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Optimal diet selection and the compliance problem: a technical note

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Optimal diet selection and the compliance problem: a technical note

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Abstract

In this paper we have posited an alternative methodology for the least-cost diet selection problem subject to a given set of nutritional constraints. In order to better address the implicit compliance problem, we have re-cast the classical diet selection problem using a two-stage approach whereby in the first stage the dietician decides on the total number of least-cost combinations which satisfy the nutritional requirements and in the second stage, the dietician finds the optimal number of diet plans that have to be formulated. The second stage basically involves finding a reduced search space n^* out of which the dietician will sequentially select one least-cost diet plan after another till a "match" is secured with the patient type to ensurecompliance, which we have argued based on prior literature to be needing more than simply finding the least-cost diet plan. Our posited approach addresses the compliance problem by making it endogenous to the optimal diet selection problem via consideration of different metabolic types.

Keywords and phrases: *least-cost diet, metabolic types, optimal stopping rule*

1. Motivation

Choosing a right combination of nutritious diet is important for most of us as we consider such choice as a voluntary attempt to control obesity (Campbell, 1994). However, there are also a large number of people

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for whom choosing a right combination of nutritious diet is not just a voluntary choice – rather it is a vital part of treating a serious medical condition such as heart failure, kidney failure, diabetes etc.

The existing literature on choosing nutritious diet focuses mainly on two importantissues: (i) on achieving the minimum cost of having a nutritious diet and (ii) on addressingthe compliance issue regarding the dietary regimens. The first issue is very important from a viewpoint of a benevolent social planner, in particular, when an economy consists of a significant number of low-income individuals. In such situation, a social planner tries to minimize the cost of having a nutritious diet. The best example is US government's Thrifty Food Plan (hereafter TFP). For given cost and nutrition constraints along with other miscellaneous constraints, the goal of TFP is to choose the food plan that minimizes the gap between the proposed food plan and the current average food consumption pattern of low-income Americans; (Wilde and Llobbera, 2009). Extant literature on choice of nutritious diet largely advocatesvarious mathematical optimization techniques.

Pioneering work in this regard was done by Stigler (1945) using linear programming. His idea was to choose the nutritional diets that minimize the costs while fulfilling the nutritional requirements of the patients. Linear programming has since been used quite often in diet optimization problems (Masset et al 2009).[1] However the major problem with such an optimization technique is that it does not always lead to feasible solutions, especially in case of very low-cost targets, as has been argued recently by Wilde and Llobbera (2009).

Furthermore, any deterministic optimization technique used in choosing nutritional diets assumes that an individual rational patient will always comply with the least cost diets. However, as was originally observed by Stigler (1945), estimated cost of nutritional diets, based on linear programming, implies that dieticians are too generous in their "cultural requirements" for palatability, variety and prestigewhich are rather difficult to encapsulate within a scientifically programmed budget (Walde and LLobrera, 2009). Although the recent literature acknowledges that cost is an important factor for the compliance of dietary regimens but it is certainly not the only factor. Other physiological and behavioural factors also play equally important roles (Cummings *et al.,* 1984). This line of

[1] Pulliam (1975), subsequent to Stigler, also used non-linear programming in solving diet optimization problems with nutrient constraints.

argument puts the compliance issue of dietary regimens within a broader framework; addressinga multitude of factors besides cost.

"Compliance" is often defined as *the extent to which anindividual's behavior (in terms of taking medication, following a diet plan or effecting life style changes) coincides with a clinically prescribed regimen* (van der Wal *et al.,* 2006). Compliance with dietary regimens involves a complex interaction of psychological factors. Brownell and Cohen (1995), in their seminal work, has an extensive survey on the various factors related to the compliance of dietary regimens, where they have identified that one of the most important factors for the compliance of dietary regimens is the "coping skill" of an individual patient to the change of dietary requirement. Existing literature has also focused on the patient's metabolism status in relation to food intake, as argued by Adams *et al.,* (2011). For example, Mousain *et al.,* (2004) and Nogovitsina *et al.,* (2005) have found that there exists a positive correlation between relative metabolic disturbances and child's developmental disorder. Indeed, different metabolic status of different people implies that the same food can have different impacts to different people. There also exists an interesting line of research carried by D'Adamoand Whitney (2002) who posited that blood group type is one of the key elements that plays a crucial role in a patient's metabolism status in relation to food intake.[2] These studies suggest that there exists a possibility of a "mismatch" between the patients and their dietary recommendations. Such mismatch could arisedue to various reasons. For example, the patients may differ in terms of their "coping skills" to the change of their dietary requirements or they may differ in terms of their metabolisms – so a "universally optimal" diet plan may not exist!

