Chapter 4
Real-World Task *Context:* Meanings and Roles

Jill P. Brown

Abstract This chapter details results of a study intended to increase understanding of the myriad meanings of real-world task context in mathematics education and their relevance to modelling. The research aim was to ascertain how context is viewed within the broader mathematics education community. Data analysis reported here followed an examination of use of the terms: context, task context and real-world in four mathematics education journals. Four samples, one from each journal, two in 2014 and two in 2017 where all papers using the term *real-world*, comprised the purposive sample used for the in-depth investigation. Whilst, often not defined by the authors, in most papers the context was *real-world task context* and, in the majority, this played an *essential*, rather than *incidental*, role.

Keywords Real-world · Context · Task context

4.1 Introduction

> That applications and modelling have been, and continue to be, central themes in mathematics education is not at all surprising. Nearly all questions and problems in mathematics education, that is questions and problems concerning human learning and the teaching of mathematics, influence and are influenced by relations between mathematics and some aspects of the *real world* [emphasis added]. (Blum et al. 2007, p. xii)

Within the mathematical modelling and applications community, the term *context* often implies a *real-world context* is being assumed. Blum et al. describe this extra-mathematical world as including the broad contexts of "the world around us," "everyday problems" and "preparing for future professions" (p. xii). However, such a meaning is not always evident both within and beyond this mathematics education community. In mathematics education research 'context' has an even greater variety of meanings—explicitly stated or not. Boero (1999), in the guest editorial for an

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ESM Special Issue on 'Teaching and Learning Mathematics in Context', noted the varied meanings of the term and, in particular, *situation context* or context for "learning, using and knowing mathematics" (p. 207) versus *task context* as articulated by Wedege (1999) as "representing reality" (p. 206). Boero describes the former as "workplace, classroom social context, computer learning environments, etc. … [and *task context*] as everyday life situations evoked in a problem-solving task" (p. vii). Wedege described *situation context* as circumstances (historical, social, psychological, etc.) in which "something happens, or, … is to be considered" (p. 206). Busse and Kaiser (2003), writing within the modelling community, describe context as "a rather nebulous concept, used by many authors in different meanings and ways" (p. 3) although its importance was not in question according to these authors. Whilst the importance of situation context is acknowledged, the focus in this chapter is task context.

In characterising the relationship between task context and the real-world, Stillman (1998) distinguished "three levels of embeddedness of context" (p. 246). These describe the extent to which the real situation remains as the situation is simplified for use in the classroom. She describes three types of problems where this embeddedness varies from almost non-existent to pseudo-real to real and the problem can be characterised as *border*, *wrapper* or *tapestry*. In *border* problems, the mathematics and task context are entirely separate. The real-world context can be ignored by the task solver. Knowing about the context is of no help to understanding or solving the problem or interpreting or validating the solution. In *wrapper* problems, the task solver must engage with the real-world context to 'find' the mathematics which is hidden within the context. Beyond that, the real-world can be ignored, or discarded as only the mathematics is needed for solving (Stillman 1998) although context can be used for checking if a solution makes sense. The third level, *tapestry*, occurs when the real-world task context and mathematics are interwoven, and task solvers need to move between the two continually crossing the boundary between the real-world and the mathematical world (Stillman 1998) throughout the solution process.

Context is often claimed to help learning, usually via fostering active engagement (Stillman 2004). A recent large study of year 2 students (50 schools) in Alaska showed that implementation of "the reform-oriented and culturally based Maths in a Cultural Context (MCC) teacher training and curriculum … significantly improved students' mathematical performance" (Kisker et al. 2012, p. 74). Previously, Langrall et al. (2006) examined the role of context knowledge in solving statistical tasks by Grade 6 Australian students, finding several important uses by students including supporting their interpretation of the data and in taking a critical stance to the data.

Smith and Morgan (2016) reviewed curriculum documents from 11 jurisdictions to ascertain the relationship between the real-word and school mathematics. They identified three orientations to real-world contexts in mathematics, a tool for everyday life, a vehicle for learning, and engagement with the real-world motivating learning. In four jurisdictions, a single main pathway was followed with variation in speed and extent of progress. However, in the other seven jurisdictions alternative pathways were offered, "with the less [mathematically] advanced pathways having a stronger emphasis on real-world contexts" (p. 42) including in assessment tasks. In these jurisdictions, if mathematics is seen as a tool for everyday life then why is this given less emphasis for students studying more advanced mathematics? If the purpose was as a vehicle for learning, or for motivation, then why is there less focus on real-world contexts in the years of schooling prior to students needing to select or embark on particular pathway options? As Smith and Morgan noted, changing the emphasis for different year levels or by nature of mathematics studied conflicts with all three of the espoused purposes.

Others claim or posit that use of real-world contexts can, or may, hinder understanding. Dapueto and Parenti (1999) note that students may face extra challenge "in relation to knowledge of the context" (p. 15). Wroughton et al. (2013) add it might be distracting to students in a statistical sampling context, whilst Zevenbergen et al. (2002) claim "there is considerable cause for concern when such a strategy [the use of contexts in school mathematics] is used simplistically" (p. 8). Cooper and Dunne (2000) have suggested that students from working class backgrounds can be misled by school mathematics questions set in everyday contexts because they misread the task as calling for an everyday response. They suggest middle class students tend to ignore the context and focus on the (esoteric) mathematical calculation required. Wijaya et al. (2014) reported that 38% of errors made by Year 9–10 Indonesian students on released PISA items were related to "understanding the context-based task". Huang (2004) found 48 Grade 4 Taiwanese students were more successful on tasks related to unfamiliar context than familiar contexts, and (perhaps not surprisingly) took longer to solve tasks with familiar contexts, suggesting that unfamiliar contexts are ignored whilst familiar ones take more time to make sense of.

The *ICMI Study on Modelling and Applications in Mathematics Education* was held in 2004 with Niss et al. (2007) suggesting the Study might "formally mark the maturation of applications and modelling as a research discipline in the field of mathematics education" (p. 29). Niss et al. define applications as being when mathematics is applied to some aspect of the extra-mathematical world for some purpose including "to understand it better, to investigate issues, to explain phenomena, to solve problems, to pave the way for decisions, …. The term 'real-world' is often used to describe the world outside of mathematics" (p. 3) and this can be in another school subject or related to personal or social issues.

The purpose of this chapter is to analyse how 'real-world' context is used or understood 'today', given 10 years have passed since the study volume was published. To achieve this, the author sampled leading mathematics education journals to ascertain what these meanings are and their purposes for different researchers. The overarching research question that is the focus of the study is: How is context viewed in the broader mathematics education community as evident in research publications? More specifically, this entailed answering for each published paper: *What are the meanings and roles of real-world task context in the learning of mathematics according to mathematics education research?*

4.2 Method

Document analysis is an analytical qualitative research method requiring "data be examined and interpreted in order to elicit meaning, gain understanding, and develop empirical knowledge" (Bowen 2009, p. 27). It can be used to complement other methods or as a sole method. In this study, the intention is to better understand how context is used in research reported in journal publications so document analysis will be used as a stand-alone method. As with all qualitative research data, "detailed information about how the study was designed and conducted should be provided" (p. 29) as will be the case here.

4.2.1 Journal Selection

In attempting to ascertain the view of context in the mainstream mathematics education research community, a review of literature was called for with a reliable method for choice of sample. Noting the variety of ways to assess the quality of academic journals (e.g., acceptance rates, prestige of editors, citations), Nivens and Otten (2017) used two journal metrics (Scopus's SCImago Journal Rank and Google Scholar Metrics h5-index) to compile a ranking of 69 mathematics education journals, after discounting Web of Science's Impact Factor as few mathematics education journals are in the relevant database. The journals considered explicitly focused on mathematics and/or statistics education. This metrics approach overcomes some limitations, such as personal opinion in earlier work by Toerner and Arzarello (2012) who compiled a ranking after surveying experts in the field.

Nivens and Otten (2017) found reasonable agreement that the top eight mathematics education journals are: *Educational Studies in Mathematics* (ESM), *International Journal of Science and Mathematics Education* (IJSME), *Journal of Mathematical Behavior* (JMB), *Journal of Mathematics Teacher Education* (JMTE), *Journal for Research in Mathematics Education* (JRME), *Mathematical Thinking and Learning* (MTL), *School Science and Mathematics* (SSM), and *ZDM: Mathematics Education* (ZDM). However, the ranking within these is less clear, although ESM was ranked in the top two in both. Six journals were in the top seven by both measures, with JRME first and fourth. MTL was in the top seven on one list but does not appear on the GSM ranking with too few papers (<100 papers in 2011–2015). These eight journals formed the original list considered for sampling and analysis.

