed to "non-round" numbers).

As an informal check on internal reliability, the questions were repeated with the same numbers in an alternate, but still traditional context, in parts (iii) and (iv).
Question 1(a). (iii)
If 2 litres of juice costs $3, how much would 9 litres cost?

Question 1(a). (iv)
If 4 kg of apples cost $7, how much would 5 kg cost?

In the present research, the responses of both groups will be noted in order to identify the various strategies employed by each and to determine whether the same type of number dependence and context dependence is demonstrated in these situations.

Question 1(b) was selected as a comparable question within the context of track gambling.

Question 1(b)
In each of the following situations,
how much can be won on a track bet if:
(i) $10 is bet at odds of 9:2
(ii) $9 is bet at odds of 3:2
(iii) $5 is bet at odds of 7:4

This question incorporates mathematical concepts in an out-of-school context that have been little researched to date. In addition, by analysing the cognitive processing of proportion tasks employed in the various parts of this question, the difficulties identified by Hart (1988) and summarised in her findings of several research studies can be recorded. Specifically these include those identified as "the importance of the numbers used, the avoidance of the manipulation of fractions, and the existence of incorrect methods of reasoning" (Hart, 1988, p. 204). In order to compare and contrast the groups, in this question the interview will be examine and record responses that relate such issues as:

1. The student's understanding of the language of the question. The research will ask what is their interpretation of the question; does it mean what they think it means?

2. The language used by the student. This refers to the four stages of language identified by Irons and Irons (1992) in the literature review. In particular the research will examine the consistency of this language. Do the students verbalise accurately the mathematical process they employ and the symbols they use? What explanations are they able to give verbally?

3. How do they go about the calculations. Do they use pencil/paper, mental computation, calculators or a combination of these? Does their choice depend on the context of the question or the numbers in the questions?
4. What strategies are used. Are these the same as those identified in the literature? Are these strategies number and context dependent as reported in the literature?

5. What approximation and estimation techniques of mental computation are employed?

Question 2. This question is a traditional comparison of fractions without context, sequenced in four parts.

Question 2
In each pair, circle the larger of the two fractions:

(a) \( \frac{1}{2} \) \( \frac{1}{3} \)
(b) \( \frac{3}{8} \) \( \frac{5}{8} \)
(c) \( \frac{1}{2} \) \( \frac{3}{5} \)
(d) \( \frac{3}{5} \) \( \frac{5}{8} \)

The importance of fraction knowledge in both proportional and probabilistic reasoning has been well documented. Fischbein (1975), for example, states that the mathematical measurement of probability requires an understanding of basic fraction concepts, including the ability to compare fractions. In this question students will be asked to give reasons for their decisions and the strategies employed will be recorded.

Category 2: Simple Probability; Comparison of Probabilities, Comparison of Odds

Question 3 involves the quantification of simple probabilities in the traditional context of drawing marbles from bags. Questions similar to those identified in the literature review will be used.

Student will be asked to express probabilities in an appropriate form, ratio or fraction, then to compare the probabilities and describe the strategy employed and the basis for the comparison.
Question 3

Suppose I select a marble at random from each of the following bags [diagram shown]:

In each of the two situations what is the probability of getting a black marble?

(a) The bag contains 3 black and 1 white
(b) The bag contains 5 black and 3 white
(c) In which of these cases, (a) or (b), is it more likely to get a black marble?

Questions of this type have been used by a number of researchers. Jones (1974), in his doctoral thesis, asked similar questions of much younger pupils. In testing intuitions of sample space the child was expected to point to or state the appropriate outcomes of experiments involving drawing balls from containers.

Pedler (1977), asked similar questions of Year 7 pupils. More recent studies of the fourth NAEP reported by Brown et al. (1988) included the quantification and comparison of simple probabilities by Year 11 students using similar questions.

Those unable to answer Question 3(c), will be asked to make easier comparisons such as:

Question 3(d)
[If unable to compare the probabilities in Question 3(c)]
Repeat Question 3(c) with:
(i) (a) 1 black and 3 white and
(b) 1 black and 4 white;
(ii) (a) 2 black and 3 white
(b) 2 black and 5 white;

For those who responded correctly to Question 3(c), a more difficult comparison will be provided:

Question 3(e)
[If able to compare the probabilities in Question 3(c)]
Question 3(c) was repeated using; 3 black marbles and 2 white marbles in one bag, and 5 black marbles and 3 white marbles in the other bag.
As with previous questions the language employed and techniques used will be noted for comparison and contrast between the groups. In addition responses will be examined to determine the use of any intuitive concepts. The consistency of the use of techniques such as the use of a common denominator to compare fractions and to compare probabilities will also be examined.

Questions 4 and 5 involve the comparison of odds. An extensive review of the literature reveals that no comparable questions within the gambling context have been asked of students of this age. Therefore in the construction of these questions, the author has drawn on his experience as a teacher, lecturer and author of text books. Green (1986), as part of a large study, included questions on the comparison of odds in situations of drawing balls from a bag by younger students (aged 7 to 11). However, Questions 4 and 5 can be interpreted as a comparison of ratios and can be answered as such without reference to the gambling context.

**Question 4**

In each of the following track betting situations, which is the better of the two odds? That is to say which gives the greater return per $1 bet, or which is the greater ratio?:

(a) 2:1 or 3:1
(b) 5:1 or 5:2
(c) 4:3 or 9:7

In this question the interview is again concerned with obtaining data relating to the language used.

- What does the question mean?
- How is it interpreted?
- What strategies are employed?
- What language is used in the students description of these strategies and how do the strategies compare between the two groups?

The responses will provide data relating to the use of intuitive reasoning or knowledge, the processing of this knowledge in the construction of algorithms, and consistency of techniques used. Probing will proceed for those unable to correctly answer Question 4(c), to examine number dependence in an out-of-school context in determining whether the choice of strategy depends on the nature of the numbers. In addition, method of computation and mental computation will again be included in the data.
Question 5
In a four horse race the odds for each horse are given as -
2:1, 5:3, 5:1, and 25:1
Which horse is thought to be most likely to win?
List the odds in order of least likely to most likely.

The cognitive processing for such ratio and proportion tasks has been well researched by Hart (1984, 1988), Karplus et al. (1972); Lunzer and Pumfrey (1966); and Streefland (1985). Data from this question will be analysed to explore the cognitive process involved, including the use of intuitive knowledge.

Category 3 Data: Misconceptions in Probabilistic Reasoning; the use of the Representativeness and Availability Heuristics

It has been noted in Chapter 5 that misconceptions in probabilistic reasoning are reported in the literature to be widespread, and that the two most commonly cited in the literature are associated with the use of the heuristics of "representativeness" and "availability."

Numerous researchers have investigated misconceptions among much younger pupils, such as the perceived difficulty of throwing a "six" with a die. (See delMas and Bart, 1986; Jones, 1974; Pedler, 1977; and Trurin, 1992). All of these researchers used questions similar to those of Questions 7(a) and (b) in the present study. Thus, these questions will explore whether this same misconception is still present in older students.

Marton (1988) has commented that phenomenography is a methodology that is particularly suited to situations such as the present research, in which misconceptions are examined. This is because this type of research does not attempt to make statements about "the world as such, but about people's conceptions of the world" (p. 145).

Question 7(a)
If I throw a single fair die is it harder (less likely) to get a six?
In general, are the numbers 1 to 6 equally likely to occur, or are some harder to get than others?

Question 7(b)
If I throw a pair of fair dice is it harder to get a pair of sixes than it is to get a pair of twos?

During interview questions students will be asked to provide answers to these questions and to give reasons for these answers in order to determine whether an availability heuristic is being used. The use of this heuristic in a variety of situations has been documented in the literature review. With Questions 7(a) and (b), students
using such a heuristic would tend to reason that it is harder to get a "six" because they can recall (availability) situations when a "six" was needed (such as in board games) but was not thrown, whereas no such conditions are easily recalled for the other numbers.

Question 7(c) involves what is essentially the same misconception involving the "availability" heuristic, but in the different context of a poker hand.

**Question 7(c)**
A poker hand consists of five cards dealt from a well shuffled deck. The Ace is the highest card. Is it harder (that is to say, is it less likely) to get three Aces than it is to get three fives?

Question 7(d) is identical to that used by Shaughnessy (1981, p.91) in a study of 80 college entry students.

**Question 7(d)**
Suppose that you and I are going to play a game with this coin (shows) in which we toss the coin just once. If the coin lands heads, I win; if it lands tails you win. But I will warn you that of the last 15 people who played this game with me only 6 won. Do you think this is unusual? Do you think the coin is not fair? Do you think the game is fair?

In this situation unfairness may be falsely justified in one of two different ways. Those using an availability heuristic would reason "people tend to lose at this type of game, "while those using a representativeness heuristic would take the shorter term frequency of the coin as representative of its long-term probability. In interviews, the understandings of those students who do not demonstrate either of these misconceptions can be probed.

[If the response to Question 7(d) is "yes"]: What if it was the case that, of last 20 people 14 lost? Is it still fair?  
[If "yes"]: How long a run would you need to suspect bias if "head" showed about two-thirds of the times?  
[If "no"]: Why not? Would you suspect bias if 10 people played and 7 lost? If 6 people played and 4 lost?

In this manner, the interview will gather data relating to the students' intuitive understanding of the need for a greater proportion of heads to tails, or the same
proportion to occur in a longer run, before unfairness can be concluded. None of the students will have adequate background in probability theory to answer these questions from a theoretical perspective, and the interview questions are designed to explore intuitive understanding.

"The gambler's fallacy" Questions 6(a) explores what is referred to in the literature as "the gambler's fallacy." This misconception is well documented, and is an example of "local representativeness" (Tversky & Kahneman, 1982a, p. 5).

Question 6(a)
In the last 200 Gold Lotto draws the number 17 has occurred more times than any other number. In future draws this number is:
(i) more likely to occur than any other number?
(ii) less likely to occur than any other number?
(iii) as equally likely to occur as any other number?

In interviews, students will be asked to provide reasons for their decisions, and the familiarity with Lotto and the publication of such information in a Sunday newspaper will be checked. For example:

If this frequency does not affect future probabilities, why do newspapers publish the frequencies of the numbers drawn and the number of weeks since the last time each number has been drawn?

From responses to such questions, the students' intuitive understanding of the concept of "independence" of probability can be determined.

Question 6(b) explores the same misconception in a context that may be less familiar to the student but is well documented in the literature.

Question 6(b)
[Describe/show roulette wheel]
In Roulette, each number is equally likely to occur. Suppose that the ball lands on Red six times in a row. On the next roll is it now more likely to land on the red, the black or are they both still equally likely?

In this situation, the representativeness heuristic can be misused in two ways. First, students make use of "local representativeness" to conclude that black is now more likely since the "deviations must balance out in the long run," when in fact such deviations do not. Second, by using the "short run" frequency as "representative" of the long run to conclude the "red" is always more likely. The use of the heuristic in
either case implies a lack of understanding of the fundamental concept of "independence."

Questions 6(c) and 6(d) examine the use of the availability heuristic further in a lottery context. Interview questions such as the following will be used to check the familiarity of students with such contexts.

**Question 6(c)**
In some lotteries the tickets are pre-numbered. If you could choose between any of the following numbers would you have any preference?
123456
619999
615472

**Question 6(d)**
In choosing six LOTTO numbers which of the following selections would you prefer?
(a) 1, 2, 3, 4, 5, 6
(b) 32, 33, 34, 35, 36, 37
(c) 9, 12, 27, 31, 35, 38

Subjects using an availability heuristic in either of these situations would reason that they cannot easily recall certain numbers or patterns, such as those of parts (a) and (b) in the above question, and would therefore regard other numbers combinations as more likely when, in fact, each combination is equally likely. Again, this demonstrates a lack of understanding of "independence." Misconceptions of this kind can then be explored in the interviews.

**Category 4 Data: The Calculation of Compound Probability in a Variety of Contexts**

Question 8 is identical to that used by Green (1983a) with a large sample of school pupils aged from 11 to 16 years (See Chapter 5).
Question 8

A robot is put into a maze which it begins to explore.

[diagram shown]
At each junction it is as likely to go down any path as any other. Where is it most likely to end up?

In interviews, students will be asked to explain why they gave the response, the aim being to determine whether the student understands that each compounding of a probability reduces the final probability, and whether the student has any technique for computing such a final probability.

Question 9(a) involves the concept of compound probability in a track gambling context.

Question 9(a)

In each of two horse races the favourite is estimated to have about a 50% chance of winning.
If you bet on both favourites, your chances of winning on both would be about:

(i) 100%
(ii) 75%
(iii) 50%
(iv) 25%
(v) 10%

Responses to this question would be expected to yield data similar to that for Question 8, and could provide answers to questions such as: Do the gamblers understand intuitively that the answer must be less than 50%? Do they have any technique for determining the compound probability? Do the techniques of the gamblers differ from those of the non-gamblers? Interview questions could be used which explore the same concept in a different contexts. For example:

If I am going to play tennis against an opponent and I have 50% chance of winning a set, what is the chance that I will win two sets in a row, assuming the probability of winning each remains at 50%?

Questions 9(b), (i) and (ii), are concerned with compound probability in the more traditional context of coin tossing. These are similar to those asked by Fischbein et al. (1991), and to those asked in the fourth NAEP and reported by Brown et al. (1988).
Question 9(b)
Two coins are tossed together.
(i) The probability that one will land a head and the other a tail is:

(a) \(\frac{1}{2}\)

(b) \(\frac{1}{3}\)

(c) \(\frac{1}{4}\)

(ii) What is the probability that at least one of the two coins will land a head?

The various parts of Question 9(c) correspond to those asked by Fischbein et al. (1991, p. 532).

Question 9(c)
(i) A pair of fair dice are tossed together.
[show pair of identical dice]
Which is more likely:
(a) a pair of fives
(b) a five and a three
(c) both are equally likely

In the interviews, those responding "equally likely" will be questioned carefully to see if their response is indicative of a "resistance" to change (as referred to by Fischbein et al., 1991, p. 533) by asking part (ii)

(ii) [repeat with different coloured dice]

Question 9(c) will also be repeated in the interview, except that one of the dice would be red and the other white. Interview questions could then also explore the students' understanding of the more general case:

(iii) In general, is it harder (less likely) to get a pair of numbers than it is to get two different numbers?
Category 5 Data: The Concept of Mathematical Expectation: Its Determination and Relationship with Mathematical Fairness

The concept of mathematical fairness was examined in the discussion of Question 7, where it was pointed out that the misuse of a heuristic may lead to a false conclusion of unfairness. Bright, Harvey, and Wheeler (1981), in a study of fair and unfair games claimed that fairness is best described by calling attention to an intuitive understanding of "unfairness." This is done in Question 10.

Question 10(a)
Suppose you and I play a game with one of these dice [show single die]. If on a single roll it is three or less you win, if it is more than three I win. If we each bet $1 and the winner gets the $2, would this be a fair game?

Question 10(b)
Suppose we change the rules so that I win if it is a three or greater. Would this still be a fair game?

In interviews, students will be asked to provide reasons for their decisions. Those who recognise that the game in Question 10(b) is no longer fair will be expected to explain their reasoning in terms of unequal probabilities and equal returns. The language that the students use in order to do this will form an integral part of the data.

The mathematical concept of fairness, as opposed to a merely intuitive understanding of the concept, requires some understanding of "mathematical expectation" or "expected return." These have been discussed in Chapter 5 where it was noted that in a mathematically fair game each participant has an equal (zero) expectation. Question 10(b) leads into an intuitive understanding of mathematical expectation. Those who recognise that the game is no longer fair will be asked:

Can we make the game fair somehow?

Those who recognise that the game can be made fair by altering the return to one of the parties would demonstrate a basic understanding of the concept of expectation. Questions can then be asked which explore the extent to which this understanding is intuitive.

For those demonstrating a basic understanding of the concept, further questioning can proceed along the lines:

How much more should I put in to make the game fair?
Those recognising that more than $2 is required would demonstrate a basic understanding of the constant product requirement discussed in Chapter 5. Again, interview questions can be asked which explore this further.

**Question 10(c)**

[If the answer to 10(b) is $2, continue with further questions]:

- How much should I put in if I choose 1-5 leaving you just the 6?
- [Repeat with drawing cards from a deck]:
  - How much should I put in if I choose spades leaving you the other three suits?
  - How much if I choose a single card of any suit? If I choose a single specific card?
  - [If correct, Repeat the questions reversing the order of "you" and "I"]

Students able to respond correctly to these questions would be considered to have a sound understanding of the concept of expectation.

**Category 6 Data: Combinatorial Situations: The Determination of Simple Combinations and Permutations in a Variety of Contexts**

Piaget and Inhelder (1975) concluded that the formation of the ideas of chance and probability depend in a very strict manner on the evolution of a combinatoric operation (p. 161). Other researchers have called for further studies in this topic. English, for example (1991) claimed that the domain of combinatorics is a particularly fertile field for research in mathematics education.

**Question 11.** This includes items involving simple combinations and permutations in both traditional and track gambling contexts. Few, if any, of the students will have studied this topic formally in school mathematics since it is not in the syllabus of earlier years. Therefore, it is expected that these questions will obtain data related to the students' intuitive understanding of the concepts. The understanding of the processes can be compared and contrasted between the two groups.

**Question 11(a)**

The "double" in horse racing consists of selecting two winners in two prescribed races. How many possible ways can the "double" occur if one race has 10 horses and the other 8? (Exclude any "dead heats")

**Question 11(b)**

If there are 6 horses in a race how many ways can the first and second places be filled? (Exclude any "dead heats")
Question 11(c)
[To be given to students who were able to answer 11(a) and (b)]
If the "quinella" consists of selecting the first two horses in a race without necessarily being in the right order, how many possible selections are there in a race with 8 horses.
[If the student is unable to answer this, proceed to 11(f)]

Question 11(d)
[To be given to students who were able to answer 11(c)]
If the trifecta consists of selecting the first three horses in the correct order, how many possible selections are there in a race with 6 horses?

Question 11(e)
If we disregard the order in the last question, how many selections are there now?

Question 11(f)
A type of new car can be bought with a choice of 6 colours and 3 engine types. How many different selections can be made?

Question 11(g)
If the same type of car can be bought with either manual or automatic transmission, how many different selections are now possible?

Question 11(h)
Suppose that you have 6 different movie vouchers but have time to go to only two movies. How many different choices can you make in selecting the two movies you will attend?

Again, interview questions will be used to examine the reasoning and the techniques of computation employed. Other questions could be used to examine the same concepts in alternate contexts.

Reliability and Validity Considerations

The importance of reliability and validity issues in assessment is well established. Shimahara (1988), for example, states that validity and reliability are crucial in all research, "regardless of disciplines and the methods employed" (p. 86).

The recording of interviews on audio tape using a micro cassette for later transcription and analysis should enable careful analysis of unusual or unexpected responses to be carried out. Although the interviews will be structured, they will nevertheless allow for variability of responses. Where necessary, the interviewer can
digress to explore a student's cognition by constructing further questions. Swanson et al. (1981) note that the variability arising from this kind of interview raises questions concerned with the reliability and validity of the data (p. 34). They maintain that there are no easy answers to these concerns, but conclude that the clinical interview, in the hypothesis testing generating stage of research, must now be viewed as an important measurement technique. As such traditional concepts of reliability and validity need to be considered (p. 35).

**Reliability Considerations**

The present author has had considerable experience in conducting clinical interviews aimed at researching the level of mathematical understanding possessed by individuals, largely through involvement in the teaching of subjects at Queensland University of Technology (QUT) in the field of the diagnosis of learning difficulties in mathematics. He has not only conducted many such diagnostic interviews himself, but also has trained both undergraduate and graduate students to conduct such interviews. Thus, the present author is familiar with the sources of unreliability which can arise in interviews, and in this study every effort will be made to minimise these.

The reliability of the research instrument would be increased using the following techniques:

1. By conducting each interview over two separate sessions, generally separated by a time period of one or two weeks, it would be possible at the second session, to perform an informal check of internal reliability by the repetition of one or two items that had been given in the first session.

2. By viewing problems and responses as items it would be possible to check the internal consistency of selected responses.

3. By ensuring that several key components can be observed in parallel form in different parts of the interview, a measure of homogeneity would be provided.

However, Swanson et al. (1981) comment that "in mathematical problem solving situations one does not necessarily expect high levels of internal consistency on the part of the subjects" (p. 36). Further, in discussing aspects of reliability in qualitative research, Hutchinson (1988) says that the question of replicability is not especially relevant, "since the point of theory generation is to offer a new perspective on a given situation that can then be tested by other research methods" (p. 132).
Validity Considerations

Swanson et al. (1981, p. 36) maintain that "the incorporation of verbal protocols supplemented by observation in the interview setting provides rich information for the validation of subject responses." This could be done in this study so that the accuracy of inferences drawn from the data collection and the analysis procedures could then be readily investigated.

Stake (1988, p. 263), in answering criticism regarding the validity of data gathered in case studies, suggested that "the primary way of increasing the validity of such data is by triangulation." He describes the technique as that of "trying to arrive at the same meaning by at least three independent approaches" (p. 263). In this research data could be validated by the triangulation of results in a number of different ways. Triangulation of subject observations, written protocols, and verbal protocols could be performed with the aim of investigating whether these all point towards the same meaning or conclusion. Triangulation of the language used, the technique of mental computation, and the context of the question, could be used to investigate whether these all point to differences in the ways that the gamblers and non-gamblers apply probabilistic concepts.

Administration of the Structured Clinical Interview

Responses to the set of predetermined questions concerned with probabilistic and related mathematical concepts generate much of the data for the present study. These data are supplemented by related interview data.

Each of the 40 students identified in Stage 1 was presented with the set of written questions shown in Appendix 2, during a structured clinical interview. These were answered one by one, together with answers to verbally administered interview questions.

Depending on the responses given to various items, the interviewer would probe, explore and digress from the predetermined questions. Examples of how this is done are given in the analysis and discussion of the results and in case study samples reported in Chapter 11 (Results IV). When responding to written questions student were, at all times, asked to verbalise their reasoning. Interview questions such as "What does this question mean?", "What are you doing now?", "Why did you do that?", "What do you mean by this?" were constantly used to probe the students' understanding. If necessary, the written question was read aloud to the student to clarify its meaning.
Background to the Interviews

Each of the 40 interviews was conducted at the school which the interviewee attended. In general the interviews were administered over two sessions, each session lasting from 20 to 40 minutes. The time period between the two sessions varied, but was generally in the order of two weeks. This was in order to avoid pupil fatigue associated with lengthy interviews. Campbell and Stanley (1963, p. 19), who describe this simply as the "growing more tired" effect, point out that this can endanger the internal validity of the interview. However, if subjects show no sign of fatigue and indicate a willingness to continue, the interview can be administered in the one session. Two of the gamblers who were highly motivated to respond were interviewed in the one session. Most of the low achieving non-gamblers did not respond to many items and were interviewed in one session.

It is recognised in the literature that some structured clinical interviews will require more time than others. In particular, in the present study, it was recognised that more time would need to be allocated with some of the gamblers in order to investigate their intuitive and more complex responses.

The physical setting in which each interview took place. This varied considerably from interview to interview. However, for each case study the student was withdrawn from a regular mathematics classroom with the permission of the regular classroom teacher. On some occasions a vacant classroom was used, on others the staff lounge or a bench under a tree outside was the venue.

Recording of Data

Students' written responses on the question sheet were kept for later analysis. At the time of the interview, quick rough notes were made of any additional formal or informal verbal responses for later transcription. The data gathering process was greatly assisted by the use of micro cassettes. In accordance with the requirements of the QUT Bio-medical Ethics Research Committee, permission to record the interviews using a micro cassette was sought from each pupil. The tape-recording permitted much greater flexibility during the interview, allowing the researcher to digress and probe without the distraction of having to record in writing the students' verbal responses.

However, on many occasions the amount of background noise in the environment interfered with the audio-taping and on these occasions it was necessary to make written notes.

After each interview, the recorded written and verbal protocols were reviewed and the responses summarised and categorised in rough form, in preparation for later
detailed analysis. A file of this "raw" data was kept for each student. At the completion of all 40 interviews the refinement of the raw data for later analysis began. This involved the collation, perusal, and the transcription of very rough notes into organised notes. These were then edited and organised for the analysis of Stage 4.

**Stage 4: Data Analysis**

Responses to all items gathered by the methodology described in Stage 2 would be categorised according to the answer given, the technique of computation, the reasoning employed, or certain other identified criteria. Categories identified in the literature such as "incorrect additive technique," "correct functional technique," and any other identifiable categories of response were recorded in each student's file. Where necessary, special categories would be constructed by the present author. These may be unique to this study: for example "greater return for lower outlay reasoning." Wherever possible, these responses would be coded numerically (see Chapter 8, Results 1) and recorded on a spreadsheet (see Appendix 3). In addition, student background data would be coded and recorded on the spreadsheet.

It is not the intent of the author to attempt to analyse all the data coded on the spreadsheet. Rather the use of the spreadsheet in data analysis would be to enable the author to select categories quickly and easily for comparison, for contrasting, and for pattern identification. For example, if the researcher wished to examine the responses of, say, high achieving female gamblers to a particular item, the responses could be first identified from a sort of the spreadsheet. However, all major analyses of data would proceed from examination of the raw data and the students' files.

Furthermore, some of the data gathered and recorded on the spreadsheet were for identification purposes only and not subsequently used in the analysis. This data related to the time of the interview and the school. Other data coded, but not analysed from the coding, included the use of language and the technique of computation. In these cases it was found to be necessary to refer to the raw interview data.

Although responses were coded numerically for ease of identification, all data are qualitative in nature and the proposed analysis is qualitative. No scores, or summation of marks would be made. Rather qualitative statements, such as "thorough knowledge as demonstrated by ..." or "little conceptual understanding as evidenced by ..." would be made. Patterns of response would be described qualitatively as would comparisons and contrasts between groups.

*In conclusion* The analysis of the data is designed to produce answers to the three major research questions that were posed. This will involve comparing and
contrasting responses of gamblers and non-gamblers, high achievers and low achievers, male and female students, and then comparing the findings with those reported in the literature. These results are reported in Chapters 8, 9, and 10.

Summary

The major research questions posed in the preceding chapter called for research into intuitive understanding of concepts, perceptions and misconceptions, the ways of processing and applying mathematical knowledge, and a determination of the pervasiveness of such knowledge. In order to gather this qualitative data, it was decided to employ the qualitative methodology that is described in this chapter.

The function of this qualitative research included the interpretation and appraisement of behaviour in relation to a variety of contextual circumstances, and the organisation of written and verbal protocols into categories for analysis.

The methodology is comprised of four distinct stages, each of which has been described in detail in this chapter:

Stage 1 - The pilot study
Stage 2 - The selection of the samples
Stage 3 - The data gathering
Stage 4 - The data analysis

The pilot study was carried out at School 1, following which 20 gamblers and 20 non-gamblers were selected for the major study. These samples were balanced with respect to gender and achievement. The criteria for selection for both categories and the composition of the samples have been clearly described.

It was decided that the investigation was best suited to clinical techniques and that the major research instrument to be used in data gathering would be the structured individual interview as described by Romberg and Uprichard (1977). Various researchers were quoted in support of this decision. One of the most important of these was Marton (1988) whose description of phenomenographic research was particularly appropriate to the present study. The construction of this instrument has been fully described with an account of the six distinct categories of interview questions. Traditional considerations of reliability and validity of the data have been taken into account.

In the analysis of data the emphasis was to be on the generation of hypotheses which, in turn, could be tested for their validity and degree of possible generalisation to the population from which they were drawn. The results obtained from the implementation of this methodology appear in the following three chapters.
CHAPTER 8

Results I

Coding and Brief Discussion of
Responses to Questions

In this chapter the responses to all of the interview questions and a brief explanation of the coding used for all the predetermined questions are given. In some situations examples of relevant written and verbal protocols are supplied in order to clarify what is meant by this coding. The results in this chapter are presented in the sequence in which the interview questions were asked with a brief discussion of the responses to all items.

It was established in the last chapter that the qualitative methodology employed was designed to generate patterns of response which are then identified in order to compare and contrast the two groups of students. In this chapter patterns of responses are identified and comparisons are made between the two groups, the gamblers and the non-gamblers. A more detailed discussion and an analysis of the responses, with reference being made to the three major research questions posed, are given in the next chapter.

In the ensuing analysis responses will often be grouped according to pertinent categories: gambler (G), and non-gambler, (NG). Many patterns of response which are related to school achievement were used with data being separated into the categories of low achiever (LA), and high achiever, (HA). Generally speaking however, since the data showed no patterns of response that related to gender, it was decided that they would not be separated for discussion according to this criterion. In the next chapter, some examples of responses to questions are analysed according to gender in order to illustrate the absence of noticeable differences between male and female students.

The Samples

The composition of the samples, by background and school achievement (see Table 2), is in accord with the methodology described in Stage 2 of the previous
chapter. In that chapter we saw also that the samples were stratified according to
gender. However, as previously stated, since no major noticeable differences in
patterns of response according to gender were observed, this division is not included
in the presentation of data in the present chapter.

Table 2
Composition of Samples by Background and Achievement

<table>
<thead>
<tr>
<th>Group</th>
<th>High Achievement</th>
<th>Low Achievement</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gamblers</td>
<td>9</td>
<td>11</td>
<td>20</td>
</tr>
<tr>
<td>Non-gamblers</td>
<td>10</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Totals</td>
<td>19</td>
<td>21</td>
<td>40</td>
</tr>
</tbody>
</table>

The Coding of Predetermined Questions

In the last chapter on methodology, it was argued that the responses to the
predetermined written questions would be coded according to the nature of the
responses and the techniques employed in order to facilitate the identification of
patterns and the comparison of the two groups. A detailed explanation of this coding
is now given. Many of the techniques used by the students described here were used
throughout the interview. When this happened the terms used had the same meaning
throughout the interview. At the outset it was decided that terms that are in common
use, such as "use of a common denominator," and "conversion to decimal," did not
require further explanation. Techniques, expressions and other terms that are unique
to this study were defined or explained in the sequence in which they arose in the
interviews.

The Coding of the Students' Responses

The basic questions which were presented to the interviewees in Question 1 were
concerned with proportional reasoning in a traditional context.

These were:

Question 1(a), (i)
A photographic negative 2 cm wide × 3 cm long is enlarged to
a photograph 9 cm wide.
How long is the photograph?
Depending on the response given by the interviewee, the following probe would be conducted in the interview. If an incorrect answer was given to Question 1(a) (i), then Question 1(a) (ii) would be given to the interviewee.

*Question 1(a), (ii)*
A photographic negative 2 cm wide and 3 cm long is enlarged to a photograph 8 cm wide. How long is the photograph?

The question was then repeated using the same numbers in alternate contexts:

*Question 1(a), (iii)*
If 2 litres of juice costs $3, how much would 9 litres cost?

and:

*Question 1(a), (iv)*
If 4 kg of apples cost $7, how much would 5 kg cost?

**Explanation of Coding in Question 1(a)**

**Code 1. Traditional Strategies**
Any traditional, school-taught multiplicative method used correctly was included in this category.

For example, $4:7 = 5:x$

$4x = 35, \ x = \frac{35}{4} = 7 \frac{3}{4}$

**Code 2. A Scalar Additive Technique**
This refers to the correct additive technique which was described in the Chapter 5. This technique was used in a variety of contexts throughout the interviews. An example of its correct use in this situation would be:
4: 7 = 5: x ;
5 = 4+1
so we must "add" a "scaled" amount
4:7

\[ \frac{7}{4}; \]

So this extra "1" means we add \[ \frac{7}{4} \]

\[ x = 7 + \frac{7}{4} = 7 + 1\frac{3}{4} = 8\frac{3}{4} \]

**Code 3. A Double/Halve Strategy**

This is the "double/halve" strategy described by Hart (1984) in the literature. An example of its use in this context would be:

\[ 2 \times 3, 4 \times 6, 8 \times 12, 1 \times 1\frac{1}{2} \text{ so } 9 \times 13\frac{1}{2} \]

**Code 4. Unitary or Functional Strategy**

Hart (1984) has used the term "unitary" to mean the same technique that Acioly and Schliemann (1989) have called "functional." The strategy involved conversion to a "unit" comparison. Although this is also a "school taught" multiplicative strategy, it is sufficiently different in nature from those of Code 1 to be classed separately. A student using this strategy might write:

\[ 4 \text{ kg cost } \$7, \text{ so each kg cost } \frac{7}{4} = 1\frac{3}{4} = \$1.75 \]

\[ 5 \text{ kg cost } \$1.75 \times 5 = \$8.75 \]

**Code 5. Incorrect Traditional/Functional Approach**

In this category the student attempts a multiplicative approach but makes an incorrect comparison or arithmetical error to obtain an incorrect answer. A student using this strategy might write:
4.7 = 5?
so 4 × 7 = 5 × ?

\[ ? = \frac{28}{5} \]

*Code 6. Incorrect Double/Halve then Add*

In this case the student attempts to use the double/halve strategy but makes an incorrect "additive" adjustment. A student using this strategy might write:

\[ 2 \times 3, 4 \times 6, 8 \times 12, \text{so } 9 \times ? \]
\[ ? = 12 + 1 = 13 \]

*Code 7. Incorrect Additive Strategy*

This is the strategy described in the literature review. A student using this strategy might write:

\[ 2 \times 3 = 9 \times ? \]
\[ 3 = 2 + 1 \]
\[ \text{so } ? = 9 + 1 = 10 \]

*Code 8. Scalar Additive with Approximation*

This produces a correct or nearly correct result. A student using this strategy might write:

\[
4:7 \\
5:7 \text{ is one more than 4} \\
4:7 \\
1: >1 \text{ but } <2 \\
\text{So } 5: ? \\
? \text{ is } >8(7+1) \text{ but } <9(7+2) \\
? \text{ is approximately } 8.5
\]

Responses to Question 1(a) are summarised in Tables 3 - 5.
Table 3  
Coding and Results: Question 1(a), (i)

<table>
<thead>
<tr>
<th>Code</th>
<th>Group of Respondent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G-HA</td>
</tr>
<tr>
<td>1. Correct Traditional</td>
<td>6</td>
</tr>
<tr>
<td>3. Correct Scalar Additive</td>
<td>1</td>
</tr>
<tr>
<td>4. Correct Unitary/Functional</td>
<td>2</td>
</tr>
<tr>
<td>5. Incorrect Traditional</td>
<td>0</td>
</tr>
<tr>
<td>6. Incorrect Double/Halve</td>
<td>0</td>
</tr>
<tr>
<td>7. Incorrect Additive</td>
<td>0</td>
</tr>
<tr>
<td>Totals</td>
<td>9</td>
</tr>
<tr>
<td>Total Correct</td>
<td></td>
</tr>
</tbody>
</table>

Brief Discussion of Data From Question 1(a), (i)

The majority (32) of the students responded correctly to this question. Seven who responded incorrectly were low achievers. Of the eight who were incorrect, six used an incorrect additive approach, and six were gamblers. The eight who responded incorrectly were asked part (ii) in order to explore whether their performance improved when one number was a factor of the other. Four then answered correctly using a traditional algorithm, four others continued to use an incorrect additive strategy. All 40 students were asked parts (iii) and (iv).

