A construction method of Atanassov's Intuitionistic Fuzzy Sets for image processing

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***Abstract*—In this work we introduce a new construction method of Atanassov's intuitionistic fuzzy sets (A-IFSs) from fuzzy sets. We use A-IFSs in image processing. We propose a new image magnification algorithm using A-IFSs. This algorithm is characterized by its simplicity and its efficiency.**

I. Introduction

Image scaling is a basic operation in digital image processing. There exist two scaling procedures: magnification and reduction. Image magnification consists in enlarging the resolution of an image [15], [17], [18] while image reduction diminishes it. In this work we focus on image magnification (also called image enlargement).

Our aim is, for a given image, to build a new image of greater dimension such that each area or block of the new image reflects the same relation of intensities than each pixel in the original image had with its neighborhood.

The most commonly used techniques for image magnification are interpolation based. They include nearest neighbour, bilinear interpolation, splines or wavelets [1]. Moreover, image magnification has also been treated with other techniques: fuzzy transform [16] or edge fitting model [14].

In this work we present a new simple and computationally efficient method for image magnification. Our proposal makes use of Atanassov's intuitionistic fuzzy sets [2], [3] and K_α operator [10]. These sets are used in image processing in fields such as segmentation [12], image contrast [8] or edge detection[11] and in pattern recognition [19].

We also present a construction method of Atanassov's intuitionistic fuzzy sets from a fuzzy set with a predetermined indeterminacy index for each element of the resulting set. We are going to understand this index as a measure of the variation of intensities in the neighborhood of each considered pixel. The relation between intensities in the original images is translated into the amplified image by means of the α parameter in K_α operator.

This work is organized as follows: first, in Section 2 we recall some preliminary definitions . In Section 3, we present the construction method of A-IFSs. Later, in Section 4 we show the image magnification algorithm. We finish with some experimental results in Section 5 and some conclusions in Section 6.

II. Preliminaries

Let U be an ordinary finite non-empty set.

An *Atanassov's Intuitionistic Fuzzy Set* (A-IFS) [2], [3] in U is an expression A given by

$$A = \{(u_i, \mu_A(u_i), \nu_A(u_i))|u_i \in U\} \tag{1}$$

where

$$\mu_A : U \to [0,1]$$

$$\nu_A : U \to [0,1]$$

satisfy the condition: $0 \leq \mu_A(u_i) + \nu_A(u_i) \leq 1$ for all u_i in U.

The numbers $\mu_A(u_i)$ and $\nu_A(u_i)$ denote respectively the degree of membership and the degree of non-membership of the element u_i in set A. We will represent as A-$IFSs(U)$ the set of all Atanassov's Intuitionistic Fuzzy Sets in U.

The following expression is defined for all $A, B \in A$-$IFSs(U)$: $A \leq B$ if and only if $\mu_A(u_i) \leq \mu_B(u_i)$ and $\nu_A(u_i) \geq \nu_B(u_i)$ for all u_i in U.

We know that fuzzy sets are represented exclusively by the membership function,

$$A_F = \{(u_i, \mu_{A_F}(u_i))|u_i \in U\}$$

where $\mu_{A_F} : U \to [0,1]$. We will denote as $FSs(U)$ the set of all fuzzy sets in U.

Atanassov defined the *intuitionistic fuzzy index* of an element u_i in A as

$$\pi_A(u_i) = 1 - \mu_A(u_i) - \nu_A(u_i)$$

Naturally, if the set A considered is fuzzy, then $\pi_A(u_i) = 1 - \mu_A(u_i) - \nu_A(u_i) = 1 - \mu_A(u_i) - 1 + \mu_A(u_i) = 0$.

Let us denote by L^* the set of all pairs such that

$$L^* = \{(x,y)|(x,y) \in [0,1] \times [0,1] \text{ and } x + y \leq 1\}.$$

The smallest element of L^* is $0_{L^*} = (0,1)$ and the greatest one is $1_{L^*} = (1,0)$.

In [4] Atanassov defined the K_α operator, that associates a fuzzy set to an Atanassov's intuitionistic fuzzy set. This

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operator has been widely studied in [10]. Let $\alpha \in [0,1]$ y $A \in A\text{-}IFSs(U)$. Then the operator K_α is given by

$$K_\alpha(A) = \{(u_i, \mu_A(u_i) + \alpha\pi_A(u_i), \nu_A(u_i) + (1-\alpha)\pi_A(u_i)) | u_i \in U\}$$

Clearly, $K_\alpha(A)$ is a fuzzy set.