From these journals, two were eliminated from the analysis on the basis of their focus being broader than mathematics education or having a narrower focus eliminating IJSME and SSM that include science education, and JMTE which focuses on mathematics teacher education. A fourth journal, ZDM, was eliminated on the basis that, unlike the other journals access to authors is by invitation only. Thus, a selection of four journals was determined. As ESM and JRME are the oldest journal in the sample, it was decided to begin with these and use that analysis to inform the subsequent analysis of JMB and MTL.

4.2.2 Initial Analysis

A text content search for each journal was undertaken electronically using the proprietary/available search engine for the terms, *context, task context*, and *real-world*. For ESM this was via Springer Link (1968–2017), JRME via JSTOR (1970–2017), MTL via Taylor and Francis Online (1999–2017), and JMB via Science Direct (1995–2017, i.e., not available for all years of publication). In addition, data about the number of papers published was also collected.

4.2.3 Detailed and In-Depth Analyses

It was decided to begin with an in-depth analysis of ESM. As 2014 was a decade after the ICMI Study on Modelling and Applications in Mathematics Education was held, it was deemed appropriate to use 2014 for an in-depth study of ESM and JRME, noting the former is based in Europe and the latter in USA. Coincidentally, 2014 provided the largest sample possible from ESM which was then used to inform the subsequent analysis. This was followed with a 2017 sample, a decade since the Study Volume was published, in each of MTL and JMB, more recently established journals, providing the most recent samples possible.

Purposeful sampling was adopted in order to find information-rich cases rather than representative cases (Patton 2002). For the years targeted, for each journal, papers that included search items context, task context and real-world/real world were selected for detailed analyses highlighting these focuses. Each paper was read in full.

Following, the detailed analyses of papers, a further in-depth analysis followed to produce an analytical summary matrix (Miles et al. 2014). Firstly, papers were classified by *type*—theoretical, commentary, document analysis, or research-based. Secondly, the paper *focus with respect to the context* being situational or a task context, or in some cases other (culture/religion) was ascertained. Thirdly, for papers with a task context focus, the *role of the real-world* was classified as incidental, pseudo-real or essential. Finally, where the role of the real-world task context was classified as essential, this was then further classified as being of a minor or major focus. Where actual tasks were included, the degree of *embeddedness of the task context* was categorised as border, wrapper or tapestry. A summary of this analysis process is presented in Fig. 4.1. The purpose was to facilitate assessing of the sub-questions by journal before aggregating into a meta-analysis across samples in Sect. 4.7.

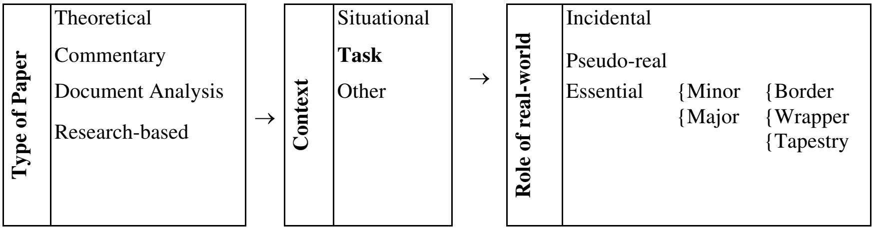

Fig. 4.1 Overview of analysis process

The role of the real-world task context was categorized as *incidental* when (i) it was one of many considerations of study either related to data collection or analysis, (ii) it was a natural part of the mathematics focus (e.g., speed) or (iii) it arose in the findings. The role of the real-world was described as *pseudo-real* when the task solver had to "suspend reality and ignore common sense" (Boaler 1994, p. 554). The role was categorised as *essential* where it played an important part in the study. However, as this importance varied, two levels *minor* or *major* were used to distinguish between being essential in the study but of low importance to being not only essential, but also intrinsic to the study. All PISA-related studies were categorized as major as the intention of PISA (even if disputed) is to assess students' mathematical literacy in a variety of contexts, which are mainly real-world contexts.

For papers where task context was *essential* the embedding of the real-world in the task context was characterized, following Stillman (1998), as *border*, *wrapper* or *tapestry*. Where multiple tasks were presented, there may have been a range of embeddedness across different tasks. For PISA-related studies, the degree of embeddedness may vary across all levels from task to task, so these papers were excluded from this level of analysis. Although Stillman's (1998) characterisations of contextualization were developed to describe more substantive tasks than appear in some of the literature surveyed, it was apparent they would be useful in distinguishing differences in the task contexts identified in the literature.

4.3 Content Analysis: ESM

4.3.1 Initial Analysis and Sample Selection

During 1968–2017 (Volumes 1–96), 2277 papers were published in ESM but the number published per volume and year varies (average 27 issues/year). The search for *context* identified 1566 papers in these years, and 595 papers in the years 2008–2017 (i.e., post 2007 ICMI Study Volume publication). Not surprisingly, there was a greater frequency of the term *context* (see Tables 4.1 and 4.2) with many uses of the term context referring to an *educational context* or *social context* (as will be discussed) rather

Table 4.1 Occurrences of search terms ESM

Search term	All years	Years by 'decade'				
	1968–2017	68–77	78–87	88–97	98–07	08–17
Context	1566	97	166	266	443	595
Task context	1277	51	120	216	373	515
Real-world	390	73	40	63	95	119
No. of papers	2277	350	322	402	531	672

Table 4.2 Search terms by year (last eight years) ESM

Search term	2010	2011	2012	2013	2014	2015	2016	2017
Context	46	57	68	79	66	59	60	60
Task context	39	44	60	73	56	48	51	58
Real-world	7	10	14	17	20	13	13	6
No. of papers	56	65	68	86	71	75	77	66

than a *real-world or extra-mathematical context* as may be expected in mathematical modelling or application specific literature.

The rate of use of the terms *context* and *task context* have steadily increased in ESM since 1968. This is evident even when the increase in papers per year and the variation in the number of papers per year are accounted for. Similarly, the term *real-world* has shown a generally increasing trend, although its use, and rate of increase are much lower. The search results for real-world resulted in 390 instances of the term *real-world* for the years 1968–2017, and 119 for the last decade (2008–17). Not only is 2014 one decade on from the ICMI Study, but also it has the maximum number of results for the term, which tail off after this year. Twenty papers (E1–E20) were in the sample. (See Appendix in electronic supplement for full details of papers sampled.)

4.3.2 Detailed Analysis

Initial exploratory analysis considered the country of author and location of study. Authors were based in 13 different countries with studies based in 11 different locations showing that the author demographic was not Euro-centric, despite the location of the publishing house. Keyword analysis showed none of the papers had real-world or context as a key word. Keywords suggestive of real-world contexts were Critical mathematics (E3), Medication dosage calculation problem-solving (E5), Drug errors, (E5), Authentic (E5) and perhaps Map tasks (E10), PISA or Mathematics literacy (E1, E7, E14), and In and out of school (E18). This should indicate a note of caution for content analyses that only look for key words.

In nine papers, the *real-world* was mentioned only once, six papers contained 2–3 mentions and in the remaining five papers 4–7 occurrences were found. The number of mentions of the term was however, not sufficient, to determine the emphasis or importance of the real-world in the paper, as is illustrated by the papers of Bratlinger (E3) and Roth (E16), both with only one mention. Bratlinger's study of high school students excluded from mainstream schooling, emphasised the real-world as he focuses on how critical mathematics, especially through classroom discourse patterns, can increase student awareness or understanding of factors impacting on their lives, that is their lived real-world. Similarly, with a significant focus on the real-world, Roth (E16) highlights the disparities between mathematics in the workplace (the real-world) and school mathematics as he reports an ethnographic study involving apprentice electrical engineers. Approaches to mathematics of conduit bending in the field, using rules of thumb, were distinctly different from trigonometry approaches in the apprentice classroom although both locations were guided by the country electrical code.

In contrast, in other papers with few mentions, the use of real-world was almost incidental, as expected. In E12 the real-world was used only to differentiate between using dynamic digital artefacts to solve abstract algebraic exercises and describing real world relationships. Similarly, in E11 McCloskey argues that the rituals of performing in school mathematics are sometimes distinct from ways of performing mathematics in the real-world. Whilst important, this received little attention in the paper. In a study of Year 7 Spanish students, the E17 authors describe a 'realistic context' of a breakfast held in the school gym with students to be seated on chairs in rows of equal length. The upper stream class students are described as using "a real world context that was exchanged for mathematical meanings". Clearly, the real-world was not needed to make sense of the task, nor was the solution reviewed in light of the real-world situation.

