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FINITE ELEMENT MODEL UPDATING USING ESTIMATION OF DISTRIBUTION ALGORITHMS

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ABSTRACT

Finite Element (FE) model updating has been attracting research attentions in structural engineering fields for over 20 years. Its immense importance to the design, construction and maintenance of civil and mechanical structures has been highly recognised. However, many sources of uncertainties may affect the updating results. These uncertainties may be caused by FE modelling errors, measurement noises, signal processing techniques, and so on. Therefore, research efforts on model updating have been focusing on tackling with uncertainties for a long time. Recently, a new type of evolutionary algorithms has been developed to address uncertainty problems, known as Estimation of Distribution Algorithms (EDAs). EDAs are evolutionary algorithms based on estimation and sampling from probabilistic models and able to overcome some of the drawbacks exhibited by traditional genetic algorithms (GAs). In this paper, a numerical steel simple beam is constructed in commercial software ANSYS. The various damage scenarios are simulated and EDAs are employed to identify damages via FE model updating process. The results show that the performances of EDAs for model updating are efficient and reliable.

KEYWORDS

Finite element, model updating, estimation of distribution algorithms.

INTRODUCTION

In mathematics, finite element method (FEM) is a numerical technique for finding approximate solutions to boundary value problems. In civil/mechanical/aerospace engineering, FEM is a widely applied method for designing and assessing structural systems. However, due to simplified assumptions of structural geometry, materials, and boundary conditions, there is unavoidable discrepancy between the numerical results from FEM and the performances of real structures. In order to minimise this discrepancy, the constructed models usually need to be adjusted via model updating. Except as a powerful method which can calibrate the FEM of real structures, model updating can also be regarded as a damage identification scheme. In this framework, usually a two-stage model updating should be performed. The first stage is the FEM calibration process based on experimental results of the "intact" structure. In the second stage, the calibrated FEM is further updated by using the test data from the "damaged" structure. Usually, the updated parameter(s) can indicate both damage location and severity.

Due to its broad application range, model updating has been a research focus in various engineering fields over a few decades. There are plenty of techniques developed for model updating, which can be generally classified into two groups: direct matrix method and indirect physical property adjustment method (Ewins 2000). In the first group, the physical meaning of the updating parameters is ambiguous and the complete mode shape vectors are required, which makes them difficult to be applied. As a result, methods of the second group, known as parameter-based methods, are widely used in updating the parameters of large and complex structures. They can be considered as an iterative process with three main elements. First, the optimization variables should be selected and bounded, which are prescribed by the degrees of uncertainty that exist in the parameters. For a real structure, the total number of such variables as geometrical properties, material properties and boundary conditions is substantially high. In order to reduce the number of updated parameters, various methods have been developed, including substructure method, damage function method, the sensitivity analysis, and so on. Second, the objective function should be determined. Friswell and Mottershead (1995) recommended that the objective function should comprise the information of frequencies and/or mode shapes. The selection of the objective function has a profound influence on the problem solving. However, it highly depends on the measurement results. In current practices, noise inevitably exists in the measurement and the errors can be accumulated when computing a complex objective function. Therefore, simple objective functions, i.e. frequency changes, modal assurance criteria (MAC) and flexibility changes, are selected in this paper. Third, the optimization algorithm should be properly selected. After the establishment of the objective function, updating the model is usually via an optimisation process, known as an inverse problem. It is usually not straightforward, since the function is nonlinear and the matrices are ill-conditioned and underdetermined. Consequently, a suitable optimization algorithm can determine the efficiency and effectiveness of the model updating results. There are numerous algorithms developed for the optimization problem, including traditional ones and 'intelligent' algorithm-based approaches. Nowadays, 'intelligent' algorithms (or evolutionary algorithms, EAs) have received more research interests. Their main advantages are the convergence speed, the robustness, the need of little information about the parameter, and the tolerance of ambiguity of the numerical model. A diverse range of intelligent algorithms have been applied to model updating by many researchers, including simulated annealing algorithm (Levin and Lieven 1998), genetic algorithm (Hao and Xia 2002), neural network (Bakhary *et al.* 2007) and clonal selection algorithm (Ou and Wang 2007).