Proposition 1: For all $\alpha, \beta \in [0,1]$ and $A, B \in A\text{-}IFSs(U)$, it is verified that

(a) If $\alpha \leq \beta$, then $K_\alpha(A) \leq K_\beta(A)$.
(b) If $A \leq B$ then $K_\alpha(A) \leq K_\alpha(B)$.
(c) $K_\alpha(K_\beta(A)) = K_\beta(A)$.

Proposition 2: For all $A \in A\text{-}IFSs(U)$, the family $\{K_\alpha(A)\}_{\alpha\in[0,1]}$ is totally ordered.

III. Construction of Atanassov's intuitionistic fuzzy sets with fixed indeterminacy index

In this section we propose a method of construction of Atanassov's intuitionistic fuzzy sets. Its importance lays on the fact that we build Atanassov's intuitionistic fuzzy sets such that the indeterminacy index for each element is fixed beforehand. Moreover, we represent with the indeterminacy index the uncertainty of experts in choosing the membership and non membership degrees of each element.

Proposition 3: The mapping

$\mathcal{F} : [0,1]^2 \times [0,1] \to L^*$ given by
$\mathcal{F}(x, y, \delta) = (\mathcal{F}_\mu(x, y, \delta), \mathcal{F}_\nu(x, y, \delta))$ where
$\mathcal{F}_\mu(x, y, \delta) = x(1 - \delta y)$
$\mathcal{F}_\nu(x, y, \delta) = 1 - x(1 - \delta y) - \delta y$

satisfies that:

1) If $y_1 \leq y_2$ then $\pi(\mathcal{F}(x, y_1, \delta)) \leq \pi(\mathcal{F}(x, y_2, \delta))$ for all $x, \delta \in [0,1]$;
2) $\mathcal{F}_\mu(x, y, \delta) \leq x \leq 1 - \mathcal{F}_\nu(x, y, \delta)$ for all $x \in [0,1]$;
3) $\mathcal{F}(x, 0, \delta) = (x, 1 - x)$;
4) $\mathcal{F}(0, y, \delta) = (0, 1 - \delta y)$;
5) $\mathcal{F}(x, y, 0) = (x, 1 - x)$;
6) $\pi(\mathcal{F}(x, y, \delta)) = \delta y$.

Theorem 1: Let $A_F \in FSs(U)$ and let $\pi, \delta : U \to [0,1]$ be two mappings. Then

$$A = \{(u_i, \mathcal{F}(\mu_{A_F}(u_i), \pi(u_i), \delta(u_i))) | u_i \in U\}$$

is an Atanassov's intuitionistic fuzzy set.

Corollary 1: In the setting of Theorem 1 if for every $u_i \in U$ we take $\delta(u_i) = 1$ then

$$\pi(u_i) = \pi(\mathcal{F}(\mu_{A_F}(u_i), \pi(u_i), 1)).$$

Notice that under the conditions of Corollary 1 the set A is given as follows:

$$A = \{(u_i, \mu_{A_F}(u_i)(1 - \pi(u_i)), 1 - \mu_{A_F}(u_i)(1 - \pi(u_i)) - \pi(u_i)) | u_i \in U\}$$

Example 1: Let $U = \{u_1, u_2, u_3, u_4\}$ and let $A_F \in FSs(U)$ given by

$$A_F = \{(u_1, 0.2), (u_2, 0.8), (u_3, 1), (u_4, 0.5)\}$$

and $\pi(u_i) = 0.2, \delta(u_i) = 1$ for all $u_i \in U$. By Corollary 1 we obtain the following Atanassov's intuitionistic fuzzy set:

$$A = \{(u_1, 0.16, 0.64), (u_2, 0.64, 0.16), (u_3, 0.80, 0.00), (u_4, 0.40, 0.40)\}$$

IV. Image magnification algorithm

In this section we propose a gray-scale image magnification algorithm that makes use of Atanassov's intuitionistic fuzzy sets and K_α operators.

In this work, we consider an image of $N \times M$ pixels as a set of $N \times M$ elements arranged in rows and columns. Hence we consider an image as a $N \times M$ matrix. We denote by q_{ij} the intensity of the pixel at the position (i, j) of the Q matrix, with $i \in \{1, \ldots, N\},\ j \in \{1, \ldots, M\}$. In this paper we are going to work with gray-scale images with intensities varying from 0 to 255. The intensity of each element of the matrix, q_{ij}, has a value in $[0,1]$ that will be calculated by normalizing it by 255.

