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A Case Study on Developing a Teacher's Capacity in Mathematical Modelling

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Abstract: Research has indicated that for teachers to facilitate mathematical modelling activities in the mathematics classroom, they need to be familiar with the process of mathematical modelling. As such, it is imperative that teachers experience the whole mathematical modelling process. This paper reports on a Multi-tiered Teaching Experiment designed to help a teacher develop his capacity in the domain of mathematical modelling. Drawing on part of a larger case-based study conducted using Design Research phases situated within the Multi-tiered Teaching Experiment framework, the purpose of the paper is to exemplify how the research design fostered growth in teacher capacity through the natural development of critical moments of learning by the teacher during interactions between the researchers and the teacher-modeller himself. The potentials of the Multi-tiered Teaching Experiment as a useful non-prescriptive teacher development approach building upon the existing repertoire of individual teachers will be discussed.

Keywords: mathematical modelling, multi-tiered teaching experiment framework, teacher development

Introduction

Mathematical modelling in Singapore curriculum

Mathematical modelling was introduced as a process component in the Singapore Mathematics Curriculum Framework in 2007 (MOE, 2007). Its introduction paves the way in keeping mathematics education relevant to the changing educational landscape where mathematical modelling has been deemed as one of the most significant goals of mathematics education (see Lesh & Sriraman, 2005; MOE, 2012) and a coveted direction in mathematical problem solving research (Lesh & Zawojewski, 2007). Figure 1 depicts a generic representation of the mathematical modelling process. Mathematical modelling begins with a real-world problem or situation. The engagement process results in the representation of such problems as a mathematical model - a simplification or abstraction of the real world problem or situation. The mathematical modelling process involves four elements, namely, *Formulate*, *Solve*, *Interpret* and *Reflect* (MOE, 2012). In the *Formulating* phase learners have to understand the problem and make assumptions that lead to attempts at representing the problem mathematically. In the Solving phase, learners use appropriate methods to solve and then present the solution. *Interpreting*, learners relate their solution as a simplified model of the real-world situation. By *Reflecting*, learners determine if their model reflects reality and represent the real-world situation and thereby accept and report the model or revise the model further. The cyclical nature suggests that initial models that are constructed need to be tested and revised to improve the model.

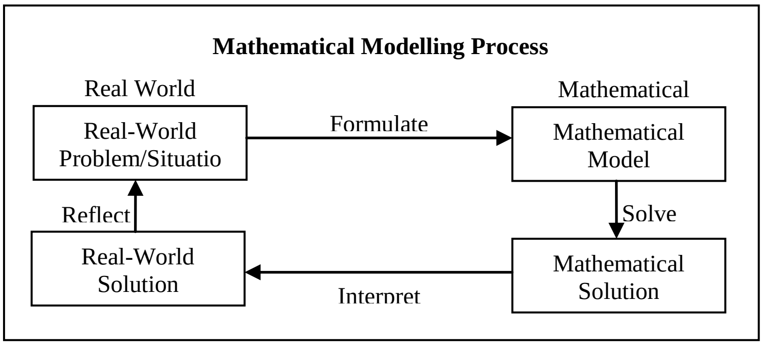

Figure 1. Mathematical modelling process

The benefits of mathematical modelling have been outlined in the Singapore mathematics syllabus since 2007 citing mathematical modelling as crucial in connecting students' learning of mathematics to the real world. Mathematical modelling has been perceived as one of the ways to develop students' mathematical competencies and reasoning through solving real world open-ended problems (MOE, 2007). Moreover, mathematical modelling is also seen as a means to develop students' mathematical literacy (MOE, 2012).

Developing teachers' capacity in mathematical modelling in Singapore schools

Since 2009 there have been concerted efforts to develop teacher readiness to incorporate mathematical modelling in primary, secondary, and pre-universities programmes (see Lee & Ng, 2015). However, interest among schools has been slow primarily because of greater focus in high stakes assessment where mathematical modelling is notably absent (Ang, 2013; Ng, 2013) as well as the lack of understanding about mathematical modelling (Ang, 2010).

Singapore research on mathematical modelling is limited. The current focus of research is predominantly on student learning outcomes (Ang, 2010, 2013; Chan, 2008, 2009, 2010, 2013; Chan, Ng, Widjaja, & Seto, 2012) and teachers' development of skills to teach mathematical modelling (Ang, 2013; Lee, 2013; Ng, 2013; Ng, Widjaja, Chan, & Seto, 2012; Ng, Chan, Widjaja, & Seto, 2013). The few pioneer Singapore research studies into mathematical modelling echoed Kaiser (2006) in emphasising the importance of teachers undergoing more in-depth professional development to adequately and meaningfully carry out mathematical modelling activities with the students. Although Ang (2013) advocates a preliminary professional development framework to help secondary school teachers advance their pedagogical content knowledge in mathematical modelling, it still remains a challenge for researchers and educators to devise effective development programmes tailoring to the different level of needs of Singapore teachers. This includes building teachers' capacities in implementing and designing appropriate modelling activities, as well as evaluating students' learning from these tasks.

This paper reports part of a larger case-based study for which the main purpose is to exemplify how the research design fostered growth in teacher capacity through the natural development of critical moments of learning by the teacher during interactions between the researchers and the teacher-modeller himself. This paper argues that the Multi-tiered Teaching Experiment can be a useful non-prescriptive teacher development approach building upon the existing repertoire of individual teachers, giving voice to teacher knowledge in research collaboration.

Theoretical Perspectives

Models-and-Modelling perspective

Mathematical modelling is widely understood as the use of mathematics to describe, represent, and solve problems that arise in real-world situations. Although there are various perspectives of mathematical modelling, this study specifically adopts a Models-and-Modelling Perspective (Lesh & Doerr, 2003) based on the use of a Model-Eliciting Activity (MEA). Model-eliciting activities are designed to be simulations of meaningful real-life problem solving situations and are meant to be thought-revealing, that is, through the process of working on such tasks, important aspects about the mathematical objects, relations, operations and patterns embedded in the modellers' underlying ways of thinking are revealed (Lesh, Yoon, & Zawojewski, 2007). Modellers are usually engaged in multiple cycles of expressing, testing and refining their models, where the initial emerging models are usually naive and unsophisticated but they become the tool for the modeller's assessment and revision towards a better model or solution (Chan, 2010; Lesh & Doerr, 2003). In this regard, the modelling process plays a central role in assisting modellers to move along a continuum of developing models in form and function where students develop a "model of" and a "model for" the modelling context presented to provide a way to better understand the problem situation (van den Heuval-Panhuizhen, 2003).