Focus has also been given in the existing literature to measure the degree of compliance. There are mainly three measurements of an individual patient's compliance and these are: (i) behavioral assessment, (ii) self-reporting assessment and (ii) physiological assessment. Cummings *et al.,* (1984) has argued that none of these measuresare free from limitations. For example, both the behavioral and self-reporting assessments have a moral hazard problem, where an individual patient tends to overestimate his compliance with recommendations. As argued by Cummings *et al.,* (1984) such overestimated compliance triggers from the fact that an individual patient has natural desire to report good behaviors, though he

[2]Although this explanation subsequently gained popularity within some circles, it was found lacking in terms of clinical evidence and thus hasn't been viewed in a favorable light by the broader scientific community.

may not be following the exact recommendations. This leads to invalidity of the compliance to dietary regimens. In that respect, physiological assessment such as seeking traces of drugs or metabolites in blood or urine samples is relatively better as it is unaffected by human judgment. However, physical measurements are costly and also the patients can be on their "best behavior" for a short while if they know beforehand that they are going to be tested (Cummings *et al.,* 1984). Furthermore, as argued by Cummings *et al.,* (1984), such physiological assessment is also subject to natural recuperative process, physical characteristics of the individual and also on the time the actual measurements are taken.

Following the above discussions, we make the following observations:

Observation 1: *Although cost is an important factor in a nutritious diet plan, it does not necessarily solve the compliance problem.*

Observation 2: *Mismatches may arise between patients and their recommended dietary intakes which may lead to a compliance problem.*

Observation 3*: A better way to address the compliance problem could be found by making it endogenous to the optimal diet selection problem.*

The above observations motivate our paper in the following ways: firstly we allow an individual dietician to choose the set of optimal dietary intakes that incur the least cost as well as satisfy the minimum nutritional intake; secondly, we allow the dietician to search sequentially within the set of optimal dietary intakes in order to identify the matched dietary intake for his patient. The sequential search approach is important (in finding the match) for two reasons: (i) as argued by DeGroot (1970), the sequential search is the best way to reduce the total risk because one can stop search as soon as one finds the match, hence it is also efficient process and (ii) this sequential approach can also partly address the problems involved in compliance regarding the dietary regimens. The following section discusses our model in details.

2. Proposed mathematical model

We consider a typical situation where there is one dietician who makes a plan of dietary intake for his patient. We assume that there are two classes of nutrients and these are S_1 and S_2 such that $S_1 := \{x_k \mid k = 1, 2 \ldots n]; k \in N\}$

and $S_2 := \{y_k \mid k = 1, 2 \ldots n]; k \in N\}$.[3] For the sake simplicity and tractability, we assume that both sets S_1 and S_2 are finite, mutually exclusive and they have the same number of elements. The intuition is as follows: suppose S_1 and S_2 represent two classes of nutrients,for example, carbohydrates and proteins; and within each class there are n items of food. Therefore, we assume that, given two different classes of nutrients, a dietician has to make a plan of dietary intake for a patient that comprises of one item of food selected from each class of nutrient. Illustratively, in the state of Jharkhand, India, the Chief Minister recentlyset up a program called the "dal-bhat" (lentil-rice) where the aim is to provide a nutritional diet to the poor people at a cost of 9 US cents (The Times of India, 16th September, 2011). Note that thisprogram consists of two items of food (i) lentils, which contains proteins and minerals and (ii) rice, which contains carbohydrates. Therefore an optimal diet plan will necessarily require an optimal combination of these two food items.