Aforementioned Classical EAs rely on the well-known two phases: selection and variation. They usually suffer from several problems. Among them, the most severe linkage problem occurs when the individual components of candidate solutions are not statistically independent of each other with regard to the objective function. For classical EAs, the variation phase cannot account for the relationships between components of solutions. This problem is traditionally solved by constructing special variation operators and by incorporating some problem-specific knowledge. Therefore, a classical EA becomes an algorithm highly specialized to the given problem, instead of a general solver. Recently, a new generation of EAs, Estimation of Distribution Algorithms (EDAs) has been developed to overcome this drawback. EDAs are population based optimization algorithms similar to genetic algorithms (GAs, Holland 1975) but which use the estimation of the distribution of selected individuals and sampling from this distribution, instead of crossover and mutation operators. The probabilistic models used by EDAs can represent a priori information about the problem structure, allowing a more efficient search of optimal solutions. New individuals are created by sampling from the probabilistic distribution. Due to the flexibility of choosing suitable probabilistic models as well as many other advantages, EDAs have received increasing attention from the optimization community, and have been applied to a variety of problems in fields such as engineering (Grosset *et al.* 2006), biomedical informatics (Larranaga *et al.* 2006), and robotics (Yuan *et al.* 2007).

In this paper, EDAs are applied to finite element model updating. First, the algorithm is introduced and employed to realize model updating process. Several objective functions, including frequency residual, flexibility residue, and their combination, are integrated into the program. Second, the program is validated by a numerical simple beam established on ANSYS. The model is used to generate various conditions including different damage numbers, damage locations and noise levels. The updating program is then employed to identify the healthy factors of all the elements. Finally, conclusions are to be drawn.

METHODOLOGY

In this section, FE model updating using EDAs is developed, as shown in Figure 1. The algorithm (EDAs) is introduced first and then the objective functions are chosen.

Estimation of Distribution Algorithms

There are a number of implementations of EDAs available, while the most practical one is the Mateda-2.0 (Santana et al 2010). This software not only provides many statistical models, but also allows users to incorporate their own methods. The flowchart of a standard EDA can be summarised in Table 1. As can be seen, its core idea is the probability distribution model, which replaces the mutation and search methods of the general EAs. Fortunately, a number of probabilistic models are available for Mateda-2.0, including Bayesian networks, Gaussian networks, Markov networks, and mixtures of distributions. In this study, the Gaussian Network model is used because of its easy implementation. The performances of other models may be discussed in the future.

Table 1. Pseudo code of a standard EDA

Step 0: Set up initial parameters, including N (population size), N_1 (size of the selected individuals, usually smaller than N), M (number of generation), and determine the selection method, probability model, and termination criteria
Step 1: Generate N random individuals as the initial population
Step 2: Select N_1<=N individuals from the first population, according to a selection method (objective function values)
Then, repeat steps 3-5 for generations i=1, 2, …, M, until the termination criteria met.
Step 3: Estimate the probability distribution model P(x) of an individual being among the selected individuals
Step 4: Sample N individuals (the new population) from P(x)
Step 5: Rank the current population according to the selection method and select N_1<=N individuals