Challenges faced by Singapore teachers in mathematical modelling instruction

Currently, there is a lack of professional development opportunities to equip teachers in the knowledge and competencies required to carry out mathematical modelling lessons. In a move to create awareness of the potential of mathematical modelling among teachers, 29 local schools and two schools from Australia and Indonesia participated in a three-day mathematical modelling event entitled the Mathematical Modelling Outreach (MMO) organized by the National Institute of Education in Singapore in 2010 (see Lee & Ng, 2015). MMO ran parallel sessions for student-participants working with pre-service teacher-facilitators trained in mathematical modelling and teacher-participants attending workshops and seminars conducted by local and overseas modelling experts. The teacher-participants articulated their perceived challenges to incorporating mathematical modelling in their classrooms such as designing an appropriate modelling task, facilitating towards mathematisation in model development and refinement, building confidence in students' handling open-ended real world problems, and balancing the demands of the syllabus with the use of open-ended real world problems in view of limited curriculum time (Chan, 2013; Lee, 2013). Besides these challenges outlined above, Ng (2010) also reported that primary school teachers showed discomfort with the openness of model-eliciting tasks. The findings of the few local studies cited here concurred with research done internationally. Blum and Niss (1991), in particular, highlighted that teachers found modelling instruction to be very complex and demanded other forms of teachers' knowledge in modelling instruction. de Oliveira and Barbosa (2013) classified the challenges of teachers' modelling experiences into three main tension areas: (i) deciding what to do (when the teacher becomes undecided about the different directions where the lesson is heading), (ii) students' involvement – when the teacher expects the students to be involved but they become indifferent and disengaged, and (iii) students' domination of mathematical content – a case when the teacher expects the students to know the mathematical content but the students show otherwise. Widjaja (2013) who provided opportunities for pre-service secondary school teachers in Indonesia to experience being modellers found these pre-service teachers themselves lack knowledge in stating assumptions and real-world considerations for making links to the mathematical model towards validation of the appropriateness of the model.

Building teachers' capacity in mathematical modelling

In the light of the issues and challenges faced by teachers such as beliefs, tensions and facilitation of mathematical modelling, researchers have attempted to provide some broad frameworks and guiding principles as a means to help develop teachers' capacity in implementing and designing modelling activities as well as evaluating student outcomes from these activities. For example, to address pedagogical dilemmas of teacher tensions, Blomhøj and Kjeldsen (2006) highlighted three aspects to be addressed, namely, to help teachers in understanding (i) the phases in the process of modelling; (ii) the goal of the modelling activity, motivation or mathematics teaching, and (iii) how to develop autonomy in students during the modelling. In learning to design modelling tasks, Galbraith, Stillman, and Brown (2010) and Lesh et al. (2003) have provided different expressions of principles as guides to ensure there would be elements of modelling that would take place when the tasks are engaged.

Lesh and Lehrer (2003) argued from a Models-and-Modelling Perspective that developing a teacher's capacity would involve on-the-job classroom-based professional development activities where the teachers' teaching experiences become productive professional learning experiences. In the professional development they encouraged teachers to interpret situations in the context of their actual practice, that is, teachers need to do some modelling themselves before they are ready to engage the class. The teachers thus play the role of a modeller and interpret the modelling process as an insider within the realm of their existing contextual knowledge so that they can build a more robust conceptual model or mental framework of what mathematical modelling involves. This experience will facilitate various reflections, modifications and revisions of the teachers' own conceptual models and tools which over time will better prepare them to anticipate the ways in which students may mathematise the real-world problems. Schorr and Koellner-Clark (2003) argued for a multi-tiered program design involving researchers, teachers and students that would provide the means by which teachers can examine and reflect on their students' modelling behaviour through naturalistic interactions between the participants of each tier. These studies provided the impetuses for the chosen research design reported here. This paper will showcase how the researchers (Tier 3) interacted with the teacher (Tier 2) to foster the teacher's own conceptual model of mathematical modelling. The paper will also present how various forms of data during the Guide-and-Support Modelling (GSM) session can elicit the teacher's critical points of learning so as to argue for the Multi-Tiered Teaching Experiment as a useful teacher development programme for incorporating mathematical modelling in Singapore primary mathematics classrooms.

Research Design

To develop teachers' capacity in mathematical modelling, we adapted the Multi-tiered Teaching Experiment (Lesh & Kelly, 2000) as our research design as it promotes collaboration between three tiers; researchers, teachers and students. We also embraced an adapted version of design research methodology (Dolk, Widjaja, Zonneveld, & Fauzan, 2010) to support the execution of the research and analysis of data. Our aim in this paper is to exemplify how the Multi-tiered Teaching Experiment supported the teacher's growing capacity and discuss its usefulness as a non-prescriptive teacher development approach.

Multi-tiered Teaching Experiment

The Multi-tiered Teaching Experiment considers the development of all participants involved in the research. Adaptive in nature, the aim is to create conditions that enhance the chances that development will occur without dictating the directions for (a) developing new conceptions of participants' (students, teachers, researchers) experiences, (b) structuring interactions to test and refine constructs, (c) providing tools that facilitate the construction of relevant models, and (d) using formative feedback and consensus building to ensure the constructs develop in productive directions (Lesh & Kelly, 2000). The main principle underlying this framework is to seek corroboration through triangulation where modelling experiences of the teacher and students and data collected are used for analysis and discussion with the researchers. In this regard, all participants or learners worked interdependently with "each of them engaged in a common goal of trying to make sense of, and learn from, their respective experiences" (English, 2003, p. 227).

Figure 2 shows the three tiers of the Multi-tiered Teaching Experiment framework where the researchers, teachers, and students are engaged differently in their own form of learning, but all of them are involved in making sense of their experiences by developing their own models (mathematical or conceptual) that are used to generate descriptions, explanations, constructions, and justifications using a variety of representational systems.

Tier 3 - Researchers	* Development of conceptual framework (model) to develop teachers' knowledge and capacity in facilitating modelling tasks in two cycles. This involved creating learning situations for teachers and students through describing, explaining, predicting teachers' and students' behaviours. * Researchers collaborate with teachers to test and review modelling activity. * Researchers reflect on their own evolving knowledge of the participants' learning experiences for the development of tools to scaffold teachers.	Data types: * Video and audio transcript on teacher-learning * Written artefacts of teacher's solutions
Tier 2 - Teachers	* Teachers collaborate with researchers to test and review modelling activity. * Teachers review feedback for designing own modelling tasks. * Teachers reflect on their own evolving knowledge of the students' learning experiences for the development of tools to scaffold their learning.	Data types: * Video and audio transcript on teacher-reflection * Written artefacts of teacher's solution
Tier 1 - Students	* Students engage in model-eliciting tasks in small groups where they will be involved in constructing and refining models that reveal their interpretation of the problem situation. They will describe, represent, explain, justify and document their mathematical constructions.	Data types: * Video and audio transcript on student-learning * Written artefacts of students' solutions

Figure 2. A three-tiered teaching experiment
(adapted and modified from Lesh & Kelly, 2000, p198)