With similar tenuous links to the real-world, Jiang et al. (E7) analysed responses to test items by approximately 350 Grade 6 students, from China and Singapore. The use of speed was said to be, in part due to its connection between the mathematics and real world. Two questions are shown here:

> Q1. A man drove at 72 km/h for 2 h, then the distance he travelled was ______km.
> Q9. On Sunday, Judy went to see her grandma who lives 150 km away. After cycling at an average speed of 15 km/h for a few hours, she got tired and took a lift from a passing truck. The truck's average travelling speed is 75 km/h. When she got to her grandma's house, she checked the time and knew that the trip took her 6 h. Find the time she cycled.

These tasks raise questions of task authenticity. Palm (2006) describes authentic tasks as those representing a real-life situation or problem, whilst Van den Heuval-Panhuizen (2005) argues authentic tasks (should) require students to think about, or imagine themselves in, the context. For Q1 the real-world could be used for checking. For Q9, we ask—is it realistic for Judy to plan a 150 km bike ride to visit her grandma? Perhaps it is in China. Certainly, in Singapore a country with approximate 'dimensions' 50 km East to West and 27 km North to South and a coastline of 193 km (source: Wikipedia), it is not. A third task where distance to a bookshop was 72 km was similarly not realistic in Singapore.

In contrast, two papers had the maximum of seven explicit references to the real-world (E6, E8). Ding and Li (E6) undertook an analysis of how the distributive property is presented (319 instances) in a Chinese textbook series. Their main focus was on ascertaining how the transition from concrete (physical or visual) to abstract occurred. They claim activating real-world knowledge or experiences can increase solving and sense-making opportunities but warn "perceptually rich but irrelevant information may distract learners' attention or may be interpreted as an essential part of the intended concepts" (p. 103). The authors convey their view of 'real world contexts' in mathematics as being dispensable. For example (p. 107):

> Find the total cost for five jackets priced at ¥65 each and five pants priced at ¥45 each. The textbook provided two solutions $(65 + 45) \times 5$ and $65 \times 5 + 45 \times 5$ to this word problem, which together illustrated the distributive property $(65 + 45) \times 5 = 65 \times 5 + 45 \times 5$.

The context is irrelevant to the task solution and its use as a *border* (Stillman, 1998) can simply be ignored and the solution is not related to cost of clothing. In E6, the use of context was generally limited to introductory tasks and portrayed very much as allowing initial activation of student knowledge and as a necessary but minimised means to accessing abstract representations of the mathematics, seen as the aim of learning.

A distinctly different view of the real-world is presented in E8. This theoretical paper is a critique of PISA. Kanes et al. argue that whilst the domain of Mathematical Literacy highly values the real-world, a student who drew on additional knowledge of the real-world, outside that provided in the question item, would receive no credit and this is contrary to what PISA claims to assess. This paper resonates with the perspective of Andrews et al. (E1) who suggest that the reason Finnish students perform well on PISA, compared to TIMMS results, is not due to an increased emphasis in teaching and learning using real-world context, but rather to students' high literacy skills allowing them to interpret what a question is asking and undertake the required calculations.

Cleary, frequency of use of the term *real-world* was no indicator of its importance or role in the papers sampled.

4.3.3 In-Depth Analysis of the ESM Sample

A summary matrix of the in-depth analysis for the ESM 2014 sample is presented in Table 4.3. Column one presents the *type* of paper, column two identifies each paper and its *context focus*. The final column classifies the *role of the real-world* for those

Table 4.3 Context focus of sample papers and categorization of task contexts (ESM)

Type of paper	Paper (context focus)	Role of real-world task context
Theoretical (4)	E2 (RW tool for analysis, mainly situation)	–
	E11 (Situation context)	–
	E12 (Situation, using digital artefacts to bridge RW and abstract MW)	–
	E19 (Situation/historical, calculus to solve RW tasks)	–
Commentary (1)	E15 (Critical commentary, situation context)	–
Document analysis (3)	E6 Text book (Task context, concrete (incl. RW) → abstract)	Minor: Border
	E8 PISA (Task context, challenging authenticity of PISA)	Major: PISA
	E9 Policy (Situation and task context—curriculum focus (PS/MM/skills) impacts task type/context)	(Incidental)
Research-based (12)	E1 (Task context, based on PISA)	Major: PISA
	E3 (Task context, critical mathematics)	Major: Tapestry
	E4 (Cultural context—religion)	–
	E5 (Task context, medicine dosage)	Major: Tapestry
	E7 (Task context, speed)	Minor
	E10 (Task context, RW application of way/path finding—‘navigation of map tasks’)	Pseudo-real
	E13 (Task context, using RW to illustrate concept (⊾—plumb bob—Pythagoras teaching experiment)	Minor
	E14 (Task context, PISA based, graphical items)	Major: PISA
	E16 (Task context, conduit bending, classroom v workplace)	Major: Tapestry
	E17 (Mainly situation—found use of RW part of discourse expectations for high ability students)	(Incidental)
	E18 (Task context, RW of leisure/work DARTS amateur/professional)	Major: Tapestry
	E20 (Task context, RW 1 of 2 dimensions in lesson observation tool)	Minor

Note In E9 and E17 the real-world was mainly situational, but there was some incidental real-world task context focus

papers identified as having a task context focus and where this is essential (minor or major), a further categorisation by the embeddedness of task contexts presented by authors.

As shown in Table 4.3, the twenty papers were theoretical (4), commentary (1), document analyses (3), and eleven had a task context focus. In most papers, the term context was not defined, but its meaning, as operationalised by the author(s), could be inferred. In eight papers (all four theoretical papers: E2, E11, E12, E19; the commentary paper: E15; one document analysis: E9, and two research papers: E4, E17), context referred exclusively, or mainly, to a situation context (including digital, historical and cultural environments) rather than to a task context, even though the sample was selected based on the term real-world. In E4, the real-world focus was religion or culture.

In the remaining 12 papers (two document analyses and 10 research), for one the task context was *pseudo real* (E10), and for the remaining 11 it was *essential*. For four of the essential, the real-world task context had a *minor* focus. In E6 (document analysis) the role of the real world was classified as *border* and for the remaining three research papers (E6, E13, E20) the embeddedness of the real-world was unable to be further classified as actual tasks were not provided. The additional seven papers had the real-world as a *major* focus. Three of these focussed on PISA tasks (E1, E8, E14) and four (E3, E5, E16, E18) used *task context* as tapestry.

Three of the four studies where the task context was *tapestry* related to the world of work or leisure (drug dosages in nursing, conduit bending in electrical work, and dart scoring). All focussed on learning mathematics in vocational education. The fourth study was a teaching experiment from a reformist critical mathematics perspective where active engagement with ‘real’ mathematics by students was viewed as partly empowering marginalized students.

With respect to context being seen as a help or a hindrance, no study claimed it to be a hindrance. Some authors (e.g., E9) in their literature reviews presented previous claims to this effect, but none did so as a result of the study being reported. For example, the authors of E9 cited research by Cooper and Dunne (2000) (see Sect. 4.1). Others, such as E14 noted that success rates on more challenging questions are lower than on less challenging questions, as one would expect. Level of challenge directly correlated with the degree of contextualization or interaction of task solver with the context.

4.4 Content Analysis: JRME

4.4.1 Initial Analysis and Sample Selection

Journal for Research in Mathematics Education was first published in 1970 with one volume per year until 1997 (Vol. 28) with six issues. Since 1998 there have been five issues published each year. A search for context, task context, and real-world identified 906, 582, and 241 instances, respectively, over the life of the journal. For 2008–2017 (i.e., *post ICMI Study Volume*), the same search terms resulted in 244, 153, and 53. See Tables 4.4 and 4.5 for additional data.