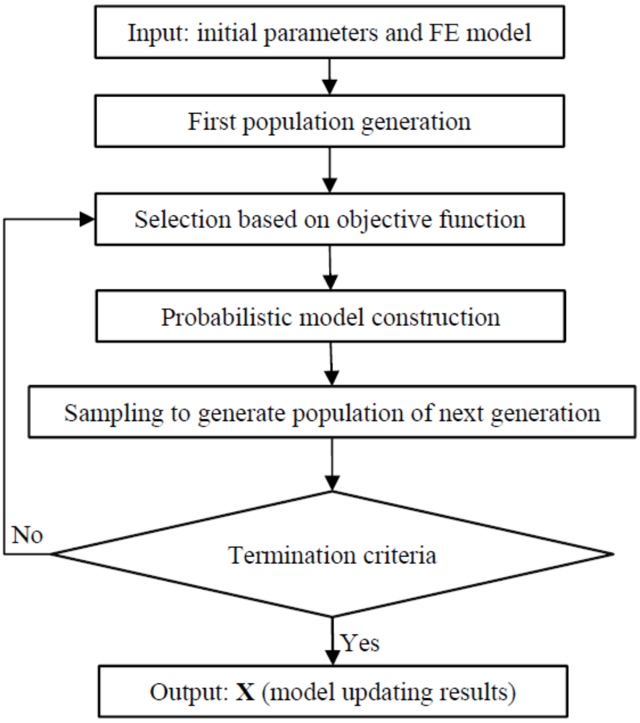

Figure 1. Flowchart of EDA based model updating

Objective Functions

The determination of the objective function has a profound effect on the final results. Based on former studies (Ou and Wang 2007; Wang *et al.* 2008), three different functions are selected, including the frequency residue, the flexibility residue and their combination.

$$J_1 = \sum_{i=1}^{m} \left(\frac{\lambda_{ai} - \lambda_{ei}}{\lambda_{ei}} \right)^2 \tag{1}$$

where λ_i represent the i^{th} natural frequency, a and e represent analytical and simulated experimental results respectively (here forth the same).

$$J_2 = \sum_{j=1}^{m} \sum_{k=1}^{m} (G_{jk})^2 \tag{2}$$

where G_{jk} is the element of the matrix G, which is equal to the difference between experimental and analytical flexibility matrices.

$$G = F_a - F_e \tag{3}$$

The flexibility matrix is defined as:

$$F = \sum_{i=1}^{n} \frac{1}{\omega_i^2} \phi_i \phi_i^T \tag{4}$$

$$J_3 = J_1 + wJ_2 \tag{5}$$

where w is the weighting parameter, which needs to be determined based on the initial results of J_1 and J_2 to make them comparable.

NUMERICAL STUDIES

The program is applied to a numerical simple beam (Ou and Wang 2007). The 6-meter long beam is discretized into 15 elements, as shown in Figure 2. The initial density, modulus of elasticity, the area and the second moment of area of the cross section of the beam are 2500 kg/m^3, 3.2×10^4 MPa, 0.05m^2 and 1.66×10^4 m^4, respectively. The FEM of the beam is established on the basis of ANSYS. Modal analysis is conducted in order to gain its first 10 natural frequency and mode shapes.

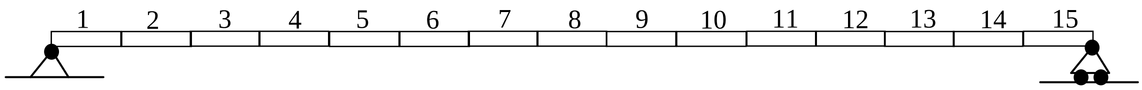

Figure 2. Plot of simple beam

The healthy factor **X**, a vector with 15 elements, is used as updating parameter. X_i is defined as the ratio between the updated and original stiffness values of the element i. When X_i >1, the updated stiffness of the i^{th} element is larger than its original stiffness, and vice versa. When X_i =1, the updated stiffness is equal to the original one. In order to allow large changes in these parameters, the upper and lower bounds of each parameter are 0.5 and 1.5, respectively. It should be noted that these bounds are just for choosing initial random parameters. In EDAs, it may adjust the bounds according to the results from the former generation.

Single Damage Case

First, the program with three different objective functions is applied to identify single imaginary damage with element 5, **X**=[1 1 1 1 0.81 1 1 1 1 1 1 1 1 1 1]. Based on initial computation results, the population and generation number are taken as 100 and 100, respectively, to obtain the balance of computational efficiency and reliable updating results. The updated healthy factors for each element by using three objective functions are shown in Figure 3 and listed in Table 2. The average difference between the updated factor and true value of each element is also shown in Table 2. Based on the updating results, the program with different objective functions reached a satisfactory level, with the maximum average difference 2.72%. The damage locations are identified by all the objective functions. However, the damage severity is underestimated as 0.856, 0.855 and 0.86 for J_1, J_2 and J_3, respectively. It can be seen that the identification results by using J_3 are not better than the other two functions.