The teacher participant of the research study worked with the researchers on two cycles of implementation of mathematical modelling tasks. In view of the focus of this paper, we report how Tiers 2 and 3 of the multi-tiered teaching experiment were enacted during Cycle 1 at the Knowledge Phase of design research (see section below). Firstly, a model-eliciting task designed by the researchers (Tier 3) was implemented with the teacher (Tier 2) who took on the role of a modeller working with another teacher-modeller during a video-recorded Guide-and-Support Modelling (GSM) session facilitated by one of the researchers. As a platform for interactions between participants in Tiers 2 and 3, the GSM session also functioned as an immersion programme that specifically required the teacher-modeller to experience the entire process of mathematical modelling and develop his own mathematical models as he engaged with the model-eliciting task. The written solutions of the teacher-modeller were collected thereafter. Prior to the GSM session, a pre-interview was conducted to find out about the teacher's beliefs on mathematics teaching and learning as well as certain pedagogical practices associated with open-ended real-world tasks. Researchers also sought a more in-depth understanding of how these beliefs were established and encouraged the teacher to become aware of his beliefs as well as how these beliefs could translate to his perception of the mathematical learning which could take place with his students during mathematical modelling. A review or reaffirmation of these beliefs in aspects related to his changing worldview of teaching and learning may result as the teacher interacts with the researchers.

Secondly, at the end of the GSM session, both the researchers (Tier 3) and teacher (Tier 2) collectively reflected on the outcomes of the session in several areas by way of a post-interview. For the researchers, the impact of the GSM session on the teacher towards fostering a meaningful experience of the entire modelling process was critically analysed based on the affordances and limitations created by the session. Inferences were then made with respect to the new knowledge acquired by the teacher-modeller concerning mathematical modelling. Reflections of the researchers would transit into subsequent more fluid approaches in working with the teacher which were adapted to the beliefs, working style, pre-requisites, and background of the teacher. The interview after the GSM session offered the teacher some opportunities to reflect upon the potentials offered by modelling tasks for his students and for himself in view of possible extension of his facilitation repertoire in problem solving tasks. Together, the collective reflections would become the knowledge input in the next cycle of design for building conceptual models for collective efforts towards enhancing the mathematical modelling experience for teachers and students (Tiers 1 and 2).

Thirdly, the subsequent revision and implementation of the model-eliciting activity by the teacher-modeller paves the way for him to function as a facilitator in conducting the modelling activity with his students (Tier 1). The focus of this paper did not involve Tier 1.

Design Research methodology

In this study, the design research methodology (Dolk, Widjaja, Zonneveld, & Fauzan, 2010) was embraced within the Multi-tiered Teaching Experiment framework to guide the analysis and interpretation of data. One key aspect of design research is its focus on the retrospective analysis that sees researchers and teachers working together to produce meaningful change in the context of classroom practice and instruction (Design-Based Research Collective, 2003). The process of the interaction involves cycles of phases comprising Knowledge (K), Design (D), Experiment (E), and Retrospective Analysis (R) as illustrated in Figure 3. The Knowledge Phase of design research (the circled K) will be exemplified in the findings.

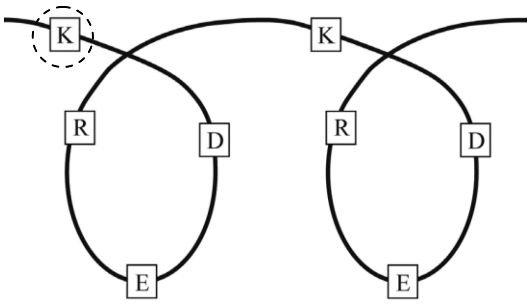

Figure 3. The cyclical process of knowledge, designing, experimenting, and retrospective analysis (Dolk et al., 2010, p.175)

The model-eliciting task

The model-eliciting task entitled “Staircase” (see Appendix) was designed by the research team who are authors of this paper to support he teacher-modeller in professional development journey. Insights from the teacher-modeller were used to refine and adapt the task to be used in his class.

Modelling design principles of Lesh et al. (2003) were adopted meeting the following criteria: (i) the task warranted sense-making and extension of prior knowledge (reality principle), (ii) the situation created the need to develop (or refine, modify or extend) a mathematically significant construct (model construction principle), (iii) the situation required self-assessment (self-evaluation principle), (iv) the situation required modellers to reveal their thinking about the situation (construct documentation principle), (v) the elicited model would be generalisable to other similar situations (construct generalisable principle) and, (vi) the problem-solving situation would be simple to carry out (the simplicity principle).

Participants

The main teacher participant was a senior teacher who had expressed interest to participate in the research for the purpose of advancing his knowledge in mathematical modelling. In this paper, he will be called James. James had some knowledge on problem-based learning (PBL), that is, experiential learning organized around the investigation of messy, real-world problems (Torp & Sage, 2002). He had implemented such tasks before although those tasks were not mathematics-specific in the content. A pre-interview was conducted with him to find out about his conceptions of working on complex mathematics task before the GSM session and a post-interview was conducted to find out about his perceptions about mathematical modelling after the GSM session.

Four participants in all worked in pairs during the GSM session facilitated by one of the researchers. James was paired with another researcher who played the role of a questioner but was not involved in the mathematising process. This was to help promote a richer discourse and would serve to develop the teacher-modeller’s capacity with the intent of having the teacher-modeller reveal his thoughts, answer questions and justify his actions. The other two participants were James’s colleagues (who were mathematics teachers) who wanted to find out more about mathematical modelling. They were not involved as target participants in the research.

Data collection and analysis

Video recording of James's modelling endeavour as well as his written solutions were collected. Pre-and-post interviews were video and audio recorded. The corpus of data collected in each tier of the Multi-tiered

Teaching Experiment has been presented earlier in Figure 2. The video and audio recordings were transcribed and James' solutions were analysed in terms of the elements of the modelling process. The descriptions of the modelling actions used in this study are adapted from the Mathematical Modelling Resource Kit (MOE, 2012) - see Figure 4. The models would be the mathematical relationships that James established between the key variables identified and presented to interpret the real-world situation.

Elements	Modelling Actions
Formulate	* Seeks to understand the problem * Makes assumptions * Identifies mathematical variables
Solve	* Uses appropriate methods/heuristics * Establishes relationship between mathematical variables * Works out a mathematical solution
Interpret	* Translates the results * Presents the initial/emerging model/solution
Reflect	* Tests the solution against the real-world context * Justifies the model/solution * Reviews and revises the model/solution if improvement is needed

Figure 4. Modelling actions of the modelling process

Findings

Findings comprising interview vignettes of James' pre-and-post perceptions of mathematical modelling as well as identified episodes of James' critical learning associated with the stages (Figure 1) and elements (Figure 4) of the modelling process are presented below. These episodes were prompted by James' interactions with the task as well as his partner. Analyses of video transcriptions mapped James' modelling actions with the corresponding elements of the modelling process. Three modelling attempts which resulted in three mathematical models were identified. James' critical moments of learning were evident through his subsequent modelling attempts that presented more robust and sophisticated mathematical models as well as his acquired knowledge about mathematical modelling based on the post-interview.