Table 4.4 JRME frequency of search terms overall and by decade

Search term	All years 1970–2017	Decades[a]				
		70–77[a]	78–87	88–97	98–07	08–17
Context	906	49	129	262	222	244
Task context	582	30	81	168	150	153
Real-world	241	17	45	122	75	53
No. of papers	2121	320	505	508	405	383

[a]*Note* 1970–1997 is less than a decade as decades calculated from 2017 back in time

Table 4.5 JRME frequency of search terms by year for recent years

Search term	2010	2011	2012	2013	2014	2015	2016	2017
Context	22	22	26	30	23	21	21	20
Task context	14	15	17	19	15	14	14	14
Real-world	4	4	6	10	7	3	4	7
No. of papers	38	31	40	45	40	34	37	36

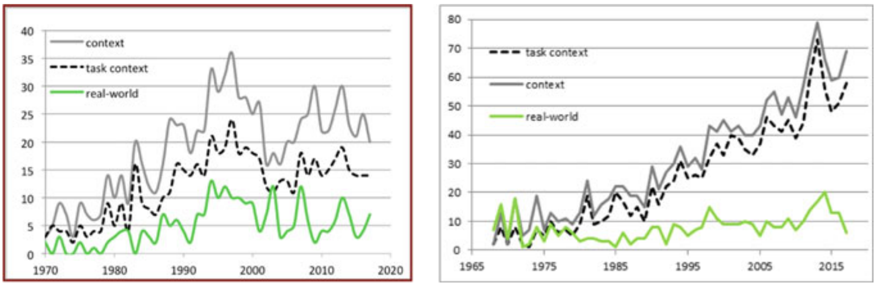

Fig. 4.2 Occurrence of search terms per year in JRME (left) and ESM (right)

Many patterns identified in the ESM data were not reflected in the JRME data. The term *context* was more frequent than *task context* and since the mid-1990s neither show evidence of the general increasing trend evident in the ESM data. Further patterns can be seen in Fig. 4.2 and clearly compared to the data from ESM. Note the vertical scale for JRME is half that for ESM. There is an increased use in terms *context* and *task context*, but substantially lower in JRME than in ESM. For the term *real-world*, both samples show a similarly low rate of increase over the journal history (approx. 0.13 per year based on linear trend line).

In 2014, the search for *real-world* identified seven papers from a total of 23 papers (30%, excluding book reviews). This proportion was similar to that of ESM (28%). Seven JRME papers were sampled (J1–J7) (see Appendix).

4.4.2 Detailed Analysis

In contrast to the geographical diversity shown by the study and author location in the ESM sample, in JRME 2014 six of the seven papers were written by authors based in USA (15 authors plus the NCTM committee of six with Lesh, a total of 22) and the research of US students or teachers. The remaining paper was written by two German researchers reporting a study of German secondary students. Hence, at least in the selected sample, the JRME data are almost exclusively from and about the USA. None of the papers had *real-world* or *context* as a key word. The only key words suggestive of real-world contexts were mathematical models, statistical models, and modelling—all in J7. The number of mentions of *real-world* was low (1–3), except for J7 with 38 instances.

Larsen et al. [J2] use the term real-world to describe the university environment where four IBL courses in which the students were taught, as they argue research-based student-centred learning can be the reality at universities [situation context]. Similarly, in J3 Mesa et al. provide a commentary on problems of mathematics instruction at US community colleges and note the disconnect between learning in class and real-world experiences with concepts. Munter [J5] details an interview-based instrument to characterize high quality mathematics instruction. The task dimension has five levels [0–4] with levels two and three referring to the real world. From level two, tasks focus beyond practising procedures and the real world can engage students, whilst problem solving and applications at level three emphasize real-world connections or prior knowledge.

For J1 the authors saw lack of explicit real-world context for negative integers as contributing to difficulties in understanding. They argue that one cognitive obstacle (subtrahend < minuend), identified both historically and in current student thinking, is in part related to the lack of real-world sense making of the notion of "removing more than one has" (p. 52). Contexts (e.g., money, elevation differences) are used in clinical interviews to provide a sense-making situation for 6–10-year-old students to develop conceptual understanding of integers—to overcome cognitive obstacles.

Moore [J4] presents one student's understanding of angle measure and trigonometric functions during participation in a teaching experiment. Tasks used included a person riding on a Ferris wheel and a bug riding on a fan blade. Real-world contexts provided a sense-making situation for the student to develop conceptual understanding of angle and the sine function (e.g., why position of bug on fan should be described relative to the length of fan blade). The author was clearly of the view that real world contexts would support student understanding, however, this was not explicitly discussed, nor was it part of the analysis reported.

In J6, the NCTM research committee report from an analysis of NCTM annual conference research pre-sessions that these sessions do not give enough attention to mathematical thinking "experiences that focus on mathematizing reality" (p. 169) from multiple areas of mathematics. They acknowledge that some such research is reported at more specialist biennial conferences such as ICTMA. To move forward, the authors propose research addressing the nature of problem-solving situations requiring mathematical thinking beyond school.

Schukajlow and Krug (J7) report on a teaching experiment to determine if encouraging multiple solutions impacted on student interest, competence, and autonomy. Students were prompted to provide multiple solutions to ill-defined real-world problems with vague conditions (e.g., not enough information). The authors clearly define the real-world as being outside the mathematical world. The vague conditions led to differing assumptions and hence different solutions. They argue that not only does solving real-world problems assist students in understanding the mathematics better, but also it allows students to "learn how they can apply mathematics and build mathematics models in their current and future lives" (p. 499). Encouraging multiple solutions had a positive effect on student interest, autonomy and competence.

4.4.3 In-Depth Analysis of JRME Sample

Table 4.6 is a summary matrix of the in-depth analysis for the JRME sample. For three papers, a *situational context* was the focus [J2, J3, J5]. Both J2 and J3 were commentary papers whilst J5 was research based. In J6 the real-world focus was *incidental* arising in the recommendations following the document analysis. The remaining three papers [J1, J4, J7] were research based with a *task context* focus.

The role of the real-world task context was categorised as *incidental* in J5, as this arose from the analysis of 900 interviews and J6 where clearly, the committee see the importance of the types of mathematical thinking inherent in solving real-world tasks, but the real-world focus was *incidental* in the arising recommendations. In both J1 and J4 the real-world task context was *minor*. Whilst J1 posited that real-world contexts would be helpful for young learners in providing integer related context, their study found that the students did not interact with the task in such a mathematical way. Rather the students reasoned about the absolute values related to the situation not using negative integers in their task solving. The teaching experiment in J7 was designed on the premise that the more realistic the task, the greater student interest and competence.

Similarly, to the ESM sample, papers in JRME, with the exception of J7 left it to the reader to infer what was implied by the real-world. All papers with a *task context* focus provided examples to illustrate their explanation. This allowed the researcher, and thus the reader, to easily infer if the role of the real-world was a major or minor focus of the tasks used and subsequently the level of embeddedness of the real-world in the task context following Stillman's categories of border, wrapper and tapestry.

Table 4.6 Context focus of sample papers and categorization of task contexts [JRME 2014]

Type of paper	Paper (context focus)	Role of the real-world task context
Commentary (2)	J2 (Brief report, situation context—undergraduate mathematics education)	–
	J3 (Research commentary, situation context community colleges)	–
Document Analysis (1)	J6 NCTM pre-session papers (Recommends increased research on RW mathematical thinking)	Incidental
Research–based (4)	J1 (Task context, learning about integers, contextual tasks one task type used)	Minor: border
	J4 [Task context, use RW (Ferris wheel, bug on fan) to illustrate concepts (radian, sine functions)]	Minor
	J5 (Focus on situation context of quality teaching, RW tasks 1 of 4 dimensions)	Incidental
	J7 (Task context, teaching experiment with RW tasks)	Major: wrapper

4.5 Content Analysis: MTL

4.5.1 Initial Analysis and Sample Selection

Mathematical Thinking and Learning (MTL), was first published in 1999 with three issues per year. A search for context, task context and real-world identified 249, 235 and 63 instances respectively over the life of the journal. Table 4.7 presents additional data. The term *context* and *task context* are found in the majority of papers. In contrast, *task context* is also found in most papers. The term *real-world* was found at lower rates (19% overall, 22% last decade) but higher than the rates for both ESM and JRME for the same time periods.

Table 4.7 shows in 2017 of 13 MTL papers, three (≈23%) included the term real-world. It must be noted that this journal published far fewer papers per year (in the last decade 183 papers, compared to 340 for JMB, 383 for JRME and 672 for ESM). Three papers (M1–M3) were sampled (see Appendix).

Table 4.7 MTL frequency of search terms

Search term	All years	2008–17	2013	2014	2015	2016	2017
Context	249	146	12	11	12	12	10
Task context	235	131	13	11	12	15	12
Real-world	63	40	5	3	0	6	3
No. of papers	330	185	13	12	15	12	12

Note Some cumulative years may include book reviews in addition to papers

4.5.2 Detailed Analysis

The authors of all papers were located in the USA as were the participants in their studies. Key words are not included on MTL papers. The papers had one (M2), four (M1) and 25 instances (M3) of the term real-world.

