

PERCEPTIONS OF FUZZY SET THEORY IN CONSTRUCTION RISK ANALYSIS

Olubukola Tokede^a and Sam Wamuziri^{b,1}

^a School of Engineering and the Built Environment, Edinburgh Napier University, Edinburgh, EH10 5DT, UK

^b University Management Research Centre, Glyndŵr University, Wrexham, LL11 2AW, UK

Over the last three decades, organizations have increasingly been taking account of risk and uncertainty in their decision-making processes. The traditional methodology for risk management involves risk identification, risk analysis and management response to risk. Common risk analysis techniques in the construction industry include sensitivity analysis, probability analysis, Monte Carlo simulation, beta analysis etc. In this regard, Monte-Carlo simulation has been a touchstone of simulation efficiency across contemporary literature on risk analysis. However, a number of concerns have been raised about the intractability and construct validity of Monte Carlo representations for construction risks. The use of Monte Carlo simulation is based on the premise that the uncertain or risk event can be defined by a known probability distribution function. Unfortunately, this is not the case as far as risk is concerned in construction projects. It is in situations such as these where precise probability density functions cannot be ascribed to uncertain events that fuzzy set analysis becomes helpful. This research investigates the perceptions of fuzzy set theory and its influence on the practice of construction risk analysis. This work is part of a larger study that aims to investigate the specific insights that fuzzy set theory could bring into the construction risk analysis discipline. Purposive sampling was employed in selection of participants for the investigation. Structured interviews were conducted with highly experienced construction professionals in the United Kingdom. The results of these studies reveal that fuzzy set theory can enhance reasoning in analysing construction risks. However, fuzzy set theory as a stand-alone mathematical tool is not sufficient to complete the risk analysis process. Furthermore, the study indicates the vulnerability, volatility and computational intensiveness of fuzzy set theory may discourage construction practitioners from embracing the principles and applications of fuzzy set theory

Keywords: decision analysis, fuzzy set theory, risk analysis, risk modelling.

INTRODUCTION

In recent years, poor performance of construction projects has provoked an increased interest into the nature and mechanism of risk analysis and management (Smith 2006). Construction risks have been mostly found to exhibit dynamism and continuity across a project's lifecycle (Chan *et al.* 2009; Nieto-morote & Ruz-vila 2010). This

¹ s.wamuziri@glyndŵr.ac.uk

conceptual framework has buttressed the need to improve the thoroughness and accuracy in the calculus of risk modelling procedures.

It has previously been considered that dynamism and continuity are attributes pragmatically suited for random and iterative manipulations (Sadeghi *et al.* 2010). This approach to risk evaluation derives its conception from probabilistic techniques. It is equally instructive that Monte Carlo simulation, a widely-known probabilistic technique is commonly employed for risk analysis procedures in the construction industry (Ayyub 2003). In many instances, Monte Carlo simulation has been found useful in evaluating outcome variance in a manner that produces the fairest summary. Of primary importance is the recognition that Monte Carlo simulation has exhibited great potential and use in many industrial risk analysis procedures. The initiative of treating every uncertainty as variability however constitutes a fatal presumption (Boussabaine & Kirkham 2004). Ayyub (1999) had reckoned that project uncertainty could result from a variety of situations among which are ambiguity and vagueness. Consequently, it is still commonly emphasized that the information potential of construction projects is not being fully harnessed (Shaopei 1998). Molenaar (2005) classified information on construction projects as "Known/Knowns", "Known/Unknowns" and "Unknown/Unknowns". Each of these categories of information requires different approaches to define and analyse at any particular time. Traditional methods usually adopt a deterministic and conservative approach which often proves insufficient in contemporary analysis and management of uncertainty. Therefore, it is considered that the utility of many risk models in fulfilling the risk analysis function is limited. In this respect, fuzzy set theory provides a useful framework for better representing the information about construction projects (Long & Ohsato 2008). Another explicit benefit in risk modelling is that fuzzy sets can better depict situational reality which ultimately assists the efficiency of decision-support for construction projects (Fayek & Oduba 2005).

Despite this crucial acknowledgement in the potential of fuzzy set theory to augment the process of risk analysis, many construction practitioners have doubts on the efficacy of fuzzy sets. Previous work carried out by Kangari (1986) was tailored at developing a conceptual model for fuzzy set theory in construction risk analysis. More recent work has advanced the mathematical rigour of risk events in established phases of construction projects (Chen & Huang 2007; Long & Ohsato 2008). Considerably, most research efforts on fuzzy set theory in construction risk analysis have focused on the development of its mathematical axioms and syntaxes. Little interest has been shown in relation to construction practitioners and the influence the attitudes in the construction sector might imply for applying fuzzy set theory to risk analysis. This study explores the perceptions of the principles of fuzzy set theory and its practical implication for construction risk analysis and management.

RESEARCH AIMS

The aim of this research exercise is to investigate the perceptions of fuzzy set theory within the context of the attitudes prevalent in the construction sector. The research questions were structured in open-ended forms. To this end, the questions were electronically sent to the interviewees ahead of the actual interview. The questions were intended to provide a basis for interaction on specific and general concerns in construction risk analysis. It also aimed to capture the informed opinions of construction experts. In effect, the research questions are stated below.