Table 4  
Coding and Results: Question 1(a), (ii) and (iii)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1(a),(ii)</td>
</tr>
<tr>
<td>1-4. Correct Multiplicative</td>
<td>4</td>
</tr>
<tr>
<td>5-6. Incorrect Multiplicative</td>
<td>0</td>
</tr>
<tr>
<td>7. Incorrect Additive</td>
<td>4</td>
</tr>
<tr>
<td>10. Not asked</td>
<td>32</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
</tr>
</tbody>
</table>

Brief Discussion of the Data From Question 1(a), (ii) and (iii)

Only the eight students who answered incorrectly to Question 1(a) part (i) were asked to answer part (ii). Of these four answered part (ii) correctly. The four who continued to respond incorrectly were four of the "adders" who continued to use this technique in part (ii), while the other two "adders" corrected to a traditional (multiplicative) technique, thus demonstrating a "number dependence" by which the
choice of strategy changed when one number was a factor of the other. The use of these techniques by Year 11 students are consistent with the results of research into proportional reasoning among younger students by Hart (1984) who found that some adders were resistant to changing their strategy while others showed a similar number dependence.

The questions of parts (i) and (iii) are "parallel forms" of the same concept. Thirty-two students answered both parts correctly. (See Table 3, Codes 1 to 4, and Table 4, Code 1). From the spreadsheet data it can be shown that of these 32 students, 27 used the same strategy for both sections. Of the eight students who responded incorrectly, five used an incorrect additive strategy for both parts. This indicates a high degree of "internal reliability" of the questions as previously discussed in the methodology.

Table 5
Coding and Results: Question 1(a), (iv)

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>*1. Correct Traditional</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>23</td>
</tr>
<tr>
<td>*2. Correct Scalar Additive</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>*4. Correct Unitary/Functional</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>6. Incorrect Double/Halve add</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>7. Incorrect Additive</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>*8. Scalar Additive Approximation</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>9. Not Sure</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Totals:</td>
<td>11</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td>Total Correct *:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>29</td>
</tr>
<tr>
<td>(exact 26, satisfactory approximation 3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Brief Discussion of Data From Question 1(a), (iv)

In all parts of Question 1(a) the "adders" tended to use this incorrect strategy consistently and those using a correct multiplicative technique also did so consistently. This resistance of the adders to changing their strategy is consistent with the findings of research reported by Hart among younger pupils (1984, p. 6).

It can be seen that in part (iv), the number of students who continued to attempt a traditional multiplicative approach (Code 1) decreased to 23. However, one student used a scalar additive technique correctly, three used a scalar additive with approximation, and two continued to use a unitary approach. This resulted in a total of 29 acceptable responses, of which 26 were exact. It can be seen that the difference
between the numbers responding correctly to the four parts of Question 1(a) does not vary greatly between the four parts.

**Question 1(b).** This question was concerned with proportional reasoning in a gambling context. The results are presented in Tables 6 and 7.

**Question 1(b)**
In each of the following situations, how much can be won on a track bet if:
(i) $10 is bet at odds of 9:2  
(ii) $9 is bet at odds of 3:2  
(iii) $5 is bet at odds of 7:4

The responses to parts (i) and (ii) are discussed first.

**Table 6**
*Coding and Results: Question 1(b), (i) and (ii)*

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Responses</th>
<th>1(b) (i)</th>
<th>1(b) (ii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct Traditional</td>
<td></td>
<td>30</td>
<td>19</td>
</tr>
<tr>
<td>3. Correct Double/Halve</td>
<td></td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>4. Correct Unitary/Functional</td>
<td></td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>5. Incorrect-Traditional/Functional</td>
<td></td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>7. Incorrect Additive</td>
<td></td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Totals Correct</td>
<td></td>
<td>30</td>
<td>29</td>
</tr>
</tbody>
</table>

**Explanation of Coding in Question 1(b), (i) and (ii)**

**Code 4.** A correct unitary approach in this context involved calculating the return on each dollar at the given odds.

For example, 9:2 is the same as $4\frac{1}{2}$: 1

So for each dollar bet $4.50 is won

If $10 is bet, $10 \times $4.50 = $45 is won
**Brief Discussion of the Data From Question 1(b), (i) and (ii)**

In part (i), we note that all 35 students who responded correctly used a traditional school technique (Codes 1 and 4). Of the 5 incorrect, 2 were High Achievers, but only one of the five was a gambler. It can be seen that a greater number of students answered this item correctly than did in Question 1, parts (i) and (iii). This may be attributed to three of the low achieving gamblers who were unable to solve the proportion task in a traditional school context, but could nevertheless cope with a similar proportion task in a gambling context.

The results of part (ii) are similar to those of part (i) but six students misused a traditional multiplicative technique, and five students used an incorrect additive technique, resulting in 11 incorrect responses altogether. This is another demonstration of the "number dependence" in which respondents select a strategy depending on whether one number is a factor of the other. The students showed the same "avoidance of fractions other than halves" that Hart (1984, p. 6) reported. Of the 11 incorrect responses, eight students were low achievers and 4 were gamblers.

Of the six students who used a unitary approach, five were gamblers. This is not unexpected since the mathematical language used in the calculations in this technique (as illustrated in the explanation to Code 4 above) is consistent with the informal language of the track. For example, "9:2 gets you $4.50 for each dollar bet." This is discussed further in the next chapter in which the use of language is examined in some detail.

**Table 7**

*Coding and Results: Question 1(b), (iii)*

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>Group of Respondent</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>*1. Correct Traditional</td>
<td>3</td>
<td>4</td>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>*2. Correct Scalar Additive</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>*4. Correct Unitary/Functional</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>5. Incorrect Traditional/Functional</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>6. Double/Halve With Approximation</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>7. Incorrect Additive</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>*8. Scalar Additive Approximation</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td><strong>Totals:</strong></td>
<td>11</td>
<td>9</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

*Total correct: 29 (20 exact, 9 with satisfactory approximation)
Explanation of Coding in Question 1(b), (iii)

**Code 7. Incorrect Additive**
Those using this strategy in this context would reason:

\[
7:4 = ?:5 \\
7 = 4+3 \text{ so } ? = 5+3 = 8
\]

**Code 8.** Those using a scalar additive with approximation in this context would reason along the lines:

7:4 means bet 4 and get 7  
So if I bet 5, I must get more than 7  
To figure out how much more.  
5 - 4 = 1  
If 4 gets 7, then this extra 1 gets me more than 1 but less than 2.  
So 5 gets me more than 8 but less than 9  
Closer to 9 than 8,  
About $8.50, or $8.75, or $8.80.

**Brief Discussion of the Data From Question 1(b), (iii)**

Here 10 of the students who gave an exact responses were gamblers. However, when the satisfactory approximations (Codes 6 and 8) with answers between $8.50 and $8.80 are accepted, then 17 of the 29 correct were gamblers. Five of the seven students who used an incorrect additive strategy were non-gamblers, and of these five students, two were consistent "adders" who used the incorrect additive strategy in all proportional calculations.

Of the 17 gamblers who responded correctly to this item, nine used either the unitary strategy or a scalar additive strategy (Codes 2, 4 and 8). In these instances the informal language used by the gamblers in track situations was consistent with the mathematical language used in describing the process and its symbolic representation. These results are discussed further in the next chapter.
The comparison of fractions in a traditional out-of-context situation.

Question 2
In each pair, circle the larger of the two fractions:

(a) \( \frac{1}{2} \) \( \frac{1}{3} \)
(b) \( \frac{3}{8} \) \( \frac{5}{8} \)
(c) \( \frac{1}{2} \) \( \frac{3}{5} \)
(d) \( \frac{3}{5} \) \( \frac{5}{8} \)

The results of Question 2 are presented in Tables 8 to 10.

Table 8
Coding and results: Question 2(a) and (b)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
</tr>
<tr>
<td>1. Correct: Any Method</td>
<td>38</td>
</tr>
<tr>
<td>2. Correct but Unsure</td>
<td>2</td>
</tr>
<tr>
<td>3. Incorrect</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 9
Coding and results: Question 2(c)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct: 1/2 of 5 is 2 1/2, which is &lt;3</td>
<td>22</td>
</tr>
<tr>
<td>2. Correct: Converts to Common Denominator</td>
<td>9</td>
</tr>
<tr>
<td>3. Correct: Convert to Decimal, 0.6 &gt; 0.5</td>
<td>6</td>
</tr>
<tr>
<td>4. Incorrect: Incorrect Generalisation</td>
<td></td>
</tr>
<tr>
<td>Compares Denominators, Halves &gt; Fifths</td>
<td>2</td>
</tr>
<tr>
<td>5. Incorrect: Guess</td>
<td>1</td>
</tr>
<tr>
<td>Total Correct</td>
<td>37</td>
</tr>
</tbody>
</table>
Brief Discussion of the Data From Question 2(a), (b) and (c)

Parts (a), (b) and (c) proved to be somewhat trivial. All students answered part (a) correctly, all but one answered part (b) correctly, and all but three answered part (c) correctly. Those who were unsure in parts (a) and (b) were asked to explain the basis of their decisions, which in both cases was a response of "It just is (the bigger)." The data from Question 2(d) proved to be more informative.

Table 10
Coding and Results: Question 2(d)

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct, Common Denominator</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>2. Correct, Converts to Decimal</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3. Correct but Incorrect Reasoning</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Guesses or 5/8 &quot;Bigger&quot; Number</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4. Incorrect, Equal</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>5. Incorrect, Ignores Numerator</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>6. Incorrect, Other/No Attempt</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Totals:</td>
<td>11</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>40</td>
</tr>
</tbody>
</table>

Total Correct: 32; Correct reasoning: 21 (14 High Achievers, 9 Gamblers)

Explanation of Coding of Question 2(d)

Code 3. This refers to those who guessed correctly, but when probed admitted that they did not know why 5/8 was bigger. Some of these replied that it "looks bigger" or has "bigger numbers" than 3/5.

Code 5. This refers to those who ignored the numerator and incorrectly reasoned "fifths are bigger than eighths."

Brief Discussion of the Data From Question 2(d)

Of the 21 students who answered correctly, 14 were high achievers indicating an expected relationship with school achievement. Not surprisingly there was no apparent relationship with gambling background. However in the more detailed analysis of the next chapter, the responses of those who used a correct strategy were examined to determine whether their strategy was used consistently in other contexts.

Question 3. This question is concerned with the quantification of simple probabilities in traditional classroom contexts followed by comparison of these probabilities. The results are presented in Tables 11 to 14.
Question 3
Suppose I select a marble at random from each of the following bags [diagram shown]:
In each of the two situations what is the probability of getting a black marble?
(a) The bag contains 3 black and 1 white
(b) The bag contains 5 black and 3 white
(c) In which of these cases, (a) or (b), is it more likely to get a black marble?

Table 11
Coding and Results; Question 3(a) and (b)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
</tr>
<tr>
<td>1. Correct</td>
<td>36</td>
</tr>
<tr>
<td>2. Incorrect</td>
<td>2</td>
</tr>
<tr>
<td>3. Unable to Attempt</td>
<td>2</td>
</tr>
</tbody>
</table>

Brief Discussion of the Data From Question 3(a) and (b)
Thirty six students quantified simple probabilities using reasoning that is equivalent to the formal definition of "number of successful ways divided by the total number of ways." Of the four incorrect/don't know responses, three were non-gamblers.

Table 12
Coding and Results; Question 3(c)

<table>
<thead>
<tr>
<th>Code</th>
<th>Group of Respondent</th>
</tr>
</thead>
<tbody>
<tr>
<td>G-LA</td>
<td>G-HA</td>
</tr>
<tr>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>1. Correct, Common Denominator</td>
<td>3</td>
</tr>
<tr>
<td>2. Correct, Decimal Conversion</td>
<td>0</td>
</tr>
<tr>
<td>4. Incorrect, Additive Comparison</td>
<td>1</td>
</tr>
<tr>
<td>6. Unable/No Attempt</td>
<td>0</td>
</tr>
<tr>
<td>7. Not Sure</td>
<td>6</td>
</tr>
<tr>
<td>8. Not Applicable</td>
<td>0</td>
</tr>
<tr>
<td>(Parts A or B Incorrect)</td>
<td>1</td>
</tr>
</tbody>
</table>

138
Explanation of Coding in Question 3(c)

Many of the terms used in the above coding have been explained in the coding of Question 1, and they are used with the same meaning here.

Code 4. Incorrect Additive Comparison

The six students who used this strategy in this context used an additive comparison, reasoning in two different ways:

1. (3 black, 1 white): The probability is 3 chances in 4,
   (5 black, 3 white): The probability is 5 chances in 8, since $8 = 5 + 3$;
   A greater amount has been "added" in the latter case, so this is the "greater" proportion (probability), or,

2. $3 = 1 + 2$ and $5 = 3 + 2$;
   In each, two has been added. Or the difference in each case is two.
   Therefore the probabilities are the same.

Brief Discussion of the Data From Question 3(c)

Twenty of the interviewees answered correctly. Of these, 10 were gamblers, and 13 were high achievers. Thus there would appear to be no relationship between ability to compare probabilities in this context and gambling background. Not unexpectedly, there is again some relationship with school achievement. Of those students responding correctly, 17 used a traditional strategy taught in school. Examination of the spreadsheet data reveals that most of the students responding correctly used the same strategies consistently, regardless of context, and it would appear that it is this consistent usage that resulted in the relationship between the ability to answer correctly and school achievement.

Depending on the response given by an interviewee to parts (a), (b) and (c), the following probe would be conducted in the interview.

Question 3(d)

(if unable to compare the probabilities in Question 3(c))
Repeat Question 3(c) with:
(i) (a) 1 black and 3 white and
    (b) 1 black and 4 white;
(ii) (a) 2 black and 3 white
    (b) 2 black and 5 white,
Question 3(c)  
[if able to compare the probabilities in Question 3(c)]

Question 3(c) was repeated using:  
3 black marbles and 2 white marbles in one bag, and 5 black  
marbles and 3 white marbles in the other bag.

Table 13  
**Coding and Results: Question 3(d)**

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct, Both Parts</td>
<td>15</td>
</tr>
<tr>
<td>2. Incorrect Both Parts</td>
<td>2</td>
</tr>
<tr>
<td>3. Incorrect Part (ii)</td>
<td>3</td>
</tr>
<tr>
<td>4. Not Applicable</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>[Able to Answer Question 3(c) Correctly,</td>
<td></td>
</tr>
<tr>
<td>or Unable to Answer Parts (a) and (b)]</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>40</strong></td>
</tr>
</tbody>
</table>

Table 14  
**Coding and Result: Question 3(e)**

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct Common Denominator</td>
<td>14</td>
</tr>
<tr>
<td>2. Correct Decimal Conversion</td>
<td>3</td>
</tr>
<tr>
<td>6. Unable/No Attempt</td>
<td>1</td>
</tr>
<tr>
<td>8. Not Asked (Q3(a), (b), or (c) Incorrect)</td>
<td>21</td>
</tr>
<tr>
<td>9. Not Sure/About the Same</td>
<td>1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>40</strong></td>
</tr>
</tbody>
</table>

**Brief Discussion of the Data From Question 3(d) and (e)**

Twenty students were asked Question 3(d) part (i). The 18 who answered correctly were then asked part (ii) and 15 answered this correctly. Of these, 10 were gamblers.

Eighteen students were asked Question 3(e) and 17 answered correctly. Of these nine were gamblers. Thus, once again, there would appear to be no noticeable relationship between the ability to compare probabilities in this context and gambling background. Not unexpectedly, there was again some relationship with school achievement. Of those students who responded correctly, all 17 used a school taught strategy. Examination of the spreadsheet data reveals that most of these used the school taught strategies consistently, regardless of context.
Questions 4 and 5. These questions are concerned with the comparison of odds in a gambling context. Informal interview questions explored each student's understanding of the relationship between betting "odds" and probability. These results are reported in the next chapter, where this relationship is discussed further. The results of the responses to Questions 4 and 5 appear in Tables 15 to 17.

Question 4(a), (b) and (c)
In each of the following track betting situations, which is the better of the two odds? That is to say, which gives the greater return per $ bet.
(a) 2:1 or 3:1
(b) 5:1 or 5:2
(c) 4:3 or 9:7

If part (c) was incorrect, the following probe would be conducted in the interview.

Question 4 (d)
In the following track betting situations, which is the better of the two odds?
That is to say which gives the greater return per $ bet.

5:2 or 9:4

Table 15
Coding and Results: Question 4(a) and (b)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents (a)</th>
<th>Number of Respondents (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>39</td>
<td>37</td>
</tr>
<tr>
<td>2. Incorrect</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>
Table 16
Coding and Results: Question 4(c)

<table>
<thead>
<tr>
<th>Code</th>
<th>Group of Respondent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G-LA</td>
</tr>
<tr>
<td>*1. Correct Common Denominator</td>
<td>3</td>
</tr>
<tr>
<td>*2. Correct Scalar Additive</td>
<td>0</td>
</tr>
<tr>
<td>*3. Correct Decimal Conversion</td>
<td>0</td>
</tr>
<tr>
<td>*4. Constructs Algorithm;</td>
<td></td>
</tr>
<tr>
<td>Less Outlay for $2 Win</td>
<td>3</td>
</tr>
<tr>
<td>*5. Correct, Functional/Unitary</td>
<td>1</td>
</tr>
<tr>
<td>*6. Scalar Additive with Approx.</td>
<td>3</td>
</tr>
<tr>
<td>7. Incorrect Additive</td>
<td>0</td>
</tr>
<tr>
<td>8. Incorrect Multiplicative</td>
<td>0</td>
</tr>
<tr>
<td>9. No Attempt</td>
<td>1</td>
</tr>
<tr>
<td>10. Not Applicable</td>
<td>0</td>
</tr>
<tr>
<td>Totals:</td>
<td>11</td>
</tr>
</tbody>
</table>

*Total Correct: 25 (18Gamblers, 11 High Achievers)

Results From Question 4(d)
This question was asked only to those who were unable to answer Question 4(c) in order to determine whether a "number dependence" was present.

The results to Question 4(d) were not coded. By referring to the raw data we find that of the 19 students, 6 gamblers and 13 non-gamblers, 7 were able to answer Question 4(d). Of these only two, both low achievers, were gamblers.

Explanations of Coding in Question 4

In this situation, both odds are converted to a common comparison. Reasoning would be:

4:3 is the same as 28:21
9:7 is the same as 27:21
So 4:3 is the greater ratio, or is therefore the better odds.

Reasoning in this situation was along the lines:
4:3 is the same as 8:6
To compare 8:6 with 9:7,
8:6 is 9.9

1: \frac{8}{6}

9: \frac{6}{6} + 1\frac{2}{6}

9: \frac{2}{6} which is greater than 9.7

**Code 3. Decimal Conversion (calculator used)**

4:3 is 1.3333... :1
9.7 is 1.2222... :1
So 4:3 is the greater ratio

**Code 4. A Context Specific Algorithm Constructed**
The reasoning used in this situation was:

4:3 is the same as 8:6,
This means bet 6 win 8, or 2 more than you bet;
9:7 means bet 7 win 9, again 2 more than you bet.
It is better to win the same amount more for a lower outlay, so 4:3 are the better odds. (This reasoning is discussed in greater detail in the next chapter).

**Code 5. Correct Functional or Unitary Strategy.**

In this situation the reasoning used was:

4:3 is 1 \frac{1}{3} :1

9:7 is 1 \frac{2}{9} :1
Code 6. Approximate Scalar Additive

The reasoning employed is the same as for code 2, but the final step was reasoned:

8:6 is > 1:1
So to compare 8: with 9:7;
8:6 is 9:?
8+1 : 6+ (> 1)
9: > 7 which is greater than 9:7


Those using this strategy in this context would tend to reason:

4.3 9:7
4 = 3+1 9 = 7 + 2
"Four is one more than three, nine is two more than 7, so 9:7 is the greater proportion"

Brief Discussion of the Data From Questions 4(a) to (d)

In these questions the gamblers performed noticeably better than the non-gamblers. In doing so, they demonstrated both an intuitive understanding of the concept of "odds" and, in processing concepts, they demonstrated an ability to construct their own context-specific strategies. These results are discussed further in the next chapter where they provide important data relating to the major research questions. Surprisingly, only two students used decimal conversion to compare odds whereas eight used this strategy to compare fractions (See Table 10).

Question 5

In a four horse race the odds for each horse are given as:
2:1, 5:3, 5:1, and 25:1
Which horse is thought to be most likely to win?
List the odds in order of least likely to most likely.
Table 17

Coding and Results: Question 5

<table>
<thead>
<tr>
<th>Code</th>
<th>Group of Respondent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G-LA</td>
</tr>
<tr>
<td>1. Correct, Common Denominator</td>
<td>4</td>
</tr>
<tr>
<td>or Equivalent Fraction</td>
<td></td>
</tr>
<tr>
<td>2. Correct, Intuitive</td>
<td>6</td>
</tr>
<tr>
<td>3. Correct, Functional/Unitary</td>
<td>1</td>
</tr>
<tr>
<td>4. Unable</td>
<td>0</td>
</tr>
<tr>
<td>5 Incorrect, Additive</td>
<td>0</td>
</tr>
<tr>
<td>Totals:</td>
<td>11</td>
</tr>
<tr>
<td>Total Correct: 30 (All 20 Gamblers and 10 Non-Gamblers)</td>
<td></td>
</tr>
</tbody>
</table>

Explanation of Coding in Question 5

Code 1. Correct Common Denominator or Correct Equivalent Ratio. An example of this kind of reasoning would be:

2:1 is equivalent to 6:3. This is a greater ratio, or "better odds" than 5:3.

Code 2. Correct Intuitive. In this context, the student recognises that 2:1 is greater than 5:3, but is unable to give a mathematically correct reason. In this case, several probing questions would then be asked. A typical response of the eight gamblers in this category was "2:1 are higher odds because you get more than at 5:3". With further probing questions such as "Why do you get more?" this response could in some instances be reclassified as Code 1 (above), or Code 3 (below). However, it was decided to leave those who responded "intuitively" initially in this category.

Code 3. Correct Functional/Unitary. Those using this strategy in this context convert the ratio to a unit comparison. For example, the gamblers might reason "2:1 give $2 for each dollar bet, 5:3 gives about $1.60 (or simply less than $2) for each dollar bet."

Code 5. Incorrect Additive. In this context the reasoning employed is that of an incorrect additive strategy: For example,

2 = 1 + 1, 5 = 3 + 2,
so the smaller amount added is in 2:1
This is therefore the smaller ratio.
Brief Discussion of the Data From Question 5

All 20 gamblers answered this question correctly. Eight of them used an
"intuitive" initial response, though with probing it was established that two of them
converted to a unitary comparison, and three used an equivalent proportion. Three
could not give a reason, saying "I just know it's (5:3) the shortest odds." An
examination of the spreadsheet data to determine the consistency of the use of each
technique is discussed in the next chapter.

Probing interview questions revealed that the gamblers, in general, had a good
understanding of the relationship between odds and probability in that they were all
able to put the odds correctly in the order of most likely to least likely. Only 8 of the
non-gamblers were able to do this. Examples of the reasoning employed are provided
in the case study transcripts in Chapter 11.

Questions 6 and 7. The analysis of the data from Questions 6 and 7 is concerned
with the prevalence of misconceptions in probabilistic reasoning. Much data were
obtained from the responses to these questions and from the subsequent probing
questions. While a detailed analysis and discussion of these data are given in the next
chapter, the summary of the responses and coding used are provided in Tables 18-20.

Question 6(a)
In the last 200 Gold Lotto draws the number 17 has occurred
more times than any other number. In future draws do you
think that this number is:
(i) more likely to occur than any other number?
(ii) less likely to occur than any other number?
(iii) as equally likely to occur as any other number?

Table 18
Coding and Results; Question 6(a)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Equal (iii)</td>
<td>33</td>
</tr>
<tr>
<td>2. More Likely (i)</td>
<td>3</td>
</tr>
<tr>
<td>4. Less Likely (ii)</td>
<td>2</td>
</tr>
<tr>
<td>5. No Idea/No Attempt</td>
<td>2</td>
</tr>
</tbody>
</table>
Brief Discussion of Data From Question 6(a)

The two interviewees who did not answer this question were non-gamblers who claimed no familiarity with Lotto. Of the remaining students, only two thought that the number was now more likely in order to "balance out the long term," and only three took the short term frequency of occurrence as "representative" of the long term, and concluded that 17 was would "always be more likely."

Question 6(b)

[Describe/show roulette wheel]
In Roulette, each number is equally likely to occur. Suppose that the ball lands on "red" six times in a row. On the next roll is it now more likely to land on the "red", the "black", or are they both still equally likely?

Table 19
Coding and Results: Question 6(b)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Equal</td>
<td>33</td>
</tr>
<tr>
<td>2. Don't Know/Not Sure</td>
<td>2</td>
</tr>
<tr>
<td>3. Red</td>
<td>3</td>
</tr>
<tr>
<td>4. Black</td>
<td>2</td>
</tr>
</tbody>
</table>

Brief Discussion of Data From Question 6(b)

The results of this item parallel those of the previous item. Those who used the representativeness heuristic in the previous question again did so for this question, concluding that the red was either more likely or less likely. Only one student who was not sure in the first part had a different response in the different context, now responding "equally likely." In questions such as these it is not possible to make comparisons by background or achievement due to the small numbers demonstrating the use of either of the heuristics described. However, the consistency of responses on this item to those on the previous one lend a measure of internal reliability to the data.

The misuse of the availability heuristic in the context of Lotto. This was examined in Questions 6(c) and (d).
Question 6(c)
In some lotteries the tickets are pre-numbered.
If you could choose between any of the following numbers would you have any preference?
123456
619999
615472

Question 6(d)
In choosing six LOTTO numbers which of the following selections would you prefer?
(a) 1, 2, 3, 4, 5, 6
(b) 32, 33, 34, 35, 36, 37
(c) 9, 12, 27, 31, 35, 38

Table 20
Coding and Results; Question 6(c) and (d)

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. No Use of Availability</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>19</td>
</tr>
<tr>
<td>2. Used Availability for Only One</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>3. Used Availability for Both</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>4. Unable to Answer</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Totals:</td>
<td>11</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>40</td>
</tr>
</tbody>
</table>

Brief Discussion of Data From Question 6(c) and (d)
The use of the availability heuristic is presumed in situations such as this when the respondent rejects numbers with sequences or patterns since occurrences of such numbers are not as easy to recall as "random" numbers. Nineteen (about 50%) of the students showed no use of availability in this context. All of the 10 students (25%) who used the heuristic in 6(c) did so again in 6(d). A further 9 students who did not use the heuristic in 6(c), used it in 6(d), resulting in a total of 19 students (approximately 50%) showing use in at least one situation.

Of the 10 students who used the availability heuristic in both situations, six were gamblers and five were high achievers. This indicates that the use of this heuristic did not relate noticeably to gambling background or school achievement.

Question 7. This question was concerned with the same misconceptions in alternate contexts.
Question 7(a)
If I throw a single fair die is it harder (less likely) to get a six than, say a four?
In general, are the numbers 1 to 6 equally likely to occur, or are some harder to get than others?

Question 7(b)
If I throw a pair of fair dice is it harder to get a pair of sixes than it is to get a pair of twos?

Question 7(c)
A poker hand consists of five cards dealt from a well shuffled deck. The Ace is the highest card. Is it harder (that is to say less likely) to get three Aces than it is to get three fives?

Table 21
Codi ng and Results: Question 7(a), (b), (c)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
</tr>
<tr>
<td>1. Correct</td>
<td>37</td>
</tr>
<tr>
<td>2. Incorrect</td>
<td>1</td>
</tr>
<tr>
<td>3. No Idea/Not Able to Answer</td>
<td>2</td>
</tr>
</tbody>
</table>

Brief Discussion of Data From Question 7(a), (b) and (c)
In these simpler or possibly more familiar contexts, the misconceptions were much less frequent, with only six students who demonstrated any misconception.

Question 7(d) examined the use of the representativeness heuristic in the determination of the concept of "fairness."

Question 7(d)
Suppose that you and I are going to play a game with this coin [shows] in which we toss the coin just once. If the coin lands heads, I win; if it lands tails you win. But I will warn you that of the last 15 people who played this game with me only six won.
Do you think this is unusual?
Do you think the coin is not fair?
Do you think the game is fair?
Table 22
Coding and Results: Question 7(d)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G</td>
</tr>
<tr>
<td>1. Basic Concept of Fairness Evident</td>
<td>18</td>
</tr>
<tr>
<td>2. Concludes Unfairness</td>
<td>2</td>
</tr>
<tr>
<td>3. Unable to Answer</td>
<td>0</td>
</tr>
</tbody>
</table>

Explanation of Coding in Question 7(d)

Code 1. In this situation the student responded that there is no reason to suspect unfairness with these numbers.

Code 2. The student uses the short term frequency of "heads" as "representative" of the long term frequency and concludes that the coin is not fair.

Brief Discussion of Data From Question 7(d)

Thirty-three interviewees thought both coin and game were fair, responding typically that "there is nothing unusual with this." Of the seven who responded differently, all seven thought the situation "unusual" and believed that the coin, and consequently the game, was not fair. None was willing to state conclusively that the coin could not be fair. Further probing questions and their responses are reported and analysed in more detail in the next chapter.

Questions 8 and 9. These questions were concerned with the concept of compound probability in a variety of contexts.

Question 8
A robot is put into a maze which it begins to explore
At each junction it is as likely to go down any path as any other. Where is it most likely to end up?
Table 23
Coding and Results; Question 8

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Uses Compound Probability, Multiplication Principle</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2. Shortest Path</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3. Not Sure, but Suspects AB</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4. Equally Likely</td>
<td>7</td>
<td>6</td>
<td>7</td>
<td>7</td>
<td>27</td>
</tr>
<tr>
<td>5. Reverses Order-AB Least Likely</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6. No Idea/Other</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Totals</td>
<td>11</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>40</td>
</tr>
</tbody>
</table>

Explanation of Coding in Question 8

**Code 1.** This response implies that the student recognised that with each compounding of probability, the overall probability reduces by the multiplication of the individual probabilities.

**Code 2.** The student selected the shortest path as the most likely without referring to the concept of compound probability at all.

**Code 4.** The student responded that all paths were equally likely.

Brief Discussion Data From Question 8

This question was the same as that asked by Green (1983a) who reported that, of a large sample of students aged from 11 to 16 years, only 13% responded correctly and that 60% responded "equally likely." Furthermore, Green reported that there was little difference between the results with either age or grade.

Of the nine who answered correctly in this study only one used reasoning that demonstrated an understanding of the basic principle of compound probability that the likelihood decreases with each compounding. The others simply reasoned "the shortest path is the most likely."

A surprising 27 students (approximately 70%) thought all paths were equally likely. This result is discussed further in the more detailed analysis of the understanding of compound probability in the next Chapter 10.
Other situations involving compound probability. Question 9 was concerned with two situations, one in the context of track gambling, and the other in the context of tossing coins.

Question 9(a)
In each of two horse races the favourite is estimated to have about a 50% chance of winning. If you bet on both favourites, your chances of winning on both would be about:
(i) 100%
(ii) 75%
(iii) 50%
(iv) 25%
(v) 10%

Table 24
Coding and Results; Question 9(a)

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 25% : Correct Multiplicative</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>2. 25% : Uses Intuitive Reasoning</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>3. 100% : Adds 50%+50%</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>4. 75% : Probability Increases</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>5. 50% : Unchanged</td>
<td>6</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>21</td>
</tr>
<tr>
<td>6. Unable to Answer/Don’t Know</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Totals:</td>
<td>11</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>40</td>
</tr>
</tbody>
</table>

Brief Discussion of Data From Question 9(a)
Six gamblers demonstrated an intuitive understanding of the concept of compound probability in so far as they recognised that the compounding of the individual probabilities must reduce the overall probability. However, they all lacked any formal technique of performing the calculation.

Most (21 students) thought that the probability remained unchanged at 50%, and only five were able to demonstrate any technique for calculating the compound probability involved. Each of these five students was a high achiever.
**Question 9(b)**

Two coins are tossed together.

(i) The probability that one will land a head and the other a tail is:

(a) 1/2  
(b) 1/3  
(c) 1/4

(ii) What is the probability that at least one of the two coins will land a head?