In M2, Stephens et al. investigated the functional thinking of 100 students, beginning in Grade 3 over three years. The authors draw on literature noting the importance of context in functional thinking, however, ‘real-world context’ used involved finding a relationship between the number of seats and number of desks being arranged at school for a party. Bargagliotti and Anderson (M1) describe statistical modelling as analogous to mathematical modelling. Solving real-world problems was one of the guiding principles for the professional learning, however, teachers used the available data to focus on developing key statistical understandings rather than solving real-world problems.

In M3, with 25 instances of *real-world* indicative of the major focus on real-world tasks, Wernet investigated interactions around context, especially those in the written curricula, in three Grade 8 classrooms. Mathematical modelling was central in the curriculum. Contextual tasks included realistic or imaginary situations whereas modelling tasks begin in the non-mathematical world and required mathematics to simplify, structure and solve the problem, which is then interpreted. Wernet classified tasks as displaying low authenticity, medium authenticity, or full alignment between the task and real-life scenario following Palm (2006). When implemented, tasks with low authenticity tended to stay low whereas those with at least medium authenticity tended to generate more discussion about context. Contrary to what is often claimed, Wernet reports that, in no instance were students observed to struggle with contextual understanding and drew appropriately upon their own everyday experiences. In fact, students mathematized with little difficulty, attributed to three years’ experience with contextual tasks in the curriculum including opportunities to discuss the contexts.

Table 4.8 Context focus of sample papers and categorization of task contexts [MTL 2017]

Type	Paper (context focus)	Role of real-world task context
Research-based (3)	M1 (Task context, RW one guiding principle for tasks developing statistical understanding)	Minor: Wrapper
	M2 (Task context, RW one considerations in task design for functional thinking)	Minor: Wrapper
	M3 (Task context, analysis of task written and enacted for real world authenticity)	Major

4.5.3 In-Depth Analysis of MTL Sample

All papers in the MTL sample were research-based and all had a *task context* focus. For two papers, the real-world task context had a *minor* focus and in both cases, the embeddedness of the real-world in the tasks was classified as *wrappe*r. For M3, where the real-world has a *majo*r focus, the author was analysing tasks used with respect to their authenticity. A summary matrix of the in-depth analysis is presented in Table 4.8.

4.6 Content Analysis: JMB

4.6.1 Initial Analysis and Sample Selection

Journal of Mathematical Behavior (JMB) was first published in 1990, but only available to search from 1995. A search for context, task context and real-world identified 621, 8 and 160 instances respectively (see Table 4.9). The term *context* is found in the majority of papers. In contrast, *task context* was rarely found. The term *real-world* was found at a similar rate (18% overall, 22% last decade) to MTL, higher than the corresponding rates for ESM and JRME.

Table 4.9 JMB frequency of search terms

Search term	All years[a]	2008–17	2013	2014	2015	2016	2017
Context	621	319	47	41	37	33	53
Task context	8	4	0	0	2	1	1
Real-world	160	74	12	10	9	5	12
No. of papers	876	340	51	43	40	39	53

[a]*Note* Data accessible 1995–2017. Some cumulative years may include book reviews

The search for *real-world* identified only 160 instances of the term 1994–2017 with 12 in 2017. A trend in this sample is difficult to discern. Twelve papers (B1–B12) were in the sample (see Appendix).

4.6.2 Detailed Analysis

The majority of authors and location of the studies were in the USA. Nine of the 12 papers had both authors and participants based in the USA. The theoretical paper (B2) had one author from Turkey and one from the USA. An additional research paper (B8) had one author and participants from Italy and two authors from Belgium. B7 had all authors and participants from Israel. As with JRME, this sample is almost exclusively from and about the US. Only one paper had *real-world* as a key word (B6) and none had *context* as a key word. The only other keywords indicative of real-world contexts were applications (B6) and mathematical modelling (B3) and possibly 'word problem solving' (B8).

In nine papers, the *real-world* was mentioned 1–3 times and in three papers (B2, B6, B8) 4–7 instances. Again, this frequency was not sufficient to determine the importance of the real-world to the authors. For four papers with few mentions the real-world was incidental. Hopkins et al. (B5) undertook a study of the role of coaches in a school district (14 primary schools) undergoing reform. Whilst arguing that 'ambitious mathematics teaching' includes providing opportunities for students to solve real-world problems, no analysis was reported specifically linked to solving real-world problems. Smith et al. (B10) researched 'instructional teacher leadership' and it was a participant who emphasised the real-world, noting she was now focussing on "making it real world to them" (p. 276). B1 reports 251 secondary mathematics teachers' "meanings for slope, measurement, and rate of change" (p. 168). In B7, the study involved 60 Grade 9 Israeli students and the extent of surprise in the solution of two abstract geometry problems. The sample lesson snippet used a real-world context of bicycle riders.

Similarly, with few mentions of the real-world, three papers had this as a minor focus. Harel (B4) undertook a teaching experiment with in-service secondary mathematics teachers on the theory of systems of linear equations. Although tasks used in the introductory unit include real-world contexts (e.g., traffic flow) and the teachers "indicated that they felt that the engagement in the unit's 'real-world' scenarios" (p. 79) enhanced their understanding, no real-world scenarios are reported as being presented in the main unit. The literature review in B11 included how understanding of whole numbers and negative integers can be grounded in real world contexts; but, in the clinical interviews, none of the questions reported were set in a real-world context, although analysis identified task solvers invoking the real-world. Wickstrom et al.'s (B12) study of pre-service primary teachers' conceptions of area, drew on literature that noted, "they demonstrate a procedural understanding of area often limited to memorized formulas disconnected from real-world applications" (p. 112) and the premise that such understanding is not sufficient for future teaching. The pre-task was abstract, but the post task was set in a real-world context, namely tiling a shower floor. Despite the authors drawing on literature related to understanding in real-world settings, this was not discussed in their analysis.

Two papers with few mentions of the real-world had this as a major focus. Paoletti and Moore (B9), undertook a teaching experiment with two pre-service undergraduate secondary mathematics teachers exploring covariational reasoning. Tasks used included bottle filling and emptying (see Swan 1985) and travelling between two towns using an applet. Results suggest real-world situations such as a Ferris Wheel moving in different directions or a car travelling to and from school will support students' parametric reasoning. Czocher (B3) compared two approaches to teaching undergraduate engineers, one emphasising decontextualized techniques to solve differential equations, whilst the other "emphasised modelling principles to derive and interpret canonical differential equations as models of real world phenomena" (p. 78). Her statistically significant results showed the modelling approach aided student learning. Data came from extensive classroom observation and three common problems on the final examination involving contextualized examples. Czocher noted the students who experienced the modelling perspective were more flexible in their thinking and better able to handle initial conditions.

The papers with more mentions of the real-world also varied in emphasis with one (B2) dismissing its usefulness. Cetin and Dubinsky's theoretical paper (B2) discusses decontextualization as one meaning ascribed to reflective abstraction. They dismiss the argument that the absence of context is what makes abstraction difficult and question use of real-world contexts to teach mathematical concepts for three reasons: "what is 'real-world'" (p. 71) varies for the individual; there is a danger students might learn something about the context but little about mathematics; and claim there is little research showing that realistic contexts help students learn decontextualized mathematics.

In contrast, in B6 and B8, the real-world was of major importance. Jones (B6) reports an exploratory study in first year calculus, arguing the majority of research in the area, focuses on kinematics and seeks to address this gap in the literature. Jones reports "applied contexts seem to bring out covariation-based thinking more than pure mathematics contexts" (p. 107). The tendency for some students to invoke time, in timeless contexts, to help with sense making, whilst sometimes helpful became problematic. Clearly, more experiences with contexts where time is not a variable would be helpful. Mellone et al. (B8) investigated whether there is a relationship between Grade 5 students' situation models and the realistic nature of their answers to problems. Clearly defining modelling as the process of creating a mathematical model from a situation model, they found working in pairs and rewording then solving led to an increase in realistic responses but for only one problem.