Table 2. Numerical experiment results using differenct objective functions

Element number	True value	J_1	J_2	J_3
1	1	1.015	0.988	1.027
2	1	0.994	1.038	0.988
3	1	0.985	0.968	0.966
4	1	0.977	0.977	0.996
5	0.81	0.856	0.855	0.86
6	1	0.978	0.998	0.972
7	1	0.958	0.952	0.96
8	1	1.003	1.016	1.002
9	1	1.047	1.03	1.046
10	1	1.008	0.992	1.014
11	1	0.94	0.989	0.936
12	1	1.022	1.019	1.007
13	1	1.017	0.997	1.031
14	1	1.007	0.969	1.016
15	1	0.99	1.021	0.978
Average difference		2.36%	2.32%	2.72%

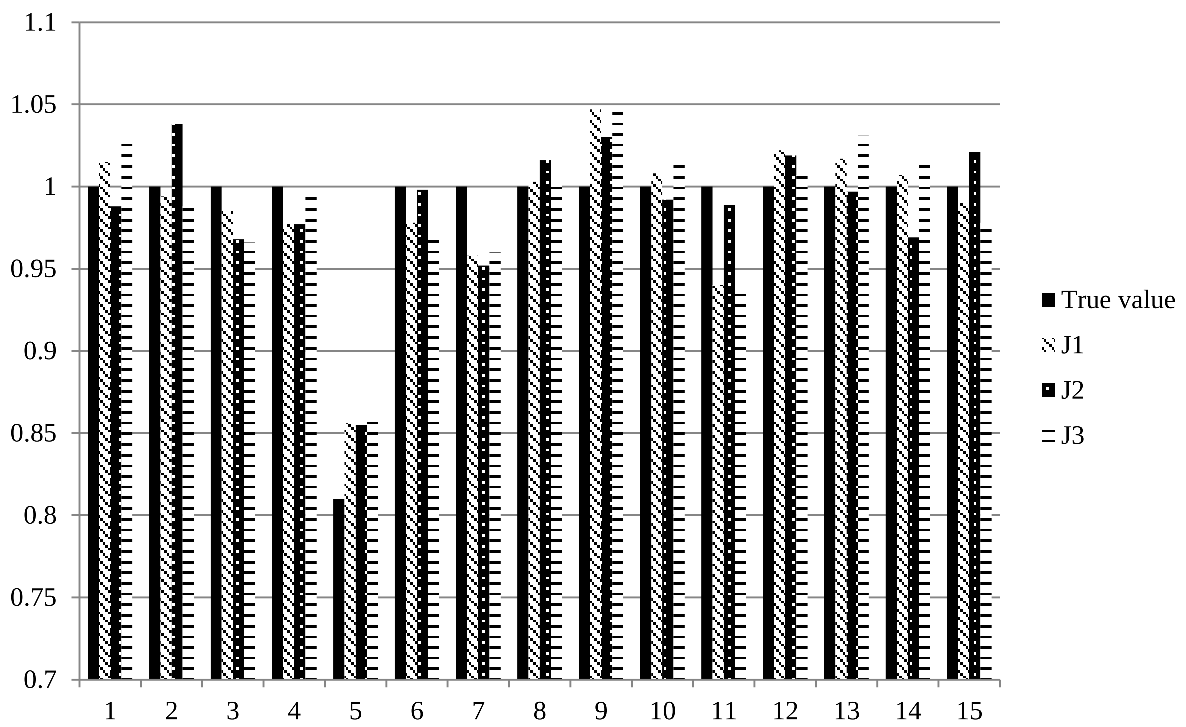

Figure 3. Performances of EDA based model updating program

Then, the program is applied to identify damage based on noise corrupted data. The updated healthy factors for each element by using three objective functions are shown in Figure 4 and listed in Table 3. Based on the updating results, the program with J_2 reached an acceptable level, with the average difference 2.94%. The correct damage location is identified while the damage severity is largely underestimated as 0.887 compared to true value 0.81. The program with J_1 and J_3 failed to identify the damage.