Pre-interview

James was asked about his beliefs on the teaching and learning of mathematics, experience with working on less-structured tasks and his perception of mathematical modelling (he had not experienced mathematical modelling before). Inferences concerning James' knowledge based on the above aspects are shown in the vignettes below.

Interview Aspect	Vignette	Inference
Beliefs on teaching and learning mathematics	*"...what we only want to do in the maths class is give them the use of mathematics, really practical use of mathematics in terms of why we use, why we learn certain topics, why certain topics are useful, how is it related to the real life situation, yah. So the authenticity of the task given must be there."*	* mathematics is a useful/practical subject * related to real life situations * mathematical task should be authentic
Experience in working with less-structured tasks	*"... there will be a problem, once they solve the problem, then I add on with a complication, yah...they design a living space in terms of for the family. So they are given, let's say a square, a house with a certain dimension that is odd shape. Irregular. So it's not regular, yah. And then they need to then make use of the space and tell me how they make use of the space and what are they going to put into the space. Then after that, then the complication comes maybe like somewhere in maybe in July. Maybe in September, August. Then there was a flood and flood in. And then how are they going to change the layout so that the house actually has got lesser damage. Yah. Or actually how they can also draw out the water from the house itself."*	* experience with problem-base learning * injects complications into task for pupils to solve
Perception on mathematical modelling	*"I think it's like a real world situation in that sense. An authentic situation where actually the pupils need to come out with reasons and assumptions to actually solve that task. So it's a problem. And then the*	* has an awareness of mathematical modelling through reading a related article

	children need to actually use their prior knowledge to actually solve the problem. Yah and then explain the problem with of course the right reasoning adequately." *"I actually read it somewhere. I did go to – yah, yah, I read a Korean article but yah it's actually mathematical models are used to interpret real world situations in mathematical formats."*	

James had the belief that mathematics teaching and learning should be related to real-life situations as mathematics is a useful subject. Mathematics tasks should be made authentic. James had facilitated problem-based learning (PBL) before and viewed PBL as injecting more complicated scenarios along the way during the problem solving. James had read up an article about mathematical modelling and had some awareness of what mathematical modelling was.

Mathematical model development

The pre-interview, enacted as part of the Multi-tiered Teaching Experiment, enabled the researchers to mediate expectations between Tiers 3 and 2 during the GSM session. As James had some experience working on less-structured tasks as well as some awareness of mathematical modelling, the GSM session saw James working quite independently in solving the modelling task and with the researchers interjecting to paraphrase what he had thought-aloud and asking questions to clarify and provoke his thinking further. Findings revealed that James developed three models during the modelling process. The three models are seen through three attempts from start to completion.

Attempt 1 – Trial-and-Error Model

Figure 5 shows a mathematical model developed through trial-and-error during the first modelling attempt. The vignettes below summarised the critical learning episodes.

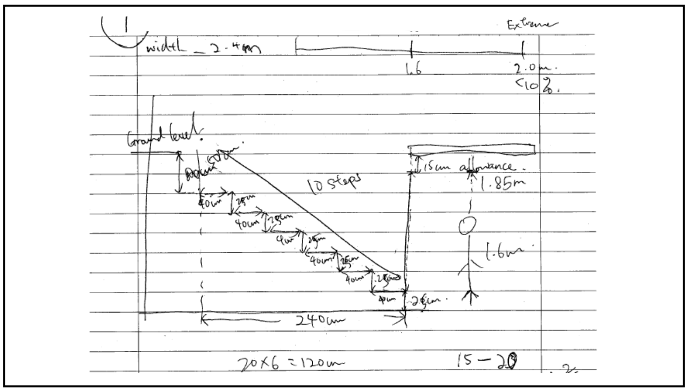

Figure 5. Model 1

Vignettes	Elements
* Established the mathematical variables to be step-space and step-height. * Assumed step-space to be 40 cm (based on his foot size) and step-height to be 20 cm (out of convenience as an initial trail). * Assumed extreme height of a person to be 1.85 m tall to allow for 15 cm gap to meet condition of not hitting the partial ceiling.	Formulate
* Drew a flight of steps based on the step-height as 20 cm and step-space as 40 cm. * Tried to relate dimensions of step-height and step-space to fit condition.	Solve
* Tested mathematical variables relationship against problem situation and conditions. * Found that the flight of stairs based on the dimensions he had used would not exist - "*So it will be 120. So it doesn't reach. So there's a need to increase this height. Because here if this were to last that height, there will still be a gap here… And so this will be a …80 cm you know*".	Interpret
* Reviewed the dimensions for step-space and step-height - "*No. So I will now have to adjust the 20. It cannot be 20 anymore. Yah, so it has to go higher. Okay so it has to go higher*"	Reflect

It is inferred that this model was developed through trial-and-error because the step-height of 20 cm and step-space of 40 cm were the initial values assumed by James without much reflection about the reasonableness of the choice of values. While the model may be drawn, its existence comes into question because the dimensions would not render such a staircase to exist. At that point, James had not realised a relationship between the number of steps and the number of step-space had to exist in order for the design of the stairs to be plausible within the given conditions of the problem (see Appendix). When he tested those dimensions as part of the interpretation process, he realised there "*was a gap*" that rendered the model unrealistic. James was mindful to revise his assumption by trying to work with a different pair of measurements for step-height and step-space, "*Yes I will have to change that assumption. But then I was thinking at it. Even if I change that assumption, so the step will be too high. I cannot increase until 30 cm. That's so high*." We inferred that in the first attempt of his model development, James managed to construct an initial model but learned that the model could not exist and this realization was only possible when he tested the dimensions against the problem situation.

Attempt 2 – Model depicting relationship between key variables

In revising his model from Attempt 1, some calculations were done without drawing the flight of stairs. The development of this second model was conceptualised based on the vignettes described below.

