CHAPTER 14.

DEAN'S GREAT DISCOVERY: MULTIPLICATION, DIVISION AND FRACTIONS*

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Multiplication, division and fractions are "hotspots" for students in the middle years with many students experiencing difficulty with these concepts (Siemon, Virgona, & Cornielle, 2001). Arrays effectively model multiplication and help children develop multiplicative thinking and learn multiplication facts (Young-Loveridge, 2005). In this article we show how an open-ended array problem enabled a Grade 5/6 student to think about the relationship between multiplication, division and fractions. In the article we describe the project and "hot spot" mathematical tasks that we used and provide some background on multiplicative thinking before presenting the case and a commentary (Western Melbourne Roundtable, 1997) of one student's exploration. This case was documented whilst we were working on a collaborative project with a team of upper primary teachers and a group of pre-service teachers at a local primary school.

Our project with Sunbury Primary

As the project began, a conversation between the teachers at the school and the university staff resulted in the idea of working together to enhance the learning of school children and pre-service teachers with a focus on the "hotspots". Pre-service teachers worked one day per week in Grade 5/6 mathematics classrooms over a five-week period. Each pre-service teacher worked with a small group of children. There were five tasks and the groups of children and their pre-service teacher rotated through these tasks over

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five lessons in the five-week program. A brief outline of these tasks is included in Table 14.1.

Table 14.1. Grade 5/6 numeracy tasks for each "hot spot".

Numeracy "hot spot"	Task
Fractions	Fraction strips An open-ended task. Students made fraction strips, found equivalent fractions and then found fractions that were less than two-thirds.
Decimals	Five cards (Beesey, Clarke, Clarke, Stephens, & Sullivan, 2001) An open-ended task in which students compared and ordered decimals using digits written on playing cards.
Multiplication	Arrays An open-ended task in which students made arrays with concrete materials for given numbers, for example, 56, and generated number sentences for the array.
Word problems	Tree Diagrams Three combinations problems and one permutation problem that required students to interpret the context and develop a strategy for finding all the possible combinations.
Algebra —problem solving	Eric the Sheep (Curriculum Corporation) A problem solving task involving generalising simple number patterns and the development of pre-algebra skills.

Multiplicative thinking

Understanding of multiplication and division is needed in order to develop sound concepts of fraction, ratio and proportion. Many students in the middle years have difficulty with these concepts and skills (Siemon, et al., 2001) and we found them to be problematic for many of our pre-service teachers too. Multiplication concepts are imbedded in each of the tasks in Table 14.1, and with the exception of the decimal task, students need to think multiplicatively to solve the problem.

Thinking multiplicatively is more than remembering multiplication table facts. It involves being able to recognise multiplication in different contexts and interpret the language that is used in the structure of the problems (Young-Loveridge, 2005). Children who think multiplicatively not only understand multiplication as repeated addition but also as Cartesian product (4 multiplied by 5, illustrated in an array), as scalar (3 times larger) and as a rate (for example, $3 for 4 tickets). Additionally they understand that division is the inverse of multiplication. Thinking multiplicatively also means that children use, perhaps unconsciously, the commutative ($7 \times 8 = 8 \times 7$), the associative ($7 \times [4 \times 2] = [7 \times 4] \times 2$) and the distributive ($7 \times 8 =$

[5 × 8] + [2 × 8]) properties when calculating mentally (McIntosh, 2002; Young-Loveridge, 2005).

The array task

Arranging Arrays

- Take 56 counters and arrange these into an array. Write down the number sentences you can find using this array.
- Make arrays for the following numbers and write the number sentences that you can find. Use:

64
72
37

Figure 14.2. Open-ended array task.

The open-ended array task used in the project is documented in Figure 14.2. Some children automatically recalled the multiplication fact and so constructed a 7 × 8 array and recorded multiplication and division facts illustrating their understanding of the commutative property and the inverse relationship with division. Children who could not recall the fact took some time to construct the array trying different numbers of counters in rows or columns until they formed the rectangular shape. We had expected some children to observe two rectangles in the array (as shown in Figure 14.3) and so record number sentences to show the distributive property of multiplication but this occurred for only a few students and only following demonstration by the university lecturer or pre-service teacher.

Of interest to us was the thinking about division that emerged in the work of one student who tackled this problem. In the following section we present a case, that is a detailed description of what occurred when this student worked on the problem, written by one of us soon after the lesson occurred.

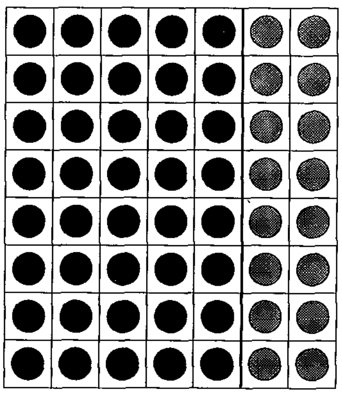

Figure 14.3. The distributive law: 7 x 8 = (5 x 8) + (2 x 8).