**Table 25(a)**

*Coding and Results: Question 9(b), (i)*

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct, Counts Cases, 2 Out of 4</td>
<td>10</td>
</tr>
<tr>
<td>2. Correct, Same or Different Equally Likely</td>
<td>17</td>
</tr>
<tr>
<td>3. Correct, Uses Tree Diagram</td>
<td>2</td>
</tr>
<tr>
<td>4. Unable to Answer</td>
<td>3</td>
</tr>
<tr>
<td>5. 1/3</td>
<td>6</td>
</tr>
<tr>
<td>6. 1/4</td>
<td>2</td>
</tr>
</tbody>
</table>

*Explanation of Coding in Question 9(b) (i)*

*Code 1.* This response involves the correct identification of a four equally likely possible outcomes, of which two contain one head and one tail.

*Code 2.* The reasoning employed here is that there are two equally likely outcomes - the same or different.

*Code 3.* A tree diagram showing the four outcomes was drawn.

*Code 5.* The reasoning employs involves an incorrect sample space of three equally likely sample points - HH, TT, HT.

*Brief Discussion of Data From Question 9(b), (i)*

Although 29 students answered correctly, responses to interview questions showed that many of those selecting Code 2 were not sure of their reasoning. Only a total of 12 students demonstrated an understanding of the compounding of probabilities, and of these, nine were gamblers.
Results for Question 9(b), (ii)

Not coded, from the raw data (only those responding correctly to the first part of this question were asked).

Table 25(b)
Results of Question 9(b), (ii)

<table>
<thead>
<tr>
<th>Response</th>
<th>Number Giving This Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/4</td>
<td>3</td>
</tr>
<tr>
<td>2/3</td>
<td>4</td>
</tr>
<tr>
<td>1/2</td>
<td>15</td>
</tr>
<tr>
<td>1/3</td>
<td>4</td>
</tr>
<tr>
<td>Other</td>
<td>3</td>
</tr>
</tbody>
</table>

Brief Discussion of Data From Question 9(b), (ii)

Only three of the 29 students probed were able to answer correctly. The majority, 15 students, continued to reason "equally likely same or different" to conclude a probability of 1/2. Four now answered 1/3 using an equally likely sample space of three.

These results are similar to, though marginally better than, those reported by Brown et al. (1988, p. 242) who stated that only 5% of year 11 students answered this same item correctly, whereas 70% selected the option of "1 in 2."

Interview questions ensured that the respondents did not misinterpret the question and that the term "at least one head" was understood.

Question 9(c)
(i) A pair of fair dice are tossed together
[show pair of identical dice]
Which is more likely:
(a) a pair of fives
(b) a five and a three
(c) both are equally likely

Table 26(a)
Coding and Results: Question 9(c), (i)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct, Reasons From Experience</td>
<td>6</td>
</tr>
<tr>
<td>2. Correct, Uses Tree Diagram or Counts Cases</td>
<td>4</td>
</tr>
<tr>
<td>3. Equal</td>
<td>27</td>
</tr>
<tr>
<td>4. No Idea How to Approach the Problem</td>
<td>3</td>
</tr>
</tbody>
</table>
Explanation of Coding in Question 9(c), (i)

Code 1. Those who gave this reason cited experience with dice games, generally reasoning from the general case that "it is harder to get doubles."

Code 2. Constructed a tree diagram or enumerated the different cases to arrive at the correct answer.

Brief Discussion of Data From Question 9(c), (i)

The majority, 27 students, thought the two outcomes were equally likely. All of the 17 who used the "same or different equally likely" reasoning to Question 9(b), (i) applied this reasoning (incorrectly) to Question 9(c). Each of the six who reasoned from experience was a gambler, resulting in an overall better performance of the gamblers on this item (8 of the 10 correct).

The following question was asked of those who had stated that getting a pair of fives had the same probability as getting a five and a three.

Question 9(c) (ii)
A pair of fair dice are tossed together.
[show pair of different coloured dice]
Which is more likely:
(a) a pair of fives
(b) a five and a three
(c) both are equally likely

Results of Question 9(c), (ii) (not coded, from raw data)

Table 26(b)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>11</td>
</tr>
<tr>
<td>2. Still Equal</td>
<td>26</td>
</tr>
<tr>
<td>4. Not Asked</td>
<td>3</td>
</tr>
</tbody>
</table>

Brief Discussion of Data from Question 9(c), (ii)

Only one student reconsidered and answered correctly. The remainder demonstrated resistance to changing their answer. This is consistent with the results of research by Fischbein et al. (1991, p. 533) who reported that "this bias is very resistant." The following probe was then conducted in the interview:
Question 9(c) (iii)
In general, is it harder (less likely) to get a pair of numbers than it is to get two different numbers?

Table 26(c)
Results of Question 9(c), (iii) (not coded, from raw data)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Yes</td>
<td>14</td>
</tr>
<tr>
<td>2. No</td>
<td>22</td>
</tr>
<tr>
<td>3. Don't Know</td>
<td>4</td>
</tr>
</tbody>
</table>

Brief Discussion of Question 9(c), (iii)
All six who reasoned from experience in Question 9(c), (i) answered this correctly. Overall, more were able to answer the general case correctly (14 students) than were able to answer the specific case correctly (10 students). These results are consistent with the results of other research reported by Fischbein et al. (1991):

At all age levels ... the percentages of correct answers are visibly higher for the generalised form of the (dice) question than for the specific one ... the higher percentage of correct answers for the generalised form is not related to the equality or non-equality of the probabilities. (p. 537)

This is discussed further in the analysis of the understanding of compound probability in the next chapter.

The concepts of fairness and expectation, or expected return. Much data were obtained from the parts of Question 10 and subsequent probing and digressing questions. These data are discussed in depth in the following chapter. The coding used and results of the predetermined questions are presented here.

Question 10(a)
Suppose you and I play a game with one of these dice [show single die]. If on a single roll it is three or less you win, if it is more than three I win. If we each bet $1 and the winner gets the $2, would this be a fair game?
Table 27
*Coding and Results: Question 10(a)*

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct: Recognises Equal Probability</td>
<td>38</td>
</tr>
<tr>
<td>2. Incorrect</td>
<td>0</td>
</tr>
<tr>
<td>3. Unable to Answer</td>
<td>2</td>
</tr>
</tbody>
</table>

*Brief Discussion of Data From Question 10(a)*

Only two students, both non-gamblers answered this question incorrectly.

The data from Question 10(b) relate to the concept of mathematical fairness.

*Question 10(b), (i)*

Suppose we change the rules so that I win if it is a three or more. Is this still a fair game?

Table 28
*Coding and Results: Question 10(b), (i)*

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>27</td>
</tr>
<tr>
<td>2. Incorrect: Does Not Use Probability to Answer</td>
<td>7</td>
</tr>
<tr>
<td>3. Unable to Answer</td>
<td>6</td>
</tr>
</tbody>
</table>

*Explanation of Coding in Question 10(b), (i)*

*Code 1.* A correct response in this situation involved the recognition of unequal probabilities.

*Code 2.* An incorrect response did not recognise that the probabilities were now unequal.

*Brief Discussion of Data From Question 10(b), (i)*

Of the 27 who responded correctly, 17 were gamblers. The relationship between students' background and the response to this question and the subsequent probes are discussed in detail in the next chapter. Depending on the response given by the interviewee, the following probe would be conducted in the interview. The 27 who recognised unfairness were asked:

*Question 10(b), (ii)*

Can we make the game fair somehow?
Results and Brief Discussion of Data From Question 10(b), (ii)

Of the 27 who recognised that the game was no longer fair, 19 demonstrated some intuitive knowledge of expectation by recognising that the game could still be made "fair" by changing the contributions of the players. This is analysed in more detail in the next chapter.

Simple permutation and combination concepts. Question 11 examined the understanding of these concepts in both traditional classroom contexts and out of school contexts. Question 11(a) required the calculation of a simple permutation of two things (horses).

Question 11 (a)
The "double" in horse racing consists of selecting two winners in two prescribed races. How many possible ways can the "double" occur if one race has 10 horses and the other 8? (Exclude any "dead heats")

Table 29
Coding and Results; Question 11(a)

<table>
<thead>
<tr>
<th>Code</th>
<th>Group of Respondent</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>G-LA</td>
<td>7</td>
<td>9</td>
<td>3</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>2. No Attempt</td>
<td>G-HA</td>
<td>2</td>
<td>0</td>
<td>5</td>
<td>6</td>
<td>13</td>
</tr>
<tr>
<td>3. Incorrect, Additive</td>
<td>NG-HA</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>4. Incorrect, Other</td>
<td>NG-LA</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td>11</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>40</td>
</tr>
</tbody>
</table>

Explanation of Coding in Question 11(a)

Code 1. A correct multiplicative procedure was employed, $8 \times 10 = 80$.

Code 3. An incorrect additive technique in this situation involved $10 + 8 = 18$.

Brief Discussion of Data From Question 11(a)

Even for this relatively simple question only half of the students answered correctly. Of these 16 were gamblers, and 12 were high achievers. Eleven non-gamblers and two gamblers did not attempt to answer the question, even when the context was explained. Responses given to informal probing questions confirmed that
the gamblers who answered correctly were familiar with betting on "the double," and this familiarity helped them to answer correctly.

*Question 11(b)*

If there are 6 horses in a race how many ways can the first and second places be filled? (Exclude any "dead heats")

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct, 6x5</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2. Correct, Enumerates Cases</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>3. Incorrect, 3x6</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>4. Enumerates Incorrectly</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>5. Unable to Attempt</td>
<td>5</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>6. Incorrect Additive</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Totals</td>
<td>11</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>40</td>
</tr>
</tbody>
</table>

*Explanation of Coding in Question 11(b)*

*Code 1.* A correct multiplicative procedure was employed.

*Code 2.* The thirty cases were enumerated correctly.

*Code 3.* A multiplicative procedure is employed, but 6x6 was calculated.

*Code 4.* In this situation the respondents attempted to enumerate all cases, but generally became confused or missed some of the possibilities.

*Code 6.* An incorrect additive procedure was used (for example, 6 + 5, or 6 + 6).

*Brief Discussion of Data From Question 11(b)*

Only 13 students (eight gamblers, 11 high achievers) answered this fairly simple permutation problem correctly. Ten did not attempt an answer, and five used an incorrect additive procedure. This apparently poor result is discussed in detail in the analysis of the next chapter.

If the response to Question 11(b) was correct, the following probe was conducted in the interview, the purpose being to explore the student's understanding of combinations.
**Question 11(c)**

[To be given to students who were able to answer 11(a) and (b)]
If the "quinella" consists of selecting the first two horses in a race without necessarily being in the right order, how many possible selections are there in a race with 8 horses.
[If the student is unable to answer this, proceed to 11(f)]

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>4</td>
</tr>
<tr>
<td>2. Incorrect</td>
<td>9</td>
</tr>
<tr>
<td>3. Not Asked</td>
<td>27</td>
</tr>
</tbody>
</table>

**Table 31**

**Coding and Results: Question 11(c)**

**Explanation of Coding in Question 11(c)**

*Code 1.* A correct answer of 28 was obtained by either enumerating all possibilities or by computing \( \frac{7 \times 8}{2} \).

**Brief Discussion of Data From Question 11(c)**

Of the 13 students who answered Question 11(b) correctly, only four answered part (c), involving the concept of a combination, correctly. Three of these were gamblers who were familiar with the "quinella" and able to use this familiarity to recognise that the 56 (= 8 \* 7) possibilities included "first and second" twice (and thus the need to divide by two). Three were high achievers. Issues arising from this low proportion of correct responses are discussed in the next chapter.

**Question 11(d).** This question returned to the concept of a permutation involving now the selection of three horses.

**Question 11(d)**

[To be given to students who were able to answer 11(c)]
If the "trifecta" consists of selecting the first three horses in the correct order, how many possible selections are there in a race with 6 horses?
Table 32
Coding and Results: Question 11(d)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>4</td>
</tr>
<tr>
<td>2. Incorrect</td>
<td>9</td>
</tr>
<tr>
<td>3. Not Asked</td>
<td>27</td>
</tr>
</tbody>
</table>

Brief Discussion of Question 11(d)

Of the 13 students probed, only four, all high achievers, were able to answer correctly. These students used a correct multiplicative procedure.

Question 11(e). This question involved the calculation of the number of combinations of three things selected from six. Only the four students who answered the previous item correctly were asked.

Question 11(e)
If we disregard the order in the last question, how many selections are there now?

Table 33
Coding and Results: Question 11(e)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>1</td>
</tr>
<tr>
<td>2. Incorrect</td>
<td>3</td>
</tr>
<tr>
<td>3. Not Asked</td>
<td>36</td>
</tr>
</tbody>
</table>

Brief Discussion of Data From Question 11(e)

Only one student, a high achiever, was able to answer correctly.

The computation of a simple permutation in an everyday context. This was required in Question 11(f).

Question 11(f)
A type of new car can be bought with a choice of 6 colours and 3 engine types. How many different selections can be made?
Table 34
Coding and Results: Question 11(f)

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>8</td>
<td>7</td>
<td>9</td>
<td>6</td>
<td>30</td>
</tr>
<tr>
<td>2. Incorrect Additive</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3. Incorrect Other/Unable</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

**Brief Discussion of Question 11(f)**

Although the question was conceptually similar to parts (a) and (b), a much greater proportion of students answered correctly.

*The calculation of a permutation involving three factors.* This was required in Question 11(g).

**Question 11(g)**

If the same type of car can be bought with either manual or automatic transmission, how many different selections are now possible?

Table 35
Coding and Results: Question 11 (g)

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>4</td>
<td>8</td>
<td>7</td>
<td>7</td>
<td>26</td>
</tr>
<tr>
<td>2. Incorrect Additive</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3. Incorrect Other/Unable</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>4. Not Applicable</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td>11</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>40</td>
</tr>
</tbody>
</table>

**Brief Discussion of Question 11(g)**

Over half (26) of the students answered this question correctly. Of these 15 were high achievers and 12 were gamblers. Thus the ability to answer correctly shows some relation to school achievement, but no noticeable relation to gambling background. This is discussed further in the next chapter.

*The computation of a combination of six things taken two at a time in an everyday context.* This is required in Question 11(h).
**Question 11(h)**

Suppose that you have 6 different movie vouchers but have time to go to only two movies. How many different choices can you make in selecting the two movies you will attend?

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct Calculation</td>
<td>3</td>
</tr>
<tr>
<td>2. Enumerates Correctly</td>
<td>1</td>
</tr>
<tr>
<td>3. Other Incorrect/Not Applicable</td>
<td>36</td>
</tr>
</tbody>
</table>

**Brief Discussion of Question 11(h)**

Only four students, all high achievers, answered this correctly. None of the forty students had been exposed to the formal topic of "combinations" in a school situation. It would appear that there is little intuitive knowledge of the topic. This is discussed further in the next chapter.

**Summary**

From this overview, a number of patterns and differences between the groups can be noticed. The gamblers demonstrated an ability to construct their own context dependent algorithms in several situations especially those involving the calculation of returns on bets at various odds. This ability however does not, in general, appear to transfer to other traditional contexts, and the results of this research support Carraher's (1988) conclusion that mathematics which is learned outside of school does not always transfer to other contexts, even when the content of the problem is familiar.

Both gamblers and non-gamblers employed a variety of strategies involving pencil/paper computation, calculator use and mental computation. However, the gamblers tended to employ context dependent estimation strategies that were very effective. Other research in which context-dependent, but effective, estimation and computation strategies have been reported include that reported by Schliemann and Carraher (1988) in their study of unschooled fishermen, Acily and Schliemann (1986) in their study of adult bookmakers, and Acily and Schliemann (1989) in their study of unschooled domestic cooks.
No noticeable differences in the understanding of basic concepts of probability and the comparison of probabilities between the gamblers and non-gamblers were apparent. Several skills appeared to relate to school achievement, but none appeared to relate to gender. In the areas of compound probability and combinatorics, the conceptual understanding of students in both groups was very low. Misconceptions in probability were not as widespread as reported in the literature. The gamblers appeared to have a better understanding of the concept of mathematical fairness and its relation to the concept of mathematical expectation and expected return.

These initial patterns and generalisations are examined in greater detail in the next two chapters. In these chapters the responses to selected items are analysed in greater detail, in order to answer the three major research questions that were posed in Chapter 6.
CHAPTER 9

Results II

Analysis, Discussion and Implications of Data
For the First Major Research Question

More detailed analyses of responses by the 40 interviewees to a selection of questions were carried out. In this Chapter responses which were pertinent to the first major research question are summarized. The generalizations from the last Chapter allowed for identification of patterns, but it was noted that certain differences and similarities for some question required more detailed analyses. The responses to these questions have been selected with reference to the major research questions posed, and their relevance to these. Since the research questions arose from a review of the literature, these analyses include references to results of other research reported in related literature.

The additional analyses of the results. In this Chapter, the main focus will be to provide answers for the first major research questions posed in Chapter 6.

The first major research question was:

Do Year 11 students of "Mathematics in Society" (a lower stream course in Queensland) whose social background includes extensive familiarity with track gambling have different intuitive probabilistic concepts and understandings from students for whom track betting is absent from their family and social background?

Intuitive Probabilistic Concepts and Understandings

These were addressed in several specific questions. Data relating to these were obtained from responses to various questions including the following:

Question 1(b)
In each of the following situations, how much can be won on a track bet if:
(i) $10 is bet at odds of 9:2
(ii) $9 is bet at odds of 3:2
(iii) $5 is bet at odds of 7:4

A condensed version of Table 7, summarizing the data for responses to Question 1(b), (iii) is reproduced from Chapter 8.
Table 7 (condensed)

*Coding and Results; Question 1(b), (iii)*

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,3,4. Correct Multiplicative</td>
<td>4</td>
<td>4</td>
<td>7</td>
<td>2</td>
<td>17</td>
</tr>
<tr>
<td>2. Correct Scalar Additive</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>5. Incorrect Traditional/functional</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>6. Double/halve with approximation</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>7. Incorrect Additive</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>8. Scalar Additive Approximation</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Totals:</td>
<td>11</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>40</td>
</tr>
</tbody>
</table>

*Further analysis of data from Question 1(b).* There are 20 "exact" correct responses, of which 10 were from gamblers, and a further 9 computational estimations resulting in a close approximation to the correct answer, of which 7 were from gamblers. Thus the overall performance of the gamblers on this item was noticeably better than the non-gamblers. Furthermore, 8 of the gamblers used a scalar additive technique (Codes 2 and 8) which was used by only three of the non-gamblers. The reasoning employed by the gamblers in the approximation technique (Code 8) tended to be along the following lines:

7: 4 means bet 4 and get 7;
So if I bet 5, I must get more than 7;
To figure out how much more:
5 - 4 = 1;
If 4 gets 7, then this extra 1 gets me more than 1 but less than 2.
[This extra "scaled" amount is estimated then added, hence the terminology employed]
So 5 gets me more than 8 but less than 9 - closer to 9 than 8, about ($8.50 or $8.75 or $8.80).

This reasoning is similar to that described by Schliemann and Magalhaes (1990) in their study of proportional reasoning employed by uneducated domestic cooks in Brazil who would add an estimated scaled amount in situations requiring similar proportional reasoning. They reported that this technique was number dependent in that other techniques would be used with different numbers. The same pattern was found in the present study where, for example, of the 11 gamblers who used this strategy in part (iii), five students used a functional/unitary strategy for part (ii), reasoning along the lines:
9.2 is the same as $4 \frac{1}{2}$

So for each dollar bet $4.50$ is won.
If $10$ is bet, $10 \times 4.50 = 45$ is won.

**Use of Intuitive Language**

The gamblers, in general, demonstrated a greater use of the unitary strategy (Table 6). When using either a unitary strategy, a scalar additive approach, or computational estimation strategies, the gamblers tended to use language in different ways from the non-gamblers.

The intuitive knowledge possessed by the gamblers identified here clearly derives from their facility with the informal language of track gambling. An important educational issue is the extent to which the informal language and experiences of the students' personal gambling world were linked cognitively to the formal language, symbols and skills associated with school mathematics (Ellerton & Clements, 1991).

This idea of linking previously disjoint elements of students' cognitive structures will be discussed in greater detail later in this Chapter, and in the final Chapter of this thesis. It suffices here to raise the question whether the separate components are informally, if not formally, associated with each other by what psychologists and educators have called *intuition* (Cobb, 1989; Fischbein, 1987; Polya, 1957). It might be expected that although the gamblers have not established cognitive links between track gambling and school mathematics, their experiences with track gambling have provided them with frames of thinking which they could attempt to use in the study of probability in Year 11 mathematics. These frames of thinking might be regarded as intuitions.

On the other hand, it could be the case, that the frames of thinking, or intuitions of the gamblers, could inhibit the learning of probability in Year 11. This might be the case, for example, if the culture of track gambling included beliefs and practices that conflicted with the central concepts of probability.

Furthermore, gamblers who used common track gambling strategies tended to avoid using pencil/paper or a calculator in the interviews. Those who did use pencil/paper or a calculator were asked probing questions to determine what they would do in a real-life situations at the track. Specific examples of responses are given in the case studies presented in Chapter 11.

*Other instances of intuitive knowledge.* These were observed in relation to the comparison of "odds" and the relationship between "odds" and probabilities. This was demonstrated in responses to Questions 4 and 5.
Question 4(a), (b) and (c)
In each of the following track betting situations, which is the better of the two odds? That is to say, which gives the greater return per $ bet.
(a) 2.1 or 3:1
(b) 5:1 or 5:2
(c) 4:3 or 9:7

Table 15, summarizing the data for responses to Question 4(a) and (b) is reproduced from Chapter 8.

Table 15
Coding and Results: Question 4(a) and (b)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>(a) 39</td>
</tr>
<tr>
<td>2. Incorrect</td>
<td>1</td>
</tr>
</tbody>
</table>

A condensed version of Table 16, summarizing the data for responses to Question 4(c) is reproduced from Chapter 8.

Table 16 (condensed)
Coding and Results: Question 4(c)

<table>
<thead>
<tr>
<th>Code</th>
<th>Group of Respondent</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>*1-3,5. Correct Traditional</td>
<td>G-LA 4 G-HA 5 NG-HA 3 NG-LA 4</td>
<td>16</td>
</tr>
<tr>
<td>*4. Constructs Algorithm, Less Outlay For $2 Win</td>
<td>G-LA 3 G-HA 2 NG-HA 0 NG-LA 0</td>
<td>5</td>
</tr>
<tr>
<td>*6. Scalar Additive With Approx.</td>
<td>G-LA 3 G-HA 1 NG-HA 0 NG-LA 0</td>
<td>4</td>
</tr>
<tr>
<td>7-10. Incorrect/Not Applicable</td>
<td>G-LA 1 G-HA 1 NG-HA 7 NG-LA 6</td>
<td>15</td>
</tr>
</tbody>
</table>

*Total Correct: 25 (18 Gamblers, 11 High Achievers)

Further analysis of data from Question 4. The results to this question are quite striking. Nineteen of the 20 gamblers were able to answer parts (a) and (b) [see Table 15], and an astonishing 18 of these students were able to make a correct comparison in part (c) using some technique. By comparison, only 7 of the non-gamblers could answer this correctly. By referring to the spreadsheet data, it can be seen that of the 18 gamblers, 11 did not compare fractions confidently (Question 2, Table 10), and 9 did not compare probabilities in a non-gambling context (Question 3(c), Table 12). Thus it would appear that nearly all the gamblers demonstrated an intuitive understanding of the terminology and nature of "odds." This knowledge was demonstrated by only seven of the non-gamblers. However, of the 13 non-gamblers
who did not make a correct comparison of odds, 11 compared fractions correctly (Table 10), and 6 compared probabilities correctly (Table 12), indicating little intuitive knowledge of the concept of odds.

Furthermore, when the data from Question 4(c) were examined, it was observed that five gamblers constructed a procedure that was number dependent, in that they used other traditional school strategies for parts (a) and (b), and context dependent, in the sense that they did not employ the strategy in other questions requiring proportional reasoning.

The ways in which this intuitive understanding of the nature of "odds" was used in the processing of probabilistic ideas is discussed further with reference to the second major research question.

In Question 5, eight gamblers gave a response that demonstrated an intuitive understanding of the concept of "odds."

**Question 5**

In a four horse race the odds for each horse are given as:

2:1, 5:3, 5:1, and 25:1

Which horse is thought to be most likely to win?

List the odds in order of least likely to most likely.

A condensed version of Table 17, summarizing the data for responses to Question 5 is reproduced from Chapter 8.

**Table 17 (condensed)**

*Coding and Results; Question 5*

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,3 Correct Multiplicative</td>
<td>5</td>
<td>7</td>
<td>7</td>
<td>2</td>
<td>21</td>
</tr>
<tr>
<td>2. Correct, Intuitive</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>4. Unable</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>5. Incorrect, Additive</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Total Correct: 30** (All 20 Gamblers and 10 Non-Gamblers)

*Further analysis of data from Question 5.* The gamblers performed noticeably better than the non gamblers (20 correct compared with 10). This may be attributed to the gamblers familiarity with the context. In those responses classified as "intuitive" (Code 2 above) the students recognised that 2:1 is greater than 5:3 but were unable to give a mathematically correct reason. Probing questions were then asked. A typical response of the eight gamblers in this category was "2:1 are higher odds because you get more than at 5:3." With further probing, using questions such as "Why do you get
more?". This response could, in some instances, have been reclassified as unitary or equivalent proportional. Unitary reasoning involved the recognition that 5:3 returned "less than 2" for the one bet, while equivalent proportional reasoning involved explaining that "at 2:1, a bet of three would win more than 5."

Thus, familiarity with the context would appear to have resulted in the intuitive understanding of some of the gamblers. Further probing questions showed that the gamblers tended to have an intuitive understanding of the relationship between "odds" and probability. Students were not asked to convert odds to numerical probabilities, but were required to demonstrate a knowledge of the relationship that the greater the odds, the less likely the probability of winning.

While all the gamblers were able to list the odds in the correct order of likelihood, only 13 non-gamblers were able to do this. Specific examples of the different kinds of reasoning are revealed in the case study transcripts of Chapter 11.

An intuitive understanding of the concept of compound probability. This was demonstrated by six gamblers in Question 9:

**Question 9(a)**
In each of two horse races the favourite is estimated to have about a 50% chance of winning. If you bet on both favourite, your chances of winning on both would be about:
(i) 100%
(ii) 75%
(iii) 50%
(iv) 25%
(v) 10%

A condensed version of Table 24, summarizing the data for responses to Question 9(a) is reproduced from Chapter 8.
Table 24 (condensed)

Coding and Results: Question 9(a)

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 25%: Correct multiplicative</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>2. 25%: Uses intuitive reasoning</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>5. 50%: Unchanged</td>
<td>6</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>21</td>
</tr>
<tr>
<td>3,4,6. Other Incorrect</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

*Further analysis of data from Question 9(a).* In Question 9(a), six gamblers demonstrated an intuitive understanding of the concept of compound probability in that they recognized that the compounding of the individual probabilities must reduce the overall probability, yet these six students lacked any formal technique for performing the calculation.

Typical reasoning in this situation was demonstrated by Case Study #1 who responded:

Well, it's got to be less than 50%. It's less likely that they will both win. But I think it's better than 10%. 25% sounds about right.

Overall, understanding of compound probability was poor, only 12 (including seven High Achievers) responded correctly, and of the non-gamblers, only three (including two High Achievers) answered correctly.

Surprisingly, more than half (21) of the students gave 50% as their response, reasoning that the probability remains constant. Further probing of this lack of understanding was done by altering the context of the question:

If I am going to play tennis against an opponent and I have 50% chance of winning a set, what is the chance that I will win two sets in a row, assuming the probability of winning each remains at 50%?

The inability of many students to determine compound probability was not noticeably affected by the context, with only two more non-gamblers and one fewer of the gamblers responding correctly.

Furthermore, of the five students who used a formally correct multiplicative procedure in both situations, all were high achievers. The intuitive approach used by the six gamblers in the gambling context did not transfer to this situation or to the other contexts of compound probability described in the remaining parts of the question.

A more detailed discussion of the students' thinking about compound probability will be provided later in this chapter under the category of "Compound Probability."
At this stage, it is sufficient to note that, as reported in the literature, understanding of the concept of compound probability is poor. However, of the 20 gamblers, three used a formal multiplicative technique, and of the 17 who lacked any formal strategy of quantifying probability, six used an intuitive method to arrive at an estimate. Only one of the 18 non-gamblers, who lacked a formal strategy, demonstrated any such intuitive knowledge.

**Intuitive Understanding of Independence**

The concept of "independence" which has been identified in the literature as important (see, for example Reys et al., 1989), is also not well understood (see Jones, 1976; Pedler, 1977; Fischbein et al., 1991; Trurin, 1992). Brown, et al. (1988, p. 243), in reporting the results of the fourth NAEP, noted that 44% of Year 11 students and 53% of Year 7 students failed to recognize the independence of the trials when a fair coin is tossed, leading them to comment that "students of all ages often fail to recognize independence of certain events." Later research by Konold et al. (1993) produced data that led them to suggest that even these NAEP figures might be optimistic. They suggested that

the percentage of secondary school students who understand the concept of independence is much lower than the latest NAEP results would lead us to believe, and, more generally, point to the difficulty of assessing conceptual understanding with multiple-choice items. (p. 393)

The use of representativeness is discussed in detail with regard to data analysis for the Major Research Question 2. At this stage, it is sufficient to note that the misconceptions resulting from the use of either the representativeness or availability heuristics were demonstrated by only 12.5% of the students. This is considerably lower than that reported in the literature and suggests that many students might have an intuitive understanding of the concept of independence in certain situations such as Lotto draws. For example, in Case Study #1, Leona answered:

"(In Lotto) The probabilities have to stay the same. They're just balls you know."

While it is possible that Australian students in general, and the gamblers in particular, might have a better intuitive understanding of the concept of independence than has been reported in the literature, this cannot be concluded from the results of this study due to the limitations noted in the methodology. This is discussed further in Chapter 12, "Implications for Further Research."
Intuitive Understanding of Fairness and Expectation

This was examined by analysis of the responses given in the interviews to Questions 7(d) and 10.

Suppose that you and I are going to play a game with this coin [shows] in which we toss the coin just once. If the coin lands heads, I win; if it lands tails you win. But I will warn you that of the last 15 people who played this game with me only 6 won.
Do you think this is unusual?
Do you think the coin is not fair?
Do you think the game is fair?

Table 22, summarizing the data for responses to Question 7(d) is reproduced from Chapter 8.

Table 22
Coding and Results: Question 7(d)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G</td>
</tr>
<tr>
<td>1. Basic Concept of Fairness Evident</td>
<td>18</td>
</tr>
<tr>
<td>2. Concludes Unfairness</td>
<td>2</td>
</tr>
<tr>
<td>3. Unable to Answer</td>
<td>0</td>
</tr>
</tbody>
</table>

Further analysis of data from Question 7. Seven of the 40 students thought the coin may not fair, that is to say, biased. Of these, three used the "representativeness" heuristic to conclude that the short term rate was representative of the long term. Two used an "availability" heuristic, reasoning that "people tend to lose at this type of game." Two were "not sure," but doubted the fairness of the coin, citing no particular reason.

Of these seven, only two were gamblers. However, it is not possible to conclude differences between the two groups from these small numbers, and the important point to note is that the misconception is low in both groups: only 10% of gamblers and 25% of non-gamblers gave incorrect responses. This rate is considerably lower than that of comparable situations reported in the literature. Shaughnessy (1981) found the use of misconception to be as high as 50% among college entry students. Tversky and Kahneman (1982a) reported the misuse to be widespread among the population and "not confined to naive subjects" (p. 5). Brown, et al. (1988) reported that 44% of Year 11 students demonstrated a misconception in this type of situation.

Depending on the response given by the interviewee to Question 7(d), the following probe would be conducted in the interview.
[If the response to part (d) is "yes"]:  
What if it was the case that, of last 20 people 14 lost?  
Is it still fair?

[If "yes"]:  
How long a run would you need to suspect bias if "head" showed  
about two-thirds of the times?

[If "no"]:  
Why not?  
Would you suspect bias if 10 people played and 7 lost? If  
6 people played and 4 lost?

All who said the coin was fair considered the game also fair. Of the 33 students in  
this category, 23 were able to conclude correctly that a longer sequence with this  
proportion of head and tails, or a greater proportion of heads to tails in this sequence  
would be needed before reasonable grounds for bias could be established. Of these 23  
students, 14 were gamblers and 9 non-gamblers. None had any formal education in  
probability theory, and interview data indicated that when they answered the question,  
they reasoned intuitively. The gamblers appeared to have a better intuitive  
understanding of the concept of fairness in this context than the non-gamblers.  
Questions 10(a) and (b) also related to the concept of fairness.

Question 10(a)  
Suppose you and I play a game with one of these dice [show  
single die]. If on a single roll it is three or less you win, if it is  
more than three I win. If we each bet $1 and the winner gets  
the $2, would this be a fair game?

Table 27, summarizing the data for responses to Question 10(a) is reproduced  
from Chapter 8.

Table 27  
Coding and Results; Question 10(a)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct: Recognises Equal Probability</td>
<td>38</td>
</tr>
<tr>
<td>2. Incorrect</td>
<td>0</td>
</tr>
<tr>
<td>3. Unable to Answer</td>
<td>2</td>
</tr>
</tbody>
</table>

Further analysis of data from Question 10(a). Thirty-eight of the forty students  
recognized this as a fair game. Only two non-gamblers (one of whom has been  
included as a case study), responded that they were unable to answer, because they  
had no basis for making a decision in such a situation.
Question 10(b), (i)
Suppose we change the rules so that I win if it is a three or more. Is this still a fair game?

Table 28, summarizing the data for responses to Question 10(b) (i) is reproduced from Chapter 8.

Table 28
Coding and Results; Question 10(b), (i)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>27</td>
</tr>
<tr>
<td>2. Incorrect: Does Not Use Probability to Answer</td>
<td>7</td>
</tr>
<tr>
<td>3. Unable to Answer</td>
<td>6</td>
</tr>
</tbody>
</table>

The following extended probe of the 27 who recognized unfairness was conducted in the interviews:

Question 10(b), (ii)
Can we make the game fair somehow?