Vignettes	Elements
* Studied the problem context again to note the conditions once more. * Made a more realistic range in the dimensions of the step-space based on shoe-size by virtue of what would be a "comfortable" steps-space. He measured his shoe size and found it to be 30.24 cm. * Provided a more realistic range for the step-height with respect to climbing the stairs comfortably - "*I feel it has to be between 15 to 23 maximum I think to be comfortable*", "*20, I think still okay… 20 you lift up a foot is not too bad. If you elevate it further, it will be difficult"*.	Formulate
* Revised step-height to be 20 cm and step-space to be 24 cm. * Found the number of steps to be 10. He worked out the number of step-space to be 24 – "*So if I need 10 steps, then this one I need to …240 to 24. Yah"*.	Solve
* Questioned if 24 cm would be a reasonable step-space and he reasoned that it might not be since that would be 6 cm short of his foot-size of about 30 cm - "*But 24, is it a bit awkward for people?...*	Interpret

No, it's just 6 cm ah. 6 cm, not so much". * Tested to find if the revised dimensions would fit the problem situation and conditions.	
* Reviewed the dimensions for step-space and step-height. * Realised that the number of step-height and step-space need not be equal – "*So this is ground. So this is one. If it's three steps height, it will be 2 height-space only… So I only need 9. I only need 9… If let's say I have 10 steps, I only need 9 steps, 9 steps space".*	Reflect

The second model was a significant improvement over the initial model. It moved towards the eliciting of more realistic dimensions for the step-height and step-space. This was prompted by the researcher who asked about the stairs in the school, "*On of the heights of the steps in the school, do you think the staircase in the school is shorter?*" to which he responded, "*I never measure, I think I will…definitely.*" The question led to the realisation of employing better measurement sense which in this case resulted in the use of the ruler to find out about the actual measurements of his foot size and thereby getting a better sense of the lengths of the step-height and step-space. He maintained that the step-height of 20 cm was "*comfortable*" and thus worked out the step-space to be 24 cm.

When James came up with 10 steps, the researcher asked the following question to prompt him to reveal his thinking further, "*Is your 10 steps related in any way to the number of step space? Did you realise (this)?*" . James related the number of step-heights to be equal to the number of step-space as evident from getting the width of the step-space to be 24 cm (i.e. 240 cm ÷ 10). This model could literally exist as compared to the first model above. James tested the second model to determine if it would be a plausible model and believed the dimensions he worked out was a better model than Attempt 1, "*From what I would think of is … after the preliminary draft, I think 10 steps can. I think 10 steps.*" We inferred that this improved model came about through a more conscious attempt to adjust the dimensions realistically to fit the conditions as well as from the scaffolding questions of the researcher to help him acquire a more realistic measurement sense through making actual measurements using the ruler.

Attempt 3 – Abstract Model

The researchers attempted to help James improve his model even further. Several key questions that prompted James to revise his second model include: "*So if 10 steps, then how many step-space would you have?*", "*Do you count this as one step? First step, right? Then this step, there's another step. No, right?*" and "*Did you see a special relationship or not in that sense, the number of steps*?" The questions were aimed at helping James conceptualise if there could be another mathematical relationship apart from number of steps being equal to the number of step-space. Taking these questions into consideration, James' gradually made further revisions to improve his model.

James' revision of the second model resulted in a third model which is depicted in Figures 6a and 6b below b.

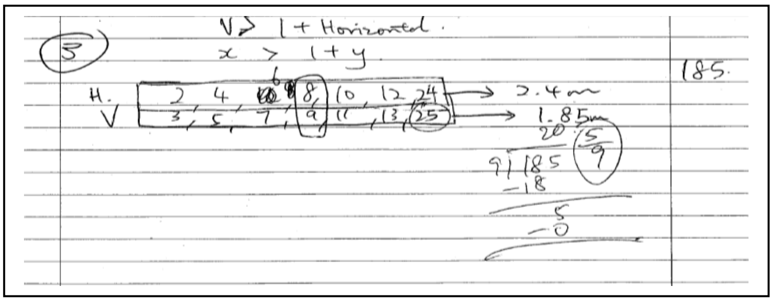

Figure 6a. Model 3

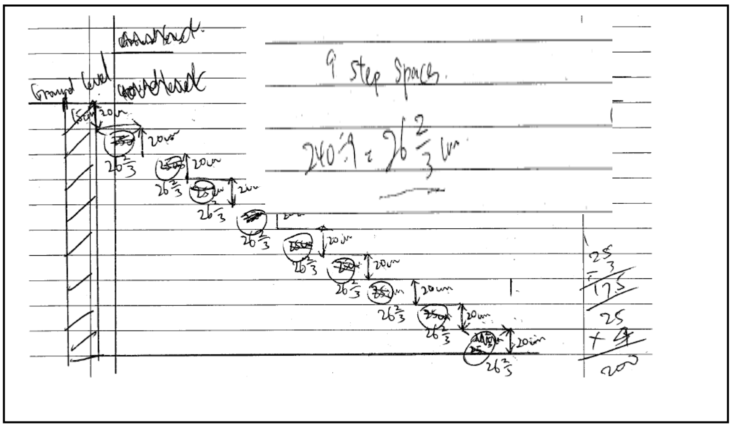

Figure 6b. Model 3

Vignettes	Elements
* *"So I only need 9. I only need 9… If let's say I have 10 steps, I only need 9 steps, 9 steps spaces… Yah, because don't count this one (ground level)"* * Established relationship between the number of step-height and the number of step-space. From Figure 6a, "V" refers to the number of vertical steps while "H" refers to the number of horizontal step-spaces. * Established the relationship as "*V > 1 + Horizontal*" in Figure 6a. * "*Because the vertical step is always one more than the horizontal because the height is always more than. So vertical is always one plus the horizontal.*"	Formulate
* Worked out the number of steps to be 10 (Figure 6a), the number of step-space to be 9 and the dimensions for the step-height to be 20 cm and step-space to be $26\frac{2}{3}$ cm (Figure 6b) - *"Yah, 27.6666666,6 okay. So 26 and 2 thirds."*	Solve
* The drawing of the flight of stairs based on the revised dimensions and the number of step-height and step-space showed the model fitted well with the problem situation and fulfilled the conditions. * "*Yah, because if let's say if I… yah, this is the one that we decided. Eh, no, this is one step higher. This is one step more. This is one step more. So we could have two solutions. One if let's say, if all right, um, the height is 25cm. We will take 10 steps, right*".	Interpret
* Related the model as the "optimised model". * "*Yah, this is what we wanted*."	Reflect

This third model appeared to be the "optimized model". James' realised that given the task parameters, this was the best model he had come up with - "*Okay this is ground level. I mean this is just the diagram.... Okay have I reached 2 metres. 2 metres ground level here. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. This is the 10th right? I think I would have hit 2 metres, right*." The third model is a more sophisticated version of the second model because the width of the step-space is wider than 24 cm, and is a better fit for the average foot length of an adult. As well, the 20 cm step-height is considered a comfortable height to climb and on the whole the dimensions of the constructed staircase did not breach the task parameters. James was also able to describe a pattern representing the relationship between the two key variables: step-space and step-height (Figure 6a) and use this pattern to predict other possible combinations of step-space and step-height. We inferred that some of the key questions that the researchers had asked prompted him to re-look and refine his second model towards obtaining a better model.