4.6.3 In-Depth Analysis of the JMB Sample

Eleven of the papers in the sample were research-based and the remaining paper theoretical. With regard to the context focus, they were more challenging to categorise than in the other two samples. For nine papers, the context was clearly a real-world task context (B1, B3, B4, B6, B7, B8, B9, B11 and B12) however for three papers (B2, B5, B10) the classification of situational or task context was not possible. The reasons for this varied, in B2 the real-world is dismissed, in B5 it relates to the goal of teaching, whereas in B10 it arose in the data collected. A summary matrix of the in-depth analysis is presented in Table 4.10.

For the nine papers, able to be classified by context focus, this was clearly on a *real-world task context* in all papers. For two, this was *incidental* (B1, B7) and the other seven *essential* (four *major* focus, three *minor* focus). For all four where the real-world context was a major focus, the embeddedness of the real-world was categorised as *tapestry*. For the three with a minor focus, one was classified as *border*. The remaining two were not classified further, as in B4 no actual tasks were reported and in B11 the real-world was evoked by the task solvers rather than the task setters who presented abstract tasks.

For B1, the real-world was classified as *incidental* as it was the teacher participants who used real-world examples (i.e., inclined planes, ski slopes) where steepness could be visualised. Similarly, in B7, the real-world was *incidental*, arising when the author compared the real-world to the mathematical world in discussing surprising situations in mathematics.

Task context was classified as having a minor focus in three papers. In B12, although the authors drew on relevant literature and had one of two tasks with a real-world context, there was no analysis or discussion related to the real-world. Similarly, in B4, the real-world was used only in the introductory unit of their study and although found helpful by teacher participants played no part in the majority of this research. The study by Whitacre et al. (B11) of school students' reasoning about integer comparisons was the only example from all samples, where the real-world context was evoked by the task solver as described by Boero (1999, p. vii). In all other cases, the real-world was *evoked by the task setter*, but here, although the task was abstract, the task solver brought in the real world to support problem solution.

Four papers (B3, B6, B8, B9) were classified as having a major focus on real-world task context, all with the embeddedness of the real-world as tapestry. Three of these had a focus at university undergraduate level, B3 with two classes of engineering students, B6 first year calculus students and B9, pre-service undergraduate secondary mathematics teachers. In contrast, B8 reported a study of Grade 5 school students.

Table 4.10 Context focus of sample papers and categorization of task contexts [JMB]

Type	Paper (context focus)	Role of real-world task context
Theoretical (1)	B2 (dismiss use of RW as they focus on abstraction)	–
Research-based (11)	B1 (Task context, inclined plane, ski slope suggested by teacher participant in study of rate of change)	Incidental
	B3 (Task context, comparison of content vs. context approach to teaching DEs)	Major: Tapestry
	B4 (Task context, real-world contexts for initial units about systems of linear equations)	Minor[a]
	B5 (Task context, goal of 'ambitious teaching includes solving real world problems)	Incidental
	B6 (Task context, moving beyond kinematics context for applications of derivatives)	Major: Tapestry
	B7 (Task context, abstract geometry problems, lesson illustrated used real-world task)	Incidental
	B8 (Task context, pair work and student rewording of tasks to increase rate of realist solutions)	Major: Tapestry
	B9 (Task context, covariational reasoning, bottle problem, car problem—driving between 2 cities)	Major: Tapestry
	B10 (Participant notes importance of real-world)	Incidental
	B11 (Task context, evoked by task solvers)	Minor
	B12 (Task context, post task item shower tiling)	Minor: border

[a]*Note* Tasks not given so no further classification possible

4.7 Discussion: Looking Across the Samples

As shown in Fig. 4.3, across the four journal samples, most times the *real-world* was mentioned (34 of 42 papers, approx. 81%) this was in reference to the *task context* rather than situation context. However, the author's purpose in just over 25% (nine papers) was *incidental* and arose in a review or discussion of the literature or in the data or its analysis or recommendations. This ranged from dismissing the

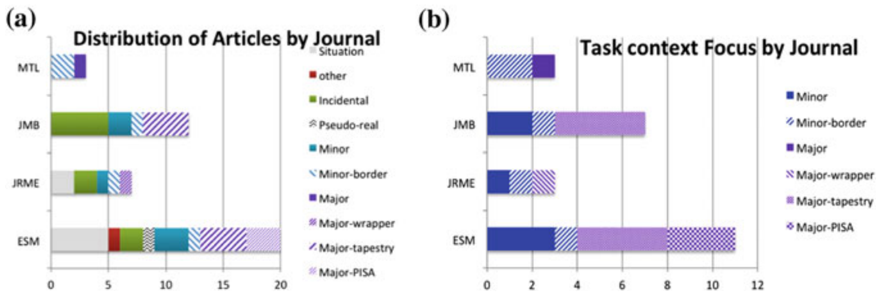

Fig. 4.3 Distribution of categories. **a** All categories, **b** task context classification only

'use of real-world contexts' to focus on abstraction, as a part of defining ambitious teaching, arising in the data or its analysis to recommendations for the use of real-world contexts. In the remaining 25 papers, one had a *pseudo-real* context, and 24 the real-world task context was *essential* with 11 having a *minor* focus and 13 a *major* focus on the real-world task context.

To answer the overarching research question, *How is context viewed?* It is helpful to consider the type of paper. Excluding the 10 non-research papers (i.e., theoretical papers or commentaries of which nine had a situation focus) and consider the 34 papers reporting research (including document analyses), all but one (E4 Cultural context) had a *task context* focus. Hence, for almost all authors reporting research, context was viewed as the *real-world task context* whereas for non-research papers, the real-world was part of the situation context. As noted, context was most often not defined although its meaning could be inferred.

In determining, *What are the meanings and role of real-world task context?* three overarching categories (incidental, pseudo-real, and essential) were defined and used in the analysis of the papers with a real-world task-context focus. Of the 33 research papers with a focus on *real-world task context*, this focus was *incidental* in eight, *pseudo-real* in one, and for the majority (24) *essential*. For 11 of these 24 research papers, the focus was *minor* and for 14 a *major* focus. So, in considering the research papers, not only is the context most likely to be a real-world task context, this focus on the real world is more likely to be *essential* than not. Furthermore, when task context had a *minor* focus and tasks could be further characterized, this tended to be as *border* or *wrapper* (not tapestry). In contrast, where the real-world task context was a *major* focus, tasks were almost exclusively classified as *tapestry* (or PISA).

Of the papers where the focus in the *real-world task context* was *essential*, seven papers (6 of 11 minor, 1 of 13 major) were unable to be further classified in terms of task context embeddedness (border, wrapper, or tapestry). The reasons for this varied. In one paper, the researchers deliberately used real-world contexts to illustrate key mathematical ideas. In another, the real-world was one dimension of the analysis but gave no further details, and in a third, the task solver(s) evoked the real-world in solving abstract tasks.

Context was at times portrayed as a hindrance, however, this only occurred when authors referred to other studies (usually very selectively) or were theorising. These authors also tended to see the real-world as (only) a pathway to the abstract mathematical world. In the actual research reported in these four purposive samples, in no study was a *real-world context* found to hinder learning. In contrast, the opposite was reported, the *real-world* helped in teaching and learning (four studies) and one reported mixed findings.

The four papers reporting positive outcomes include the teaching experiment comparing modelling versus decontextualized approaches to teaching differential equations in first year calculus in terms of performance on the final examination. Both low and high achievers performed significantly better in the class with the modelling perspective, being more flexible in their thinking and better able to handle initial conditions. At the secondary level, two papers reported *real-world context* as helpful. In Grade 8, rich contexts, particularly when teachers supported sense-making discussion about the context and the mathematics, supported student engagement with tasks of high cognitive demand. Requiring Grade 9 students to provide multiple solutions to authentic real-world problems had a positive effect on student interest and competence. In the fourth paper, it was the secondary teacher participants in the study who reported the usefulness of the *real-world contexts* in supporting their understanding.

A further 12 research studies had *real-world task contexts* as an inherent part of their study, from which it is inferred the authors had the expectation that real-world contexts are supportive of teaching and/or learning. For some, this was an integral part of the mathematics that was the focus of the study (e.g., primary: speed, mapping; secondary: trigonometry; tertiary: (first year calculus) derivatives, (nurse education) drug dosages, (teacher education) co-variational reasoning; and in-service teachers: statistics). Given over half of all papers and over 70% of the research papers sampled considered the *real-world task context* as playing an *essential role*, this author concurs with Niss et al. (2007) suggesting the maturation of the applications and mathematical modelling research discipline.