Further analysis of data from Question 10(b), (ii). Of the 27 who recognized that the game was no longer fair, 19 demonstrated some intuitive knowledge of expectation by recognizing that the game could still be made fair by changing the contributions of the players.

Question 10(b), (iii)
How much should I put in to make the game fair?

Data from Question 10(b), (iii) is presented in Table 37.

Table 37
Coding and Results; Question 10(b), (iii)

<table>
<thead>
<tr>
<th>Code</th>
<th>Group of Respondent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G</td>
</tr>
<tr>
<td>1. $2</td>
<td>8</td>
</tr>
<tr>
<td>2. &gt;$1, But Not Sure How Much</td>
<td>6</td>
</tr>
</tbody>
</table>

Analysis of the data from Question 10(b), (iii). Of these 19 students, ten (eight gamblers) were able to recognize that since one probability is twice the other, the returns must be in that ratio, thus demonstrating an intuitive understanding of the basic multiplicative principle underlying mathematical fairness.
Question 10(c) further probed the concept of expectation. The understanding of the "multiplicative" nature of the concept of expectation, as discussed in Chapter 5, was researched in this section. By asking the probing question that reversed the roles of the players ("you" and "I"), this concept was examined in some detail for the few students who demonstrated such an understanding. An interesting case study, in which the concept of expectation features strongly, is provided in the next chapter.

*Question 10(c)*

[If the answer to Question 10(b) is $2, continue with further questions]

How much should I put in if I choose the numbers 1 through 5, leaving you just the 6?

[Repeat with drawing cards from a deck]

How much should I put in if I choose spades leaving you the other three suits?

How much if I choose a single card of any suit? How much if I choose a single specific card?

[If correct, repeat the questions reversing the order of "you" and "I"]

Data from Question 10(c) is presented in Table 38.

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Able to Answer Only Some Parts Correctly</td>
<td>4</td>
</tr>
<tr>
<td>2. Able to Answer All Parts Correctly</td>
<td>6</td>
</tr>
<tr>
<td>3. Not Asked</td>
<td>30</td>
</tr>
</tbody>
</table>

*Analysis of data from Question 10(c).* Of the 10 who responded correctly to the first part, 6 were able to answer all parts correctly.

**Further Analysis of Data Relating to the Concept of Expectation**

Table 39 summarize the above results, and also incorporates findings from the preceding analysis of all parts of Question 10, and data from the actual student transcripts.

In the table, students who seem to have no knowledge of mathematical expectation were regarded as being in Category 1. These students were unable to answer Question 10(b) and would tend to reason: "A game can only be fair if each player has the same chance of winning." Two "non-gamblers" admitted to having no basis on which to make decisions of fairness.
Students allocated to Category 2 showed some intuitive knowledge of the use of expectation in the determination of fairness, but did not use the formal mathematical language of "expectation." For example, a satisfactory response to Question 10(b) (ii) would be:

"Well you have four chances to my two, so you should put in twice as much as me to make the game fair."

Students allocated to Category 2 answered Question 10(b), correctly and were able to recognize that a game could be made fair by varying the amounts paid, at least in simple situations. Category 2 students however, were unable to answer the more complex questions that followed.

Category 3 students demonstrated a thorough knowledge of the concept of mathematical expectation. This was demonstrated by the interviewee's correct responses to all questions in this section. To qualify for inclusion in this category, the student had to be able to demonstrate that in all of the situations, fairness could be established by each player contributing an amount that is in inverse relation to the probability (the constant product requirement). In addition, the students had to be able to compute the amounts correctly. For example, in response to the last Question 10(c), it was not adequate merely to reason: "I have more than twice the chance you have, so I should put in more than twice as much," but it was required that the student reason: "I have $\frac{36}{16}$ times the chance that you do, so I must contribute $\frac{36}{16} \times \$2.25$ for each $\$1$ you put in."

### Table 39

<table>
<thead>
<tr>
<th>Evidence of Knowledge of the Concept of Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Category</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>1. No Knowledge Evident</td>
</tr>
<tr>
<td>2. Some Knowledge of Concept</td>
</tr>
<tr>
<td>3. A Through Knowledge</td>
</tr>
</tbody>
</table>

In order to examine the relationship between knowledge of expectation and school achievement, Table 40 was constructed.
Table 40
*Separation of the Data Relating to the Knowledge of the Concept of Expectation by School Achievement*

<table>
<thead>
<tr>
<th>Knowledge</th>
<th>Achievement Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>1. None:</td>
<td>11</td>
</tr>
<tr>
<td>2. Some:</td>
<td>5</td>
</tr>
<tr>
<td>3. Complete:</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td>19</td>
</tr>
</tbody>
</table>

Entries in Table 40 do not suggest that there is any strong relationship between the understanding of mathematical expectation and school achievement.

*Knowledge of Expectation and Gender*

In order to examine the relationship between knowledge of expectation and gender, Table 41 was constructed.

Table 41
*Separation of the Data Relating to the Knowledge of the Concept of Expectation by Gender*

<table>
<thead>
<tr>
<th>Knowledge</th>
<th>Gender</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Female</td>
</tr>
<tr>
<td>1. None:</td>
<td>12</td>
</tr>
<tr>
<td>2. Some:</td>
<td>6</td>
</tr>
<tr>
<td>3. Complete:</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td>21</td>
</tr>
</tbody>
</table>

Data in Table 41 do not suggest that there is any strong relationship between the understanding of mathematical expectation and gender.

*Implications of the Results of the Analysis of Data From all Parts of Question 10 Relating to Expectation*

These results have two important implications. First, the concept of expected return is not part of the regular school curriculum at this level – they do not use the term "expectation" but construct a procedure that recognizes the inverse relationship between probability and return. In this sense they are constructing knowledge in the manner described by the constructivists mentioned in the literature.

Second, since there is a noticeable difference in this conceptual understanding between the two groups relating to gambling background and not relating to either school achievement or gender, it may be conjectured that this understanding results
from their familiarity with the related concept of "return on track bets at given odds." Further research may be necessary to test this hypothesis generated.

Nevertheless, we may conclude that one field of probabilistic reasoning in which the gamblers demonstrated an intuitive understanding and the non-gamblers did not, was that of the concept of fairness and its relationship to mathematical expectation and expected return.

**Absence of Intuition**

Generally speaking, students from both groups were able to quantify simple probabilities without difficulty, but the ability to compare probabilities related more to school achievement and to the ability to compare fractions in traditional school contexts, than to gambling background. There appears to be little, if any, difference between the two groups in intuitive understanding of basic simple probability.

Very few students in either groups demonstrated any intuitive understanding of the basic principles of the combinatoric knowledge required to answer Question 11. Although the mathematics of track betting on the "quinella" and the "trifecta" involve combinatoric concepts, the gamblers showed no better intuitive understanding of these concepts than the non-gamblers. One possible reason for this is that all bets on the trifecta and the quinella are placed through the TAB in which all calculations are performed automatically (see Appendix 5).

In Chapter 3, Gambling in the Australian Social Context, it was noted that over the past few years there has been an increase in TAB betting and a corresponding decrease in bookmaker betting. This is discussed in greater detail in the next chapter, but here it should be noted that this shift in betting habits may have implications for future education research in this field. Since all TAB betting is done without the bettor knowing the exact odds, it is likely that these bettors perform fewer calculations regarding the comparison of odds and the amounts of payouts than do those betting through bookmakers.

Furthermore, the relative decrease in track gambling has been attributed to a corresponding increase in Casino and Poker machine gambling. This could result in there being less active involvement by the gambler in the mathematics of gambling situations, and, if this is indeed the case, it would reasonable to expect a corresponding decrease in any intuitive knowledge. Misconceptions, such as the use of "representativeness," are well documented among casino gamblers (see, for example Tversky & Kahneman, 1982a).
Summary

Patterns of responses by individuals in the two groups led to the following generalisations which enable the first major research question to be answered. Year 11 students of "Mathematics in Society" (a lower stream course in Queensland) whose social background includes extensive familiarity with track gambling have intuitive probabilistic concepts and understandings which differ from those of students for whom track betting is absent from their family and social background, in the following ways:

1. The gamblers tended to use a common language or jargon that is specifically associated with track gambling. They did so in a number of gambling contexts, particularly in the computation of return on bets placed at varying odds. The use of such language was intuitive, and consistent in the sense that the gamblers' informal language generally agrees with the mathematical and the symbolic language used in such calculations.

2. These gamblers used this informal language, and the experiences of the students' personal gambling to form intuitive links between previously separated elements of their cognitive structures with the formal language, symbols and skills associated with school mathematics.

3. Some of the gamblers had a good intuitive grasp of the concept of independence. These students also demonstrated a very good intuitive understanding of the concept of fairness and its relationship with mathematical expectation, and were able to calculate the expected returns from betting situations in which varying odds were available. Although evident in only some of the gamblers, these understanding and abilities were not to be found in the non-gamblers to any great extent.

4. Many of the gamblers demonstrated an intuitive understanding of the concept of proportion in gambling contexts by the use of computational estimation strategies and approximations. These strategies were often context dependent, in the sense that the gamblers did not use them in non-gambling contexts, and that they were not used by the non-gamblers.

5. Some of the gamblers demonstrated a limited, but intuitive, understanding that, in the context of track betting, the compounding of events reduces the final probability.
CHAPTER 10

Results III

Analysis, Discussion and Implications of Data
For the Second and Third Major Research Questions

In the additional analyses of the results in this Chapter, the main focus will be to provide answers for the second and third major research questions posed in Chapter 6.

The Second Major Research Question

It will be recalled from Chapter 6 that the second major research question was:

What are the cognitive processes employed in the application of probabilistic and related mathematical concepts in traditional classroom situations and in out-of-school contexts involving track gambling; do these processes differ between the groups and what are the differences in the ways individuals in the groups tend to process these concepts?

Much data were obtained relating to this question. Before analysing specific items, some generalisations can be made regarding overall responses.

Generalisations Regarding Major Research Question 2

Comments on Responses According to Gender

An examination of the data from the spreadsheet (see Appendix 3) showed that for most items there were no markedly noticeable gender effect on the quality of responses. This is not unexpected, since, as was noted in Chapter 5, there is no gender bias of achievement in this course at any of the schools involved in the study, or in the State in general.

Table 42 shows the correct responses according to gender for a number of key interview questions.
Table 42

Correct Responses to Selected Items According to Gender

<table>
<thead>
<tr>
<th>Question</th>
<th>Number of Correct Responses (%)</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>a(i)</td>
<td>16 (84%)</td>
<td>16 (76%)</td>
<td></td>
</tr>
<tr>
<td>b(iii)</td>
<td>12 (63%)</td>
<td>17 (81%)</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>10 (52%)</td>
<td>11 (52%)</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>8 (42%)</td>
<td>12 (57%)</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>14 (74%)</td>
<td>11 (52%)</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>6 (32%)</td>
<td>6 (29%)</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>6 (32%)</td>
<td>4 (19%)</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>10 (52%)</td>
<td>10 (48%)</td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>15 (79%)</td>
<td>15 (71%)</td>
<td></td>
</tr>
</tbody>
</table>

The composition of the samples (Table 2) included 21 females and 19 males. Of the 21 females, 11 were high achievers, and of the 19 males eight were high achievers. It can be seen from Table 42 that the proportion of correct answers is approximately the same for males as it is for females for most items. In some instances (Question 4(c) for example) males performed noticeable better, but in other instances (Question 1(b), (iii), for example), females performed better. Overall, there was no strong difference between male and female performance, and it is not the intent of the present research to examine any of the minor differences observed.

Comments on Responses According to School Achievement

The quality of responses to many items, especially the "traditional" classroom questions, showed a noticeable relationship with school achievement level. As would be expected, high achievers tended to perform better than low achievers.

Analysis by Category

The analysis of the data with respect to this research question is concerned with the differences in the ways in which the members of the two groups apply and process the concepts in various contexts. For this purpose, the data are analysed and discussed for each of the six categories determined in the methodology.
Category 1 Data: Proportional Reasoning, Fraction Knowledge and Construction of Rational Number Algorithms

Questions 1 and 2 were concerned with the concepts in this category. Question 1(a) was in a traditional context and Question 1(b) was in a gambling context.

A condensed version of Table 5, showing the relevant information for responses to Question 1(a), (iv) is repeated.

Table 5 (condensed)
Coding and Results; Question 1(a), (iv)

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>*1-4. Correct Multiplicative</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>26</td>
</tr>
<tr>
<td>6-7. Incorrect /not sure</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>*8. Scalar Additive Approximation</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Totals:</td>
<td>11</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td>Total correct *:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>29</td>
</tr>
<tr>
<td>(exact 26, satisfactory approximation 3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Discussion and Analysis of Data From Question 1(a)

In the Chapter 8 it was reported that in Question 1(a), (i) there were 32 correct responses, and in Question 1(a), (iv) there were 28 correct responses. Fewer students (28 compared with 32) responded correctly to part (iv) which required a computation involving numbers that did not have a common factor. In all parts, high achievers performed better than low achievers. As noted in the last chapter, those who used an additive strategy did so consistently in these situations.

One feature that stands out in part (iv) is the use by some gamblers of a scalar additive strategy with approximation. These students reasoned along the lines:

"4 kg cost $7. One more kg cost more than $1 but less than $2, so the total cost is between $8 and $9."

Estimates such as $8.50, $8.75 and $8.80 were then given.

It will be recalled from Chapter 8 that Question 1(b) involves proportional reasoning in a track gambling situation.
**Question 1(b)**

In each of the following situations, how much can be won on a track bet if:

(i) $10 is bet at odds of 9:2
(ii) $9 is bet at odds of 3:2
(iii) $5 is bet at odds of 7:4

In the Chapter 8 it was reported that there were no major differences between the proportion of correct responses of the gamblers and the non-gamblers to the items of parts (i) and (ii). However, as expected, the high achievers performed better than the low achievers. For part (iii) a different pattern of response was observed.

A condensed version of Table 7, summarising the data for responses to Question 1(b), (iii) is repeated.

Table 7 (condensed)

<table>
<thead>
<tr>
<th>Code</th>
<th>Group of Respondent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G-LA</td>
</tr>
<tr>
<td>1.-4. Correct</td>
<td>5</td>
</tr>
<tr>
<td>5.-7. Incorrect</td>
<td>1</td>
</tr>
<tr>
<td>8. Scalar Additive Approximation</td>
<td>5</td>
</tr>
</tbody>
</table>

**Analysis of Data From Question 1(b)**

Eleven of the 21 "exact" correct responses, came from the gamblers. However, when we consider the "scalar additive with approximation" as resulting in a correct response, there are 29 correct, of whom 17 are gamblers. This was noted in the earlier analysis in relation to the first major research question.

The results of this item are worthy of further investigation. It was noted that the "scalar additive" procedure was used by eight gamblers - two using an exact method and six with approximation. Three non-gamblers also used this technique.

**Differences Between the Groups in the Use of Language**

Responses to Question 1 raised issues associated with what might be called "cognitive linking." The informal language of track gambling was used frequently and confidently by the gamblers, and there can be little doubt that the patterns of reasoning of these students in gambling contexts were influenced, if not determined, by this usage of language. The same students had all studied fraction and ratio concepts in school mathematics contexts. The crucial issue, however, with which the present study is fundamentally concerned, is whether the informal language and experiences of the students' personal gambling worlds were linked cognitively to the formal language.
symbols and skills associated with school mathematics. It is possible that connections between information obtained and the skills acquired in the two areas of endeavour (gambling and school mathematics) had never been established in the students’ minds (Ellerton & Clements, 1991). This is discussed further in the final Chapter of this thesis.

Irons and Irons (1992) have identified four distinct, but overlapping stages of language development as this applies to mathematical reasoning. These were outlined in Chapter 5. According to Irons and Irons (1992), students’ inability to use language consistently throughout the stages is a major source of difficulty in problem solving, and derives from the fact that the students’ “mathematical” or “symbolic” language is not consistent with the “informal” language they use in describing a problem situation.

However, in the gambling context described in Question 10(b), eight gamblers who used their own computational technique, also used consistent and appropriate language throughout.

*Intuitive and informal language.* The gamblers informal language includes terms such as "gets me", "returns" and "find out." For example, several students stated: "At 4:7, $4 gets me $7 if I win. I need to find out how much I will get if I have bet $5."

*Material language.* This related to money and included terms such as "more", "how much more", and "extra." For example, several students reasoned: "$4 gets me $7, so $5 will get me more. How much more will I get from the extra one dollar?"

*Mathematical language.* The more formal mathematical language included traditional mathematical terms, such as "plus" and "between." For example, several students reasoned: "If 4 gets 7, then the extra one will get me more than one but less than two. Thus the five dollars gets me seven plus the extra - between eight and nine, closer to nine, about $8.60."

*The symbolic language.* The use of traditional mathematical symbols to record the steps in the reasoning was noted. In many instances, the symbols used were in agreement with the informal language of the gamblers in that the symbols accurately described what had been said. Also, the language employed by the gamblers who used a unitary strategy was consistent with the symbolic recording of the reasoning processes. These symbols are shown in the triangulation diagram following.
**Triangulation of Results**

The second major research question is concerned with differences in ways by which concepts are processed, the language used, and the computational techniques employed. An attempt to triangulate the analysis of the data will be made with the responses to Questions 1(a), (iv) [Table 5], and 1(b), (iii) [Table 7]. In these questions the numbers and concepts are the same but the context is different. The differences in context were accompanied by differences in the students' language and techniques of computation. These three differences all pointed to the same conclusion, namely that there are differences in the way in which individuals within the two groups process mathematical concepts. It must be emphasised however that the differences identified referred to differences between individuals within the groups and not to all members of the groups. For example, seven of the 20 gamblers used a "traditional" school technique in the gambling context, and three of the 20 non-gamblers used a "scalar additive" technique in both gambling and non-gambling contexts. Figure 9 illustrates similarities and differences.
Concept: Proportional Reasoning

**Gambling**
- Odds are 7:4
- $5 bet wins?

**Non-gambling**
- 4 kg. costs $7
- 5 kg. costs ?

**Context**

**Use of Language**

**Gamblers**
- 7:4 is shorter than 2:1
- At 2:1 $5 wins $10
- At 7:4, $4 wins $7
- $5 gets me more than $7 but it won't get me $10
- At 7:4, $1 returns 7/4
- So $5 gets me the same back as bets of $4 and $1
- That's $7 and a dollar seventy-five
- $8.75
- (or That's $7 and under 2. Under $9)

**Non-gamblers**
- If 4 kg costs $7
- that's less than $2 a kilo
- If 4 kg costs $7
- 5 kg will cost more
- to find out how much
- it's 5 times 7 over 4
- 35 divided by 4
- 8 and three-quarters
- $8.75

**Technique of Computation**

**Gamblers**
- 7:4
- $4 → $7
- $1 → $7/4
- 5 = 4 + 1
- $7 + $ 7/4
- $8.75
- (or $7 + $2 < $9)

**Non-gamblers**
- 4:7 = 5: ?
- $4 → $7
- $1 → $7/4
- 5 = 4 + 1
- $7 + $7/4
- $8.75

Figure 9: Triangulation of the results of Question 1.
**Implications of the Results From Question 1(a), (ii) and Question 1(b), (iv)**

The analysis of the results of these items contributed significantly to the answer to the second major research question in that there is a distinct and noticeable difference between the members of the two groups in the ways in which they process concepts in proportional reasoning. These differences were in the language used and in the techniques of computation employed.

**Traditional Comparison of Fractions**

The results of Question 2 parts (a), (b) and (c) proved to be somewhat trivial since all students answered part (a) correctly, all but one answered part (b) correctly, and all but three part (c) correctly. The analysis of the results to part (d) proved to be more interesting.

**Question 2**

In each pair, circle the larger of the two fractions:

- (a) 1/2 1/3
- (b) 3/8 5/8
- (c) 1/2 3/5
- (d) 3/5 5/8

A condensed version of Table 10, summarising the data for responses to Question 2(d) is reproduced from Chapter 8.

**Table 10 (condensed)**
**Coding and Results: Question 2(d)**

<table>
<thead>
<tr>
<th>Code</th>
<th>Group of Respondent</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>*1-2. Correct</td>
<td></td>
<td>3</td>
<td>6</td>
<td>8</td>
<td>4</td>
<td>21</td>
</tr>
<tr>
<td>3. Correct but Incorrect Reasoning</td>
<td></td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>Guesses or 5/8 &quot;Bigger&quot; Number</td>
<td></td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4-6. Incorrect</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals:</td>
<td></td>
<td>11</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td>Total Correct: 21 (14 High Achievers, 9 Gamblers)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Further Analysis of Data From Question 2**

Only slightly more than 50% of the students were able to compare all the fractions correctly and confidently (Codes 1 and 2). As noted in the last Chapter, the ability to compare fractions related noticeably to school achievement. Of the Low Achievers, we see that only 7 (33%) were correct, whereas for the High Achievers, 14 (67%) were correct. There was no noticeable difference in the proportion of correct questions between gamblers and non-gamblers.
Eight students were totally unable to compare 3/5 and 5/8, and 11 students (8 of whom were Low Achievers) guessed or were unsure, reasoning "5/8 looks bigger." Thus, it would appear that even among Year 11 students there is a significant proportion for whom fraction knowledge is limited, confirming the comment by Davis (1989) that "many people in the community today are totally unaware of how one can compare fractions like 3/5 and 5/8" (p. 40).

Since it has been claimed in the literature that probabilistic reasoning is dependent on fractional knowledge (Fischbein, 1975), it is useful to use the spreadsheet data to cross reference the analysis of these results with those of Category 2, Simple Probability.

**Category 2 Data: Simple Probability; Comparison of Probabilities; Comparison of Odds**

Question 3 was concerned with the concept of the quantification of simple probabilities in traditional classroom contexts followed by comparison of probabilities.

**Question 3**

Suppose I select a marble at random from each of the following bags [diagram shown]:

In each of the two situations what is the probability of getting a black marble?

(a) The bag contains 3 black and 1 white
(b) The bag contains 5 black and 3 white
(c) In which of these cases, (a) or (b), is it more likely to get a black marble?

Tables 11 and 12, summarising the data for responses to Question 3 are reproduced from Chapter 8.

**Table 11**

**Coding and Results: Question 3(a) and (b)**

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td>1. Correct</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>2. Incorrect</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3. Unable to Attempt</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
Table 12
Coding and Results: Question 3(c)

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>*1. Correct, Common Denominator</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>*2. Correct, Decimal Conversion</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>4. Incorrect, Additive Comparison</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>6. Unable/No Attempt</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>7. Not Sure</td>
<td>6</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>8. Not Applicable (Parts A or B Incorrect)</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>*10. Correct, Other</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Further Analysis of Data From Question 3

Nearly all students (36 or 90%) were able to quantify simple probabilities correctly. This figure is noticeably higher than that reported by Brown et al. (1988) in reporting the results of the Fourth NEAP in the United States:

Students were asked to select the probability of picking a red object from a jar containing 2 red and 3 blue objects. About half the seventh-graders and two-thirds the eleventh graders answered correctly (p 242)

This lower success rate reported in the Brown et al. (1988) study may have been due to the fact that a multiple-choice format was used in that study. According to Scholz (1991) much of the research on the acquisition of probabilistic concepts has methodological flaws, because tasks have been posed in a multiple-choice format, and this has meant that often the students' reasoning processes have not been revealed.

The noticeably higher success rate of students in the present study on simple probability questions might be a more reliable measure of the students' ability to quantify simple probabilities. Even so, only 20 of these 36 students were able to explain correctly which probability was greater. This tends to support the claim by Brown et al. that (Year 11) students have difficulty with all probability items except those involving very simple concepts or skills.

Of the 20 students who were able to compare simple probabilities correctly, 10 were gamblers and 13 were high achievers. Of the 13 who used a common denominator to compare fractions (Table 10, Code 1), 12 were able to compare probabilities in both Questions 3(c) and (e) using the same technique. Of the eight who used decimal conversion to compare fractions, only two used this strategy to compare probabilities.
Of the 19 students who were unable to compare fractions confidently (see Table 10, Codes 3 to 6), only four were able to compare probabilities correctly.

Implications of the Data From Question 3 Relating to the Comparison of Probabilities

There were no apparent differences in the ways by which the gamblers and non-gamblers quantified or compared probabilities in traditional classroom situations or non-gambling context. Both groups tended to use methods which they had been taught at school to do this, and in consequence, this ability related noticeably to school achievement. In particular, those who had learned the traditional common denominator strategy were much more likely to compare probabilities successfully, irrespective of gambling background. These results would support comments by Hart (1988), that school taught fraction concepts and operations remain important.

Comparison of Odds in a Gambling Context

Questions 4 and 5 were concerned with the comparison of odds in a gambling context. Informal interview questions explored the students' understandings of the relationships between betting "odds" and probability (see examples in Chapter 10).

Question 4

In each of the following track betting situations, which is the better of the two odds? That is to say which gives the greater return per $1 bet, or which is the greater ratio?.

(a) 2:1 or 3:1
(b) 5:1 or 5:2
(c) 4:3 or 9:7

A condensed version of Table 16, summarising the data for responses to Question 4(c) is reproduced from Chapter 8.

Table 16 (condensed)
Coding and Results; Question 4(c)

<table>
<thead>
<tr>
<th>Code</th>
<th>Group of Respondent</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>*1,3,5</td>
<td>Correct Multiplicative</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>*2.</td>
<td>Correct Scalar Additive</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>*4.</td>
<td>Constructs Algorithm;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less Outlay for $2 Win</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>*6.</td>
<td>Scalar Additive with Approx.</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>7.</td>
<td>Incorrect, Additive</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>8,9,10.</td>
<td>Incorrect, Other</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>10</td>
</tr>
</tbody>
</table>

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**Question 5**

In a four horse race the odds for each horse are given as:

2:1, 5:3, 5:1, and 25:1

Which horse is thought to be most likely to win?
List the odds in order of least likely to most likely.

A condensed version of Table 17, summarising the data for responses to Question 5 is reproduced from Chapter 8.

**Table 17 (condensed)**

*Coding and Results; Question 5*

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,3: Correct, Traditional</td>
<td>5</td>
<td>7</td>
<td>5</td>
<td>4</td>
<td>21</td>
</tr>
<tr>
<td>2: Correct, Intuitive</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>4,5: Incorrect</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

**Further Analysis of Data From Questions 4 and 5**

As for the first major research question, the gamblers were, in general, better able to answer these items. Of the 25 students who answered all parts of Question 4 correctly, 18 were gamblers, and of these nine constructed their own context dependent strategy for part (c), (see Codes 4 and 6, Table 16). The reasoning employed by the five gamblers who used the strategy of Code 4 was along the lines:

4.3 is the same as 8.6;
This means bet 6 win 8, 2 more than you bet;
9:7 means bet 7 win 9, again 2 more than you bet.
It is better to win the same amount more for a lower outlay, so the 4:3 are the better odds.

Mathematically, this reasoning is correct although it is not referred to in the earlier extensive review of the literature on strategies employed in proportional reasoning. Its use by the gamblers in the present study was highly context dependent, in the sense that none of the gamblers who used it in this context did so in any other contexts. Its use was also number dependent in the sense that none of the gamblers used it in parts (a) or (b).

The technique would appear to be unique to gamblers and, as such, represents a distinct difference in the way in which proportional reasoning is processed by individuals within groups. The form of reasoning employed by the four gamblers who used the Code 6 strategy was along the following lines:
4:3 is equivalent to 8:6;
To compare this with 9:7, 8:6 is more than 1:1;
So 8:6 is equivalent to 8+>(1):6+1 [scalar addition]
So 8:6 > 9:7

This scalar additive technique was used in other gambling contexts, but only rarely in non-gambling contexts (see Tables 5, 7, and 12).

Fewer students were able to compare fractions confidently (See Table 10), than made comparable comparisons of odds in a gambling context (21 compared with 25). Furthermore analysis of the spreadsheet data showed that of the 20 students who compared probabilities correctly (Table 12), only 16 compared fractions equally well. Of the 21 students who compared fractions correctly, only 15 compared all the odds of this question correctly.

This pattern results from the fact that some gamblers who were unable to compare fractions in a traditional classroom context could nevertheless compare odds correctly by constructing their own context-dependent procedure. The results of informal interview questions showed that all 20 gamblers could list the odds in order of most likely to least likely, however only 12 non-gamblers were able to do this.

**Implications of the Data From Questions 4 and 5 Relating to the Comparison of Odds**

These results indicate that further research is needed to determine the extent to which the techniques used by the gamblers in the context of the calculation of odds are applied in other contexts; the extent of the number dependence, that is to say the extent to which the strategy relies on the numbers involved; and whether the strategy can be used by the gamblers in alternate contexts. These issues are discussed further in the final chapter.

**Category 3 Data: Misconceptions in Probabilistic Reasoning; the use of the Representativeness and Availability Heuristics**

The extent to which different problem-solving process are used in the situations described in Questions 6 and 7, is now discussed.

**Question 6(a)**
In the last 200 Gold Lotto draws the number 17 has occurred more times than any other number. In future draws this number is
(i) more likely to occur than any other number?
(ii) less likely to occur than any other number?
(iii) as equally likely to occur as any other number?
Results and Analysis of Data From Question 6(a); [Table 18]

Thirty-two of the forty students showed no misconception in this situation. Two used representativeness to conclude that "17" is now less likely, thereby demonstrating the misconception that deviations average out over the long run ("local representativeness" or "the gamblers' fallacy"). Three used representativeness to conclude that the short term frequency for "17" was representative of the long term. Reasoning from the short term (200 times), they concluded that this number would always be more likely to occur than any other.

Question 6(b)
[Describe/show roulette wheel]
In Roulette, each number is equally likely to occur. Suppose that the ball lands on Red six times in a row. On the next roll is it now more likely to land on the red, the black or are they both still equally likely?

Results and Analysis of Data From Question 6(b); [Table 19]

Thirty-three students gave correct answers to this question, showing no misconception. Two used local representativeness to conclude that red is now less likely (black is more likely), demonstrating the gamblers' fallacy. Three used representativeness to conclude that the short term frequency for red is representative of the long term. Reasoning from the short term (6 times in a row), they concluded that red was more likely to occur than black.

Question 6(c)
In some lotteries the tickets are pre-numbered.
If you could choose between any of the following numbers would you have any preference?
123456
619999
615472

Question 6(d)
In choosing six LOTTO numbers which of the following selections would you prefer?
(a) 1, 2, 3, 4, 5, 6
(b) 32, 33, 34, 35, 36, 37
(c) 9, 12, 27, 31, 35, 38

Table 20, summarising the data for responses to Questions 6(c) and (d) is reproduced from Chapter 8.
Table 20
*Coding and Results: Question 6(c) and (d)*

<table>
<thead>
<tr>
<th>Code</th>
<th>Group of Respondent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G-LA</td>
</tr>
<tr>
<td>1. No Use of Availability</td>
<td>5</td>
</tr>
<tr>
<td>2. Used Availability for Only One</td>
<td>3</td>
</tr>
<tr>
<td>3. Used Availability for Both</td>
<td>3</td>
</tr>
<tr>
<td>4. Unable to Answer</td>
<td>0</td>
</tr>
<tr>
<td>Totals</td>
<td>11</td>
</tr>
</tbody>
</table>

**Further Analysis of Data from Questions 6(c) and (d)**

Nineteen of the 40 students in the samples used the availability heuristic, and of these 20, 10 (or 25% of the sample) used the heuristic in both instances. The extensive accounts of the use of availability reported in the literature provide no specific reference to lotto numbers such as were asked in this question. However research by three Polish mathematicians studying the equivalent Polish Liczyrzerka, suggest that Polish bettors made extensive use of the availability heuristic in the selection of numbers. (See reports by Seth Gregory, *Courier Mail* (Brisbane), June 19, 1993). Gregory (1993) reported:

What intrigued the mathematicians was that the number of winners rarely matched the number the predictions made using the laws of probability. On some weeks there were more than 50 times as many winners than expected; on other weeks there were only one-tenth the number of expected winners. Puzzled by what appeared to be a breakdown in the laws of probability, the mathematicians ... found that players preferred certain sequences to others. For example, players shunned consecutive sequences (such as 1,2,3,4,5,6) ... as well as number progressions. (p. 8)

In both the present study and in real life LOTTO, it would appear that those who used an availability heuristic reasoned that sequences and patterns such as 1, 2, 3, 4, 5, 6, were less likely to occur since their are few instances of these patterns that can be recalled as having occurred in Lotto draws.

**Question 7(a)**

If I throw a single fair die is it harder (less likely) to get a six than, say a four?
In general, are the numbers 1 to 6 equally likely to occur, or are some harder to get than others?
Question 7(b)
If I throw a pair of fair dice is it harder to get a pair of sixes than it is to get a pair of twos?

Question 7(c)
A poker hand consists of five cards dealt from a well shuffled deck. The Ace is the highest card. Is it harder (that is to say less likely) to get three Aces than it is to get three fives?

Very little evidence of the use of the availability heuristic was shown in the responses to these questions. Table 21, summarising the data for responses to Questions 7(a), (b) and (c) is reproduced from Chapter 8.

Table 21
**Coding and Results; Question 7(a), (b), (c)**

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
</tr>
<tr>
<td>1. Correct</td>
<td>37</td>
</tr>
<tr>
<td>2. Incorrect</td>
<td>1</td>
</tr>
<tr>
<td>3. No Idea/Not Able to Answer</td>
<td>2</td>
</tr>
</tbody>
</table>

Question 7(d) was concerned with the use of the same heuristic in determining fairness, and has been discussed under Major Research Question 1. Further discussion occurs in Category 5. Table 22, summarising the data for responses to Question 7(d) is reproduced from Chapter 8.