A case: Dean's great discovery

I was working in a small group with Dean. He had not been showing much interest in the maths activities for the last few weeks. In fact, he was disengaged and distracting other students both in our group and in the other groups. This particular week the pattern was the same. We started by talking about the task in our small group. It was arrays this week, and the girls in the group set about laying out the 56 counters and answering the questions on the task sheet.

Dean did nothing. So I said "Come and sit down here on the floor and we'll do it together." Dean reluctantly laid out the 56 counters in an array and, as we began to talk about the questions, I wrote the number sentences that Dean stated in my book:

$$7 \times 8 = 56$$
$$8 \times 7 = 56$$
$$56 \div 7 = 8$$
$$56 \div 8 = 7$$

Then Dean asked: "What's 7 divided by 56?"

"Oh," I said in a slightly startled voice, "Let's work it out."

We put seven counters out on the floor and I asked, "If we want to divide these seven counters by 56 how many would we have to divide each counter into?"

After some thinking Dean tentatively suggested eight, and I asked: "What would you call that?" Even more tentatively Dean said "one-eighth?" and I said "Yes!" and I wrote it down in my book:

$$7 \div 56 = \frac{1}{8}$$

I must say, at this point I was getting excited because I realised that I was learning something too! Surely I must have known this—but did I really? I felt as though I had just understood something that I had not known before. Dean, however, was not sure. He ran to get a calculator so that he could check. He punched in 7 ÷ 56 = and was puzzled to get the answer 0.125. "We were wrong," he announced. "Are you sure?" I asked. "Maybe the fraction one-eighth is the same as the decimal 0.125? What do you think?"

This was getting tricky but we stuck with it. Using strips of folded paper we talked about how eight times one eighth would equal one:

$$8 \times \frac{1}{8} = 1$$

and then worked out that eight times 0.125 would also equal one:

$$8 \times 0.125 = 1.$$

Dean was looking perplexed but he was not about to stop there. He rushed over to show his class teacher, Bill, what he had found out. Bill raised his eyebrows and smiled with surprise, "I wonder whether that would work with other numbers too?" he asked.

Dean returned and we tried the idea with a few more numbers:

$$8 \div 56 = \frac{1}{7}$$

Then we turned to the number 72, the next number on the task sheet.

$$9 \times 8 = 72$$

$$9 \div 72 = \frac{1}{8}$$

Dean said "I'll check that one too." And to his great surprise he punched in $9 \div 72 =$ into the calculator and the answer was 0.125! "That's the same answer as we got for seven divided by 56 equals," he said rushing over to show Bill again.

When he returned I cajoled Dean into recording his discoveries by copying the things I had recorded in my book into his Mathematics book and adding this new finding. He was not keen but he did it. In writing down what we had done we wondered whether there might be a pattern. We had:

$$7 \div 56 = 0.125 \text{ and}$$
$$9 \div 72 = 0.125$$

We had recorded the number needed to divide seven and nine to give 0.125 but what number would work for eight?

$$7 \div 56 = 0.125$$
$$8 \div ? = 0.125$$
$$9 \div 72 = 0.125$$

We worked out the difference between 56 and 72 was 16. What if we tried half way? And "bingo," eight divided by 64 equaled 0.125. Dean excitedly wrote down the new discovery in the gap to make a series of number sentences:

$$7 \div 56 = 0.125$$
$$8 \div 64 = 0.125$$
$$9 \div 72 = 0.125$$

He decided to find the next number sentence in the series and so continued his great discovery:

$$10 \div 80 = 0.125$$
$$11 \div 88 = 0.125...$$

By the end of the lesson, and many pages later, Dean had reached:

$$400\,000 \div 3\,200\,000 = 0.125$$
$$400\,001 \div 3\,200\,008 = 0.125$$

He was showing no signs of tiring.

Commentary

The first thing to notice is that Dean automatically recalled the multiplication and division facts that were represented by the array of 56 counters and he had no difficulty recording number sentences for other arrays, nor using multiplication facts to generate the list of number sentences. What was significant is that when prompted by the teacher to see if it worked for other numbers he was able to recognise a relationship and used this to generate a series of number sentences to fit the pattern, in this case a constant ratio.

For $9 \div 72 = 0.125$, he was able to see that 72 worked because you used $9 \times 8 = 72$, and so 72 needed to be the divisor:

$$9 \div (9 \times 8) = 0.125$$

He could then continue the pattern by using $10 \div (10 \text{ x } 8) = 0.125$ and so on for 11, 12 and up to the really large six-digit numbers in his pattern.