If we denote the nutrient value of dietary intake as q, then we can formally write $q = \varphi(x_k, y_k)$ (i.e.q can be stated as a "production function" of the food items). We assume that the function φ(.) satisfies the Inada conditions i.e. (i) φ(0,yk) = φ(xk,0) = 0 and (ii) φ(xk,yk) is subject to diminishing returns (Inada, 1963). A dietician makes his decision in two stages: Stage 1: the dietician has to choose the set of least-cost combination of foods that satisfies the nutritional requirement; and Stage 2: the dietician has to search sequentially for the matched dietary intake within the optimal set of least-cost combination of foods as obtained in Stage 1. The following sections explain these two stages.

2.1. *Least-cost combination of foods and nutritional requirements*

A dietician has to choose the combination of dietary intakes that not only incur the least cost but also satisfy allnutritional requirements. Given the two classes of nutrients, the total possible combinations will be n^2. For example, let $S_1 := \{x_1, x_2\}$ and $S_2 := \{y_1, y_2\}$ so that each class of nutrient has two items of food i.e. $n = 2$. Therefore, the total number of possible combinations will be $2^2 = 4$ and these are $\{(x_1, y_1), (x_1, y_2), (x_2, y_1), (x_2, y_2)\}$. The problem with the dietician is to choose the least-cost combinations of food items that also satisfy the nutritional requirement;among the n^2 total number of possible combinations. If we assume that the per unit of prices

[3] $\mathbb{N}$ refers to set of natural numbers.

of x_k and y_k are P_{xk} and P_{yk} respectively then an dietician faces the following objective function:

$$\begin{aligned} &\min_{\{x_k,\, y_k\}} C = P_{x_k} x_k + P_{y_k} y \\ &s.t. \qquad x_k \geq 0\,;\, y_k \geq 0 \\ &\text{and} \qquad \phi(x_k, y_k) \geq r \end{aligned} \tag{1}$$

In the above problem, $\varphi(x_k, y_k) \geq r$ represents the nutritional constraint, where r is the minimum nutritional requirement. By multiplying objective function with –1 and using the *Lagrangian multiplier* λ, we obtain the following function, which needs to be maximized:

$$\max_{\{x_k,\, y_k\}} \mathcal{L} = P_{x_k} x_k - P_{y_k} y_k + \lambda[\phi(x_k, y_k) - r] \tag{2}$$

Using the *Kuhn-Tucker conditions*, one can observe that $\lambda > 0$, $x_k > 0$ and $y_k > 0$ (Kuhn and Tucker, 1951). The condition for an "interior solution" is as follows:[4]

$$\frac{\phi_{x_k}(x_k, y_k)}{P_{x_k}} = \frac{\phi_{y_k}(x_k, y_k)}{P_{y_k}} \tag{3}$$

With the help of (3), one can derive the optimal amount of x_k^* and y_k^*, which are as follows:

$$\begin{aligned} x_k^* &= f\left(r, \frac{P_{y_k}}{P_{x_k}}\right) \\ y_k^* &= f\left(r, \frac{P_{x_k}}{P_{y_k}}\right) \end{aligned} \tag{4}$$

Where, both x_k^* and y_k^*, are increasing with respect to r and cross prices, but decreasing with respect to their own prices. The dietician will then sequentially choose from the set of least-cost combination items that satisfies the nutritional requirement i.e. $S_i^* := \{\varphi(x_k^*, y_k^*)\}$, where $i = \{1, 2 \ldots t\}$. Note thatwe assume that $t = n^2$ without loss of generality.[5] The following figure graphically illustrates the least-cost

[4]This is a standard result; hence we have omitted the details of the calculation.

[5]While $t < n^2$ is possible, this will not qualitatively change our fundamental results.