Table 22
**Coding and Results; Question 7(d)**

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G</td>
</tr>
<tr>
<td>1. Basic Concept of Fairness Evident</td>
<td>1</td>
</tr>
<tr>
<td>2. Concludes Unfairness</td>
<td>2</td>
</tr>
<tr>
<td>3. Unable to Answer</td>
<td>0</td>
</tr>
</tbody>
</table>

Further Analysis of Data From Questions 6 and 7
There was a significant use of availability demonstrated in Questions 6(c) and (d), where approximately 50% of students used availability at least once. There was no noticeable difference between the gamblers and non-gamblers, nor between the high and low achievers in these questions. Results for the similar Question 6(b), showed that only 5 of the 40 students, or 12.5%, answered incorrectly, and that in Question 7(d) referring to the toss of a single coin only 7 students, or 18%, came to an incorrect conclusion.
Are the Data for Questions 6 and 7 Relating to Misconceptions Consistent with Data From Other Studies?

In the present study the occurrence of the misconceptions associated with the use of the representativeness and availability heuristics is noticeably lower than that reported in the literature.

Brown et al. (1988) found the misuse of the representativeness heuristic among Year 11 students on similar tasks to be as high as 44%, and Shaughnessy (1991) found the incidence to be as high as 50% among college entry students. Tversky and Kahneman (1982a) reported that both representativeness and availability misconceptions were widespread among the population and that the misconceptions were "not limited to naive subjects" (p. 5).

However Kapadia (1984) has questioned the validity of the findings of much of this research and suggested that some of the misconceptions reported may actually refer to misinterpretations of the questions, arising from the fact that all of the above researchers used multiple-choice tests and large groups of students.

Konold et al. (1993) also commented on the difficulty of assessing conceptual understanding in probabilistic reasoning with multiple-choice items. The results of the present study were obtained from structured individual clinical interviews, in which students' understandings of the questions were checked. In fact, the results obtained in the present study would seem to support the views of Kapadia and Konold et al., however further research is required to confirm the conclusions which have been reached.

The extent of misuse of the availability heuristic reported in the literature in situations involving the throwing of dice by Jones (1974), Pedler (1977), Anderson and Pegg (1988), and Trurin (1992) was not in evidence among the students of the present study. Note however, that most of the studies referred to in the literature were of younger students, whereas those in the present study were in Year 11. Thus, it would appear that the conclusion of these researchers that children perceive throwing a "six" to be more difficult than throwing other numbers on the die, does not apply to older (Year 11) students. Further research may be needed to confirm the generalisability of this claim, or to determine precisely at what stage in a child's cognitive development such misconceptions are corrected.
Category 4 Data: The Calculation of Compound Probability in a Variety of Contexts

Questions 8 and 9 were concerned with concepts in compound probability.

**Question 8**
A robot is put into a maze which it begins to explore (diagram)
At each junction it is as likely to go down any path as any other. Where is it most likely to end up?

A condensed Table 23, summarising the data for responses to Question 8 is reproduced from Chapter 8.

**Table 23 (condensed)**
*Coding and Results; Question 8*

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uses Compound Probability, Multiplication Principle</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Shortest Path</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>Equally Likely</td>
<td>8</td>
<td>6</td>
<td>7</td>
<td>7</td>
<td>28</td>
</tr>
<tr>
<td>3, 5, 6. Other</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Totals</td>
<td>9</td>
<td>13</td>
<td>12</td>
<td>12</td>
<td>40</td>
</tr>
</tbody>
</table>

**Further Analysis of Data From Question 8**

This question was the same as that asked by Green (1983a), who reported that of a large sample of students aged from 11 to 16 years only 13% responded correctly. Green reported that 60% responded "equally likely."

Of the eight students who answered correctly in the present study, only one used reasoning that demonstrated an understanding of how the probabilities can be compounded by the multiplication principle. The others simply reasoned that "the shortest path is the most likely."

An astonishing 28 (70%) thought all paths were equally likely. This is virtually the same figure as that cited by Green for students who were from one to six years younger than the students in the present study. Furthermore, examination of these 28 respondents (Code 4, Table 23) revealed no noticeable relationship with gambling background, achievement or gender. The misconceptions are as frequent among the gamblers as among the non-gamblers. Interestingly, Green reported little difference in achievement on this item between students of different ages.

These results of the present study provide support to the statement by Fischbein et al. (1991) that "the concept of compound probability raises some difficulties" (p. 532).
Fischbein et al. (1991), who reported no improvement in performance on compound probability questions with age, stated that two unequally likely outcomes are considered equivalent at all age levels by most of the subjects ... There is also no improvement with instruction. On the contrary, there are less correct answers in older children who received certain instruction than in younger children who did not receive any instruction in probability. (p. 534)

The concept of compound probability was examined further in Question 9.

**Question 9(a)**

In each of two horse races the favourite is estimated to have about a 50% chance of winning. If you bet on both favourites, your chances of winning on both would be about:

(i) 100%
(ii) 75%
(iii) 50%
(iv) 25%
(v) 10%

A condensed version of Table 24, summarising the data for responses to Question 9(a) is repeated.

**Table 24 (condensed)**

_Coding and Results: Question 9(a)_

<table>
<thead>
<tr>
<th>Code</th>
<th>Group of Respondent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G-LA</td>
</tr>
<tr>
<td>1. 25%: Correct multiplicative</td>
<td>0</td>
</tr>
<tr>
<td>2. 25%: Uses intuitive reasoning</td>
<td>4</td>
</tr>
<tr>
<td>5. 50%: Unchanged</td>
<td>6</td>
</tr>
<tr>
<td>3,4,6. Other Incorrect</td>
<td>1</td>
</tr>
<tr>
<td><strong>Totals:</strong></td>
<td><strong>9</strong></td>
</tr>
</tbody>
</table>

Further analysis of data from Question 9(a). Twenty-eight of the 40 students responded incorrectly to this question. Of these, 21 responded that the probability remained unchanged, thereby indicating little understanding of a compound event. Of the 28 incorrect responses, 10 came from gamblers who were well familiar with the context. This tends to support research by Carraher (1988a) that learning mathematics outside of school does not always lead to correct responses even when the content of the problem is familiar. Of the 12 students who responded correctly to this question, five used a school-based approach, either drawing a tree diagram, or employing the
multiplication principle. It is worth noting that most of the 40 students would have encountered questions of this nature in earlier years at school.

However seven gamblers answered correctly, reasoning along the lines: "It must be less than 50% and 10% is too low." This would imply some understanding that the probability of a compound event must be less than the individual probabilities even though no algorithm was employed to calculate the probability.

**Question 9(b)**

Two coins are tossed together.

(i) The probability that one will land a head and the other a tail is:

(a) 1/2
(b) 1/3
(c) 1/4

(ii) What is the probability that at least one of the two coins will land a head?

Table 25(a), summarising the data for responses to Question 9(b), (i) is reproduced from Chapter 8.

**Table 25(a)**

*Coding and Results; Question 9(b), (i)*

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct, Counts Cases, 2 Out of 4</td>
<td>10</td>
</tr>
<tr>
<td>2. Correct, Same or Different Equally Likely</td>
<td>17</td>
</tr>
<tr>
<td>3. Correct, Uses Tree Diagram</td>
<td>2</td>
</tr>
<tr>
<td>4. Unable to Answer</td>
<td>3</td>
</tr>
<tr>
<td>5. 1/3</td>
<td>6</td>
</tr>
<tr>
<td>6. 1/4</td>
<td>2</td>
</tr>
</tbody>
</table>

*Further analysis of data from Question 9(b), (i).* While 29 students answered Question 9(b), (i) correctly, 17 of these 29 reasoned incorrectly that the probabilities were either the same or different, and that each of these possibilities was equally likely.

Only 10 students used the method of considering an equally likely sample space of four in which two are different, and a further two used a tree diagram with the multiplication principle. Six used an equally likely sample space of three to respond incorrectly, 1/3.

Table 25(b), summarising the data for responses to Question 9(b), (ii) is reproduced from Chapter 8.
Table 25(b)
Results to Question 9(b), (ii)

<table>
<thead>
<tr>
<th>Response</th>
<th>Number Giving This Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/4</td>
<td>3</td>
</tr>
<tr>
<td>2/3</td>
<td>4</td>
</tr>
<tr>
<td>1/2</td>
<td>15</td>
</tr>
<tr>
<td>1/3</td>
<td>4</td>
</tr>
<tr>
<td>Other</td>
<td>3</td>
</tr>
</tbody>
</table>

Further analysis of data from Question 9(b), (ii). Only those 29 students who
answered the first part of Question 9(b) correctly were asked to answer this question.
Only three (about 10% of those asked, or 12.5% of the full sample) answered
correctly. This is somewhat higher than the 5% reported by Brown et al. (1988), who
also noted that 70% responded one-half, compared with 50% in the present study.

This difference may be attributed to the fact that Brown et al. reported results
obtained from multiple-choice, pencil and paper test items, while in the present study
the clinical interview was employed. Nevertheless, the results for this item are similar
to those reported by Brown et al. in that the proportion demonstrating an
understanding of this relatively easy compound event is very low. Furthermore, an
examination of the results when the students’ backgrounds were taken into account
showed no noticeable difference between the gamblers and the non-gamblers. Nor was
there any noticeable difference by school achievement or gender.

Question 9(c)
(i) A pair of fair dice are tossed together.
   [show pair of identical dice]
   Which is more likely:
   (a) a pair of fives
   (b) a five and a three
   (c) both are equally likely
(ii) [repeat with different coloured dice]
(iii) In general, is it harder (less likely) to get a pair of numbers
     than it is to get two different numbers?

Table 26, summarising the data for responses to Question 9(c), (i) is reproduced
from Chapter 8.
Table 26
Coding and Results; Question 9(c), (i)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct, Reasons From Experience</td>
<td>6</td>
</tr>
<tr>
<td>2. Correct, Uses Tree or Counts Cases</td>
<td>4</td>
</tr>
<tr>
<td>3. Equal</td>
<td>27</td>
</tr>
<tr>
<td>4. No Idea How to Solve the Problem</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
</tr>
</tbody>
</table>

Further analysis of data from Question 9(c), (i). Seventeen students in Question 9(b), (i) reasoned that the outcomes were either the same or different, and that each of these was equally likely. Of these, 15 students continued to use this reasoning in Question 9(c), although it is incorrect in this situation.

All of the 10 who used the method of considering a sample space in Question 9(b), (i) attempted to use the same method to answer this item, but only four were successful. The others became confused when trying to enumerate the outcomes. Of these four, two were gamblers. All four were high achievers. Six students (all gamblers) answered correctly citing experience - "Doubles are harder to get." The results of this item indicate a relationship with gambling background, and all six students who gave a correct response cited experience with dice games as a factor.

Question 9(c) (ii)
A pair of fair dice are tossed together.
(show pair of different coloured dice)
Which is more likely:
(a) a pair of fives
(b) a five and a three
(c) both are equally likely

Table 26(b), summarising the data for responses to Question 9(c), (ii) is reproduced from Chapter 8.

Table 26(b)
Coding and Results; Question 9(c), (ii)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>11</td>
</tr>
<tr>
<td>2. Still Equal</td>
<td>26</td>
</tr>
<tr>
<td>4. Not Asked</td>
<td>3</td>
</tr>
</tbody>
</table>
Further analysis of data from Question 9(c), (ii). The 27 students who responded "equally likely" to part (i) were further probed to see if they exhibited "resistance" as defined by Fischbein et al. (1991, p. 533). These were asked whether the results would be the same if different coloured dice were used. As reported by Fischbein et al., the misconception that both are equally likely is very resistant to change. Only one student changed her view from equally likely, to doubles being less likely.

Question 9(c), (iii) was concerned with the general situation:

**Question 9(c), (iii)**

In general, is it harder (less likely) to get a pair of numbers than it is to get two different numbers?

Table 26(c), summarising the data for responses to Question 9(c), (iii) is reproduced from Chapter 8.

Table 26(c)
Results of Question 9(c), (iii) (not coded, from raw data)

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Yes</td>
<td>14</td>
</tr>
<tr>
<td>2. No</td>
<td>22</td>
</tr>
<tr>
<td>3. Don't Know</td>
<td>4</td>
</tr>
</tbody>
</table>

Further analysis of data from Question 9(c), (iii). Data generated by the more general question showed that the "equally likely" misconception is not as common in this situation. The number of students who responded correctly rose from 10 to 14 (25% to 35%). These all claimed that doubles were, in general, less likely than different numbers. The 14 consisted of the same 10 who responded correctly to the first part, and four who changed their answer in the general case. Each of these four was asked whether this answer would cause them to reconsider their response to part (i), but none changed their response, confirming that "this bias is very resistant to change" (Fischbein et al., 1991, p. 533). Further, we note that more than half (22 students) still answered "no", that is to say "equally likely."

The observed increase in the number of correct response in the general case (from 25% to 35%) is consistent with the findings of Fischbein et al. (1991), although the improvement in the present study was not as large as might have been expected from the data of Fischbein et al. They reported that studies of 13 and 14-year-old students showed that less than 10% of these students answered the specific dice question correctly, but 38.4% answered the general dice question correctly. Fischbein et al. (1991) commented on the responses of the students to several similar questions:
At all age levels ... the percentages of correct answers are visibly higher for the generalised form of the question than for the specific one. This is a fundamentally new finding. First, let us remark that the higher percentage of correct answers for the generalised form of the questions is not related to the equality or non-equality of the probabilities. (p. 537)

The smaller increase observed in the present study may be attributed to the greater percentage of correct responses to the specific case resulting from the correct response of six (30%) of the gamblers.

**Implications of the Data From Questions 9 Relating to Compound Probability**

Overall, data from the responses to items in this category pointed to similar conclusions to those reached by Brown et al. (1988). The students investigated in the study reported by Brown et al. were of the same age and grade level as those in the present study. Brown et al. concluded that "knowledge of all but the simplest of probabilistic questions is extremely limited" (p. 242). The result of the present study would confirm that knowledge of all but the simplest of probabilistic questions is very limited. The basic concept of compound probability is not well understood by Year 11 students in any of the contexts, gambling or non-gambling and this inability does not relate noticeably to gambling background or school achievement. As Fischbein et al. (1991) commented, there appears to be "no natural intuition for evaluating the probability of a compound event" (p. 534).

**Category 5 Data: The Concept of Mathematical Expectation: its Determination and Relationship with Mathematical Fairness**

These data have been analysed in the last chapter in the discussion of Research Question 1, and will not be repeated here.

**Category 6 Data: Combinatorical Situations: The Determination of Simple Combinations and Permutations in a Variety of Contexts**

Data from the various parts of Question 11 were concerned with computations of permutations and combinations in gambling and non-gambling contexts. Condensed versions of Tables 29 to 36 summarising the data for responses to Question 11 are reproduced from Chapter 8.
Question 11 (a)

The "double" in horse racing consists of selecting two winners in two prescribed races. How many possible ways can the "double" occur if one race has 10 horses and the other 8? (Exclude any "dead heats")

Table 29 (condensed)

**Coding and Results: Question 11(a)**

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>7</td>
<td>9</td>
<td>3</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>2-4. Incorrect</td>
<td>4</td>
<td>0</td>
<td>7</td>
<td>9</td>
<td>20</td>
</tr>
<tr>
<td>Totals</td>
<td>11</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>40</td>
</tr>
</tbody>
</table>

Question 11(b)

If there are 6 horses in a race how many ways can the first and second places be filled? (Exclude any "dead heats")

Table 30 (condensed)

**Coding and Results: Question 11(b)**

<table>
<thead>
<tr>
<th>Code</th>
<th>G-LA</th>
<th>G-HA</th>
<th>NG-HA</th>
<th>NG-LA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2. Correct</td>
<td>1</td>
<td>7</td>
<td>4</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>3-6. Incorrect</td>
<td>10</td>
<td>2</td>
<td>6</td>
<td>9</td>
<td>27</td>
</tr>
<tr>
<td>Totals</td>
<td>11</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>40</td>
</tr>
</tbody>
</table>

Question 11(c)

[To be asked only of those students who were able to answer both Questions 11(a) and (b) correctly]

If the "quinella" consists of selecting the first two horses in a race without necessarily being in the right order, how many possible selections are there in a race with 8 horses.

[If the student is unable to answer this, proceed to Question 11(f)]

Table 31 (condensed)

**Coding and Results: Question 11(c)**

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>4</td>
</tr>
<tr>
<td>2. Incorrect</td>
<td>9</td>
</tr>
</tbody>
</table>
**Question 11(a)**

The "double" in horse racing consists of selecting two winners in two prescribed races. How many possible ways can the "double" occur if one race has 10 horses and the other 8?

(Exclude any "dead heats")

**Table 29 (condensed)**

**Coding and Results: Question 11(a)**

<table>
<thead>
<tr>
<th>Code</th>
<th>Group of Respondent</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G-LA</td>
<td>G-HA</td>
</tr>
<tr>
<td>1. Correct</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>2-4. Incorrect</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Totals</td>
<td>11</td>
<td>9</td>
</tr>
</tbody>
</table>

**Question 11(b)**

If there are 6 horses in a race how many ways can the first and second places be filled? (Exclude any "dead heats")

**Table 30 (condensed)**

**Coding and Results: Question 11(b)**

<table>
<thead>
<tr>
<th>Code</th>
<th>Group of Respondent</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G-LA</td>
<td>G-HA</td>
</tr>
<tr>
<td>1.2. Correct</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>3-6. Incorrect</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>Totals</td>
<td>11</td>
<td>9</td>
</tr>
</tbody>
</table>

**Question 11(c)**

[To be asked only of those students who were able to answer both Questions 11(a) and (b) correctly]

If the "quinella" consists of selecting the first two horses in a race without necessarily being in the right order, how many possible selections are there in a race with 8 horses.

[If the student is unable to answer this, proceed to Question 11(f)]

**Table 31 (condensed)**

**Coding and Results: Question 11(c)**

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>4</td>
</tr>
<tr>
<td>2. Incorrect</td>
<td>9</td>
</tr>
</tbody>
</table>
**Question 11(d)**
[To be asked only of those students who were able to answer both Questions 11(a) and (b) correctly]
If the "trifecta" consists of selecting the first three horses in the correct order, how many possible selections are there in a race with 6 horses?

Table 32 (condensed)

**Coding and Results; Question 11(d)**

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>4</td>
</tr>
<tr>
<td>2. Incorrect</td>
<td>9</td>
</tr>
</tbody>
</table>

**Question 11(e)**
[To be asked only of those students who were able to answer both Questions 11(d)]
If we disregard the order in the last question, how many selections are there?

Table 33 (condensed)

**Coding and Results; Question 11(e)**

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>2</td>
</tr>
<tr>
<td>2. Incorrect</td>
<td>2</td>
</tr>
</tbody>
</table>

**Question 11 (h)**
If you have 6 different movie vouchers each entitling you to a different movie, but you have time to go to only two of them, how many different choices can you make?

Table 36 (condensed)

**Coding and Results; Question 11(h)**

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct</td>
<td>4</td>
</tr>
<tr>
<td>2. Incorrect</td>
<td>36</td>
</tr>
</tbody>
</table>
In the gambling contexts of parts (c), (d), and (e), the proportion of correct responses was too low to enable any meaningful comparisons to be made between the two groups.

Only the 13 students who answered Question 11(b) correctly (a permutation) were asked to compute combinations. Of these, eight were gamblers familiar with the contexts of the quinella and trifecta, and yet only three of these (and only one non-gambler) gave a correct answer to Question 11(c), (the quinella), and Question 11(d), (the trifecta). Subsequent interview questions with the gamblers (see examples in Chapter 11) revealed that although they were familiar with the quinella and trifecta betting, all such betting was done through the TAB, where all of the calculations are done by the TAB. For example, if one places a bet on the quinella or the trifecta for "the favourite and the field," the TAB computes the number of possibilities required (See Appendix 5). Thus, although the mathematics of gambling involves combinatorics, the gamblers may or may not have reflected on these computations.

Questions 11(e) and (h) required the computation of a combination of three things. Only two and four students respectively computed a correct answer to these parts. For the majority of students, these questions were too difficult even to attempt.

**Implications of Data From Question 11 Relating to Combinatorics**

Further research into why the general level of performance on these items was so low is required. According to Piaget and Inhelder (1975), "the formation of the ideas of chance and probability depend in a very strict manner on the evolution of a combinatoric operation" (p. 161). However, the data from the questions in Category 3 indicate that nearly all students were able to calculate simple probabilities. The poor performance of these Year 11 students on the combinatoric questions raise serious mathematics curriculum issues. It would appear that there is a need for further research into the role of combinatoric ideas in the development of probabilistic concepts, and further consideration is needed on how combinatoric concepts might be best in the school mathematics curriculum. This is discussed further in the final chapter.

**Summary of the Analysis of Data Pertaining to the Second Major Research Question**

The ways in which individuals in the two groups processed the concept of proportional reasoning differed noticeably. Depending on the context (track gambling, or traditional school mathematics), the gamblers employed different strategies of computation, and used different language from the non-gamblers in processing concepts of proportional reasoning in track gambling contexts. Their use of these
strategies in traditional school mathematics contexts was limited to the use of a scalar additive strategy by some of the gamblers in proportional computations. The significance of the use of different language is discussed further later in this chapter in the answer to the third major research question.

The cognitive processes employed in the application of probabilistic and related mathematical concepts both in traditional classroom situations and in out-of-school contexts involving track gambling, include many of the techniques and strategies identified in the literature. However, in the present study, a number of alternate strategies used by the gamblers were identified. These included:

1. The use of a scalar additive technique in gambling contexts. This use was number dependent, in the sense that it was generally used only when one number was not a factor of the other. For some gamblers it was context dependent in the sense that they used it in a gambling context but not in other contexts. However some gamblers also used the strategy in non-gambling contexts.

2. The construction of a specific computational procedure to compare ratios in gambling context (odds) that has not been previously identified in the literature (see Chapter 8, Table 16, Code 4). This procedure also is highly number and context dependent.

3. The use of computational estimation and approximation strategies. These strategies were often effective only within a gambling context.

The marked difference between the concepts of the gamblers and the non-gamblers in regard to fairness, independence, and mathematical expectation and return, have been noted in the discussion of the first major research question. It was also noted in Chapter 9 that some of the gamblers showed an intuitive, but limited understanding of some concepts in compound probability. However, in general, no strong difference between the two groups were observed in the processing of basic concepts in simple and compound probability, or combinatorics.

Analysis of Data with Respect to the Third Major Research Question

The third major research question posed in Chapter 6 was:

Is the knowledge acquired by the gamblers as a result of socially induced cognitive interactions in gambling contexts sufficiently pervasive to be regarded as a form of ethnomathematics in the sense that this term has been described in the review of the literature?
In Chapter 1 the term "ethnomathematics" was defined as:

The mathematics which is practised by identifiable cultural groups, such as national tribal societies, labor groups, children of a certain age bracket, professional classes and so on. (D'Ambrosio, 1985a, p. 45)

It was further noted that "the term [ethnomathematics] itself is not well defined" (Bishop, 1988c, p. 180). Thus in formulating an answer to this question from an analysis of the data collected in the present study, some flexibility should be allowed. More recently, D'Ambrosio (1992) in an attempt to develop a research program has reiterated his concept of "ethnomathematics" which he redefines as "the study of generation, organisation, transmission, dissemination, and the use of ... jargons, codes, styles of reasoning, practices, results and methods" (p. 10).

Thus, the following points need to be considered in the formulation of an answer to the third major research question:

1. The generation of practices and styles of reasoning.
2. The organisation of codes and practices.
3. The status of "different mathematics."

While answering the second major research question, the point was made that there are noticeably different practices and styles of reasoning employed by the gamblers from the non-gamblers in proportional reasoning in gambling contexts. Thus, these data support the conclusion that a form of ethnomathematics does exist. However, the practices and styles of reasoning were employed by only some individuals of the group and were not universal among the group members. Some of these individuals used number dependent strategies, such as a scalar additive strategy, effectively in both gambling and non-gambling contexts but these practices and styles of reasoning did not, in general, transfer to traditional school contexts, and therefore were of limited applicability.

Nevertheless, it can be concluded that the proportional reasoning employed in the gambling contexts, the language used in the computation of betting returns, and the comparison of odds in gambling contexts, together form practices and styles of reasoning that do, in fact, constitute a type of ethnomathematics as defined by D'Ambrosio (1992).

The use of language was identified earlier in this chapter as a major difference between the two groups. Many individuals in the gambling group employed certain jargons and codes that are unique to the context of track gambling. These jargons and codes include the use of informal mathematical language and the experiences of the students' personal gambling world that might provide cognitive links with the formal
language, symbols and skills associated with school mathematics (Ellerton & Clements, 1991).

It is contended that the use of the informal language of track gambling which includes jargons and codes contributes to the body of identifiable knowledge which may be described as "ethnomathematics."

Further research is needed to determine the status of this mathematics among the broader gambling public and this is discussed further in Chapter 12.

Summary

In this chapter data associated with the second and third major research questions were analysed. It was noted that, in general, there was no noticeable gender effect on the quality of responses, and that the responses to many items, especially the "traditional" classroom questions, showed a noticeable relationship with school achievement level with high achievers performing consistently better than low achievers. This, surely, was not surprising.

In order to formulate an answer to the second major research question, data were analysed according to the six categories of mathematical concepts identified in Chapter 6. Differences between the responses of members of the two groups were noted. Data concerned with proportional reasoning in gambling and non-gambling contexts, the language used by the gamblers and the non-gamblers, and the different techniques of computation which were employed were triangulated to confirm that there was a distinct difference between the ways members of the two groups processed concepts in this topic.

An answer to the third major research question was formulated by the synthesis of data from the first two major research questions. This involved the recognition of the different practices, codes, jargons and styles of reasoning employed by the gamblers in the context of track gambling. Although this language, methods of computation and ways of thinking associated with track gambling are learned in the sense that they are acquired through participation in a particular sub-culture, they are not formally studied. Often their application is more an unconscious than conscious act. Bishop (1988a) has called this "small m" mathematics, as opposed to "capital m" mathematics and it was in this sense that the conclusion was reached that the informal mathematical methods, codes and language associated with track gambling, do indeed constitute a form of ethnomathematics.
Chapter 11

Results IV - Case Studies

In this chapter the responses to the interview questions by several selected students are presented. These students and their responses were selected from each of the four categories; Gambler-Low Achiever, Gambler-High Achiever, Non-Gambler-Low Achiever, Non-Gambler-High Achiever. The responses of one of the students from each of these categories were selected to illustrate and highlight certain response patterns that were representative of students in that category. Other responses from selected students are then presented in order to illustrate some of the points made in earlier chapters.

Case 1. Gambler, Low Achiever

Study #1, Leona.

Background. Leona was a pleasant student, very friendly and willing to take part in the study. She stated that she enjoyed most things about school, but not her mathematics classes which she described as "boring and a waste of time." She obtained a "Sound Achievement" assessment for Year 10 Ordinary Mathematics, but her teacher classified her as a Low Achiever in Semester 1, Year 11 "Maths in Society". She agreed with this assessment, admitting that she avoided doing mathematics as much as possible.

Both of her parents were avid punters who followed the forms from the newspapers, betting regularly, both on track and at the TAB. She thought that they were fairly successful, though the stakes were "not great." She followed her parents' betting but did not herself place bets. She said that her brother was a jockey and her father had at one time owned race horses. She reported that she accompanied her family to the track "regularly" — once or twice a month, attending both horse racing and trotting events. These occasions were viewed as "social" events rather than money making affairs, a view that was held by most of the other "gamblers" who attended track events with their families.

Leona stated that she played card games "regularly" with friends and family. These games included poker and blackjack, which were played "mostly for fun," though sometimes for small stakes. When playing for money the emphasis was on "fun rather than winning." In this, she was representative of those gamblers who
played card games for money. She also played various board games that involved rolling dice "about once a week."

**Leona's Responses to Interview Questions**

(Note that in all transcripts in this chapter, in relation to the conversations which occurred between the author and the students, the author will be denoted by R and the student by the first letter of his or her name).

**Question 1(a), (i)**
A photographic negative 2 cm wide and 3 cm long is enlarged to a photograph 9 cm wide.
How long is the photograph?

L: I'm not too sure about these. I think

\[ \frac{9}{2} = 4.25 \text{ [mental computation with error]} \]

\[ \text{then } 3 \times 4.25 = 12.75 \text{ [mental computation]} \]

Parts (ii) and (iii) of Questions 1(a) were completed correctly with the same correct functional reasoning but without mechanical error.

A calculator was used in part (iv) to compute \( \frac{7}{4} \times 5 \).

**Question 1(b)**

In each of the following situations, how much could be won on a track bet if:

(i) $10 is bet at odds of 9:2
(ii) $9 is bet at odds of 3:2
(iii) $5 is bet at odds of 7:4

Questions 1(b), parts (i) and (ii) were completed using the correct multiplicative approach and mental computation, but Leona made the same computational error as in part (a), that is to say, \( \frac{9}{2} = 4.25 \)

For part (iii), Leona reasoned "Seven to four is the same as three and a half to two, so $2 gets $3.50; and so $5 gets $3.50" A calculator was used to compute the answer correctly.
R: If you were at the track without a calculator, how would you do this calculation?
L: Well at the track, the bookie writes on the ticket the amount you get if you win.
R: Is this the amount you win or the amount you get back?
L: It's the total you will get back. Like if you bet $5 at 2:1, the bookie writes $15 on the ticket.
R: So if you bet $5 at 7:4 and wanted to make sure that the bookie wrote the right amount on the ticket, what would you do if you didn't have a calculator with you?
L: Let's see ... At 7:4, $4 will win $7, so $5 will win ... more than $8. So the bookie writes $13 something.
R: What's the something?
L: Bookies don't write the cents, they just put strokes for each twenty-five cents.
R: So how many strokes on this ticket?
L: It would be $13 /// like that. [writes]
R: What if you bet $5 at 9:4 at the track without a calculator, how would you calculate what you'd win.
L: Let's see ... 9:4 is more than 2:1, so you'd win more than $10, about $11, and the bookie would write $16 something.
R: What if you bet on the TAB?
L: The TAB doesn't give you odds like 7:4. It tells you how much you get back for each $1 bet.
R: If the bookies were paying 7:4, what would the TAB be showing.
L: Well it's not always the same. The TAB could be different, you know.
R: In what way?
L: Different odds.
R: Which is better, the bookies or the TAB?
L: It depends. Sometimes, if you get in early, you can get better odds with the bookies. Other times, the TAB pays out more.
R: So, if a bookie was paying 7:4, what would you expect the TAB to be paying?
L: About $2 ... a bit less, $1.80 or so.
R: And 9:4 ?
L: More than $2.
R: How much more?
L: About $2.20.

Leona employed mental computation in all parts. For simple numbers she used a school learned multiplicative approach, but the strategy changed to a scalar additive approach with approximation, or to a unitary approach for more difficult questions. When the calculator was used, she employed a correct multiplicative approach.

Throughout the interview, the language that Leona used was consistent with the symbols employed.
Question 2
In each pair, circle the larger of the two fractions:

(a) \(\frac{1}{2}\) \(\frac{1}{3}\)
(b) \(\frac{3}{8}\) \(\frac{5}{8}\)
(c) \(\frac{1}{2}\) \(\frac{3}{5}\)
(d) \(\frac{3}{5}\) \(\frac{5}{8}\)

Parts (a), (b), and (c) were answered correctly. In part (c) Leona responded:

\[ \frac{2}{5} \]

L: One-half is (writes) \(\frac{2}{5}\) This is less than three-fifths.

But in part (d), she reasoned incorrectly:

L: 5 is closer to zero than 8. That is 5 is less than 8. So fifths are bigger than eights. Three fifths will be closer to 1 than five-eighths.
R: What about three-fifths or seven-eights
L: Oh, seven-eights is closer to 1.
R: How do you know that?
L: It just is.

Leona did not use either of the school-taught algorithms of finding a common denominator or conversion to decimals to compare fractions, and instead relied on intuitive, but imperfect strategies.
Question 3

Suppose I select a marble at random from each of the following bags [diagram shown].
In each of the two situations what is the probability of getting a black marble?
(a) The bag contains 3 black and 1 white  
(b) The bag contains 5 black and 3 white  
(c) In which of these cases, (a) or (b), is it more likely to get a black marble?

Leona had no difficulty in answering these correctly as "3 chances out of 4, and 5 chances out of 8." With regard to part (c), the following conversation took place:

L: [hesitates] I'm not sure about this one. I think (a) has the better chances.
    Yes, 3 out of 4 is better than 5 out of 8.
R: Why do say that?
L: I just think it is?
R: Let's look at the next part.

Question 3(d)

[if unable to compare the probabilities in Question 3(c)]
Repeat Question 3(c) with:
(i) (a) 1 black marble and 3 white marbles  
    (b) 1 black marble and 4 white marbles

(ii) (a) 2 black and 3 white  
    (b) 2 black and 5 white;

L: The first is better, 1 out of 4 is a better chance than 1 out of 5.
R: What about 2 out of 5 compared with 2 out of 7?
L: The 2 out of 5 is a better chance.
R: Let's look at another one.

Question 3(e)

[if able to compare the probabilities in Question 3(c)]
Question 3(c) was repeated using;
3 black marbles and 2 white marbles in one bag, and 5 black marbles and 3 white marbles in the other bag.

L: (a) has three-fifths and (b) has five-eighths, they're about the same. One might be more likely than the other. I can't really tell.

Note that although in Question 3 Leona demonstrated a clear understanding of basic probability concepts, she was unable to compare probabilities except in very simple situations.
This is consistent with her limited ability to compare fractions. Lacking school based mathematics skills, she was unable to answer the question in this context, although, as will become clear, in the more familiar contexts of Questions 4 and 5, she was able to compare odds.

**Question 4**
In each of the following track betting situations, which is the better of the two odds? That is to say which gives the greater return per $1 bet, or which is the greater ratio?:

(a) 2:1 or 3:1
(b) 5:1 or 5:2
(c) 4:3 or 9:7

Parts (a) and (b) were answered confidently. She was well acquainted with the terminology. In part(c), Leona was one of the gamblers who constructed their own context dependent procedure, reasoning:

L: 4:3 is the same as 8:6, that is bet 6 win 8, 2 more than you bet.
9:7 means bet 7 win 9, that is also 2 more than you bet.
So 8:6 is better since you win the same amount more for a smaller outlay.