The purpose of this algorithm is: given an image Q of dimension $N \times M$ we are going to build a new image of dimension $N' \times M'$ with $N' = n \times N$, $M' = n \times M$ and $n \in \mathbb{N} - \{0\}$.

The algorithm consists of the following steps:

(A) We choose a parameter $\delta \in [0,1]$.
(B) For each pixel (i, j) in the image Q, we are going to fix a net (matrix) of dimension $n \times n$ centered on that pixel.
(C) For each pixel we are going to calculate the difference between the biggest and the smallest intensities in the net, that we denote by W.
(D) To each pixel (i, j) of the image we are going to associate the intuitionistic pair :

$$\mathcal{F}(q_{ij}, W, \delta) = (q_{ij}(1 - \delta \cdot W), 1 - q_{ij}(1 - \delta \cdot W) - \delta \cdot W).$$

In Figure 1 we show an example. For the 5×5 image, we want to build a magnified image of dimension 15×15 ($n = 3$). We fix $\delta = 1$. For the pixel $(2, 3)$ (marked in dark gray), we fix a net of dimension 3×3 around it (marked in light gray).

0.7	0.6	0.65	0.4	0.41
0.66	0.59	0.6	0.5	0.46
0.62	0.7	0.6	0.52	0.48
0.6	0.68	0.55	0.53	0.5
0.63	0.62	0.52	0.5	0.5

Fig. 1. Example: original image

We calculate W:

$$W = \vee(0.6, 0.65, 0.4, 0.59, 0.6, 0.5, 0.7, 0.6, 0.52) - \wedge(0.6, 0.65, 0.4, 0.59, 0.6, 0.5, 0.7, 0.6, 0.52) = 0.7 - 0.4 = 0.3$$

Then the intuitionistic pair associated to pixel $(2,3)$ is given by:

$$\mathcal{F}(0.6, 0.3, 1) = (0.42, 0.28)$$

Remark. If we consider pixels in the first or the last row, or in the first or the last column, we choose the net as depicted in Figure 2.

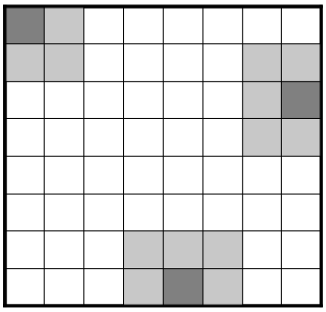

Fig. 2. Example: net for pixels in the first or last row / column.

(E) In this step we are going to use the result of Proposition 4. We consider that this is the justification of our algorithm.

Proposition 4: In the setting of Proposition 1, if we take $\alpha = x$, then

$$K_x(\mathcal{F}(x, y, \delta)) = x$$

for all $x, y, \delta \in [0,1]$.

Proof. $K_x(\mathcal{F}(x,y,\delta)) = K_x((x(1-\delta y), 1-x(1-\delta y)-\delta y)) = x(1-\delta y) + x\pi(\mathcal{F}(x,y,\delta)) =$

From condition 6) in Proposition 3

$K_x(\mathcal{F}(x,y,\delta)) = x(1-\delta y) + x\delta y = x$ □

Next we are going to expand each pixel (i,j) in image Q building a new block of dimension $n \times n$ centered in (i,j).

In Figure 3 we show the result of this expansion for the pixel at $(2,3)$.

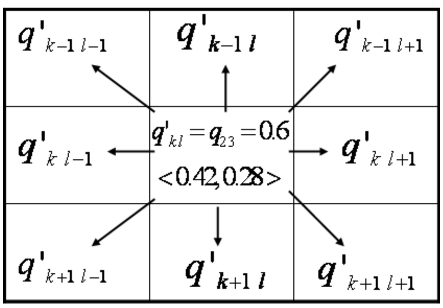

Fig. 3. Expanded block for pixel (23)

By Proposition 4 we have $q'_{kl} = q_{ij}$, that is, in the new construction we keep the value of the original central pixel

$$0.6 = q'_{kl} = K_{q_{ij}}((0.42, 0.28)) = 0.42 + q_{ij}0.3 = 0.6$$

Hence to keep the intensity of the central pixel Proposition 4 settles that α must be equal to the value of the intensity of that pixel. We extend this reasoning for the rest of elements in the block.