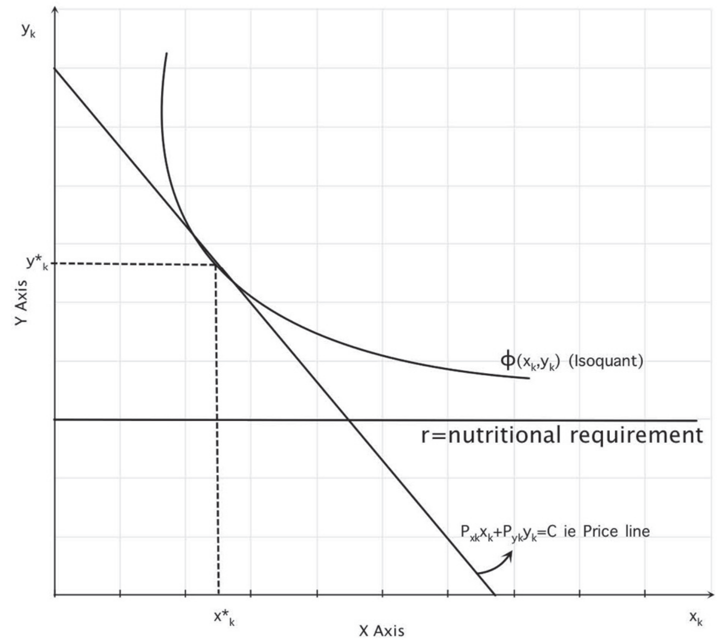

Figure 1

Least-cost combination of food items satisfying the nutritional requirements

combination of food items that satisfy the nutritional requirements as an isoquant.

2.2. *Sequential search and optimal stopping*

The set of least-cost combination of food items that satisfy the nutritional requirements have been already got in Stage 1 as follows:

$$S_i^* := \{\varphi(x_k^*, y_k^*)\}, \text{ where } i = \{1, 2 \ldots n^2\} \text{ and } k = \{1, 2 \ldots n\} \tag{5}$$

After obtaining relevant information from an individual patient, a dietician recommendsa diet combination $S_g \in S_i^*$, where $g \in i$. However, the dietician's recommendation is based on a sequential search i.e. he will recommend one combination of diet at a time, each of which are chosen from his least-cost combination diets i.e. S_i^*. The idea is that a dietician will try with the first combination of diet – if it fails then he will try the second one and so on. However, such sequential search process is based on the dietician's belief systemas is posited below.

2.2.1. Dietician's belief system (two-state)

We assume that each least-cost combination of nutritional diet has twostates: $\{s_1, s_2\}$ which is a random process i.e. each state has a certain probability to occur; and an individual patient belongs to either of two metabolic types: $\{t_1, t_2\}$. We shallcall it a matched dietary regimen if $\{s_i, t_i\}$ where $i = \{1, 2\}$, otherwise we call it a mismatch.This implies that a dietician has to find out the match through sequentialsearch, which is similar to a clinical trials process. Since the dietician does not know the state as well as the type of an individual patient, he forms the following belief: the probability that first state will occur is p and the probability that second statewill occur is (1-p) i.e. $\Pr\{s_1\} = p$ and $\Pr\{s_2\} = (1 - p)$

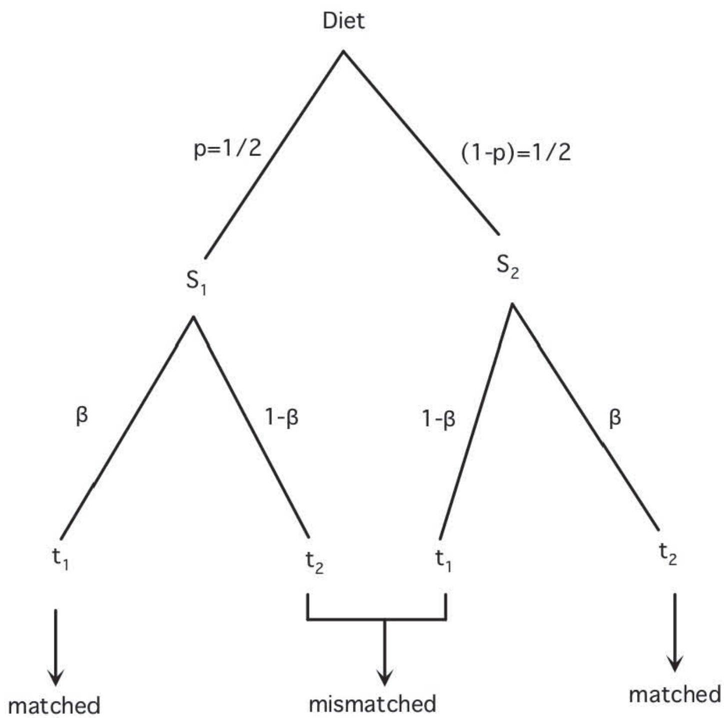

Figure 2

A schematic representation of the dietician's prior belief system (two-state)

respectively. For simplicity, we assume that $p = 1/2$. Furthermore, the patient will be matched with probability β i.e. $\Pr\{s_i, t_i\} = \beta$; hence the probability of mismatch will be $\Pr\{s_i, t_j\} = 1 - \beta$, where $i = j = \{1, 2\}$ and $i \neq j$. The following figureschematically represents thisprior belief system.