Post-Interview

James was asked to give his thoughts about mathematical modelling after the GSM session. His responses mainly compared features of mathematical modelling with his previous experience in facilitating problem-based learning (PBL) as shown in the vignettes below:

Interview Aspect	Vignette	Inference
View of mathematical modelling	"*Actually I find it's a more simpler task than the PBL. Because PBL ah, you have to insert problems. You have to insert problems to an already set of problems itself. A set of tasks already. You have to inject two or three problems into it to actually get them. ... This (mathematical modelling) to me is actually much simpler because there is no injection of anymore complications.*" "*But to me I think it's also freely structured because there are conditions. So I'm grappling between the thought of what is a structured problem sum or what is a (ill) structured task compared to a structured task.*" "*But there's only a few assumptions that they can make. It's still limited in the sense. To me, to me when I see it, yah, rather than for the one I mentioned about which I really did with the class. Really totally open*." *"And actually, this one we are looking at in terms there's only two models. But if you look at a problem based learning, there can be more than two models. Example, how would you first, if let's say you have a house. You give them a certain height or*	* mathematical modelling is simpler than PBL * mathematical modelling is more structured because conditions are given but not so with PBL. * the structuredness limits the number of models that can be constructed in mathematical modelling and limits the number of assumptions pupils can make compared to PBL.

	there's really not much limitation in terms of height. They can give you a two storey house. They can give you a three storey house. Then they can see that okay the structure of the house it has to be a wider base below and a slightly smaller space on top. Or equal space. But the top should not be bigger than the smaller space."	

In the post-interview, we inferred that James had acquired some knowledge about the features of mathematical modelling as comprising conditions and the conditions made the modelling task more structured and simpler to work on. The structuredness limited the number of assumptions that could be made and the number of models that could be constructed in his comparison of mathematical modelling and PBL.

In summary, the modelling experience that James went through as a first-timer saw him mathematising the problem situation through establishing variable relationships, testing them in the light of the problem situation and parameters, revising the solutions and re-interpreting the solutions against the real-world situation. James went through three modelling attempts that began with the construction of an emerging model that eventually evolved into an "optimised" model of which the improvement of his models were enabled by some facilitative questions raised by the researcher. It was evident that the modelling process James had gone through comprised elements of formulating, solving, interpreting, and reflecting before he reached his "optimised" solution. Such an experience was invaluable for him to understand the modelling process and the cyclical nature of solving modelling problems. As well, James was able to articulate some features of mathematical modelling tasks as comprising conditions, making assumptions and having a structure that could limit the number of models to be constructed.

Discussion

For teachers who are new to mathematical modelling, they need to be familiar with the process of mathematical modelling and experience the process themselves to be able to teach and facilitate modelling activities well (Lesh & Lehrer, 2003; Schorr and Koellner-Clark, 2003; Cheah, 2008; Tan & Ang, 2013). Model-eliciting tasks such as the one used in this study place greater demand on learners' abilities as the task is distinctly different from the structured word problems typically used in mathematics classrooms. In this case study, the research design focused on an collaborative approach between researchers and teachers that afforded James to show what he was capable of given such a novel complex model-eliciting task to solve. James exhibited modelling actions comprising making assumptions, identifying mathematical variables, establishing relationships between the variables, testing, interpreting and revising the models. The modelling actions mapped to the elements of the modelling process suggest how James has had to go through several modelling cycles to obtain the "optimised" model. The experience has elicited his capacity to engage in mathematical modelling. Moreover, the interview data also suggest that the experience enabled him to enquire more into understanding designing mathematical modelling activities, for example, whether it made better sense for numerical figures to be presented in .5 instead of .666667 - "*So I think you know, so when we craft the question, I was thinking we need to come up with a optimal number that can lead to either a point-five or a zero. A point-five can still be a mixed number because like half, as long as it's between the conventional one that is half, one quarter, I think it's still fine.*" Thus the experience gained as a modeller has implications on one's view of mathematics; the teacher sees how mathematics is used in the real-world and the meaningfulness of mathematics (Mousoulides, 2009).

The affordances of the Multi-Tiered Teaching Experiment design are manifold from the perspectives of the participants of each tier. Firstly, it provided opportunities for the teacher (Tier 2) to experience being a modeller first-hand under the tutelage of the researchers (Tier 3). The teacher was able to mediate his own conceptual model or mental framework of mathematical modelling with those more experienced. Secondly, the teacher was able to articulate his plans for the incorporation of mathematical modelling in his own class based on discussions with the researchers In the case of this study, James thought about how he could facilitate the modelling session with his class to help his students better understand the real-world situation. He articulated his thoughts with a clear mental procedure of how he was going to introduce the task to his students. This could be seen when he wanted his students to liken themselves as architects in dealing with the problem so that they could relate to the real world problem better – "*So I will also be probably like you know in terms of getting them to imagine what they are going to be like that they are architects… Yah, so that they know that this is tied to the real world... So I'm going to do something like a role play for them... So they will zoom in on they must create the steps. Then how they can make it more practical in that sense.*" In this regard, James was taking his learning experience and advancing it to another level where he would have to plan and manage the learning experience of his students.

Thirdly, one other asset of the Multi-tiered Teaching Experiment lies in helping the modeller to experience the cyclic nature of mathematical modelling. In developing James' growing capacity, we contend that the questions raised by the researchers during James' modelling endeavour (as presented in the earlier section) over the three attempts were not to assist him to find a predetermined solution but to maintain and nurture the diversity of the learners' approaches to solve the problem (Lesh & Doerr, 2003). The questions targeted at heightening the teacher-modeller's measurement sense as well as improving the model. In a sense, rresearchers play the role of the more knowledgeable others (Vygotsky, 1978) in extending the thinking of the modeller. Lastly, from the perspective of the researcher, the Multi-Tiered Teaching Experiment provided a valuable opportunity for the researcher to gain insights into the journey of a teacher beginning his venture into mathematical modelling from with insider knowledge.

In this paragraph, we summarise the workings of the Multi-tied Teaching Experiment as pathways to show the interaction between different tiered participants for developing the teacher's capacity in mathematical modelling (see Figures 7 and 8). Figure 7 provides us with a clear picture how the research design has taken the teacher-modeller through the initial phase of his learning journey in mathematical modelling. The development of the teacher's capacity seen in the light of the research design suggests that the teacher acquires the knowledge and awareness about mathematical modelling through interactions between the teacher-modeller in Tier 2 and the researchers in Tier 3 of the Multi-tiered Teaching Experiment. The interaction between the tiers plays a part in enabling the teacher-modeller to complete the modelling experience successfully. The mathematical modelling experience and the knowledge acquired would serve to help the teacher become familiar with the students' evolving ways of thinking about important ideas and abilities that he would want the students' to develop (Carpenter, Fennema & Romberg, 1993) when the interaction between Tiers 2 and 1 comes about. To add on, the interaction between the teacher-modeller and the researchers (Tiers 2 and 3) is also seen as a model-development process for putting the theoretical framework into practice and reviewing how each party is learning through the express-test-revise cycles of the multi-tiered teaching experiment.