Table 3. Numerical experiment results under 1% noise

Element number	True value	J_1	J_2	J_3
1	1	0.973	0.994	0.972
2	1	0.935	1.054	0.936
3	1	1.035	0.967	1.026
4	1	0.878	0.991	0.911
5	0.81	0.949	0.887	0.907
6	1	1.022	1.006	1.025
7	1	1.009	0.963	1.033
8	1	1.022	1.041	1.025
9	1	1.014	1.027	0.987
10	1	0.979	1.034	0.991
11	1	0.852	0.99	0.879
12	1	0.979	0.961	0.932
13	1	1.012	1.046	1.034
14	1	1.098	1.012	1.065
15	1	1.161	1.004	1.264
Average difference		6.32%	2.94%	6.56%

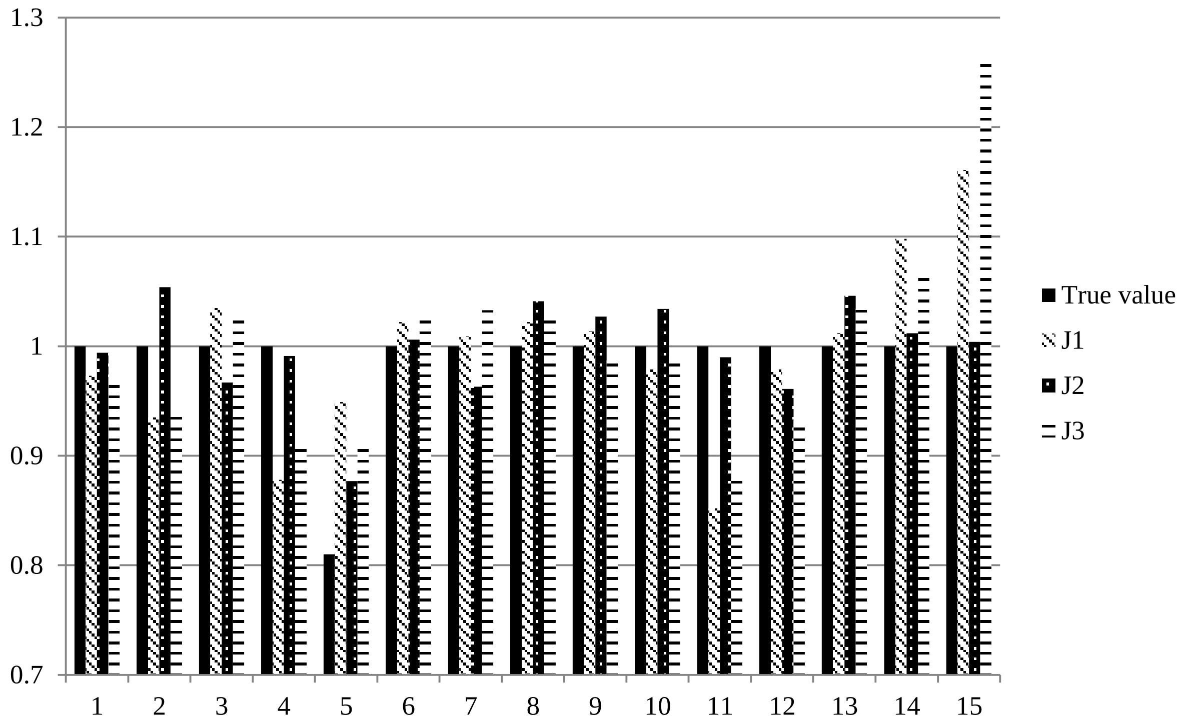

Figure 4. Performances of EDA based model updating program

Multiple Damage Case

At last, the program with three objective functions is applied to identify two imaginary damages with element 2 and element 8, **X**=[1 0.912 1 1 1 1 1 0.718 1 1 1 1 1 1 1]. The updated healthy factors for each element by using three objective functions are shown in Figure 5 and listed in Table 4. Based on the updating results, the program with different objective functions reached an acceptable level, with the maximum average difference 3.99%. The damage locations are identified by all the objective functions. For J_1, the larger damage (0.718) at element 8 is accurately identified as 0.71, while the smaller damage (0.912) at element 2 is identified as 0.955. For J_2, the damages are identified as 0.746 and 0.908, respectively, indicating that the smaller damage is identified more accurately than the larger one by using J_2. For J_3, both damage severities are overestimated (0.708 and 0.893).