In this way, following Proposition 4 we take:

- $\alpha = q_{i-1j-1}$. Then $q'_{k-1l-1} = 0.42 + q_{i-1j-1}0.3 = 0.42 + 0.6 \cdot 0.3 = 0.6$
- $\alpha = q_{i-1j}$. Then $q'_{k-1l} = 0.42 + q_{i-1j}0.3 = 0.42 + 0.65 \cdot 0.3 = 0.615$
- $\alpha = q_{i-1j+1}$. Then $q'_{k-1l+1} = 0.42 + q_{i-1j+1}0.3 = 0.42 + 0.4 \cdot 0.3 = 0.54$
- $\cdots$
- $\alpha = q_{i+1j+1}$. Then $q'_{k+1l+1} = 0.42 + q_{i+1j+1}0.3 = 0.42 + 0.52 \cdot 0.3 = 0.576$

In Figure 4 we show the expanded block for pixel $(2,3)$ of Figure 1.

0.6	0.615	0.54
0.597	0.6	0.57
0.63	0.6	0.576

Fig. 4. expanded block of pixel q_{23}

In Figure 5 we depict the result of applying the previous method to all the pixels in the image and we show the magnified image.

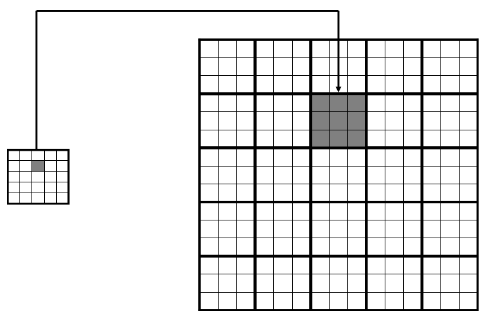

Fig. 5. Magnified image

Next we present an scheme of the proposed algorithm:

(A) Choose a parameter $\delta \in [0,1]$.
(B) For each pixel (i,j) fix a net of dimension $n \times n$ centered on it.
(C) For each pixel (i,j) calculate W.
(D) To each pixel (i,j) associate the intuitionistic pair

$$(\mu_{ij}, \nu_{ij}) = \mathcal{F}_{T_P}(q, W, \delta).$$

(E) For each pixel (i,j) build a block of dimension $n \times n$. For each element q' of the block
  (E1) Take α as the value of the intensity of the element placed in the net in the same position than q'.
  (E2) Take $q' = K_\alpha((\mu_{ij}, \nu_{ij}))$.

Algorithm 1

V. Experimental results

In this section we are going to implement Algorithm 1 on three images and we are going to study the results that we obtain. We follow these steps:

1. We start from images of 255×255 and we reduce them to a 85×85 size using the reduction algorithm proposed in [9].
2. Next, with Algorithm 1, we magnify the images to a 255×255 size.
3. Finally, we compare the images that we obtain with the original ones.

We are also going to study the effect of Algorithm 1 on the considered images when we take the values $\delta = \{0, 0.25, 0.5, 0.75, 1\}$.

On the other hand, to compare the images we use the measures developed in [7], and more specifically, the expression

$$S(Q,Q') = \frac{1}{N \times M} \sum_{i=1}^{N} \sum_{j=1}^{M} 1 - |q_{ij} - q'_{ij}| \qquad (2)$$

where Q, Q' are two images of dimension $N \times M$.

In Figure 6 we show the original images (first column) and their reductions (second column).

A. Specific case: $\delta = 0$

By Proposition 1, we have that

$$F(x,y,0) = (x, 1-x)$$

for all $x, y \in [0,1]$. We also know that

$$K_\alpha((x, 1-x)) = x$$

for all $\alpha \in [0,1]$. In this way, when we apply the magnification method taking $\delta = 0$ we build blocks in which all the elements take the value of the central pixel (see Figure 7).

Observe that visually the quality of the images that we have obtained is not very good. This is due to the fact that for $\delta = 0$ we loose information from the neighbourhood, that is, the reconstructed blocks do not keep the same relation than the original pixel with the surrounding ones.