Based on the above probability tree i.e.prior belief system, an individual dietician forms his "reward function" R which is as follows: if the patient's type is matched with the state the reward will be y i.e. $R(s_i, t_i) = y$. However, if the patient's type is mismatched then the reward will be $R(s_i, t_j) = y - c$, where c is a *coefficient of confidence* of the dietician forgetting a match at the first go. If the *coefficient of confidence* is a small value then the dietician will compensate for that by formulating more number of diet plans as "back-up". On the other hand if the coefficient of confidence is a large value then the dietician will formulate fewer diet plans as he will be "more confident" of getting a match. Without any loss of generality or intuitive validity we make an assumption that $(y - c) \geq 0$.

Given the above prior beliefs, an individual dietician calculates the expected reward functions (for both matched and mismatched cases) from his posterior beliefs, which are as follows:

$$E(R(s_i, t_i)) = \frac{1}{2}\beta y \text{ (when it is matched)} \tag{6}$$

$$E(R(s_i, t_j)) = \frac{1}{2}(1 - \beta) \cdot (y - c) \text{ (when it is mismatched)} \tag{7}$$

This implies that irrespective of his posterior beliefs, the expected reward from the mismatch to the dietician will be always the same.

2.2.2. Deriving an optimal stopping rule for the dietician's search

An individual dietician, will *ex-ante* choose the optimal number of least-cost combination of nutritional diets i.e. n. Let us assume that Φ is the probability that a dietician will recommend the next best least-cost combination to hispatient when the previously chosen oneturns out to be a mismatch.Therefore $\Phi = 0$ if the diet which was chosen originally is a match, otherwise $\Phi = 1$. Since the dietician chooses the diet sequentially to find out the match, thisex-antesearch problem can be solved intuitively using adynamic programming approach as follows:

$$V^e_{l+2}(n) = \max_n \left\{R^e_t + \phi V^e_{t+1}(n)\right\} \tag{8}$$

Equation (8) indicates that the expected value of the objective function in $t+2$ period, i.e. $V^e_{t+2}(n)$, is equal to the maximum of the current expected value R^e_t plus the probability that it will go to next period where the expected value is $V^e_{t+2}(n)$. This implies that if the match occurs at period t then Φ will be zero and the maximum expected return *ex-ante* will be $\frac{1}{2}\beta y$. However, if there is no match in that case $\Phi = 1$ and $R^e_t = \frac{1}{2}(1-\beta)(b-c)$ i.e. the dietician has to choose a different least-cost combination of food items in the next period. In the next period, the remaining number of the least-cost combinations is $n^2 - 1$ and there are again two possible outcomes i.e. either a matchor a mismatch.Therefore, it is binomial process where the expected number of matches from choosing x_i samples is always obtainable as $\sum x_i f(x_i) = \sum x_i \binom{n^2-1}{x_i} \beta^{x_i}(1-\beta)^{(n^2-1)-x_i} = (n^2-1)\beta$. Therefore, viewed as a binomial process,the expected value function becomes $V^e_{t+1} = \frac{1}{2}(n^2-1)\beta y$.

The *ex-ante* dietician's reservation value of n (also the *optimal stopping rule* when a mismatch occurs in the very first period) will be as follows:

$$R^e_t = V^e_{t+1} \Rightarrow \frac{1}{2}(1-\beta)(y-c) = \frac{1}{2}(n^2-1)\beta y$$

$$\Rightarrow n = \sqrt{1 + \frac{(1-\beta)(y-c)}{\beta y}} \equiv n^* \tag{9}$$

Equation (9) tells us the optimal number of least-cost combination of nutritional diets that a dietician should formulate for each of his patients. As long as $n^* < n^2$,this will increase the dietician's operating efficiency by reducing the overall search space. Of course, if the probability of getting a match first time is one i.e. $\beta = 1$ then $n^* = 1$ as per intuitive logic.