**Question 5**
In a four horse race the odds for each horse are given as -
2:1, 5:3, 5:1, and 25:1
Which horse is thought to be most likely to win?
List the odds in order of least likely to most likely.

L: 5:3 are shorter odds, so that one's more likely to win. It would probably be the favourite.
R: What do you mean by "shorter odds"?
L: The lowest pay out. You get less back.
R: How do you know that 5:3 is "shorter" than 2:1?
L: Well if you bet $1 at 2:1 and won, you'd win $2 and get $3 back, but at 5:3 you'd win less than $2 and get back less than $3.
R: How much would you win at 5:3
L: About $1.60
R: How did you get that?
L: It would be a third of $5. I think that's about $1.60.
R: Which horse is least likely to win?
L: The 25:1
R: So could you put the odds in order of most likely to least likely?
L: Yes that's easy 5:3, 2:1, 5:1 and 25:1
Leona showed absolutely no misuse of availability or representativeness in any parts of the questions relating to these. She was very confident when answering these questions and thought some of them were so simple as to be trivial.

Question 6(a)
In the last 200 Gold Lotto draws the number 17 has occurred more times than any other number. In future draws do you think that this number is:

(i) more likely to occur than any other number?
(ii) less likely to occur than any other number?
(iii) as equally likely to occur as any other number?

L: It's still the same. It doesn't make any difference.
R: The fact that it's occurred the most often doesn't matter?
L: Of course not. [laughs]. Lotto doesn't work that way.
R: Do you buy Lotto tickets?
L: No.
R: Do your parents buy Lotto tickets?
L: Sometimes. Not very often, I don't think so anyway.
R: The Sunday paper publishes these frequencies of occurrence for people to see. Why do you think the newspapers publish these numbers then?
L: I'm not sure. I never really thought about it.
R: How do you know that the probability remains the same with Lotto numbers?
L: It has to. They're just balls you know.

Question 6(b)
[Describe/show roulette wheel]
In Roulette, each number is equally likely to occur.
Suppose that the ball lands on "red" six times in a row. On the next roll is it now more likely to land on the "red", the "black", or are they both still equally likely?

L: They're still the same. It makes no difference.
R: It doesn't matter how many times the Red occurs?
L: No, not if each number is equally likely.
Question 6(c)
In some lotteries the tickets are pre-numbered. If you could choose between any of the following numbers would you have any preference?
123456
619999
615472

L: No, I wouldn't really care.
R: Are they all equally likely?
L: Of course. All the tickets have the same chance of being drawn, don't they?

Question 6(d)
In choosing six LOTTO numbers which of the following selections would you prefer?
(a) 1, 2, 3, 4, 5, 6
(b) 32, 33, 34, 35, 36, 37
(c) 9, 12, 27, 31, 35, 38

L: I wouldn't really have any preference.
R: Are they all equally likely?
L: Yes, all Lotto numbers have the same chance.

It can be seen that Leona has a very good concept of "independence" of events. This concept is not well understood, even by university students (see Shaughnessy, 1989; Scholz, 1987), and the misuse of representativeness is "not confined to naive subjects" (Tversky & Kahnemann, 1982a, p. 5).

Leona had no difficulty with any of Questions 7(a), (b), or (c) and again viewed them as quite trivial.

Question 7(d)
Suppose that you and I are going to play a game with this coin [shows] in which we toss the coin just once. If the coin lands heads, I win; if it lands tails you win. But I will warn you that of the last 15 people who played this game with me only 6 won.
Do you think this is unusual?
Do you think the coin is not fair?
Do you think the game is fair?
L: No, there's nothing unusual about this. Why wouldn't the coin be fair?
R: Is the game fair?
L: I suppose so.
R: What if I told you that of the last 20 people I played 14 lost? Do you think the coin is still fair.
L: I'm not sure. It might be. If it kept coming up heads like this I don't think it would be fair.

It can be seen that Leona was able to recognise that a long run would be necessary before unfairness or bias could be suspected.

*The concept of mathematical expectation.* Leona's understanding of this concept, its determination and relationship with mathematical fairness is shown in the transcripts of the interview for Question 10.

*Question 10(a)*

Suppose you and I play a game with one of these dice [show single die]. If on a single roll it is three or less you win, if it is more than three I win. If we each bet $1 and the winner gets the $2, would this be a fair game?

L: Yes, this is fair.

*Question 10(b), (i)*

Suppose we change the rules so that I win if it is a three or greater is this still a fair game?

*Question 10(b), (ii)*

Can we make the game fair somehow?

*Question 10(b), (iii)*

How much should I put in to make the game fair?

L: Don't be silly. The game's not fair now, you've got a better chance now.
R: Can we make the game fair somehow, with these new rules?
L: Sure, you've got to pay me more if I win.
R: Can you figure out how much more?
L: Well you've got four chances to my two. That's twice as much, so you've got to put in twice as much to make it fair.
R: If you win that is?
L: Yes, if I win. But see, my chances of winning are less than yours, so I need to win more to make it fair.
R: If you were to play this game, with the new rules, which would you rather take; the 1,2 or the 3, 4, 5, 6?
L: It doesn't really matter. The game's fair.
R: Suppose we look at another game. How much should I put in if I win with the numbers 1-5 leaving you just the 6 to win on?
L: Let's see. You've got five numbers and I've got one, so you put in five dollars for my one.
R: Suppose we draw cards from a well shuffled deck. I win if a spade is drawn leaving you to win if it's one on the other three suits. How much now?
L: $3
R: What if I take the 16 coloured cards and leave you the rest?
L: This is harder. You've got 16 cards so I've got [uses pencil and paper to compute 52-16] 36 cards. So you've got \(\frac{36}{16}\) [stops] Hmm. That's two and a half.
R: Do you want to check that with your calculator?
L: [gets her calculator and computes \(\frac{36}{16} = 2.25\)] Oh, it's two twenty five. Yes, you put in $2.25.
R: What if I take one card of any suit?
L: It's the same thing. You have 12 cards to my one, so you put in $12 to my $1.
R: If I take one specific card?
L: And I get the rest?
R: Yes
L: $51 then.

From this transcript it can be seen that Leona has clearly constructed her own definition of fairness. Although she did not use the formal mathematical language of expectation, she recognised that the product of return and probability must be the same for each player in order to make the game fair and computed this product correctly in all situations using a multiplicative technique. This concept of expectation is further illustrated by her responses to the following probing questions.

R: You said earlier that you sometimes bet small amounts on the track. When you bet at the track, do you bet on the favourites, or do you go for outsiders?
L: I don't go for outsiders. They generally don't have much chance of winning. What you've got to find is one that you know has got a real good chance and get on with a bookie early at good odds.
R: What sort of odds?
L: Like 3:1, 2:1, or maybe 6:4, but if it drops to evens or odds-on, I wouldn't bet.
R: Why not?
L: The return is too small.
R: How do you know when a horse has a real good chance?
L: [laughs] That's the trick, isn't it? I generally let my dad choose.
R: Do you generally win or lose?
L: Mmm. I'd say its about 50-50.
R: Does it worry you, if you lose?
L: Well .... you want to win you know. But you know that you're not going to win all the time. And anyhow, I don't bet much. It's just for fun really.
R: But do you think you come out ahead in the long run?
L: Probability a little bit ahead. Yes.
R: Do you bet on the TAB?
L: No, because you don't know what the odds are going to be. They might start out O.K. but end up real short.

Leona clearly demonstrated an intuitive understanding of the concept of mathematical expectation in recognising that betting on a "good chance" at "good odds" would result in coming out ahead in the long run, even though she might lose on any individual race. She also understood that if the odds were "too short" her expectation would be too low to bet. Her responses to further probing questions indicated that she also understood how the concept of expectation related to fairness.

R: Let's go back to the card games. If you were to play the game with me in which one person draws a single card, would you choose the single card or any of the 51 others?
L: I don't think I'd want to play this game. I couldn't afford to pay you $51 if you won on the single card, and if I choose the single card I'd probably lose my $1.
R: True, but you might win $51.
L: Yes, but that's not very likely.
R: What's the probability of that happening?
L: One in 52.
R: And the game is still fair?
L: Yes
R: Suppose I offered you more than $51 to try to draw a single card for $1, how much would you want to win before you'd be willing to play?
L: It doesn't matter, I'm too likely to lose my dollar.
R: $100?, $200??
L: Yes, for $200 I'd risk my dollar.
R: But would this be a fair game?
L: No, it's not really fair for you. You could lose a lot of money.

From this transcript, it can be seen that in recognising an "unfair" situation Leona has demonstrated a very sound understanding of "fairness." Even in situations where she "expects" to loose, she was able to recognise that the game favoured her.
The concept of compound probability: Leona's lack of understanding of this concept in a variety of contexts was revealed by her responses to Questions 8 and 9. In Question 8 Leona correctly chose paths A and B but was not confident, reasoning:

L: They're the closest, so they're the most likely.
R: So where is the robot most likely to end up?
L: Here or here (points to A and B)
R: Because they're closest?
L: Yes.

Although Leona had previously demonstrated a sound knowledge of basic probability, she did not have any mechanism to compute compound probabilities.

Question 9(a)
In each of two horse races the favourite is estimated to have about a 50% chance of winning.
If you bet on both favourites, your chances of winning on both would be about:
(i) 100%
(ii) 75%
(iii) 50%
(iv) 25%
(v) 10%

L: Well it has to be less than 50%, 10% seems a bit small so I'd say about 25%

Although Leona had shown no mechanism to compute compound probability, she did recognise that the compounded probability must be less than the individual probabilities. In doing so, she has demonstrated some intuitive knowledge of compound probability within the gambling context. When the same concept was assessed in a different context, she did not demonstrate this skill.

R: If I am going to play tennis against an opponent and I have 50% chance of winning a set, what is the chance that I will win two sets in a row, assuming the probability of winning each remains at 50%?
L: 50%?
R: It would stay the same?
L: I think so. I'm not sure. Maybe it would be less.
Question 9(b)

Two coins are tossed together.
(i) The probability that one will land a head and the other a tail is:

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{4}$

(ii) What is the probability that at least one of the two coins will land a head?

L: I'd say a half.
R: Are you sure?
L: No, but I think it would be a half.
R: Why? Any reason?
L: Well they'd either be the same or different, so there's a half chance for each.
R: What is the probability that at least one of the two coins will land a head?
L: Well, it's got to be more than before. So, more than one-half, about three-quarters.

Again, Leona demonstrated some intuitive understanding of compound probability and although she answered all parts correctly, she apparently did not use any computational technique to find the compound probability.

Question 9(c)

(i) A pair of fair dice are tossed together.
[show pair of identical dice]
Which is more likely:
(a) a pair of fives
(b) a five and a three
(c) both are equally likely

(ii) [repeat with different coloured dice]

(iii) In general, is it harder (less likely) to get a pair of numbers than it is to get two different numbers?

L. [Thinks carefully, examines the dice for a few moments] I'm not sure about this. I suppose they're equally likely.
R: What about if we change the dice to one red and one green?
L: No. That wouldn't make any difference. They're still the same.
R: In general, is it harder to get a pair than it is to get different numbers.
L: Yes, I think a pair is harder to get than different numbers
R: Any reason?
L: There's less of them.
R: Does this affect your response to part (c)?
L: No, because this is just fives and threes - not all of them. There's the same number of each now.

Although she was familiar with the throwing of dice. Leona demonstrated the equally-likely misconception in this situation, and she was resistant to changing her view of the specific case, even when she was able to answer the general case correctly from experience.

**Combinatoric computations.** Leona's understanding of combinatorics and the determination of simple combinations and permutations in a variety of contexts is illustrated by her responses to Question 11. Parts (a) and (b) were answered correctly using a correct multiplicative technique.

**Question 11(a)**
The "double" in horse racing consists of selecting two winners in two prescribed races.
How many possible ways can the "double" occur if one race has 10 horses and the other 8?
(assume that there are no dead heats).

**Question 11(b)**
If there are 6 horses in a race, what is the total number of ways in which the first and second places be filled?
(Ignore the possibility of a dead heat)
[If the student is unable to answer this, proceed to Question 11(d)]

**Question 11(c)**
[To be asked only of those students who were able to answer both Questions 11(a) and (b) correctly]
If the "quinella" consists of selecting the first two horses in a race without necessarily being in the right order, how many possible selections are there in a race with 8 horses.
[If the student is unable to answer this, proceed to Question 11(f)]

L: There's a lot, but I don't know how to figure it out.
R: Do your parents place bets on the quinella?
L: Yes, sometimes. But generally not. The quinella goes through the TAB. We generally use bookmakers.
R: What sort of bets can you place on the quinella?
L: Well you can choose a pair of horses or you can go for "the favourite and the field."
R: What's that involve?
L: You get the favourite and all the other horses.
R: So if there were 8 horses in the race, how much would that cost?
L: Well you get the favourite and the other 7 so it cost you $7.
R: So in an 8 horse race, there's lots of possible ways the quinella could occur. Could you calculate how many?
L: No, I don't think so.
R: In part (b) you said that there were $6 \times 5 = 30$ ways in which the first two places could be filled in a 6 horse race. How many ways would there be in an 8 horse race?
L: Oh, yes. There'd be $8 \times 7$, 56.
R: Is this the number of quinellas?
L: Yes, I suppose that would be it.

Although Leona was unable to answer Question 11(c) correctly, the interview proceeded with Question 11(d).

**Question 11(d)**

[To be asked only of those students who were able to answer both Questions 11(c) correctly]
If the trifecta consists of selecting the first three horses in the correct order, how many possible selections are there in a race with 6 horses?

L: There's a lot more with the trifecta. It pays more than the quinella.
R: Do your parents bet on the trifecta?
L: Yes, sometimes. If there's a sure thing, you can put it with the field on the trifecta.
R: What happens then?
L: You put the one horse to win. It has to win or you'll lose, but if it wins you get the trifecta because you've got all the rest of the field covered.
R: You mean you've got all the other possibilities.
L: Yes.
R: How much would that cost you if there were 6 horses in the race?
L: The TAB tells you what it will cost.
R: But can you figure it out before you place the bet?
L: I think so. If you take the favourite to win then there's five others left to come second and third. So, I think this will cost $20. I'm not too sure.
R: How did you figure that.
L: It's $5 \times 4$.
R: So if we go back to the original question, can you figure out the total number of trifectas.
L: No, I'm not really sure.
Questions 11(f) and (g) were answered correctly, but no attempt was made at Question 11(h).

**Question 11(f)**
A type of new car can be bought with a choice of 6 different colours and 3 different engine types. What is the total number of selections possible.

**Question 11(g)**
If the same type of car as in the last question can also be bought with either manual or automatic transmission, how many selections are now possible?

**Question 11(h)**
Suppose that you have 6 different movie vouchers but have time to go to only two movies. How many different choices can you make in selecting the two movies you will attend?

Despite a thorough familiarity with the context of the betting, Leona’s computational techniques were very limited. She demonstrated a basic understanding of the multiplicative principle in the simple situation involving a permutation of only two things, both in gambling and non-gambling situations. However, although she calculated that there are 56 ways in which the first two places of an 8 horse race can be filled, and she did not recognise that there would be fewer quinellas in which the order did not matter. Nor was she able to extend the multiplicative principle to the trifecta involving the first three horses.

**Summary of Leona’s responses.** Leona’s computational skills were limited and she exhibited the difficulties with fractional numbers that the literature identifies as being common among low achieving students. However, her reasoning in all situations involving proportion was multiplicative, and she did not use an incorrect additive strategy in any situations. Furthermore, she was able to employ mental computation and estimation skills with some success. She compared odds using either a multiplicative procedure for simple numbers or her own context dependent procedure. She had an sound understanding of basic simple probability, but was restricted in her use of this by her limited mathematical skills. Nevertheless she showed an exceptionally good understanding of the concept of independence of events and did not show any of the misconceptions associated with the use of the representativeness or availability heuristics that are reported in the literature as being widespread. She was able to recognise mathematically fair and unfair situations and showed an intuitive understanding of the concept of mathematical
expectation. In the computation of the return from bets at various odds and the comparison of odds he used language that was consistent with the mathematical procedures employed.

Case 2. Gambler, High Achiever

Study #17, Troy.

Background. Troy was a mature student, who left school at the end of Year 10 and worked as an apprentice for two years before returning to school. He was a pleasant student, friendly and willing to take part in the study. He enjoyed school and was doing well in all subjects. He chose to do the lower level mathematics because "it is more practical," and found the work "quite easy." He was classified by his teacher as a High Achiever.

Both of Troy's parents were avid punters following the forms from the newspapers, betting regularly both on track and at the TAB. His elder brother owned and trained race horses. He described his grandfather as "a real professional" who won consistently at the track, often large amounts; $2000 was not unusual.

Troy followed his parents', grandfather's and brother's betting. He himself bet "occasionally," placing bets of between $1 and $5, using either the TAB off track, or bookmakers at the track. He accompanied his father and brother to the track "regularly" - about once a week at either horse racing, trotting or dog racing. Troy viewed these occasions as "social" rather than as money-making affairs, although he reported that his father and brother had expectations of winning. He played card games "occasionally - generally at weekends" with friends and family. These games included poker and blackjack which were played "mostly for fun," though sometimes for small stakes. When playing for money, the emphasis was on "fun rather than winning."

Troy's Responses to Interview Questions

Troy worked quickly and efficiently through all parts of Questions 1(a) and (b) using mental computation for most and a calculator for some parts, such as \( \frac{7}{4} \times 5 \). Pencil and paper computations were not performed, and a pencil was used only to record results. The traditional multiplicative technique was used consistently both in gambling and non-gambling contexts. Without a calculator, Troy used a correct multiplicative procedure and mental computation.

R: How would you do the \( \frac{7}{4} \times 5 \) if you were at the track and didn't have a calculator?
T: Um, it'd be 35 divided by 4, that's 8 and three-quarters.
R: And if you bet $5 at 9:4 how much could you win?
T: 45 divided by 4, 11 and one-quarter.

In Question 2, Troy used the traditional conversion to decimal method to compare fractions correctly, employing mental computation with confidence for the first three parts and using a pencil and paper method to convert 5/8 to 0.625.

**Question 3**
Suppose I select a marble at random from each of the following bags [diagram shown]:
In each of the two situations what is the probability of getting a black marble?
(a) The bag contains 3 black and 1 white
(b) The bag contains 5 black and 3 white
(c) In which of these cases, (a) or (b), is it more likely to get a black marble?

Troy answered these questions confidently and correctly.

T: (a) is the more likely.
R: Why?
T: The probability is three-quarters, that's greater than five-eights.
R: How do you know that?
T: Three-quarters is six-eights. That's greater than five-eights.

The interview probed this understanding further:

**Question 3(c)**
(if able to compare the probabilities in Question 3(c)]
Question 3(c) was repeated using;
3 black marbles and 2 white marbles in one bag, and 5 black marbles and 3 white marbles in the other bag.

T: Three-fifths or five-eights? Five-eights is bigger.
R: How do you know?
T: 0.625 is bigger than .6.

Troy used the common denominator technique to compare 3/4 and 5/8 correctly, and then used a decimal comparison for 3/5 and 5/8. In this question he demonstrated a clear understanding of basic probability concepts. Troy had been out of school for
two years and stated that he had no recollection of any school work in probability. Nevertheless, he was able to compare probabilities using either the common denominator method, or by the technique of conversion to decimals which he used to compare fractions in Question 2.

**Question 4(a), (b) and (c)**

In each of the following track betting situations, which is the better of the two odds? That is to say, which gives the greater return per $ bet.

(a) 2:1 or 3:1
(b) 5:1 or 5:2
(c) 4:3 or 9:7

Parts (a) and (b) of this question were answered confidently. He was well acquainted with the terminology. In part (c) he constructed a context dependent procedure, and reasoned:

\( T: 4:3 \) is the same as 8:6 that is bet 6 win 8, 2 more than you bet. 9:7 means bet 7 win 9, that is also 2 more than you bet. So 8:6 is better since you win the same amount more for a smaller outlay.

This was the same procedure that Leona and three other gamblers used.

**Question 5**

In a four horse race the odds for each horse are given as - 2:1, 5:3, 5:1, and 25:1

Which horse is thought to be most likely to win?
List the odds in order of least likely to most likely.

This question was answered confidently. Again a common denominator method was employed.

\( T: 2:1 \) is the same as 6:3.
5:3 are shorter odds, that means it's more likely to win.
\( R: \) Can you list the odds in order of most likely to least likely?
\( T: \) Yes, that's easy.

Troy showed no misuse of availability or representativeness in any parts of the Questions 6 and 7.
Question 6(a)
In the last 200 Gold Lotto draws the number 17 has occurred more times than any other number. In future draws do you think that this number is:

(i) more likely to occur than any other number?
(ii) less likely to occur than any other number?
(iii) as equally likely to occur as any other number?

T: I'm not too sure here. It might be more likely.
R: Why might it be more likely?
T: If the 17 keeps coming up, maybe it's more likely.
R: You mean the Lotto numbers might not be really equally likely?
T: Yes, that's right.
R: Do you or your parents ever buy Lotto tickets.
T: No.
R: If we were sure that the numbers in Lotto all really do have the same chance of being drawn each time, what would your answer be now.
T: Equally likely.

In Question 7(d) Troy recognised that a longer run would be necessary before unfairness or bias could be suspected.

Calculation of compound probabilities. Troy's lack of understanding of the concept of compound probability in a variety of contexts was revealed by his responses to Questions 8 and 9. In Question 8 Troy correctly chose paths A and B but was not confident, using the "closest path" reasoning. Although he had already demonstrated a sound knowledge of basic probability, he did not here demonstrate the ability to compound probabilities. In response to Question 9(a) he reasoned intuitively, indicating that he understood that the compounding of probabilities reduced the final probability. He also answered the question in the alternative context in the same manner:

T: There's only a 50% chance for each so for the two to win, it will be less than 50%, probably about 25%.
R: If I am going to play tennis against an opponent and I have 50% chance of winning a set, what is the chance that I will win two sets in a row, assuming the probability of winning each remains at 50%?
T: That's the same thing. It's less, about 25% again.
Question 9(b)
Two coins are tossed together.
   (i) The probability that one will land a head and the other a tail is:

   (a) $\frac{1}{2}$

   (b) $\frac{1}{3}$

   (c) $\frac{1}{4}$

T: (a)
R: Why?
T: They're either the same or different.

Question 9(b)
   (ii) What is the probability that at least one of the two coins will land a head?

T: It's more now.
R: Can you figure out the probability?

T: Between $\frac{1}{2}$ and 1, so about $\frac{3}{4}$.

Question 9(c)
   (i) A pair of fair dice are tossed together.
       [show pair of identical dice]
       Which is more likely:
       (a) a pair of fives
       (b) a five and a three
       (c) both are equally likely

   (ii) [repeat with different coloured dice]

   (iii) In general, is it harder (less likely) to get a pair of numbers than it is to get two different numbers?

T: I'm not too sure about here. I think they're equally likely.
R: What about if we change the dice to one red and one green?
T: No. That wouldn't make any difference. They'd still be the same.
R: In general, is it harder to get a pair than it is to get different numbers.
T: Yes, I think pairs are harder to get than different numbers.
R: Any reason?
T: There's not as many of them.
R: Does this affect your response to part (c)?
T: No, I don't think so.

Although Troy was familiar with the throwing of dice, he demonstrated the "equally-likely" misconception in this situation, and the resistance to change his view of the specific case. Again, although Troy demonstrated some intuitive knowledge of compound probability, he lacked any overall grasp of the concepts involved.

*Mathematical expectation.* Troy's understanding of the concept of mathematical expectation, its determination and relationship with mathematical fairness was demonstrated by his responses to the interview questions in Question 10. Many of Troy's responses to these questions were essentially the same as Leona's in the previous case study cited. Troy was another one of the five gamblers who demonstrated a sound understanding of these concepts (see Table 39).

*Combinatoric situations.* Troy's understanding of combinatorics and the determination of simple combinations and permutations in a variety of contexts is illustrated by his responses to Question 11. Parts (a), (b), (f) and (g) were answered correctly using a correct multiplicative technique. Although Troy was familiar with the context of Question 11(c) and answered correctly, he was not confident with his answer. Only four students were able to answer this question correctly.

*Question 11(c)*

[To be asked only of those students who were able to answer both Questions 11(a) and (b) correctly]
If the "quinella" consists of selecting the first two horses in a race without necessarily being in the right order, how many possible selections are there in a race with 8 horses.
[If the student is unable to answer this, proceed to Question 11(f)]

T: There'd be 56
R: $8 \times 7$
T: Yes
R: But the quinella doesn't need them to be in the correct order.
T: Oh right, so it's less than 56. Is it half as much?
R: What do you think?
T: I'm not sure, but I think that's right. Yes there'd be 28
Question 11(d)
[To be asked only of those students who were able to answer both Questions 11(c) correctly]
If the trifecta consists of selecting the first three horses in the correct order, how many possible selections are there in a race with 6 horses?

Question 11(e)
[To be asked only of those students who were able to answer both Questions 11(d)]
If we disregard the order in the last question, how many selections are there?

T: There’s lots here - let’s see, two horses $6 \times 5$, 30. Three horses $30 \times 4$?
   I’m not sure, but I think 120.

R: If we disregard the order in the last question, how many selections are there now?

T: A lot less. Divide by $2!$, $3!$ 40 sounds about right.

Only one student was able to answer this question correctly. Troy’s response was a credible attempt, but despite familiarity with the context, he was not correct.

Question 11(h)
Suppose that you have 6 different movie vouchers but have time to go to only two movies. How many different choices can you make in selecting the two movies you will attend?

Troy attempted to enumerate the total number of possibilities but did not arrive at a correct answer.

Summary of Troy’s responses. Troy was competent and generally confident with his computation in all situations. Although he had been out of school for two years, he used school-taught multiplicative techniques for all proportional questions. He made extensive use of mental computation and estimation techniques and little use of pencil and paper procedures. He was one of the very few students who answered most of the combinatoric questions correctly. He demonstrated no misuse of heuristics and the only misconception that he showed was that of the equal likelihood of the same and different numbers on the throw of a pair of dice. He was able to quantify and compare simple probabilities, but lacked any real understanding of the concept of compound probability. In all the questions in gambling contexts he used the language and jargon of the gambling situation, and used mathematics consistent with this language.
Case 3. Non-gambler, High Achiever

Study # 25, Ruth.

Background. Ruth was one of seven students selected from School 3 as part of the non-gambling group. She was totally unfamiliar with gambling in any situation and unfamiliar with the terminology used. She was unaware of the composition of a card deck and did not play any games involving the throwing of dice or any other games of chance. She said that her parents did not approve of gambling of any type.

She received a "High Achievement" in Year 10 mathematics, but claimed that she had not studied any probability at all in Year 10 or earlier years. Her Year 11 teacher described her as a "hard working, high achieving student." Ruth said that she enjoyed most things about school including her mathematics classes. She selected the lower level mathematics course because it was "more appropriate to her needs."

Ruth's Responses to Interview Questions

Ruth answered all parts of Questions 1 and 2 correctly and confidently using both mental computation and a calculator. Pencil and paper were used only to record results. In all parts of Question 1 she used a traditional multiplicative strategy. Although she was unfamiliar with the terminology of the gambling questions, when this was explained to her, she used a traditional school-taught strategy to obtain correct answers. To compare fractions she used the conversion to decimal strategy using a calculator for part (d).

Ruth's responses to interview questions in Question 3 demonstrated that she had limited knowledge of even the most fundamental concepts in probability.

Question 3
Suppose I select a marble at random from each of the following bags [diagram shown]:
In each of the two situations what is the probability of getting a black marble?
(a) The bag contains 3 black and 1 white  
(b) The bag contains 5 black and 3 white  
(c) In which of these cases, (a) or (b), is it more likely to get a black marble?

R: In the first one there's three black and one white so you're more likely to get a black.
R: You mean that you're more likely to get a black marble than a white one in this case?
Ru: Yes.
R: Can you say what your chance of getting a marble black is?
Ru: Three.
R: Three out of how many?
Ru: Three to one.
R: Can you express this in any other way?
Ru: No, I don't think so.
R: What about part (b)?
Ru: Yes, there's still more black than white, so the black's more likely.
R: Is it more or less likely than with the first bag?
Ru: I don't know. It 'd be about the same wouldn't it?
R: You mean the black is more likely for each bag and you've got about the
same chance in each?
Ru: Yes.
R: If we have 1 black and 3 white in the first bag and 1 black and 4 white in
the second, in which is it more likely now to get a black?
Ru: The first.
R: Can you say how much more?
Ru: One more.
R: Can you express this any other way?
Ru: No, it's just one more.

Probing at this stage was discontinued since the student has demonstrated that
she had only a basic understanding of very elementary probability, and that she
had no mechanism by which to quantify or compare probabilities. This was
despite the fact that she had demonstrated a sound knowledge of fraction
concepts, the ability to compare fractions and the ability to calculate and
compare proportions.

**Question 4**

In each of the following track betting situations, which is
the better of the two odds? That is to say which gives the
greater return per $1 bet, or which is the greater ratio?:

(a) 2:1 or 3:1
(b) 5:1 or 5:2
(c) 4:3 or 9:7

Ru: I'm not sure about these.
R: If you think of them as ratios, which is the greater ratio in each case.
Ru: Oh, 3:1 is greater than 2:1 and 5:1 is greater than 5:2 and ... I'm not
sure about the next one, I think it will be 9:7.
R: Why's that?
Ru: There's two difference between 9 and 7 and only one between 4 and 3.
R: Is that what you did in the first two parts?
Ru: Yes, ... no, maybe not. I'm not really sure about this one.
In this situation Ruth used an incorrect additive comparison that she did not use elsewhere.

In Question 5, Ruth responded:

Ru: The smallest ratio?
R: Yes. 2:1 means that you win $2 if you bet $1 and win whereas 5:3 means that if you bet $3 and won you would win $5.
Ru: Well 2:1 would be 6:3 and that's bigger than 5:3.
R: Can you put the odds in order of likelihood?
Ru: You mean biggest to smallest?
R: Or smallest to biggest.
Ru: O.K., 5:3 is the smallest- we've figured that out already, next would be 2:1 then 5:1 then 25:1.

In this situation, Ruth reverted to the correct traditional multiplicative comparison that she used in other proportion comparisons. Ruth's lack of familiarity with gambling of any form and her lack of skills in probabilistic reasoning are illustrated by the following interview transcript.

Question 6(a)
In the last 200 Gold Lotto draws the number 17 has occurred more times than any other number. In future draws this number is:
(i) more likely to occur than any other number?
(ii) less likely to occur than any other number?
(iii) as equally likely to occur as any other number?

Ru: I don't know.
R: Do you know what Lotto is?
Ru: Oh yes, at least I've heard about it. I don't know much about it though.
R: Do you know that it involves the drawing of numbered balls?
Ru: Yes.
R: So if 17 has occurred more times than any other in the last 200 draws, what do you think?
Ru: I suppose it's more likely then isn't it?
R: Do you think that the 17 will always be more likely then?
Ru: Yes, I suppose so.

Question 6(b)
[Describe/show roulette wheel]
In Roulette, each number is equally likely to occur.
Suppose that the ball lands on "red" six times in a row. On the next roll is it now more likely to land on the "red", the "black", or are they both still equally likely?

R: What about with the roulette wheel?
R: Do you think that the Red would be more likely?
Ru: No, not really.

*Question 6(c)*

In some lotteries the tickets are pre-numbered. If you could choose between any of the following numbers would you have any preference?

123456
619999
615472

Ru: Well I wouldn't buy a Lotto ticket.
R: But if you did, or if someone were to give you one, would you have any preference?
Ru: No, not really, I wouldn't care.

*Question 6(d)*

In choosing six LOTTO numbers which of the following selections would you prefer?

(a) 1, 2, 3, 4, 5, 6
(b) 32, 33, 34, 35, 36, 37
(c) 9, 12, 27, 31, 35, 38

R: Would you have any preference here?
Ru: No, I wouldn't care.
R: Do you think they are all equally likely?
Ru: I don't know. I suppose so. I wouldn't really care which one I got.

Although it could be argued that Ruth employed some use of availability in the first part of this question, she did really have an adequate base in simple probability to make informed decisions using either probabilistic knowledge or a heuristic. Tversky and Kahneman (1982a) have noted that the use of the availability and representativeness heuristics requires an intuitive judgement of probability. Ruth did not demonstrate any intuitive understanding of probability concepts. The following interview transcripts illustrate the difficulty of classifying Ruth's responses in this category.

*Question 7(a)*

If I throw a single fair die is it harder (less likely) to get a six than say, a four?
In general, are the numbers 1 to 6 equally likely to occur, or are some harder to get than others?
Question 7(b)
If I throw a pair of fair dice is it harder to get a pair of sixes than it is to get a pair of twos?

R: Why's that?
R: I really don't know.

Question 7(c)
A poker hand consists of five cards dealt from a well shuffled deck. The Ace is the highest card. Is it harder (that is to say less likely) to get three Aces than it is to get three fives?

R: I don't know. I suppose some cards would be harder to get than others.
R: Would the Aces be harder to get?
R: I don't know.

Question 7(d)
Suppose that you and I are going to play a game with this coin [shows] in which we toss the coin just once. If the coin lands heads, I win; if it lands tails you win. But I will warn you that of the last 15 people who played this game with me only six won.
Do you think this is unusual?
Do you think the coin is not fair?
Do you think the game is fair?

R: It doesn't sound fair. I don't know if that's unusual or not.
R: Why doesn't it sound fair?
R: Well, your winning more often, like it's not equal, so it's not fair.
R: Is this because the coin is not fair?
R: Could be, I don't really know.

Ruth has no mechanism with which to compare or quantify probabilities, and it can be seen here that she has no basis on which to make judgements of fairness. Because she lacked these basic skills, no time was spent on the sections involving mathematical expectation or compound probability.