What is really interesting about this case is the question that Dean asked in the first place: "What's 7 divided by 56?" To scaffold his thinking he was asked about sharing 7 between 56, at least that is the language that we should have been using. A visual image of the problem was prompted by the selection of seven counters. Dean could see that each counter would need to be divided into eight equal pieces. But he was not convinced that one-eighth was the same as 0.125. This conflict arose for Dean because he did not recognise 0.125 as equivalent to $\frac{1}{8}$, and perhaps even that every fraction could be represented by an equivalent decimal. He may have known this about quarters and halves and recorded decimals as tenths and hundredths but he had not encountered eighths as decimals before.

Furthermore he did not seem to be thinking that division could also be represented symbolically as a fraction:

$$7 \div 56 = \frac{7}{56}$$

As Clarke (2006) and Gould (2005) have observed this representation of division is not familiar to children, even though children readily use division on the calculator to find the decimal equivalent of a fraction as Dean did in this case. If fractions were modeled to show this meaning and division represented in this form, Dean may have been able to apply his knowledge of equivalent fractions, explored in the fraction strips task, to this problem to realise that:

$$\frac{7}{56}=\frac{1}{8}$$

Writing Dean's thinking using fraction notation helps to show why the answer would be $\frac{1}{8}$:

$$9 \div 72 = 9 \div (9 \times 8) = \frac{9}{9\times8}=\frac{9}{9}\times\frac{1}{8}=1\times\frac{1}{8}$$

Powerful learning occurred for Dean because he persisted by trying other numbers and discovered a constant answer for equivalent ratios. Crucial for Dean's discovery was the openness of the task, the classroom teacher's suggestion that he try it with different numbers and the university teacher's persuasion to record his findings systematically. The calculator was a powerful tool that Dean used to confirm the pattern that emerged through his investigation.

The relationship between division and fractions that he discovered may have been more easily visualised if we had used square counters, and therefore could represent seven shared between 56 using the array as shown in Figure 14.4. Certainly Dean's finding could be shared with other children and they could be challenged to find similar relationships.

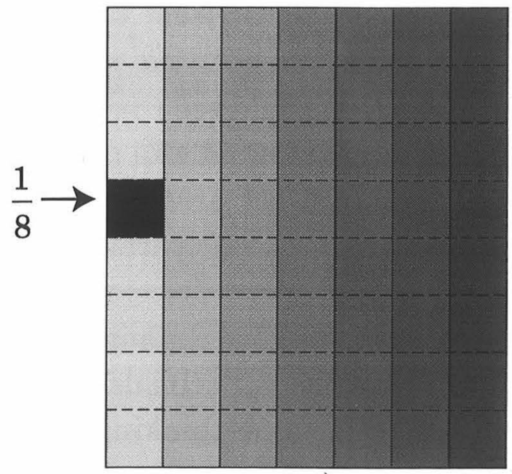

Figure 14.4. $7 \div 56 = \frac{1}{8}$.

Conclusion

This case illustrates the importance of discussion with students while they are working on open-ended tasks. The teacher prompted further exploration and the university teacher supported this through questions and suggestions that engaged him in the problem solving process. This case also shows that the open-ended array task enabled one student to generate a number pattern to show the relationship between multiplication, division and fractions. With modelling by the teacher, children could discover this pattern and relationship for other composite numbers. For this to be a successful learning experience for children, and for children to develop their understanding of fractions, they need to be introduced to the idea that a fraction is not only a part of a whole and a part of a collection, but that a fraction also means division and finding a ratio.

References

Beesey, C., Clarke, B., Clarke, D., Stephens, M., & Sullivan, P. (2001). *Effective assessment for mathematics: CSF Levels 4-6.* South Melbourne: Longman.

Clarke, D. (2006). Fractions as division: The forgotten notion? *Australian Primary Mathematics Classroom, 11*(3), 4–10.

Curriculum Corporation. *Maths300.* Retrieved 14 March 2005 from http://www.curriculum.edu.au/maths300/

Gould, P. (2005). Really broken numbers. *Australian Primary Mathematics Classroom, 10*(3), 4–10.

McIntosh, A. (2002). Common errors in mental computation of students in grades 3–10. In B. Barton, K. Irwin, M. Pfannkuch, & M. Thomas (Eds) *Mathematics education in the South Pacific* (Proceedings of the 25th Annual Conference of MERGA, pp. 457–472). Auckland, NZ: MERGA.

Siemon, D., Virgona, J., & Cornielle, D. (2001) *The Middle Years Numeracy Research Project: 5–9, Final Report.* Department of Education, Employment and Training, Victoria, Catholic Education Commission of Victoria and Association of Independent Schools of Victoria. Retrieved 14 March 2005 from http://www.sofweb.vic.edu.au/mys/MYNRP

Western Melbourne Roundtable (1997). *Teachers write: A handbook for teachers writing about changing classrooms for a changing world.* Ryde, NSW: National Schools Network.

Young-Loveridge, J. (2005). Fostering multiplicative thinking using array-based materials. *Australian Mathematics Teacher 61*(3), 34–39.