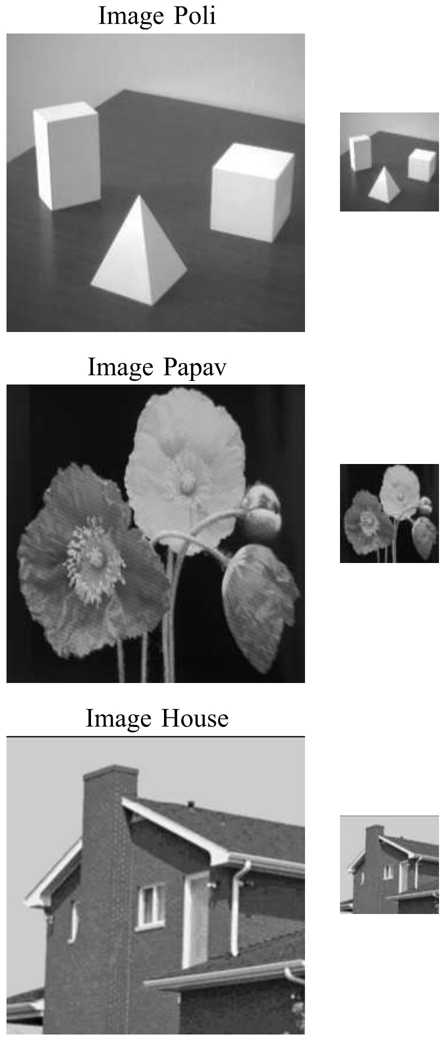

Fig. 6. Original images and reductions

B. Effect of increasing the value of δ

In this subsection we study how the obtained images vary when we increase the δ parameter in Algorithm 1. We analyze four specific cases: $\delta = 0.25$, $\delta = 0.5$, $\delta = 0.75$ y $\delta = 1$.

In Figure 8 we show all the test images magnifications for the considered δ values.

Notice that when δ increases, so does the indeterminacy index of the intuitionistic pair associated to each pixel. In this way, it also increases the range in which the intensities of pixels in each reconstructed block vary. Observe that if we take $\delta = 1$ the quality of areas with large intensity changes (edge zones) diminishes.

To compare the obtained images with the original one we use the comparison index given in Equation 2. In Table I we show the results. We observe that the best solutions are obtained when we take intermediate values of δ, that is, values of δ close to0.5.