Tier 3 - Researchers	Asks questions to enable the teacher-modeller to think aloud, clarify thoughts and think more deeply during the Guide-and-Support Modelling (GSM) session.
Tier 2 - Teachers	Exposure to the real-world problem Formulates a mathematical solution Tests the solution Revises the solution Validates the solution as the "ideal" solution Offers it as a solution to the real-world problem

Figure 7. Interaction between Tiers 2 and 3 in building teacher capacity

As a step forward, the researchers would want to help the teacher-modeller think about the guiding role of the teacher in designing and facilitating mathematical modelling activities with his students. As shown in Figure 8, this would begin with a reflection of the modelling experience the teacher has gone through to suggesting what real-world situations that could be designed as a model-eliciting task that would incorporate mathematics that the students could use. Legitimate concerns to be reflected upon would also include the explicitness of the guidance a facilitator would provide to the students as well as the questions to be asked in invoking mathematical inquiry for model development. The capacity-building effort in the next round of interaction between the researchers and the teacher-modeller is shown in Figure 8 as a more expansive view of the multi-tiered teaching experiment framework.

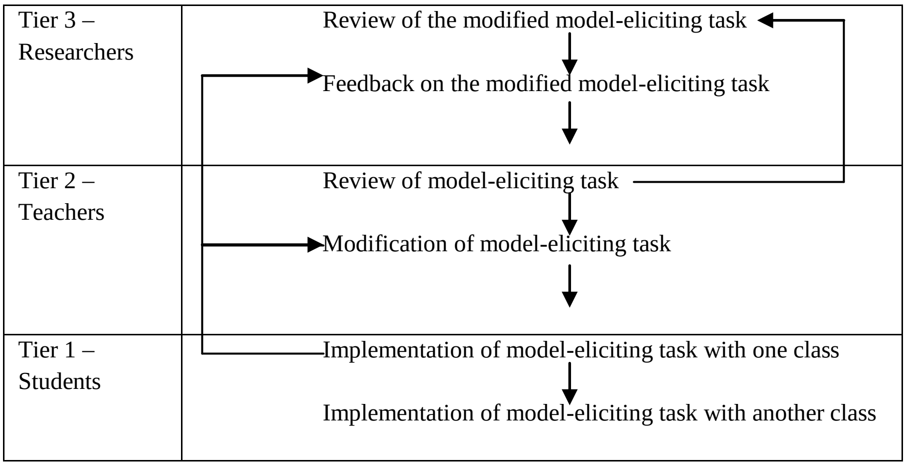

Figure 8. The express-test-revise cycle from a macro level view of modelling

Beginning with Tier 2, “Review of model-eliciting task” (the original ‘Staircase” task as the starting point), the consequence of the GSM session is for the teacher-modeller to modify the model-eliciting task in consultation with the researchers, and to refine the task. Interactions between Tiers 3 and 2 would continue until the task is deemed suitable to be tested with the students at Tier 1. In implementing it with the students, it is expected that there would be varied responses and outcomes with respect to how the students would manage the task. This in turn becomes “thought-revealing conceptual tools” (Lesh & Lehrer, 2003, p.119) for the teacher to further review, modify and refine the task for the next implementation with another class. The multiple express-test-revise cycles at play at this macro level is deemed to enhance the teacher’s learning and development further.

Concluding Thoughts

This case study suggests that the Multi-tiered Teaching Experiment that has been put in place is serving its goal in building one teacher’s capacity in mathematical modelling. The interactions between Tiers 3 and 2 saw the teacher-modeller display modelling actions towards successfully completing the modelling task and constructing models with some facilitative questions rendered by the researchers. Interviews revealed his knowledge acquired about the features of mathematical modeling and how he would embed the mathematical modeling session for his class with some modifications. Further growth of the teacher-modeller is a logical consequence when he takes his learning into a stage where he has to learn to develop conceptual tools to manage the student learning and his own learning as a teacher teaching mathematical modelling as illustrated in Figure 8. As a case study, we note that learning outcomes would differ from teacher to teacher when we do GSM sessions with other teachers. Nonetheless, implementing the GSM with more teachers would provide greater opportunities to equip and empower them to take the bold step in engaging meaningful and authentic learning experiences in the mathematics classroom. As pockets of such research gradually emerge to inform of the benefits that mathematical modelling could bring, it is hoped that the findings of such research will provoke teachers to see the potential and essence of mathematical modelling to want develop their capacities in this domain as well as their students' capabilities in mathematical modelling.

References

Ang, K. C. (2013). Real-life modelling within a traditional curriculum: Lesson from a Singapore experience. In G. Stillman, G. Kaiser, W. Blum, & J. Brown (Eds.), *Teaching* m*athematical modelling: Connecting research to practice* (pp. 131-141). New York: Springer.

Ang, K. C. (2010). Mathematical modelling in the Singapore curriculum: Opportunities and challenges. In A. Araújo, A. Fernandes, A. Azevedo & J. F. Rodrigues (Eds.), *Educational interfaces between mathematics and industry: Proceedings of the EIMI 2010 Conference* (pp. 53-62). New York: Springer.

Blomhøj, M., & Kjeldsen, T. (2006). Teaching mathematical modelling through project work - experiences from an in-service course for upper secondary teachers. *ZDM, 38*(1), 163 – 177.

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects – state, trends, and issues in mathematics instruction. *Educational Studies in Mathematics*, *22*(1), 37-68.

Carpenter, T., Fennema, E., & Romberg, T. (1993). *Rational numbers: An integration of research.* Hillsdale, NJ: Erlbaum.

Chan, C. M. E. (2013). Initial perspectives of teacher professional development. In Stillman, G., Kaiser, G., Blum, W., & Brown, J. (Eds.), *Teaching mathematical modelling: Connecting to research and practice* (405-413). New York: Springer.

Chan, C. M. E. (2010). Tracing primary 6 pupils' model development within the mathematical modelling process. *Journal of Mathematical Modelling and Application, 1*(3), 40-57.

Chan, C. M. E. (2009). Mathematical modelling as problem solving for children in the Singapore *Mathematics Classroom. Journal of Science and Mathematics Education in Southeast Asia, 32*(1), 36-61.

Chan, C. M. E. (2008). Using model-eliciting activities for primary mathematics classrooms. *The Mathematics Educator, 11*(1/2), 47-66.

Chan, C. M. E., Ng, K. E. D., Widjaja, W., & Seto, C. (2012). Assessment of primary 5 students' mathematical modelling competencies. *Journal of Science and Mathematics Education in Southeast Asia, 35*(2), 146-178.

Cheah, U. H. (2008). Introducing mathematical modelling to secondary school teachers: A case study. *The Mathematics Educator, 11*(1/2), 21-32.

de Oliveira, A.M.P., & Barbosa, J. (2013). Mathematical modeling, mathematical content and tensions in discourses. In G.A. Stillman, G. Kaiser, W. Blum, & J.P. Brown (Eds.), *Teaching mathematical modeling: Connecting to research and practice* (pp. 67-76). Dordrecht, The Netherlands: Springer.

Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher, 32*(1), 5-8.

Dolk, M., Widjaja, W., Zonneveld, E., & Fauzan, A. (2010). Examining, teacher's role in relation to their beliefs and expectations about students' thinking in design research In R. K. Sembiring, K. Hoogland & M. Dolk (Eds.), *A decade of PMRI in Indonesia* (pp. 175-187). Bandung, Utrecht: APS International.

English, L. D. (2003). Reconciling theory, research, and practice: A models and modelling perspective. *Educational Studies in Mathematics, 54*, 225-248.

Galbraith, P., Stillman, G., & Brown, J. (2010). Turning ideas into modeling problems. In R Lesh, P. Galbraith, C. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies* (pp. 133-144). New York: Springer.

Kaiser, G. (2006). Mathematical modelling as a bridge between school and university. *ZDM, 38*(2), 196-208

Lee, N. H. (2013). Teacher professional development on mathematical modelling: Problem posing and task design. In G. Stillman, G. Kaiser, W. Blum, & J. Brown (Eds.), *Teaching* m*athematical modelling: Connecting research to practice* (pp. 415-426). New York: Springer.

Lee, N. H., & Ng, K. E. D. (Eds.). (2015). *Mathematical modelling: From theory to practice* . Singapore: World Scientific.

Lee, N. H., & Ng, K. E. D. (2015). Introduction: Mathematical modelling outreach in Singapore. In N. H. Lee & K. E. D. Ng (Eds.). *Mathematical modelling: From theory to practice* (pp. 1-18). Singapore: World Scientific.

Lesh, R., Cramer, K., Doerr, H., Post, T., & Zawojewski, J. S. (2003). Model development sequences. In R.A. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning and teaching* (pp. 35-58). Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R., & Doerr, H. M. (2003). Foundations of a models and modeling perspective in mathematics teaching and learning. In R.A. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning and teaching* (pp. 3-34). Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R., & Kelly, A. (2000). Multitiered teaching experiments. In A. Kelly, R. Lesh (Eds.), *Research design in mathematics and science education*. (pp. 197-230). Lawrence Erlbaum Associates, Mahwah, New Jersey.

Lesh, R., & Lehrer, R. (2003). Models and modeling perspectives on the development of students and teachers. *Mathematical Thinking and Learning, 5*(2&3), 109-129.

Lesh, R., & Sriraman, B. (2005). John Dewey revisited - pragmatisim and the models-modeling perspective on mathematical learning. In A. Beckmann, C. Michelsen, & B. Sriraman (Eds.). *Proceedings of the 1st international symposium of mathematics and its connections to the arts and sciences* (pp. 7-31). The University of Education, Schwöbisch Gmund, Germany.

Lesh, R., Yoon, C., & Zawojewski, J. (2007). John Dewey revisited: Making mathematics practical versus making practice mathematical. In R. A. Lesh, E. Hamilton, & J. J. Kaput (Eds.), *Foundations for the future in mathematics education* (pp. 315-348). Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 763-803). Charlotte, NC: Image Age Publishing.

Ministry of Education [MOE]. (2007). *Ministry of Education Mathematics Syllabus – Primary*. Singapore: Curriculum Planning and Developmental Division.

Ministry of Education [MOE]. (2012). *Mathematical modelling resource kit*. Singapore: Curriculum Planning and Developmental Division.

Mousoulides, N. G., (2009). Mathematical modeling for elementary and secondary school teachers. In Kontakos, A. (Ed), *Research & theories in teacher education*. Greece: University of the Aegean.

Ng, K. E. D. (2013). Teacher professional development on mathematical modelling: Facilitation and scaffolding. In G. Stillman, G. Kaiser, W. Blum, & J. Brown (Eds.), *Teaching* m*athematical modelling: Connecting research to practice* (pp. 427-436). New York: Springer.

Ng, K. E. D. (2010). Initial experiences of primary school teachers with mathematical modelling. In B. Kaur & J. Dindyal (Eds.), *Mathematical Applications and Modelling: Yearbook 2010 Association of Mathematics Educators* (pp. 129-148). Singapore: World Scientific.

Ng, K. E. D., Chan, C. M. E., Widjaja, W., & Seto, C. (2013, March). *Fostering teacher competencies in incorporating mathematical modelling in Singapore primary mathematics classrooms*. Paper presented at Innovations and exemplary practices in mathematics education: 6th East Asia Regional Conference on Mathematics Education, Phuket, Thailand.

Ng, K. E. D., Widjaja, W., Chan, C. M. E., & Seto, C. (2012). Activating teacher critical moments through reflection on mathematical modelling facilitation. In Brown, J. & Ikeda, T. (Eds.) *ICME 12 electronic pre-conference proceedings TSG17: Mathematical applications and modelling in the teaching and learning of mathematics* (pp. 3347-3356). Seoul, Korea: ICME12.

Schorr, R. Y., & Koellner-Clark, K. (2003). Using a modeling approach to analyze the ways in which teachers consider new ways to teach mathematics. *Mathematical Thinking & Learning, 5*(2), 191-210.

Tan, L. S., & Ang, K. C. (2013). Pre-service secondary school teahcers knowledge in mathematical modelling – A case study. In G. Stillman, G. Kaiser, W. Blum, & J. Brown (Eds.), *Teaching* m*athematical modelling: Connecting research to practice* (pp. 373-384). New York: Springer.

Torp, L., & Sage, S. (2002). *Problems as possibilities: Problem based learning for K-16 education* (2nd ed.). Alexandria, VA: Association for Supervision and Curriculum Development.

van den Heuvel-Panhuizen, M. (2003). The didactical use of models in Realistic Mathematics Education: An example from a longitudinal trajectory on percentage. *Educational Studies in Mathematics, 54*(1), 9-35.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Widjaja, W. (2013). Building awareness of mathematical modelling in teacher education: A case study in Indonesia. In G. Stillman, G. Kaiser, W. Blum, & J. Brown (Eds.), *Teaching mathematical modelling: Connecting research to practice* (pp. 415-426). New York: Springer.

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Appendix

Stairs

In a shopping centre, there is a basement that will lead to a train station. However, a flight of stairs is to be constructed to link the basement to the ground floor. **In this activity, you will refer to the diagram and help design a flight of stairs between the Ground Level and the Basement.**

Think about some considerations that you will take to ensure that a person of average height 1.6m can go up the stairs from the Basement to Ground Level comfortably.
There are two conditions to note:
(i) There needs to be a safe vertical height space of at least 15 cm between the ceiling and an adult when he is <u>only</u> on the first step (at the basement).
(ii) Each step must be of the same height and each step-space must be of the same width.