4.8 Concluding Remarks

It appears the nature of the construct: *context* previously described as nebulous (Busse and Kaiser 2003) has become more focussed in recent times. Although, drawing on the analysis of the overall data and the purposive samples, the construct *context* is still used in multiple ways as previously noted by Boero (1999). At times the construct was not explicitly defined although its meaning in the sample analysed could be inferred. It is incumbent on the modelling and applications community and in fact all mathematics education researchers to clearly articulate when the real-world is an important aspect of their research.

Stacey (2015) in articulating the way PISA "theorises and operationalises the links between the real world and the mathematical world" (p. 57) notes that using real-world contexts is considered essential in the teaching and learning of mathematics. Context in PISA "refers specifically to those aspects of the real world that are used in the item" (p. 74). This *essential* use of context was evident in the majority of papers in the purposively selected samples reported in this chapter. What constituted the *real-world* (Niss et al. 2007), the authenticity of the context (Palm 2006; Van den Heuvel-Panhuizen 2005), and the degree of embeddedness of the real-world task context (Stillman 1998) varied greatly. Clearly, when researchers had the real-world context as a *major* focus, this degree of embeddedness was higher with tasks characterised as *tapestry* (or PISA) whereas if only a *minor* focus, the embeddedness tended to be lower, tasks characterised as *border* or *wrapper.* As the level of challenge for students generally directly correlated with the degree of contextualization or interaction of task solver with the context, it is important all students have opportunities to interact with 'tapestry type tasks' (Stillman, 1998). Notwithstanding the challenges inherent in solving tasks of high cognitive demand, in part due to the real-world task context (Dapueto and Parenti 1999), no studies reported findings where the real-world context hindered learning. Researchers focusing on *real-world task contexts* do consider these as critical and hence need to be understood, at least in order to understand the problem, if not throughout the solution process. In contrast, a minor focus on the real-world generally saw trivial contexts or those that the task solver could ignore entirely, showing that this essential use of real-world contexts is not accepted by all in the mathematics education community.

Whilst some papers reported research where context helped learning, none concluded context was a hindrance, and rather more papers were not even considering this question as important. Perhaps this question has, for most, become too simplistic to consider as the complexities of learning, particularly when engaging with real-world tasks, are well understood by researchers who see this engagement as essential and are more focused on other aspects of learning assuming the real-world is an intrinsic part of this process.

Knowing mathematics means learners can use their mathematics to solve real-world problems (e.g., Freudenthal 1973; Gravemeijer et al. 2017; Pollak 1969). Further research is recommended in school mathematics classrooms, ascertaining ways in which teachers should be aspiring to support learners in knowing more about the world in which they live and analysing how the real-world contexts support student learning of mathematics and maintaining the high cognitive demand of such tasks. The real-world is a complex and messy place, thus real-world task contexts should reflect this reality and the embeddedness of the task should, following Stillman (1998), be at least *wrapper*—where task solvers must consider the context—if not at the highest level of *tapestry*—where the real-world and mathematics are interwoven, and both must be engaged with throughout the solution process. Finally, researchers must acknowledge that such tasks involve higher order thinking and are necessarily more challenging and demanding of learners. Engagement by learners with such tasks is a critical part of mathematics for all learners at all levels of schooling and beyond.

Appendix/Online Supplementary Material

ESM Sample

E1 Andrews, P., Ryve, A., Hemmi, K., & Sayers, J. (2014). PISA, TIMSS and Finnish mathematics teaching: An enigma in search of an explanation. *Educational Studies in Mathematics, 87*, 7–26.

E2 Artigue, M., & Mariotti, M. A. (2014). Networking theoretical frames: The ReMath enterprise. *Educational Studies in Mathematics, 85*, 329–355.

E3 Brantlinger, A. (2014). Critical mathematics discourse in a high school classroom: Examining patterns of student engagement and resistance. *Educational Studies in Mathematics, 85*, 201–220.

E4 Chan, Y.-C., & Wong, N.-Y. (2014). Worldviews, religions, and beliefs about teaching and learning: Perception of mathematics teachers with different religious backgrounds. *Educational Studies in Mathematics, 87*, 251–277.

E5 Coben, D., & Weeks, K. (2014). Meeting the mathematical demands of the safety-critical workplace: Medication dosage calculation problem-solving for nursing. *Educational Studies in Mathematics, 86*, 253–270.

E6 Ding, M., & Li, X. (2014). Transition from concrete to abstract representations: The distributive property in a Chinese textbook series. *Educational Studies in Mathematics, 87*, 103–121.

E7 Jiang, C., Hwang, S., & Cai, J. (2014). Chinese and Singaporean sixth-grade students' strategies for solving problems about speed. *Educational Studies in Mathematics, 87*, 27–50.

E8 Kanes, C., Morgan, C., & Tsatsaroni, A. (2014). The PISA mathematics regime: Knowledge structures and practices of the self. *Educational Studies in Mathematics, 87*, 145–165.

E9 Lerman, S. (2014). Mapping the effects of policy on mathematics teacher education. *Educational Studies in Mathematics, 87*, 187–201.

E10 Logan, T., Lowrie, T., & Diezmann, C. (2014). Co-thought gestures: Supporting students to successfully navigate map tasks. *Educational Studies in Mathematics, 87*, 87–102.

E11 McCloskey, M. (2014). The promise of ritual: A lens for understanding persistent practices in mathematics classrooms. *Educational Studies in Mathematics, 86*, 19–38.

E12 Morgan, C., & Kynigos, C. (2014). Digital artefacts as representations: Forging connections between a constructionist and a social semiotic perspective. *Educational Studies in Mathematics, 85*, 357–379.

E13 Moutsios-Rentzos, A., Spyrou, P., & Peteinara, A. (2014). The objectification of the right-angled triangle in the teaching of the Pythagorean Theorem: An empirical investigation. *Educational Studies in Mathematics, 85*, 29–51.

E14 Olande, O. (2014). Graphical artefacts: Taxonomy of students' response to test items. *Educational Studies in Mathematics, 85*, 53–74.

E15 Radford, L. (2014). On the role of representations and artefacts in knowing and learning. *Educational Studies in Mathematics, 85*, 405–422.

E16 Roth, W.-M. (2014). Rules of bending, bending the rules: The geometry of electrical conduit bending in college and workplace. *Educational Studies in Mathematics, 86*, 177–192.

E17 Straehler-Pohl, H., Fernández, S., & Gellert, U. (2014). School mathematics registers in a context of low academic expectations. *Educational Studies in Mathematics, 85*, 175–199.

E18 Swanson, D., & Williams, J. (2014). Making abstract mathematics concrete in and out of school. *Educational Studies in Mathematics, 86*, 193–209.

E19 Tall, D., & Katz, M. (2014). A cognitive analysis of Cauchy's conceptions of function, continuity, limit and infinitesimal, with implications for teaching the calculus. *Educational Studies in Mathematics, 86*, 97–124.

E20 Walkowiak, T. A., Berry, R. Q., Meyer, J. P., Rimm-Kaufman, S. E., & Ottmar, E. R. (2014). Introducing an observational measure of standards-based mathematics teaching practices: Evidence of validity. *Educational Studies in Mathematics, 85*, 109–128

JRME Sample

J1 Bishop, J. P., Lamb, L., Philipp, R. Whitacre, I., Schappelle, B., & Lewis, M. (2014). Obstacles and affordances for integer reasoning: An analysis of children's thinking and the history of mathematics. *Journal for Research in Mathematics Education, 45*(1), 19–61.

J2 Laursen, S. L., Hassi, M.-L., Kogan, M., & Weston, T. (2014). Benefits for women and men of inquiry-based learning in college mathematics: A multi-institution study. *Journal for Research in Mathematics Education, 45*(4), 406–418.

J3 Mesa, V., Wladis, C., & Watkins, L. (2014). Research problems in community college mathematics education: Testing the boundaries of K-12 research. *Journal for Research in Mathematics Education, 45*(2), 173–192.

J4 Moore, K. (2014). Quantitative reasoning and the sine function: The case of Zac. *Journal for Research in Mathematics Education, 45*(1), 102–138.

J5 Munter, C. (2014). Developing visions of high-quality mathematics instruction. *Journal for Research in Mathematics Education, 45*(5), 584–635.

J6 NCTM Research Committee. (2014). The NCTM research presession: A brief history and reflection. *Journal for Research in Mathematics Education, 45*(2), 157–172.