Table 4. Numerical experiment results using differenct objective functions

Element number	True value	J_1	J_2	J_3
1	1	0.971	1.005	1.019
2	0.912	0.955	0.908	0.893
3	1	0.951	0.983	0.995
4	1	1.023	0.976	1.009
5	1	0.969	1.042	1
6	1	1.066	0.997	0.992
7	1	0.955	0.96	0.999
8	0.718	0.71	0.746	0.708
9	1	1.096	0.991	1.015
10	1	0.926	1.021	1.036
11	1	1.038	0.994	0.974
12	1	1.001	0.991	1.021
13	1	1.018	1.016	1.008
14	1	0.973	0.97	0.948
15	1	1.043	1.008	1.038
Average difference		3.99%	1.82%	1.82%

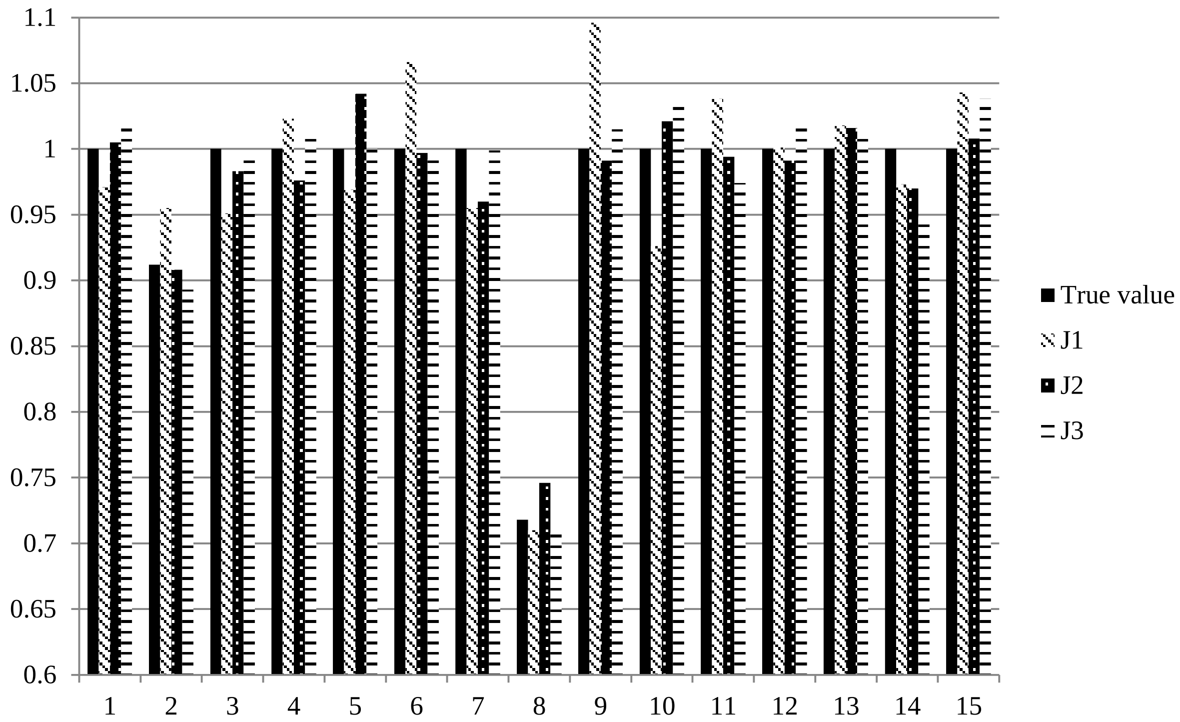

Figure 5. Performances of EDA based model updating program

Discussions

Based on above results, the updating performance of the flexibility residue is reliable under various conditions. However, the simple combination of different objective functions may not yield satisfactory results.

In the second stage of model updating, since most elements have been calibrated in the first stage, usually only a few damaged elements should be updated, while others should remain their calibrated values. The inherent sparse information can be exploited, which has recently been studied by Wang and Hao (2013). Further development on model updating by using sparse information will be explored in future works.

CONCLUSIONS

In this paper, a new optimisation algorithm, EDA, is introduced to finite element model updating process. The model updating program with three objective functions are developed, defined in terms of frequency residue, flexibility residue, and their combination. Numerical studies on damage identification of a simple beam under different damage scenarios are conducted. It is found that the flexibility residue provides the best identification results even when the frequencies are smeared with noises or when there are multiple damages. Based on the results, it can be concluded that EDA is a promising tool for such complex optimization problems as model updating.

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