Combinatoric situations. Ruth was able to compute some simple combinations in non-gambling contexts using correct multiplicative techniques [Questions 11(f), (g)]. Although unfamiliar with the gambling contexts, when these were explained, she was able to use the same multiplicative procedures to answer some of these correctly [Questions 11(a), (b), and (c)]. Questions 11 (d), (e) and (h) were not attempted.
Summary of Ruth's responses. Of all the students in this study, Ruth demonstrated the least ability in probabilistic reasoning. This was despite her overall mathematical competence in performing computations in proportional reasoning and a high level of school achievement. However, when the context was explained, she was able to transfer school taught multiplicative techniques to proportional questions in a gambling context. Her lack of any intuitive understanding of probability made any assessment of the use of the availability and representativeness heuristics difficult. (In Chapter 8, she was coded as "unable to answer" or "no attempt" on these items [Tables 18 - 22].) Interview questions relating to the assessment of the more advanced concepts of compound probability, mathematical fairness and expectation were not asked. However, her performance on the combinatoric questions was better than most others, and she answered most parts of Question 11 correctly.

Case 4. Non-gambler, Low Achiever

Study #39, Tim.

Background. Tim was one of the seven non-gamblers from School 3. He was a pleasant student, who indicated that he was willing to participate in the study. He said that he enjoyed school and liked his mathematics classes despite the fact that he was not achieving well. Some of the work was "hard" but he thought that his achievement would improve later. He attended Year 10 at the same school and obtained a "Sound Achievement." He said that he could not recall studying any probability in previous years and had no experience with gambling in any form. Although his parents did not participate in any form of gambling, Tim said that their attitude to things like Lotto was "fairly neutral."

Tim's Responses to Interview Questions

Tim answered Question 1(a) correctly using a traditional multiplicative technique and a calculator. However, in the gambling context of part (b), he first ignored the second term in the proportion, multiplying the first two terms. When probed, he indicated that he knew that this was incorrect, but failed to correct it, instead reasoning:

T: $90, $27, and $35
R: You've multiplied, $9 \times 10$?
T: Yes.
R: If the odds were 9:1, what would your answer be?
T: $90
R: Would it be the same answer at 9:2?
T: No, I'm wrong aren't I? It would be $180.
R: You multiply by the two?
T: Yes.
T: 27 times 2, that's $54.

Tim compared all fractions correctly using the school taught technique of converting to a common denominator and using pencil/paper for the computation of parts (b), (c), and (d). Part (a) was answered correctly without computation, but Tim was unsure of his answer until he returned to this part and converted $\frac{1}{2}$ to $\frac{3}{6}$ and $\frac{1}{3}$ to $\frac{2}{6}$. Tim's understanding of basic probability concepts was demonstrated by the responses to the following interview questions.

**Question 3**
Suppose I select a marble at random from each of the following bags [diagram shown].
In each of the two situations what is the probability of getting a black marble?
   (a) The bag contains 3 black and 1 white
   (b) The bag contains 5 black and 3 white
   (c) In which of these cases, (a) or (b), is it more likely to get a black marble?

R: Which is the greater probability 3 chances in 4, of 5 chances in 8?
T: I'm not sure. I think it's the 3 in 4.
R: Why's that?
T: There's only one difference, 5 is three from 8, so that's less chance.

Tim quantified the probabilities correctly, but was not sure how to compare them. Although he was able to compare fractions using a common denominator strategy, he reverted to an "additive" comparison to compare probabilities. However, in probing questions, he was able to compare simpler probabilities such as $\frac{1}{4}$ with $\frac{1}{5}$ and $\frac{2}{5}$ with $\frac{2}{7}$.

**Question 4**
In each of the following track betting situations, which is the better of the two odds? That is to say which gives the greater return per $\$1$ bet, or which is the greater ratio?:
   (a) 2:1 or 3:1
   (b) 5:1 or 5:2
   (c) 4:3 or 9:7
When the context was explained, Tim answered parts (a) and (b) correctly, but he was incorrect in part (c).

T: I don't know about this one.
R: Which ratio is the greater?
T: No, I can't tell.
R: What if we had 5:2 and 9:4. Which of these is the greater?
T: No, I'm still not sure.

**Question 5**
In a four horse race the odds for each horse are given as -
2:1, 5:3, 5:1, and 25:1
Which horse is thought to be most likely to win?
List the odds in order of least likely to most likely.

T: I don't know.
R: Can you compare 5:1 and 2:1 ?
T: Yes, 2:1 is smaller.
R: What about 5:1 and 5:3 ?
T: 5:3 is the smaller
R: And 2:1 compared with 5:3 ?
T: I'm not sure. I think 2:1 is bigger.
R: So 5:3 is the smallest?
T: Yes, it would be.

**The use of the availability and representativeness heuristics.** Tim was one of the students who demonstrated extensive use of both the representativeness heuristic and the availability heuristic in the situations of Question 6.

T: The 17 is more likely.
R: Are you familiar with Lotto? Do you know what it's about?
T: Yes. But I don't follow it.

*To Question 6(b), [the roulette context], Tim responded:*

T: The red is more likely.
R: Why do you think that?
T: It's the one that comes up most often, isn't it?

*Here Tim has used the short term frequency of the occurrence of red as representative of its long term frequency, to incorrectly conclude that the red is more likely. (Note that in Chapter 8, Tables 18 and 19, it was recorded that only three students demonstrated this misconception in these two situations.*
In other Lotto contexts, Question 6(c):

T: The 615472.
R: Why's that?
T: I don't like the look of the other numbers.
R: Do you mean you think that they'd be less likely?
T: Yes.

To Question 6(d):

T: The third.
R: For the same reason as before?
T: Yes, it's more likely than the others.

Tim was one of the 10 students who used the availability heuristic in both of these situations (see Table 20). However, he did not use the availability heuristic in the contexts of parts (a), (b), or (c) of Question 7.

**Question 7(a)**
If I throw a single fair die is it harder (less likely) to get a six than, say a four?
In general, are the numbers 1 to 6 equally likely to occur, or are some harder to get than others?

**Question 7(b)**
If I throw a pair of fair dice is it harder to get a pair of sixes than it is to get a pair of twos?

**Question 7(c)**
A poker hand consists of five cards dealt from a well shuffled deck. The Ace is the highest card. Is it harder (that is to say less likely) to get three Aces than it is to get three fives?

However in part (d), he was one of the six students who used the representativeness heuristic to conclude the unfairness of the situation.

**Question 7(d)**
Suppose that you and I are going to play a game with this coin [shows] in which we toss the coin just once. If the coin lands heads, I win; if it lands tails you win. But I will warn you that of the last 15 people who played this game with me only six won.
Do you think this is unusual?
Do you think the coin is not fair?
Do you think the game is fair?
T: I don't know if that's unusual but it doesn't look fair.
R: Do you mean the coin?
T: Yes, the coin's not fair.
R: Is the game fair.
T: No, because the coin isn't fair- it comes up heads too often.
R: What if I told you that of the last 10 people who played me 7 lost. Would the coin still be unfair?
T: Seven out of ten? I don't think so, but I suppose it could be.
R: How about if I played it 6 times and I won 4 of them. Might it be a fair game then?
T: Yes, that'd be O.K. I suppose that might be fair.

Tim has had no practical experiences with games of chance involving coins or dice, and he claimed to have had no formal education in probability. Nevertheless, he demonstrated an understanding of the basic concept of probability, though little understanding of the concept of "independence." Furthermore, he used the short term frequency of as few as 7 out of 10 as representative of the long term frequency and concluded unfairness of the situation.

*Concepts in compound probability.* Not unexpectedly, Tim demonstrated no knowledge of compound probability and selected paths A and B in Question 8 and did not answer any parts of Question 9 correctly.

*The concepts of fairness and mathematical expectation.* Tim was able to recognise the fairness of the simple situations of Questions 10(a) and (b). He did not attempt the further questions on the concept of expectation.

*Combinatorial situations.* Tim demonstrated little knowledge in this category. Question 11 (f) was the only part of Question 11 that he answered correctly. Other parts were either answered using an incorrect additive strategy or not attempted.

*Summary of Tim's responses.* Although classified as a Low Achiever, Tim's basics competencies in computing proportions and comparing fractions in traditional classroom contexts was sound. However, in the unfamiliar gambling contexts, he was not able to apply this knowledge. His knowledge of probability was limited to the quantification of basic probabilities, and he as unable to reason correctly in probabilistic situations in any of the contexts. He did not understand the basic concepts of independence or fairness, and further to this unfamiliarity with probabilistic reasoning, he made extensive misuse of the heuristics of availability and representativeness.
Other Cases: Selected Responses

The background and selected responses of three other students (all gamblers) are presented here to illustrate some of the characteristics of the gamblers.

Case 5. Gambler, Very Low Achiever

Study # 10, Todd.

Background. Todd was a very sociable student who said that he was very pleased to have been selected for the interview and was willing to talk very openly about his involvement in track gambling. In fact, Todd volunteered a great deal of information about his track gambling activities, offering to attend the track with the present author. This offer was declined only because of the ethical factor that Todd as not of legal age to be placing bets as he did (There is no legal age requirement regarding track attendance). Todd said that he enjoyed school and was doing "alright" in most subjects but "not too well" in mathematics. His teacher classified him as a "a pleasant student" but "very lazy" and a "Very Low Achiever." Todd was one of the few students who did not agree with the teacher assessment. He did admit that he was "a bit lazy", but maintained that he would "do better towards the end of the year" and end up with a "Sound Achievement."

Todd played card games "regularly, each weekend with friends." These games included blackjack and poker which were sometimes played for small stakes, although generally just for fun. He also played cards sometimes "at lunch times at school," although on these occasions no stakes were involved. He also played board games that involved the throwing of dice and sometimes played "under/over 7" for small stakes.

He attended the track "regularly - Wednesdays to the horses, and Saturdays to the trotting," generally missing school on Wednesday afternoon (sports day) to do so. In addition, he would go some evenings to the dog races. Of all the gamblers in the present research, Todd was the most involved in the actual betting situations, and he was the only one who had an expectation of winning. He studied the forms in the newspapers, discussed these with older friends and his older brother. Todd said that he regularly bet amounts of $1 to $5, and maintained that overall he came out ahead, winning more than he lost. He said that he preferred to bet on the dog races, claiming that "it is easier to follow the form of the dogs" and the "returns are better." He preferred to place bets with bookmakers at the track, rather than with the TAB because "you can shop around for the odds if you get in early." Although not of legal age, he said that he was generally able to place his own bets without being questioned.
His track attendances were generally in the company of friends and his older brother. Surprisingly, he said that his parents were "not much interested in the track."

**Todd's Responses to Selected Interview Questions**

In traditional classroom situations involving proportion Todd consistently used an incorrect additive strategy. He worked quickly and carelessly, displaying little interest in the questions.

**Question 1(a), (i)**

A photographic negative 2 cm wide and 3 cm long is enlarged to a photograph 9 cm wide. How long is the photograph?

T: 10 cm
R: How did you get that?
T: It's one more than 9.

**Question 1(a), (ii)**

A photographic negative 2 cm wide and 3 cm long is enlarged to a photograph 8 cm wide. How long is the photograph?

T: 9 cm
R: One more than 8?
T: Yes.

**Question 1(a), (iii)**

If 2 litres of juice costs $3, how much would 9 litres cost?

T: I don't know how to do these types of question.
R: How much would you expect it to cost?
T: Well, it's generally cheaper to buy it in big containers, so probably about 8 or 9 dollars.
R: Suppose that the juice cost the same per litre no matter how much you bought. How would that affect your answer to the question?
T: It would be more, about $10
R: Are you sure about that?
T: Yes, that seems O.K.
Question 1(a), (iv)
If 4 kg of apples cost $7, how much would 5 kg cost?

T: About $8

However, in a gambling context Todd abandoned the additive strategy and employed a unitary approach.

Question 1(b)
In each of the following situations, how much could be won on a track bet if:
(i) $10 is bet at odds of 9:2
(ii) $9 is bet at odds of 3:2
(iii) $5 is bet at odds of 7:4

Parts (i) and (ii) were answered correctly using a unitary strategy with approximation in part (ii).

T: 9:2. That’s $4\frac{1}{2}$ to 1, so $10 will get you $40 ... plus $\frac{1}{2}$ ... that is $45, plus your $10 back.
R: If you placed that bet with a bookmaker at the track would you do that calculation yourself?
T: No, the bookie writes the amount on the ticket.
R: Would you check it?
T: Well, you always check your ticket. Generally to make sure you've got the right one. The bookies don't generally make any mistakes with the money.
R: But do you check the amount anyway?
T: I always look at it, but I generally don't need to do any calculation.
R: Because it looks right?
T: Yeah, it's always O.K.
R: What about $9 at 3:2?
T: 3:2 is easy. You win $1.50 for each dollar.
R: So for $9?
T: 9 \times $1.50. About $14 [mental computational estimation]
R: Could you give an exact amount?
T: No, there's no need to. The bookie writes the total on the ticket.
R: And how much would that be?
T: More than $20. About $22 or $23.
In part (iii) Todd used a scalar additive strategy employing both mental computation and estimation.

T: 7.4. That's harder. It's less than 2:1, so $5 will return a bit less than $15.
R: Could you figure it out a bit closer?
T: Well $4 would win you $7. The extra $1 would win you a bit less than $2, so you'd win about $9. You'd get back about $9 and $5, $14.

Comparison of fractions and probabilities. Todd compared the fractions in Question 2 correctly, but could not give a reason for his choice in parts (c) and (d). In part (c) he guessed that \( \frac{3}{5} \) is bigger than \( \frac{1}{2} \) and in part (d) \( \frac{5}{8} \) looks like it's bigger.

Todd quantified the probabilities in Question 3 correctly, but lacked any formal strategy for comparison other than in the simple situations. Nevertheless, he answered part (c) correctly showing some intuitive understanding by reasoning:

T: Three chances in four has to be better than five chances in eight.
R: Why does it have to be better?
T: I don't know, it just is.
R: Can you give me a reason?
T: No, I just know it is.

In Question 4(c) Todd was another one of the gamblers who constructed the "smaller outlay" strategy that was used by Leona and Todd.

Todd did not use the representativeness heuristic at all but used the availability heuristic in the context of Lotto, Questions 6(c) and (d). Todd also thought, in Question 7(c) that it was harder to get three aces than three fives (availability). This was despite extensive familiarity with playing card games.

Todd was one of the gamblers who did not demonstrate any understanding of the concepts of fairness or expectation.

He demonstrated little understanding of compound probability in that he selected the "equally likely" option in Question 8 and thought that the probabilities of Questions 9 (a) and (b) remained unchanged at 50%, again despite familiarity with the context. However, in the questions relating to the throwing of a pair of dice, he was able to conclude that the probability of getting a pair of fives was less than the probability of getting a three and a five and that in general it was harder to get a pair than different numbers. In these questions he cited experience and familiarity with throwing dice as the basis for his decision.
Like many of the low achievers, Todd demonstrated a very limited knowledge of combinatorics.

**Summary of Todd's responses.** Todd's responses to the traditional classroom questions confirmed his classification as a Low Achiever, demonstrating limited skills in proportional reasoning, fractional operations or combinatoric procedures. In the gambling contexts, he appeared to be highly motivated and used a correct unitary strategy in simple situations, and constructed other context dependent and number dependent strategies in the more difficult situations. His limited abilities in school taught procedures resulted in his inability to compare probabilities. However, he constructed strategies to compare odds correctly in gambling contexts. Despite familiarity with cards, dice and other gambling contexts, he had little intuitive probabilistic knowledge and no understanding of compound probability. Todd showed some use of the availability heuristic and had little understanding of mathematical fairness or expectation.

**Case 6. Gambler, Very Low Achiever**

Study #5, Justin.

**Background.** Justin was identified by the Special-Needs teacher as a student with an intense interest in gambling, particularly at the local track which he attended regularly including at times when he should have been at school. Thus, he had a very high rate of absenteeism and a very negative attitude towards school. Although he received a "Sound Achievement" in Year 10 mathematics, he received a Very Low Achievement in Term 1 of Year 11 "Maths in Society" and subsequently dropped mathematics.

His track attendance included dog racing every Saturday night with either his parents or friends, some of whom were school friends. He also attended mid-week horse racing "two or three times a month" and the trotting races "occasionally - once or twice a month." He followed the forms from the newspapers and published guides and bet regularly at the track with bookmakers and off-track on the TAB. He claimed that bookmakers generally gave the better odds "if you know what you're doing and get in early." He said that he would compare bookmakers odds before placing bets, generally of the order of $1 to $5.

His parents followed the races and bet regularly "not large amounts - not more than $10 or so." His cousin was a jockey, and other family members had worked in various capacities in the racing industry.

Although Justin was described as a "problem student" who had been suspended from school on a number of occasions, he was very open and friendly during the interview and straightforward with his responses.
Justin's Responses to Selected Interview Questions

In proportional reasoning, Justin showed a similar pattern of response to Todd, in that he consistently used an incorrect additive strategy in non-gambling contexts. In gambling situations he used a correct unitary technique using mental computation and pencil/paper methods.

**Question 1(b)**

In each of the following situations, how much could be won on a track bet if:

(i) $10 is bet at odds of 9:2
(ii) $9 is bet at odds of 3:2
(iii) $5 is bet at odds of 7:4

J: At 9:2 you would win $4.50 for each dollar bet. So that's $45 for a $10 bet. At 3:2 you win $1.50 for each dollar, so that's ... this is harder. [Picks up pencil and performs pencil/paper computation $1.50 \times 9. This takes some time, but he gets the correct answer.]

R: If you were doing this calculation at the track how would you do it?

J: You could go $2 gets $3, so $4 will get $6, and $8 gets $12. So $9 will get about $13, you'll win about $13.

R: What about $5 at 7:4. How would you do this at the track?

J: $4 will get $7, so $5 will win you more than $8.

R: Can you say how much more?

J: No, it would be a bit more, not much though.

Here Justin used an "additive" strategy with estimation in order to arrive at a reasonable approximation. He also used a unitary approach, again with approximation, in other gambling contexts such as:

**Question 4**

In each of the following track betting situations, which is the better of the two odds? That is to say which gives the greater return per $1 bet, or which is the greater ratio?:

(a) 2:1 or 3:1
(b) 5:1 or 5:2
(c) 4:3 or 9:7

J: At 4:3 you win a bit more than $1 for each dollar bet; 9:7 is pretty close to evens, not much better, so it wouldn't be as good. Anyhow, bookies don't give odds of 9:7.

R: Could you give me an example of odds that are less than 4:3?

J: Well even money would be less.

R: Are there an odds that are better than even money, but less than 4:3?

J: There might be. 4:3 is the same as 8:6, so 7:6 would be less.
The concepts of fairness and expectation. Justin was another of the gamblers who did not demonstrate any intuitive understanding of the concepts of fairness or expectation. He demonstrated little understanding of compound probability in that he selected the "equally likely" option in Question 8 and thought that the probabilities of Questions 9(a) and (b) remained unchanged at 50%, again despite familiarity with the context. In the questions relating to the throwing of a pair of dice, he was able to conclude that the probability of getting a pair of fives was less than the probability of getting a three and a five and that in general it was harder to get a pair than different numbers. Justin also demonstrated a very limited knowledge of combinatorics.

Summary of Justin's responses. Overall, Justin's mathematical skills confirmed his standing as a Very Low Achiever. The limited skills and techniques that he employed in a gambling context did not apparently transfer to other contexts. For example, Justin was unable to compare the fractions of Question 2(c) and (d) or compare the probabilities of Question 3. In gambling contexts, Justin's mathematical achievements were much better, but nevertheless, his abilities were limited. He made extensive use of approximation to arrive at "reasonable" estimates.

Case 7. Gambler, Very High Achiever
Study #15, Lisette.

Background. Lisette was a pleasant student, very friendly and willing to take part in the study. She was a year older than most students in her class having spent a year out of school. She enjoyed school, had a very positive attitude to her mathematics classes which she described as "fairly interesting and easy." She obtained a High Achievement in Year 10 Advanced Mathematics but selected the lower level course because "it is more interesting." She was classified as a Very High Achievement in Semester 1, Year 11 by her teacher. She agreed with this assessment.

Her father and older brother were "very interested in track racing." Her mother was "somewhat interested." She had a boyfriend who was a friend of her brother and several years older than she. She attended all three forms of track racing "regularly" - at least once a week, sometimes two or three times. Both her boyfriend and her brother bet on these occasions, following the forms and guides, though she herself rarely bet her. She followed the bets placed, often checking different bookmaker odds. On these occasions she thought that their betting was "fairly successful," although the stakes were "not great."
She said that she played card games "regularly" with friends and family. These games included poker and blackjack which were played "mostly for fun," though sometimes for small stakes. When playing for money, the emphasis was on "fun rather than winning." She said that she also played various board games that involved the rolling of dice.

Listette's Responses to Selected Interview Questions

Questions involving proportional reasoning were completed correctly using traditional functional techniques, mental computation and calculator use. In Question 1, part (iii) a correct scalar additive technique was employed to arrive at an exact correct answer, reasoning:

\[ L: \text{7:4 means bet 4 win 7 so, the one more bet 1 will win } \frac{1}{4} \text{ of 7, that's } 1\frac{3}{4}. \]

So \$5 will win \( 7 + \frac{3}{4} = 8\frac{3}{4} \)

This scalar additive technique was used again in Question 4(iii), reasoning:

\[ L: \text{4:3 means bet 3 win 4. So bet 6 win 8. One more wins } \frac{8}{6} \text{, that's } 1\frac{2}{6}. \text{ So}
\]

\( 7, (6+1), \text{ wins } 8 + 1\frac{2}{6} = 9\frac{2}{6}, 9\frac{1}{3} \). This is a bit better than 7:9, since you get more for the same outlay, but not much more.

Probabilistic concepts. Lisette had no problem with any parts of Question 3. She had an very good understanding of basic probability and was able to compare probabilities by conversion to decimals. Without a calculator, she used mental computation confidently. She said that she recalled doing this type of question in Year 10 mathematics.

\[ L: 2:1 \text{ is the same as 6:3. 5:3 are shorter odds, that means its more likely to win.} \]
In the questions involving misconceptions in probabilistic reasoning, Lisette showed no misuse of availability or representativeness in any parts. She answered these questions very confidently.

**Question 6(a)**
In the last 200 Gold Lotto draws the number 17 has occurred more times than any other number. In future draws do you think that this number is:
(i) more likely to occur than any other number?
(ii) less likely to occur than any other number?
(iii) as equally likely to occur as any other number?

L: Of course not.
R: Why do you think the newspapers publish these numbers then?
L: I suppose some people think it matters—what's gone before.
R: How do you know that the probability remains the same?
L: It doesn't matter what's happened before. It makes no difference.

Lisette demonstrated a very good knowledge of the concept of "independence" of events. She said that she recalled this from her Year 10 work, and in Question 7, she was able to recognise that a longer run would be necessary before unfairness or bias could be suspected.

*Fairness and expectation.* Lisette's understanding of the concept of fairness is illustrated by her responses to the various parts of Question 10 (b).

**Question 10(b), (i)**
Suppose we change the rules so that I win if it is a three or greater is this still a fair game?

**Question 10(b), (ii)**
Can we make the game fair somehow?

**Question 10(b), (iii)**
How much should I put in to make the game fair?

L: No the game's not fair now. You have a much better chance of winning.
R: How much better?
L: Twice as much
R: How do you know that?
L: Well, you've now got four numbers to my two. That's twice as many, so you've got twice the chance.
R: O.K. Let's look at the next part.
L: Yes, $2
R: How does that make the game fair?
L: Well you've got twice as much chance as me so to make it fair I should win twice as much as you.
R: If you win that is?
L: Yes, if I win. But see, my chances of winning are less than yours, so I need to win more.
R: If you were to play this game, with the new rules, which would you rather take; the 1,2 or the 3, 4, 5, or 6?
L: It doesn't really matter. The game's fair.
R: Suppose we look at another card game.
L: Yes, this is the same sort of thing. You've got three suits to my one. So I get three times as much if I win.

Lisette has constructed a correct definition of fairness. Although she did not use the formal mathematical language, she recognised that the product of return and probability must be the same for each player. Further evidence in support of this understanding is provided in the following transcript of her responses to further questions.

R: One card of any suit?
L: It's the same thing. You have 12 cards to my one, so you put in $12 to my $1.
R: One specific card?
L: $51
R: If you were to play this game with me which would you choose the single card or any of the 51 others?
L: I don't bet this much money.
R: So you'd want to take the single card and loose $1 rather than risk paying out $51.
L: Definitely.
R: But the game is still fair?
L: Yes.
R: On the occasions when you bet at the track, do you generally bet on long shots or go for the favourites?
L: Generally on a favourite or close favourite. Long odds don't often win.
R: But when they do they pay more.
L: Yes, but that's pretty rare. We generally go for the favourites.

*Concepts compound probability.* In Question 8, Lisette correctly chose A and B using the multiplication principle.

R: Why A or B?
L: For these it's a half times a half, that's a quarter. For the others it's less - you've got to multiply by a half at each junction.
In Question 9 she again employed the multiplication principle for compound probability. Few students answered this item correctly and most of those who did were also High Achievers.

**Concepts in combinatorics.** Lisette was one of only two students to answer all parts of Question 11 correctly. In part (h), she enumerated the number of cases and encountered some confusion, but eventually arrived at the correct answer. Other parts were answered using a correct multiplicative strategy. Despite familiarity with the context of parts (c) and (d), she was not confident about her answers.

**Question 11(c)**

[To be asked only of those students who were able to answer both Questions 11(a) and (b) correctly]

If the "quinella" consists of selecting the first two horses in a race without necessarily being in the right order, how many possible selections are there in a race with 8 horses.

[If the student is unable to answer this, proceed to Question 11(f)]

L: There's lots, but I'm not sure. It's less than like in the last one - it's less than $8 \times 7$, 56. I think it's half as much. That's 28.

R: Have you done this type of question before?

L: I can't remember.

R: Are you familiar with betting on the quinella?

L: Oh yes, but the TAB tells you how much it costs and figures out the amount when you win.

R: So in this situation if you took all possible quinellas, how many would there be?

L: 28.

**Question 11(d)**

[To be asked only of those students who were able to answer both Questions 11(c) correctly]

If the trifecta consists of selecting the first three horses in the correct order, how many possible selections are there in a race with 6 horses?

L: There's a lot more trifectas. There'd be ... is it $6 \times 6 \times 6$? No, I think it's $6 \times 5 \times 4$, 120. I'm not sure, but I think that's it.

Although she was thoroughly familiar with the context of these questions, she was no more confident with her answers to these combinatorial questions in a gambling context than in the traditional contexts of the other parts of Question 11.
Summary of Lissette's responses. Lisette was a high achiever who answered nearly all of the interview questions correctly. She used school taught strategies consistently in traditional contexts. In gambling contexts, she used both traditional strategies and a correct scalar additive strategy. Lisette understood the multiplicative nature of compound probability, and had a sound understanding of the concepts of independence, fairness and mathematical expectation.

Summary

In this chapter the detailed responses of four of the students, one from each of the four categories, have been presented in order to illustrate the use of the structured individual interview in the clinical study and to show examples of characteristic responses of students from each category. The backgrounds of these four students together with those of three other gamblers have also been presented to help illustrate characteristics of the gamblers. From an examination of the responses of the gamblers, it is clear that they do, in fact, possess an intuitive base for probabilistic reasoning. This base includes the use of an informal mathematical language associated with the placing of bets at various odds. However, it appears that this base does not necessarily provide the gamblers with a link to the more formal school mathematics. The implications of this are discussed further in the final chapter.

It can be seen from the interview transcripts that, in some instances, the present author was able to use the gambler's intuitive understanding to direct and guide the student in the right direction to find an answer to the question. However, in many situations, this understanding was inadequate to form a base from which further reasoning could proceed. This inadequacy was noticeable in questions involving the concept of compound probability and computations in combinatorics.
CHAPTER 12

Summary, Conclusions, Implications, and Recommendations for Further Research

The Structure of this Final Chapter

The preceding chapters have described an investigation into the probabilistic reasoning of two groups of Year 11 students. Qualitative data gained from the individual structured interviews were gathered, categorised and analysed to provide answers to three major research questions.

This final chapter provides a summary of the thesis and an account of the main findings. Implications of the findings for teachers, tertiary educators, researchers and those interested in curriculum development are suggested and recommendations for further research are made.

Summary of the Thesis

After the introductory Chapter 1, a detailed rationale for the study was developed in Chapter 2, where it was established that there is a need to research the mathematical skills and concepts that a segment of the population bring to the school environment as a result of the influence of gambling within their social background. The role and importance of probabilistic concepts and relationships in school mathematics curricula were also clearly established.

In Chapter 3, on "Gambling in the Australian Social Context," it was established that the phenomenon of track gambling is widespread within Australian culture and is an important aspect of everyday life for a large segment of the Australian population.

Chapter 4 provided a summary of the history of the development of probability. The historical relationships between gambling and probability were summarised, and it was noted that the study of how people acquire probabilistic concepts is a relatively recent field of research in mathematics education.

In Chapter 5, literatures relating to key mathematics education research areas that were likely to impinge on the proposed study were reviewed. In particular, the study brought together literatures on (a) ethnomathematics, including social and cultural factors influencing the learning of mathematics; (b) constructivism in mathematics education; and (c) the development of probabilistic reasoning and related mathematical skills and concepts.
In Chapter 6, the major research questions were formulated. These arose from the reviews of the literatures in Chapter 5.

The selection process employed in obtaining the two groups of students was described in Chapter 7. Reasons for the development and adoption of the methodology employed for the purpose of investigating the research questions were given, together with a detailed account of the actual methodology itself.

Data gathered in the study were summarised and discussed in Chapter 8. Detailed analyses of selected data were provided in Chapters 9 and 10, the primary purpose being to answer the three major research questions which had been posed in Chapter 6.

Illustrations of specific student responses were presented in Chapter 11. These illustrative data were placed in the contexts of (a) four case studies, and (b) the selected responses of three other gamblers. In this chapter interview transcripts formed an important part of the data.

In the remaining sections of this final chapter, the main findings of the study, the implications of these findings and recommendations for further research, together with hypotheses generated by the results, are presented.

The Main Findings of the Study – A Summary

The three major research questions posed in Chapter 6 were investigated through analyses of the data which are summarised in Chapters 8, 9 and 10. The following synthesis of these analyses constitute the main findings of the study. Conclusions are drawn from the various data arising from the topics in each of the six categories. The implications of these conclusions are discussed in detail later in this chapter.

Differences Between the Gamblers and the Non-Gamblers in the Understanding of Probabilistic Concepts

Expectation and Fairness

One of the major findings of this study was that there were noticeable differences between the two groups in their understandings of the concept of mathematical expectation and its relationship with the concept of mathematical fairness.

From Table 39 it can be seen that five of the 20 gamblers demonstrated a thorough knowledge of basic mathematical expectation, compared with only one
Conclusion. The possession of intuitive understandings of the important concept of mathematical expectation by a majority of the gamblers gave rise to one of the most noticeable differences between the two groups, and represents one of the major findings of the study.

The Use of Language

It was observed that the gamblers employed language and jargon in gambling contexts that the non-gamblers did not use. This use of "gambling" language was another of the major differences between the two groups and represents another major finding of the study.

Conclusion. The consistent use of intuitive, informal language by the gamblers in the context of betting at varying odds formed a link between the intuitive probabilistic concepts of the familiar real world situation of track gambling and the symbolic manipulations employed in the mathematical processes. However, strong links between this informal mathematics and the formal mathematics of the classroom were not necessarily present. The need to develop curricula aimed at fostering such links, and to provide appropriate professional development programs aimed at helping teachers to create learning environments likely to assist students to make these links, is discussed later in this chapter.

Mental Computation and Estimation

Conclusion. The gamblers employed a number of mental computational techniques effectively in gambling contexts, exhibiting a strong number sense in these contexts. These techniques often incorporated computational estimation and approximation.

The strategies included a scalar additive with approximation technique, and a unitary strategy with approximation. Such strategies, in general, were not used by the non-gamblers. However, the use of these techniques by the gamblers did not, in general, transfer to traditional non-gambling contexts.

It was also reported in the review of the literature that research into the use of estimation and mental computation in mathematics generally is very limited. The findings from this study suggest that, provided students can establish links with their own personal worlds, then they are capable carrying out even complicated estimations and approximations. However, the challenge for educators is to develop
approaches which are likely to result in students with isolated number sense skills being able to connect these skills with a much broader range of contexts.

**Misconceptions in Probabilistic Reasoning**

Overall, the extent of the use of the representativeness and availability heuristics by students in the investigation which has been described was found to be much lower than that reported in the literature. However, it cannot be concluded from the present study that this difference can be attributed to the influence of the gamblers' social backgrounds.

The concept of independence of events in probabilistic situations was, in general, better understood by the gamblers. However, the limitations of the study prevent strong conclusions being made regarding the effect of social background.

**Simple and Compound Probability**

It was noted in Chapter 10 that the results support the findings of other research into students' understanding of the concepts of simple and compound probability.

**Conclusion** Overall, data from the responses to items in this category pointed to similar conclusions to those reached by Brown et al. (1988). The students investigated in the study reported by Brown et al. were of the same age and grade level as those in the present study. Brown et al. concluded that "knowledge of all but the simplest of probabilistic questions is extremely limited" (p. 242) and data gathered in the present study points towards the same conclusion.

In particular, the basic concept of compound probability was not well understood by Year 11 students, gamblers and non-gamblers alike, in any of the contexts. This was true for both high and low achieving students. As Fischbein et al. (1991) commented, there appears to be "no natural intuition for evaluating the probability of a compound event" (p. 534).