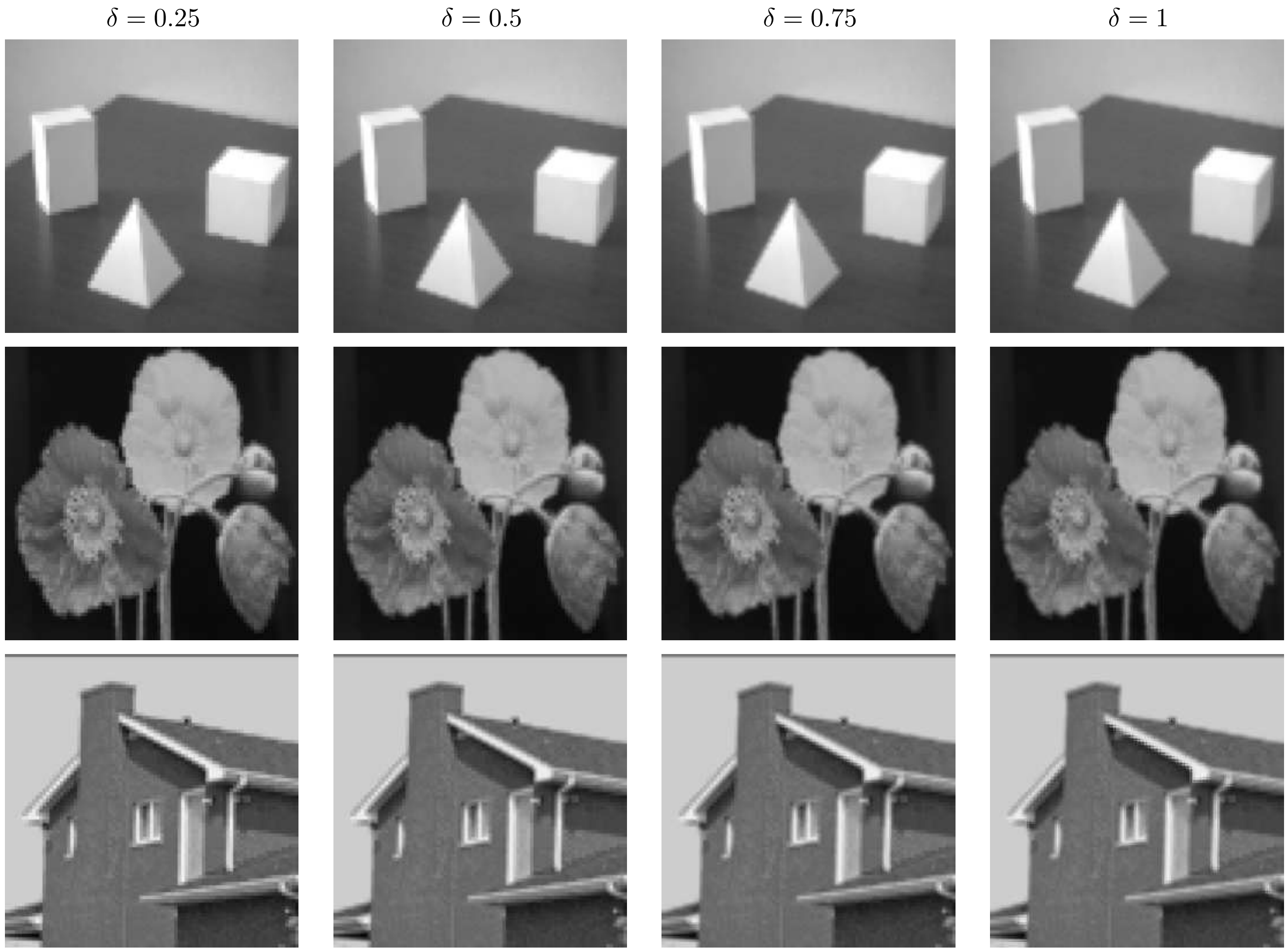

Fig. 8. Reconstructed images with different values of the parameter δ

TABLE I
COMPARISON OF THE RECONSTRUCTED IMAGES WITH THE ORIGINAL ONE

	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 0.75$	$\delta = 1$
Poli	0.9887	0.9893	0.9895	0.9895	0.9893
Papav	0.9785	0.9795	0.9799	0.9799	0.9796
House	0.9724	0.9738	0.9743	0.9742	0.9735

C. Further study on the values of δ

Next we carry on a deeper analysis of our study. Up to now, we have worked with 5 values of δ uniformly distributed between 0 and 1. We have refined our study working with 100 values of δ uniformly distributed between 0 and 1. In Figures 9, 10 and 11 we see the accuracy of our solutions. In the abscissa axis we show the values of parameter δ. In the ordinate axis we show the similarities between the reconstructed images and the original ones.

With the resulting graphics we see that the best results are obtained for $\delta = 0.6$ for Poli image, $\delta = 0.65$ for Papav image and $\delta = 0.57$ for House image. We also observe that reconstructions loose quality as δ goes to zero or one, as it has been experimentally shown in subsections A y B.

VI. CONCLUSIONS

In this work we introduce a new method of construction of Atanassov's intuitionistic fuzzy sets from fuzzy sets. We use this construction to develop a new image magnification algorithm. Contrary to the case of most of the published methods, this algorithm is not based on interpolation. Moreover, it is remarkable because of its simplicity and efficiency in time.

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Fig. 7.   Images built with $\delta = 0$

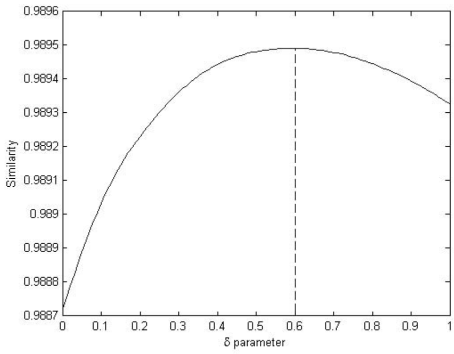

Fig. 9.   Comparison of Poli image reconstructed with different δ

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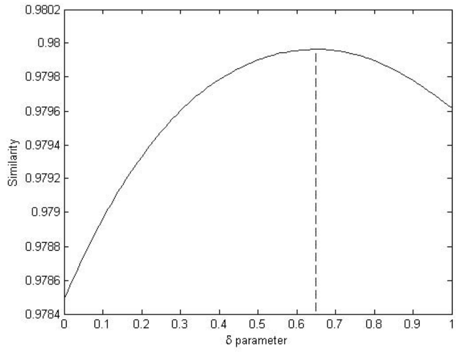

Fig. 10.   Comparison of Papav image reconstructed with different δ

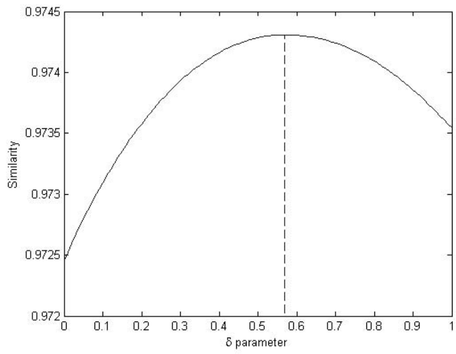

Fig. 11.   Comparison of House image reconstructed with different δ

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