J7 Schukajlow, S. & André Krug. (2014). Do multiple solutions matter? Prompting multiple solutions, interest, competence, and autonomy. *Journal for Research in Mathematics Education, 45*(4), 497–533

MTL Sample

M1 Bargagliotti, A. E., & Anderson, C. R. (2017). Using learning trajectories for teacher learning to structure professional development. *Mathematical Thinking and Learning, 19*(4), 237–259.

M2 Stephens, A. C., Fonger, N., Strachota, S., Isler, I., Blanton, M., Knuth, E., & Gardiner, A. M. (2017). A learning progression for elementary students' functional thinking. *Mathematical Thinking and Learning, 19*(3), 143–166.

M3 Wernet, J. L. (2017). Classroom interactions around problem contexts and task authenticity in middle school mathematics. *Mathematical Thinking and Learning, 19*(2), 69–94.

JMB Sample

B1 Byerley, C., & Thompson, P. W. (2017). Secondary mathematics teachers' meanings for measure, slope, and rate of change. *Journal of Mathematical Behavior, 48*, 168–193.

B2 Cetin, I., & Dubinsky, E. (2017). Reflective abstraction in computational thinking. *Journal of Mathematical Behavior, 47*, 70–80.

B3 Czocher, J. A. (2017). How can emphasizing mathematical modeling principles benefit students in a traditionally taught differential equations course? *Journal of Mathematical Behavior, 45*, 78–94.

B4 Harel, G. (2017). The learning and teaching of linear algebra: Observations and generalizations, *Journal of Mathematical Behavior, 46*, 69–95.

B5 Hopkins, M., Ozimek, D., & Sweet, T. M. (2017). Mathematics coaching and instructional reform: Individual and collective change. *Journal of Mathematical Behavior, 46*, 215–230.

B6 Jones, S. R. (2017). An exploratory study on student understandings of derivatives in real-world, non-kinematics contexts. *Journal of Mathematical Behavior, 45*, 95–110.

B7 Koichu, B., Katz, E., & Berman, A. (2017). Stimulating student aesthetic response to mathematical problems by means of manipulating the extent of surprise. *Journal of Mathematical Behavior, 46*, 42–57.

B8 Mellone, M., Verschaffel, L., & Van Dooren, W. (2017). The effect of rewording and dyadic interaction on realistic reasoning in solving word problems. *Journal of Mathematical Behavior, 46*, 1–12.

B9 Paoletti, T., & Moore, K. C. (2017). The parametric nature of two students' covariational reasoning. *Journal of Mathematical Behavior, 48*, 137–151.

B10 Smith, P. S., Meredith L. Hayes, M. L., & Lyons, K. M. (2017). The ecology of instructional teacher leadership. *Journal of Mathematical Behavior, 46*, 267–288.

B11 Whitacre, I., Azuz, B., Lamb, L. L. C., Bishop, J. P., Schappelle, B. P., & Philipp, R. A. (2017). Integer comparisons across the grades: Students' justifications and ways of reasoning. *Journal of Mathematical Behavior, 45*, 47–62.

B12 Wickstrom, M. H., Fulton, E. W., & Carlson, M. A. (2017). Pre-service elementary teachers' strategies for tiling and relating area units. *Journal of Mathematical Behavior, 48*, 112–136.

References

Blum, W., Galbraith, P., Henn, H.-W., & Niss, M. (Eds.). (2007). *Modelling and applications in mathematics education*. New York, NY: Springer.

Boaler, J. (1994). When do girls prefer football to fashion? An analysis of female underachievement in relation to 'realistic' mathematic contexts. *British Educational Research Journal, 20*(5), 551–564.

Boero, P. (1999). Guest editorial. *Educational Studies in Mathematics, 39*(1), vii–x.

Bowen, G. A. (2009). Document analysis as a qualitative research method. *Qualitative Research Journal, 9*(2), 27–40.

Busse, A., & Kaiser, G. (2003). Context in application and modelling: An empirical approach. In Q.-X. Ye, W. Blum, I. D. Huntley, & N. T. Neil (Eds.), *Mathematical modelling in education and culture* (pp. 95–107). Chichester, UK: Horwood.

Cooper, B., & Dunne, M. (2000). *Assessing children's mathematical knowledge: Social class, sex and problem-solving.* Buckingham, UK: Open University.

Dapueto, C., & Parenti, L. (1999). Contributions and obstacles of contexts in the development of mathematical knowledge. *Educational Studies in Mathematics, 39*(1), 1–21.

Freudenthal, H. (1973). *Mathematics as an educational task*. Dordrecht, The Netherlands: Reidel.

Gravemeijer, K., Stephan, M., Julie, C., Lin, F.-L., & Ohtani, M. (2017). What mathematics education may prepare students for the society of the future*? International Journal of Science and Mathematics Education, 15*(Supplement 1), 105–123.

Huang, H. (2004). The impact of context on children's performance in solving everyday mathematical problems with real-world settings. *Journal of Research in Childhood Education, 18*(4), 278–292.

Kisker, E. E., Lipka, J., Adams, B. L., Rickard, A., Andrew-Ihrke, D., Yanez, E. E., & Millard, A. (2012). The potential of a culturally based supplemental mathematics curriculum to improve the mathematics performance of Alaska native and other students. *Journal for Research in Mathematics Education, 43*(1), 75–113.

Langrall, C., Nisbet, S., & Mooney, E. (2006). The interplay between student's statistical knowledge and context knowledge in analyzing data. In A. Rossman & B. Chance (Eds.), *Proceedings of the Seventh International Conference on Teaching Statistics, Salvador, Brazil [CD-ROM]*. Voorburg, The Netherlands: International Statistical Institute. Available: https://iase-web.org/Conference_Proceedings.php?p=ICOTS_7_2006

Miles, M., Hubermann, M., & Saldaña, J. (2014). *Qualitative data analysis: A methods sourcebook and the coding manual for qualitative researchers*. Thousand Oaks, CA: SAGE.

Niss, M., Galbraith, P., & Blum, W. (2007). Introduction. In W. Blum, et al. (Eds.), *Modelling and applications in mathematics education* (pp. 3–30). New York, NY: Springer.

Nivens, R., & Otten, S. (2017). Assessing journal quality in mathematics education. *Journal for Research in Mathematics Education, 48*(4), 348–368.

Palm, T. (2006). Word problems as simulations of real-world situations: A proposed framework. *For the Learning of Mathematics, 26*(1), 42–47.

Patton, M. Q. (2002). *Qualitative research and evaluation methods* (3rd ed.). Thousand Oaks, CA: SAGE.

Pollak, H. O. (1969). How can we teach applications of mathematics? *Educational Studies in Mathematics, 2*(2–3), 393–404.

Smith, C., & Morgan, C. (2016). Curricular orientations to real-world contexts in mathematics. *The Curriculum Journal, 27*(1), 24–45.

Stacey, K. (2015). The real world and the mathematical world. In K. Stacey & R. Turner (Eds.), *Assessing mathematical literacy* (pp. 57–84). Cham, Switzerland: Springer.

Stillman, G. (1998). The emperor's new clothes? Teaching and assessment of mathematical applications at the senior level. In P. Galbraith, W. Blum, G. Booker, & D. Huntley (Eds.), *Mathematical modelling: Teaching and assessment in a technology-rich world* (pp. 243–253). Chichester, UK: Horwood.

Stillman, G. (2004). Strategies employed by upper secondary students for overcoming or exploiting conditions affecting accessibility of applications tasks. *Mathematics Education Research Journal, 16*(1), 41–70.

Swan, M. (1985). *The language of functions and graphs*. Nottingham, UK: Shell Centre.

Toerner, G., & Arzarello, F. (2012, December). Grading mathematics education research journal. *EMS Newsletter*, 52–54.

Van den Heuvel-Panhuizen, M. (2005). The role of context in assessment problems in mathematics. *For the Learning of Mathematics, 25*(2), 2–9, 23.

Wedege, T. (1999). To know or not to know—Mathematics, that is a question of context. *Educational Studies in Mathematics, 39*(1), 205–227.

Wijaya, A., van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2014). Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors. *The Mathematics Enthusiast, 11*(3), 555–584.

Wroughton, J., McGowan, H., Weiss, L., & Cope, T. (2013). Exploring the role of context in students' understanding of sampling. *Statistics Education Research Journal, 12*(2), 32–58.

Zevenbergen, R., Sullivan, P., & Mousley, J. (2002). Contexts in mathematics education: Help? Hindrance? For whom? In P. Valero & O. Skovsmose (Eds.), *Proceedings of the 3rd International MES Conference* (pp. 1–9). Copenhagen, Denmark: CRLM.