One minor difference between the two groups which was noted was that in some gambling contexts several of the gamblers demonstrated an intuitive, but limited understanding of compound probability concepts. The need for further research into why the understanding of the concept of compound probability is so low is discussed in more detail later in this chapter.
The Cognitive Processes Employed in the Processing of Chance and Data Concepts

It was noted at the outset in Chapter 1 that the study would attempt to identify the mathematical knowledge implicit in the ad hoc practices of the gamblers, and especially knowledge they possess in the social context of participation in track events.

Patterns of differences between individuals within the two groups were observed in the construction of algorithms, in the methods of computation, and in the language used in track gambling contexts. It was observed that these differences depended largely on the contexts defined by the questions, and the differences were recorded according to the categories of probabilistic knowledge and related mathematical concepts that arose from the review of the literature.

Differences Between the Two Groups According to Category of Question

The different patterns of response according to the six categories identified are summarised.

Category 1

This category contained questions involving proportional reasoning, fraction knowledge and the use of rational number algorithms in both gambling and non-gambling contexts. There were noticeable differences observed in the strategies employed in gambling contexts, and several alternate strategies used by the gamblers were identified. These strategies were number dependent, in the sense that the gamblers would employ different strategies for different numbers, and context dependent in the sense that the gamblers would not necessarily use the same strategy for the same type of question in a non-gambling context.

Conclusions. The gamblers tended to use different strategies from those of the non-gamblers in the calculation of the return from bets placed at various odds (see Table 7 in Chapter 8). Among the strategies effectively used by the gamblers was a scalar additive technique, and this was often accompanied by effective approximation.
Category 2

This category contained questions involving simple probability, and the comparison of probabilities and odds. The ability to quantify and compare simple probabilities accurately did not relate noticeably to gambling background. The noticeable differences observed here were in the comparison of odds.

Conclusions. The gamblers tended to use different strategies from those of the non-gamblers in the comparison of odds (see Table 16 in Chapter 8). Strategies used by the gamblers included a unitary strategy, and gamblers often constructed algorithms that were highly context dependent, in the sense that they did not use the same strategy in any other context. Some of the gamblers constructed context-dependent procedures that were not found in the review of the literature relating to proportional reasoning (see Table 16, Code 4 in Chapter 8). It was noted that TAB returns on bets are always quoted as a return per dollar bet, and this may have influenced the gamblers' choice of strategy to compare odds. The implication of this for further research is discussed later in this chapter.

Category 3

This category contained questions that were concerned with the use of the representativeness and availability heuristics in situations involving probabilistic reasoning.

Conclusions. The proportion of students using these heuristics was noticeably lower than that reported in the literature. Although a slightly lower proportion of gamblers than non-gamblers used the heuristics, this difference cannot necessarily be attributed to their gambling backgrounds.

Category 4

This category was concerned with the calculation of compound probability in a variety of contexts. The results in this category have been discussed earlier in this chapter where it was noted that the concepts of compound probability are not well understood by students of both groups.
Category 5
This category contained questions relating to the concept of mathematical expectation, to the calculation of expected returns, and to relationships with mathematical fairness. It has been noted earlier in this chapter that there were major differences between students in their responses to questions in this category, with the gamblers demonstrating noticeably better understandings of the concepts involved.

Category 6
This category contained questions relating to computations in combinatorial situations and the determination of simple combinations and permutations in a variety of contexts.

Conclusion. Overall performance on these questions was poor and there were no noticeable differences between the responses of the two groups.

Gambling as a Form of Ethnomathematics

In Chapter 2 it was stated that the research would draw attention to the learning of mathematics in circumstances unique to Australian culture. In Chapter 3 it was argued that there is a very high level of social acceptance of gambling in Australia, and that therefore it makes sense to talk about a gambling sub-culture within Australia. The findings of this study have shown that these circumstances do, in fact, influence the ways in which some Australian secondary school students think about probabilistic and related mathematical concepts.

It was found that the gamblers used special linguistic codes and jargons that were not used by non-gamblers. They employed computational strategies that were clearly associated with the computation of returns from bets placed at different odds in track gambling contexts, and also with the comparison of odds.

Conclusion. It would be reasonable to argue that these special codes, jargons and computational practices do, in fact, constitute a form of ethnomathematics as defined by D'Ambrosio (1985a), although it should be recognised that it is reported in the literature that this term is not well defined (Bishop, 1988c; Zepp, 1989).
Other Findings of the Study

In addition to the answers to the three major research questions, other findings of the research relating to gender issues, school achievement and motivation can be reported.

Gender Issues

Despite the fact that much research into gender differences in mathematical achievement has shown that males perform noticeably better than females on higher level formal probability tasks, there were no strong overall performance differences according to gender in the present study. This finding was not unexpected, however, as there were a number of possible reasons for this. Within the "Mathematics in Society" classes at the schools used in the present study, males and females were equally represented, and the students' achievements according to school-based assessment showed no gender bias. The sampling procedure described in the methodology resulted in an approximately equal number of male and female gamblers being selected in the main sample of gamblers, and it could reasonably have been expected, therefore, that gender-based performance differences would not have been found in the study.

Conclusion. The present study has shown that within the limitations of the methodology, interest in track gambling among Year 11 Maths in Society students did not show any gender bias. Furthermore, achievement on traditional questions relating to probabilistic reasoning and the associated mathematical skills and concepts showed no strong gender related differences, and the same was true of performance on various questions in the context of track gambling. The equal involvement of female students in what is often viewed as a "male oriented" activity (track gambling) has some implications that are discussed later in this chapter.

School Achievement

Not unexpectedly, achievement on many items of the present study showed a noticeable relationship with school achievement.

Conclusion. Students classified as "High Achievers" in the school-based assessments performed noticeably better than those classified as "Low Achievers" on questions relating to the computation of proportions, the comparison of fractions and probabilities, and some combinatoric questions. No strong relationships with
school achievement were observed, however, on questions relating to the understanding of the concepts of mathematical expectation and fairness.

Motivation for Gambling

One of the objectives of the present study was to determine why the gamblers participated in gambling activities. It was thought that if the motivations for gambling could be identified then this could assist educators to identify and influence cognitive processes involved in the activities.

Conclusion. The overwhelming motivation for participation in gambling activities among the gamblers was that of enjoyment of the activities as "social" events. Rarely was profit, or an expectation of long term winning, a motivating factor.

Implications of the Findings of the Study

Implications for Mathematics Educators in General

It has been noted in Chapter 2 that teachers and educators need to be take account of the knowledge which children bring to the school environment as a result of their cultural backgrounds, in general, and their out-of-school experiences in particular. The findings of this study have shown that in Australian society there are cultural practices in gambling that generate the development of intuitive knowledge in the area of probabilistic reasoning. These findings support the claim by Clements (1988) that "often in Australia there are unique factors influencing how children learn mathematics" (p.5). They also support Bishop's (1988a) contention that mathematics is not a culture free phenomenon.

Implications for Teachers of Secondary Mathematics

The findings of this study have a number of implications for practising classroom teachers. The most significant of these relates to the use of language.

Language and linkage. D'Ambrosio (1985b) has noted that there is a need to incorporate features arising from the study of ethnomathematics into the curriculum in order to avoid the "psychological blockade" that is so common in school mathematics for many students. One of the features identified in the present study that might be effectively used for this purpose is the informal mathematical language used by the gamblers, which seems to assist the communication and understanding of informal and formal probabilistic concepts. Teachers need to consider how this language can be meaningfully incorporated into present school
practices in order to assist students to link their informal intuitive knowledge with the formal mathematics that they are expected to learn.

Teachers need to explore the relationship between the students' perceptions of probability based on their informal out-of-school experiences, curriculum design, and teaching methodology, the aim being to maximise student learning. Towards this end, Ellerton and Clements (1991) call for teachers to provide classroom experiences that assist students to make the cognitive links indicated in Figure 10:

![Diagram showing the relationship between familiar real world concepts, formal mathematical language, and symbolic manipulation.](image)

*Figure 10: Linking mathematics with the personal worlds of learners (from Ellerton & Clements, 1991, p. 14).*

The present research has shown that a significant proportion of senior secondary school students are well acquainted with track gambling activities. Participation in these activities has facilitated the acquisition of a wealth of intuitive language that provide links to the formal mathematical language and the formal symbolic manipulations of school mathematics.

The gambling experiences of the gamblers who were studied were meaningful to these students, and in the context of formal school mathematics some of them often constructed their own informal context-dependent strategies for computations involving proportions.

It is not easy for teachers to develop an ethnomathematical approach to their teaching, however, because they themselves are the product of a mathematics education subculture which encourages them to emphasise isolated mathematical facts, skills, and outcomes. Ellerton and Clements (1991) have noted that

often teachers think they are providing learning environments that encourage students to construct meaning in mathematics, when in fact, the children are, ever so subtly, being required to respond to teacher initiatives most of the time and are being led along comparatively rigid paths towards preset goals. (p. 15)
The present research has shown that for many students the informal mathematics associated with gambling is part of their personal worlds, but rarely do curriculum developers and teachers take account of this.

*Compound probability.* Clearly data from the present study indicate that classroom teachers need to become more aware that even at the Year 11 level, very few students have any understanding of the concept of compound probability.

*Gender issues.* Teachers need to be aware that although the topic of gambling is often viewed as a male-oriented activity, the results of this study have suggested that female students are as interested in gambling as males and participate in gambling activities to the same extent as males.

**Implications for Curriculum Development in Topics in Probability**

It was reported in Chapter 2 that many of the concepts and topics included in the present study have an important place in current secondary mathematics curricula and feature prominently in the "Chance and Data" strand of *A National Statement on Mathematics for Australian Schools*. With respect to the findings of the present study and school curricula, Watson's (1992) concern that recent initiatives in curriculum development "have been taken without the benefit of previous educational research in Australia on the learning of probability" (p. 1) needs to be noted.

The present study has added to the research base that is needed to enable curriculum planners to make suggestions and recommendations that will assist Australian teachers with the implementation of the syllabus objectives as they relate to probabilistic concepts. As Watson (1992) has stated:

> In Australian school systems teachers are currently implementing the Chance and Data curriculum using the best resources and advice they can get from educators and curriculum planners, all of whom are operating without the luxury of a local research base. (p. 5)

There have been recent attempts to remedy this situation. We have seen the production of curriculum materials such as those produced as part of the Mathematics Curriculum and Teaching Program (Lovitt & Clarke, 1988; Lovitt & Lowe, 1993). The results of the present research suggest that the development and use of resources of this nature in the senior mathematics curriculum should be a priority in national curriculum development. In this context, it should be noted that at the time this thesis was about to be presented, materials aimed at assisting the professional development of Australian teachers in the implementation of topics in
"Chance and Data" were published by the Australian Association of Mathematics Teachers (Watson, 1994; Watson & Reeves, 1993).

The results of the present research regarding the concepts of mathematical expectation and fairness have particular implication for curriculum development. It was noted in Chapter 2 that these concepts feature prominently both in the Queensland senior mathematics syllabus and in the National Statement. In the past, the inclusion of these concepts has been confined to the more academic courses in senior secondary mathematics. In Table 39, in Chapter 9, it was reported that eight of the 11 Low Achieving gamblers had at least some understanding of these concepts. Thus this study has shown that these relatively sophisticated mathematical concepts can be understood, at least in part, by a good proportion of non-academic students.

This finding provides strong support for the inclusion of these concepts in the mathematical education of all students, regardless of social background or prior school achievement in mathematics. Further, it is possible that the use of gambling contexts to introduce and develop these concepts could prove to be highly effective. This is especially likely to be the case if the concepts are introduced in practical ways which link school mathematical concepts with meaningful practical activities. However, with respect to this point, the deep-seated concerns about the morality of various forms of gambling, within some well defined groups in the Australian community, need to be noted and respected.

*Independence of events.* All secondary school students should receive instruction in the important concept of independence of events. Professional development programs for teachers aimed at assisting teachers to link the difficult concept of probabilistic independence with the personal worlds of learners, should be implemented.

**Recommendations for Further Research in Probabilistic Reasoning and Hypotheses Generated by the Findings**

The findings of the present study have raised many questions some of which, in turn, have generated hypotheses for further testing. Recommendations for further research are made in the following areas:
Misconceptions in Probabilistic Reasoning

The occurrence of misconceptions relating to the use of representativeness and availability heuristics was much lower in the study which has been described than would have been expected from the literature.

Even so, the availability heuristic was used in the context of Lotto at least once by 50% of the students. It was noted that research in Poland by three mathematicians studying the equivalent Polish Lotto (Liczyrzepka), reported in the Brisbane Courier Mail by Seth Gregory (1993, June 19, p. 8), found behaviour that would suggest extensive use of the availability heuristic in the selection of numbers by gamblers in Poland. There is evidence that the same situation applies in Australia (Lotto number frequencies are published weekly, for example). However, it was noted in the present study that the gamblers tended to use the availability heuristic in slightly lower proportion than the non-gamblers.

Further research. It is recommended that further research aimed at determining the extent of the use of the availability and representativeness heuristics among (a) the general public, (b) people from gambling backgrounds, (c) people from non-gambling backgrounds, and (d) secondary school students, be carried out. Such research would be aimed at testing the following hypotheses generated by the present study:

Hypothesis 1
The use of the availability heuristic in the selection of Lotto numbers is widespread by those members of the Australian population who buy Lotto tickets.

Hypothesis 2
The use of the representativeness heuristic in the selection of Lotto numbers and in Casino gaming activities is common by those members of the Australian population who participate in these activities.

Hypothesis 3
The concept of the independence of events in games of chance and lotteries is not well understood by those members of the Australian population who buy Lotto tickets or participate in Casino gaming.

It would appear to be the case, also, that conclusions of other researchers regarding misconceptions of independence of events and use of availability such as the perceived difficulty of throwing a "six" compared with other numbers were not held by Year 11 students in Queensland.
*Further research.* It is recommended that further research aimed at determining the generalisability of the findings of the present study in the area of misconceptions be carried out. Also research is needed to establish at what point in children's cognitive growth such misconceptions are corrected, and how it happens that the misconceptions are changed.

It was noted that Kapadia (1984), and Konold et al. (1992) have questioned the quality of much of the research which has been reported in the literature. They have suggested that some of the "misconceptions" reported may merely be misinterpretations of questions on multiple-choice tests. The results of the present study were obtained from structured individual clinical interviews, in which any misinterpretations of the questions were corrected during the interviews. The methodology used in the present study would not therefore be subject to the criticisms of Kapadia (1984) and Konold et al. (1992), and hence the findings should be carefully noted.

*Further research.* It is recommended that further research aimed at determining the proportions of students demonstrating misconceptions involving the use of the availability and representativeness heuristics be carried out. In order to determine the generalisability of the results of the present study, such research should involve larger samples of students from more varied backgrounds and employ interview techniques aimed at generating rich forms of data.

**Compound Probability**

In Question 8, (the robot and the maze task), 70% thought all paths were equally likely. This is virtually the same figure as that cited by Green for students who are up to six years younger. Furthermore, so far as performance on this task was concerned, there was no noticeable relationship with gambling background, achievement, or gender. A low proportion of students answered other questions on compound probability correctly. These results give support to the statement by Fishbein et al. (1991) that concepts in compound probability are poorly understood by most secondary school students.

*Further research.* It is recommended that further research aimed at determining why students have such difficulties with compound probability concepts be carried out. Also the question of why there is no apparent difference in performance between students of such varying ages on such tasks needs to be investigated.
**Expectation and Fairness**

The data show that many of the gamblers demonstrated an intuitive understanding of the inverse relationship between the product of "return from a win" and "probability of winning" for betting on track events. These findings have two important implications.

First, the concept of expected return is not part of the regular school curriculum - students do not use the term "expectation" but construct a procedure that recognises the inverse relationship between probability and return. In this sense the students construct knowledge in the manner described by the constructivists in the mathematics education literature.

Second, it was noted that there is a marked mean performance difference between the gamblers and the non-gamblers on Question 10 which was concerned with the concept of expectation (see Table 39, in Chapter 9). However, no noticeable performance difference between high and low school achievers, nor any strong gender differences were observed (see Tables 40 and 41 in Chapter 9).

It was noted in Chapter 5 that the concept of mathematical expectation requires an understanding of the inverse relationship between probability and return. It would therefore be reasonable to conjecture that it is this familiarity with track betting, which involves gamblers in the computation of the returns from placing bets at various odds, which leads to the development of intuitive understandings of the concepts of mathematical expectation and expected return.

**Further research.** It is recommended that further research to test the following hypothesis be carried out.

**Hypothesis 4**

Familiarity with track betting situations which involve the computation of the return from placing bets at various odds leads to the development of intuitive understandings of the concepts of mathematical expectation and expected return.

**Combinatorics**

Performance on all questions in this category was noted to be much poorer than would be expected in light of research reported by English (1991, 1992). Furthermore the gamblers performed noticeably better than the non-gamblers on only one item in a gambling context. Although the mathematics of track betting in the contexts of the "quinella" and the "trifecta" involve combinatoric concepts, the gamblers showed little, if any, intuitive understanding of these concepts. One
possible reason for this is the fact that all such bets are placed through the TAB at which all calculations are performed automatically (see Appendix 5).

Further Research. It is recommended that further research aimed at determining why the general level of performance on combinatoric items was so low be carried out.

Piaget and Inhelder (1975) have said that "the formation of the ideas of chance and probability depend in a very strict manner on the evolution of a combinatoric operation" (p. 161). It was shown by the data obtained from Category 3 that nearly all students were able to express simple probabilities as fractions and many could correctly compare such probabilities. However, many of these same students were not able to handle combinatoric operations such as the computation of simple permutations and combinations.

Further Research. It is recommended that further research aimed at clarifying the relationship between probabilistic concepts and combinatoric operations be carried out.

Computational Strategies of the Gamblers

In the research methodology described in Chapter 7 it was noted that to be selected as a gambler, a student had to demonstrate an extensive interest in track betting. However, it was reported in Chapter 3 that although the amount of money spent on track betting in Australia has continued to increase over the past years, the proportion of this money spent on bookmaker betting at the track is declining (because the proportion spent on TAB betting is increasing). Furthermore, the proportion of money spent on Casino and Lotto betting is also increasing. In TAB betting, and in Casino and Lotto betting, the bettor has seldom any choice of odds at which to place bets. However, bookmaker odds in any situation may vary from bookmaker to bookmaker, and with the time at which the bet is placed. Thus, when betting with bookmakers, the bettor is constantly comparing odds. In other betting situations the bettor does not necessarily perform the computations that are associated with such comparisons.

Further Research. It is recommended that further research aimed at determining the mathematics employed by gamblers at TAB and Casino gaming be carried out. Such mathematics would include an investigation of the strategies for computational estimation. An examination of the probabilistic concepts employed and the misconceptions which are common would also be useful.
Issues in Ethnomathematics Raised by the Study

It was noted in the methodology, that the study had several limitations. The first of these was the size of the sample. Further research involving larger samples of students from a greater variety of schools is needed to determine whether the use of the various computational strategies identified in the present study is widespread. Even if the strategies are widely used, albeit unconsciously, by some secondary mathematics students, further research is needed to determine how teachers of mathematics can capitalise on this intuitive knowledge, given that the present research suggests that many students do not possess the same intuitions. The typical teacher of Year 11 mathematics often has to cope with classes of up to 30 students, and almost certainly many of the students in these classes would not have backgrounds of strong involvement in track gambling. The present research raises the issue of what the classroom teacher should do to capitalise on the "ethnomathematical" knowledge of those students who do come from track gambling families.

This educational question could be generalised to apply to students with other "ethnomathematical" knowledge, such as that associated with various sports, or forms of music, or patterns of life associated with a wide range of social backgrounds.

Reporting of Results

Findings of this study will be reported to researchers in mathematics education through seminars given at The Centre for Mathematics and Science Education, Queensland University of Technology, and through publications of this Centre. The findings will also be made known through publications of the Mathematics Education Research Group of Australia (MERGA), and through papers presented at educational research conferences such as those conducted by the Australian Association for Research in Education. Mathematics teachers will be made aware of the results through the inclusion of the topic of probability in in-service workshops conducted by this writer, publications in The Australian Mathematics Teacher and in Teaching Mathematics, the journal of the Queensland Mathematics Teachers Association. Results will also be presented at the conferences of the International Group for the Psychology of Mathematics Education (PME) and at conferences of other interested organisations such as the National Council of Teachers of Mathematics, in the U.S.A.
Concluding Remarks

All senior secondary students belong to cultures and subcultures. Within the schools involved in the present study some students came from wealthy families but others came from families in which no person has paid employment. The students had many different ethnic origins (Anglo-Saxon, Chinese, Vietnamese, etc.) and different religious backgrounds (Christian, Moslem etc.). In addition, they came from families in which there were a wide range of lifestyles, with different interests in sport, music, literature, etc.

It could be argued that the term that the term "ethnomathematics" could be applied to that particular informal, even unconscious mathematics, that is implicit in the everyday activities and ways of thinking of any of the reasonably well-defined cultural or sub-cultural group represented by students in secondary mathematics classes. For example, children who regularly assist parents who are small business proprietors would be expected to engage frequently in activities involving calculations, classification and measurement which would not be unlike the activities of the students in the present study.

In this sense then, the typical secondary mathematics class represents the coming together of a whole range of different kinds of ethnomathematics: there is the ethnomathematics of sport; of music; of small business operation; of track gambling activities, and so on. The problem for the curriculum developer, text book writer, teacher and examiner then is how best to identify and take advantage of the intuitive, unconscious understandings brought in to the learning environment by students from backgrounds that include these interests.

Often the classroom teacher may be unaware of students' backgrounds and interests and therefore have no knowledge of the mathematical concepts that their students develop as a result of these backgrounds. Indeed, it would be unrealistic to expect teachers to be aware of all of the ethnomathematical understandings of their students. One possible response to the dilemma raised by the multiplicity of student backgrounds is for educators to identify and study the nature of major ethnomathematical understandings and features associated with various cultures and sub-cultures. Having done this teachers can then work at identifying ways of incorporating ideas from these understandings in curriculum development, teaching methodologies, and professional development programs for teachers. Perhaps, though, the simplest and most effective way for teachers to proceed is for them to attempt to achieve a greater understanding of the predominant ethnomathematical forms represented in the classes they teach.
Quite clearly, within Australia gambling forms an important part of the personal daily lives of a sufficiently large number of families to justify the funding of additional studies into the educational implications of the ethnomathematics of gambling. The findings of these studies should be incorporated into curriculum development and should ultimately influence the teaching and learning of probability and related mathematical concepts in secondary schools.

A possible strategy for implementing the above recommendations would be: (a) that funding be made available for additional research of the type described in this study, (b) that implications of such research for mathematics education be identified, and (c) that these implications be acted upon in curriculum development and teacher professional development projects.

Mathematics education research is nothing more than an academic exercise unless mathematics educators take deliberate steps to make the results of such research known to those having teaching and curriculum development responsibilities for school mathematics in Australia. The challenge to mathematics educators presented by the present study is to make its findings known to curriculum developers and teachers so that ultimately more students will be able to link their personal worlds with the school mathematics they are expected to learn.
Appendix 1

Identification Questionnaire

MATHS IN SOCIETY - YEAR 11

Please circle the appropriate response and answer the questions.

1. Do you play any card games?
   Yes          No

   If "yes", indicate which ones and roughly how often you play (Every day, once a week, occasionally....etc).

   Name of Game            How Often

2. Do you play any games that involve throwing dice?
   Yes          No

   If "yes":

   Name of Game            How Often

3. Do you ever go to:

   (a) the horse races:
   (b) the trotting races:
   (c) the dog races

   No Never    Occasionally    Regularly

4. To what extent would you say that your parents are interested in horse racing:

   (a) very much
   (b) fairly interested
   (c) a little bit
   (d) not at all

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Appendix 2

Predetermined Interview Questions

The following questions are those described in Chapter 7, the Research Methodology, which were asked in the individual structured interviews. As was noted then, not all questions were asked of all students. Those probing questions which were asked of only selected students in a verbal form (according to their response to previous questions) are indicated by an asterisk. Unless indicated in this manner, all other question were presented to all students in a written format accompanied with some verbal explanation or clarification if necessary. Examples of this interaction appear in Chapter 11, Case Studies.

Question 1

Question 1(a), (i)
A photographic negative 2 cm wide and 3 cm long is enlarged to a photograph 9 cm wide.
How long is the photograph?

*Question 1(a), (ii)
A photographic negative 2 cm wide and 3 cm long is enlarged to a photograph 8 cm wide.
How long is the photograph?

Question 1(a), (iii)
If 2 litres of juice costs $3, how much would 9 litres cost?

Question 1(a), (iv)
If 4 kg of apples cost $7, how much would 5 kg cost?

Question 1(b)
In each of the following situations, how much could be won on a track bet if:
(i) $10 is bet at odds of 9:2
(ii) $9 is bet at odds of 3:2
(iii) $5 is bet at odds of 7:4
Question 2

In each pair, circle the larger of the two fractions:

(a) $\frac{1}{2}$  $\frac{1}{3}$
(b) $\frac{3}{8}$  $\frac{5}{8}$
(c) $\frac{1}{2}$  $\frac{3}{5}$
(d) $\frac{3}{5}$  $\frac{5}{8}$

Question 3

Suppose I select a marble at random from each of the following bags:

In each of the two situations what is the probability of getting a black marble?
(a) The bag contains 3 black and 1 white
(b) The bag contains 5 black and 3 white
(c) In which of these cases, (a) or (b), is it more likely to get a black marble?

* Question 3(d)  
[If unable to compare the probabilities in Question 3(c)]
Repeat Question 3(c) with:
(i) (a) 1 black marble and 3 white marbles
    (b) 1 black marble and 4 white marbles
(ii) (a) 2 black and 3 white
     (b) 2 black and 5 white;

* Question 3(e)  
[If able to compare the probabilities in Question 3(c)]
Question 3(c) was repeated using;
3 black marbles and 2 white marbles in one bag, and 5 black marbles and 3 white marbles in the other bag.
**Question 4**

In each of the following track betting situations, which is the better of the two odds? That is to say, which gives the greater return per $ bet.

(a) 2:1 or 3:1  
(b) 5:1 or 5:2  
(c) 4:3 or 9:7  

*Question 4 (d)*  
[To be asked if part (c) is incorrect]  
In the following track betting situations, which is the better of the two odds?  
That is to say which gives the greater return per $ bet.  
5:2 or 9:4

**Question 5**

In a four horse race the odds for each horse are given as:  
2:1, 5:3, 5:1, and 25:1  
Which horse is thought to be most likely to win?  
List the odds in order of least likely to most likely.

**Question 6**

**Question 6(a)**  
In the last 200 Gold Lotto draws the number 17 has occurred more times than any other number. In future draws do you think that this number is:

(i) more likely to occur than any other number?  
(ii) less likely to occur than any other number?  
(iii) as equally likely to occur as any other number?

**Question 6(b)**

In Roulette, each number is equally likely to occur. Suppose that the ball lands on "red" six times in a row. On the next roll is it now more likely to land on the "red", the "black", or are they both still equally likely?
Question 6(c)
In some lotteries the tickets are pre-numbered. If you could choose between any of the following numbers would you have any preference?

123456
619999
615472

Question 6(d)
In choosing six LOTTO numbers which of the following selections would you prefer?
(a) 1, 2, 3, 4, 5, 6
(b) 32, 33, 34, 35, 36, 37
(c) 9, 12, 27, 31, 35, 38

Question 7

Question 7(a)
If I throw a single fair die is it harder (less likely) to get a six than, say, a four?
In general, are the numbers 1 to 6 equally likely to occur, or are some harder to get than others?

Question 7(b)
If I throw a pair of fair dice is it harder to get a pair of sixes than it is to get a pair of twos?

Question 7(c)
A poker hand consists of five cards dealt from a well shuffled deck. The Ace is the highest card. Is it harder (that is to say less likely) to get three Aces than it is to get three fives?

Question 7(d)
Suppose that you and I are going to play a game with this coin [shows] in which we toss the coin just once. If the coin lands heads, I win; if it lands tails you win. But I will warn you that of the last 15 people who played this game with me only six won.
Do you think this is unusual?
Do you think the coin is not fair?
Do you think the game is fair?

* [If the response to part(d) is "yes"]: What if it was the case that, of last 20 people 14 lost? Is it still fair?
* [If "yes"]:
How long a run would you need to suspect bias if "head" showed about two-thirds of the times?

* [If "no"]:
Why not?
Would you suspect bias if 10 people played and 7 lost? If 6 people played and 4 lost?

**Question 8**

A robot is put into a maze which it begins to explore

At each junction it is as likely to go down any path as any other. Where is it most likely to end up?

**Question 9**

**Question 9(a)**
In each of two horse races the favourite is estimated to have about a 50% chance of winning.
If you bet on both favourites, your chances of winning on both would be about:

(i) 100%
(ii) 75%
(iii) 50%
(iv) 25%
(v) 10%
Question 9(b)
Two coins are tossed together.
(i) The probability that one will land a head and the other a tail is:

(a) 1/2
(b) 1/3
(c) 1/4

(ii) What is the probability that at least one of the two coins will land a head?

Question 9(c)
(i) A pair of fair dice are tossed together.
[show pair of identical dice]
Which is more likely:
(a) a pair of fives
(b) a five and a three
(c) both are equally likely

* Question 9(c) (ii)
A pair of fair dice are tossed together.
[show pair of different coloured dice]
Which is more likely:
(a) a pair of fives
(b) a five and a three
(c) both are equally likely

* Question 9(c) (iii)
In general, is it harder (that is to say less likely) to get a pair of numbers than it is to get two different numbers?

Question 10

Question 10(a)
Suppose you and I play a game with one of these dice [show single die]. If on a single roll it is three or less you win, if it is more than three I win. If we each bet $1 and the winner gets the $2, would this be a fair game?

Question 10(b), (i)
Suppose we change the rules so that I win if it is a three or greater is this still a fair game?

Question 10(b), (ii)
Can we make the game fair somehow?
* Question 10(b), (iii)
   How much should I put in to make the game fair?

* Question 10(c)
   [If the answer to 10(b) is $2, continue with further questions]
   How much should I put in if I choose the numbers 1 through 5, leaving you just the 6?
   [Repeat with drawing cards from a deck]
   How much should I put in if I choose spades leaving you the other three suits?

   * How much if I choose a single card of any suit?
   How much if I choose a single specific card?
   * [If correct, repeat the questions reversing the order of "you" and "I"]

Question 11

Question 11(a)
   The "double" in horse racing consists of selecting two winners in two prescribed races.
   How many possible ways can the "double" occur if one race has 10 horses and the other 8?
   (assume that there are no dead heats).

Question 11(b)
   If there are 6 horses in a race, what is the total number of ways in which the first and second places be filled?
   (Ignore the possibility of a dead heat)
   [If the student is unable to answer this, proceed to Question 11(f)]

* Question 11(c)
   [To be asked only of those students who were able to answer both Questions 11(a) and (b) correctly]
   If the "quinella" consists of selecting the first two horses in a race without necessarily being in the right order, how many possible selections are there in a race with 8 horses.
   [If the student is unable to answer this, proceed to Question 11(f)]

* Question 11(d)
   [To be asked only of those students who were able to answer both Questions 11(c) correctly]
   If the trifecta consists of selecting the first three horses in the correct order, how many possible selections are there in a race with 6 horses?
* Question 11(e)
[To be asked only of those students who were able to answer both Questions 11(d)]
If we disregard the order in the last question, how many selections are there?

Question 11(f)
A type of new car can be bought with a choice of 6 different colours and 3 different engine types. What is the total number of selections possible.

Question 11(g)
If the same type of car as in the last question can also be bought with either manual or automatic transmission, how many selections are now possible?

* Question 11(h)
If you have 6 different movie vouchers each entitling you to a different movie, but you have time to go to only two of them, how many different choices can you make?
Appendix 3

Coded Spreadsheet Data
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Appendix 4

Consent Form

CONSENT FORM

I ................................................ hereby indicate that I am willing to participate in the proposed study which will involve my responding to written and verbal questions.

I understand that the purpose of this study is for educational research and that the results of the questions will not be used for any other purposes and that my responses will be treated in confidence.

I understand that I need not answer any questions that I choose not to, and that I may withdraw from the study at any time I so choose.

The interview may/may not be recorded on audio tape.

Signed
Appendix 5

Trifecta Betting (TAB Guide)

TRIFECTA Betting

A Trifecta requires you to select the three runners that will finish FIRST, SECOND and THIRD, in the CORRECT order.

When you Box a Trifecta your selections can finish FIRST, SECOND and THIRD, in ANY order.

e.g. If you BOX the runners 1, 2 and 3 you receive 6 combinations with which you can win the TRIFECTA.

1 - 2 - 3  1 - 3 - 2
2 - 1 - 3  2 - 3 - 1
3 - 1 - 2  3 - 2 - 1

Cost

Box Trifecta

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A Standout Trifecta is where a runner selected to finish FIRST (a STANDOUT) is combined with two or more other runners to finish SECOND and THIRD in any order.

e.g. If you STANDOUT number 1 to finish FIRST with numbers 2, 3 and 4 to finish SECOND and THIRD, you receive 6 combinations with which you can win the TRIFECTA.

1 - 2 - 3  1 - 3 - 2
1 - 2 - 4  1 - 4 - 2
1 - 3 - 4  1 - 4 - 3

Cost

Standout Trifecta

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A Multiple Trifecta is where two or more runners selected to finish FIRST, are combined with a number of other runners to finish SECOND and THIRD.

e.g. If you select numbers 1 and 2 to finish FIRST with numbers 1, 2, 3 and 4 to finish SECOND and THIRD, you receive 12 combinations with which you can win the TRIFECTA.

1 - 2 - 3  2 - 1 - 3
1 - 3 - 2  2 - 3 - 1
1 - 2 - 4  2 - 1 - 4
1 - 4 - 2  2 - 4 - 1
1 - 3 - 4  2 - 3 - 4
1 - 4 - 3  2 - 4 - 3

Cost

Multiple Trifecta

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<table>
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*All costs are for $1 (costs can be halved if the investment is for 50 cents).

NOTE: The runners stand-out for 1st are also included in the second and third selections.
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