3. Numerical analysis and conclusion

We present the result of our numerical analysis in a graphical form in the following figure where β is increased from 0.01 to 1 in step sizes of 0.01 on the horizontal axis and the corresponding values of n^* are plotted on the vertical axis for different levels of c which is varied as a percentage of y in our analysis from 0% to 100% in step sizes of 5%.

The maximum size of n^* (corresponding to $\beta = 0.01$) is obtained as 10 for $c = 0\%$ of y (i.e. for $c = 0$) and the minimum size of n^* is obtained as 1 for $c = 100\%$ of y. This implies that so long as the total number of

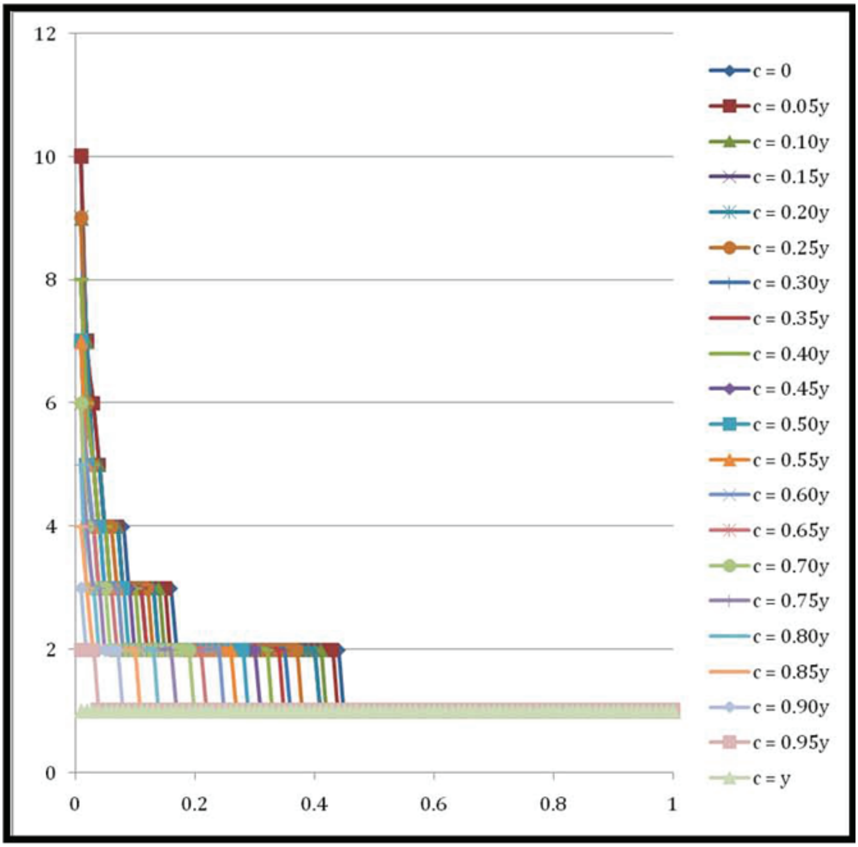

Figure 3

Graphical plot of numerical analysis result with c ranging from 0% to 100% of y

least-cost combinations of food items available to the dietician that can satisfy the nutritional requirements (obtained in Stage 1) is $t(= n^2) > 10$, then the dietician will only formulate a maximum of ten different diet plans if the probability of getting a match the very first time is close to zero (e.g. $\beta = 0.01$ in our analysis). However this will be the case when the coefficient of confidence of getting a match the first time is at its lowest (i.e. $c = 0$).If this *coefficient of confidence* is at its highest point, then n^* reduces to 1, which is what one will expect as per intuitive logic. We have posited an alternative methodological approach to the classical diet selection problem. Our approach is distinct from earlier ones in the literature as we have set it up as a two-stage process whereby the dietician determines all possible least-cost combinations of the food items that satisfy the required nutritional constraints in the first stage and subsequently in the second stage, formulates n^* optimal number of diet plans out of all the possible least-cost combinations. The benefit is that it better addresses the compliance problem by making it endogenous to the overall diet selection problem and also increases the operational efficiency of the dietician so long as $n^* < n^2$ by reducing the space within which the dietician has to execute a sequential search. The proposed model is limited to a *two-state version* and an interesting future research direction could be to extend this to more number of states for diets and also metabolic types so as to enhance applicability.

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