1. Does fuzzy set theory represent a valid mathematical modelling technique for analysing risk in construction projects?
2. If positive response is given to question 1(above), in what aspect(s) of the construction project lifecycle does fuzzy set theory have validity and utility?
3. Are there any peculiarities that limit the application of fuzzy set theory in the construction sector?
4. What comparative significance exists between fuzzy set theory and other established risk analysis tools and techniques in the construction sector?
5. Are construction professionals adequately trained to apply the concepts of fuzzy set theory?
6. Does adoption of fuzzy set theory by construction organizations translate into any major advantage in the competitiveness of construction products?
7. Does fuzzy set theory provide sufficient guidance to support robust decision-making functions?

PRINCIPLES OF FUZZY SET THEORY

Fuzzy set theory was formally introduced by Zadeh (1965) as a calculus that can be used in formalizing our intuitions about composition of graded categories (Kim *et al.* 2006; Chan *et al.* 2009). Fuzzy set theory is a branch of modern mathematics and belongs to the wider family of concepts termed “fuzzy logic” (Belohlavek *et al.* 2009). In other words, fuzzy set theory encompasses the classical set theory – where degree of belonging is either complete or null as well as other sets whose degree of membership is partial and not well defined (Zadeh 2008). It equally follows that fuzzy set theory is a superset of the classical binary or Boolean conventional logic. Fuzzy set theory is not a theory of concept in itself; rather it is a mathematical approach that can be used to build models for different applications (Belohlavek *et al.* 2009). Fuzzy set theory cuts across the entire spectrum of mathematical modelling languages. Zadeh (2008) posits that mathematical modelling languages include probability theory, differential equations, difference equations and functional analysis. All of these are based on bivalent logic. Fuzzy set theory is however not restricted to this bivalence (Zadeh 2008). It does not have a uniquely defined mathematical form (Zimmerman 2001) but an entire range of multi-valued logic (Chan *et al.* 2009).

Zadeh (1965) postulated that fuzzy set theory was developed to model the vagueness existent in human cognitive processes. Unarguably, fuzziness is prevalent in all areas in which human judgment, evaluation and decision-making is required (Zimmerman 2001). Kosko (1990) indicates that fuzziness has both physical and sociological consequences. In the physical realm, fuzziness connotes a gradual transition between possible states. Sociologically, fuzziness implies the possibility of an infinite degree of relationship between elements of a set as opposed to just being “completely related” or “non-related”. In recent years, fuzzy set theory has been found remarkable in explicitly evaluating the fuzziness in dynamic systems. More recently, this conceptual setting has also been embraced to enhance risk and uncertainty analysis in construction projects. Some of the crucial features of fuzzy set theory are discussed.

Fuzzy set operations.

Set operations are logical mathematical formulations that guide the interaction between elements of a set (Nieto-morote & Ruz-vila 2010). Since the concept of fuzzy

set theory espouses gradual transition between elements of a set; the operations are interpreted as the interactions between membership functions of the sets rather than just the single elements within the sets. The fuzzy set operations applicable for two hypothetical sets, A and B on the universe, X with a given element, x is shown below.

The *UNION* of two sets is the maximum value of the membership function of the particular element(s) in either of both sets.

According to Ross (2004), the union of two sets using its corresponding membership functions, $\mu_{\bar{A}}(x)$ and $\mu_{\bar{B}}(x)$ are given below as:

$$\mu_{\bar{A} \cup \bar{B}}(x) = \max [\mu_{\bar{A}}(x), \mu_{\bar{B}}(x)]$$

The *INTERSECTION* of two sets is the minimum value of the membership function of the particular element(s) in either of both sets.

According to Ross (2004), the intersection of two sets using its corresponding membership functions, $\mu_{\bar{A}}(x)$ and $\mu_{\bar{B}}(x)$ are given below as:

$$\mu_{\bar{A} \cap \bar{B}}(x) = \min [\mu_{\bar{A}}(x), \mu_{\bar{B}}(x)]$$

Mathematically, the *COMPLEMENT* of a set, A, with a given element, x is shown as:

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

The elements of a fuzzy set can also undergo fuzzy arithmetic operations if the elements in the fuzzy set are represented by fuzzy numbers. Fuzzy numbers are approximate numerical values which lack clearly defined boundaries (Ayyub 2001).

Membership Functions.

The concept of a membership function is not unique to fuzzy set theory. A membership function is equally not restricted to numerical values (Ayyub 1999). Membership functions embody the description of a set (Ross 2004). The purpose of a membership function is basically to express the degree of belonging of an element in a particular set (Long & Ohsato 2008). A membership function equally provides an effective way to translate subjective terms into mathematical measure (Kim *et al.* 2006). In a more practical sense, a membership function basically represents the degree of similarity of different objectives of a defined parameter (Shaopei 1998).

The most common shapes of the membership functions of fuzzy sets in the literature of construction management are triangular and trapezoidal shapes (Fayek & Oduba 2005). Other shapes that could suffice in contemporary literature include camel-back, tent, spire or steeple (Byrne 1997). The shape of a membership function is however a graphical approximation of the membership values. Real-life problems might have more complex patterns that may be more difficult to represent in simplified diagrammatic forms (Fetz *et al.* 2005). It is noteworthy that the operational essence of a fuzzy set lies in the descriptive precision of its membership function. Hence enormous attention and commitment must be devolved into configuring the membership functions of fuzzy sets.

APPLICATION OF FUZZY SET THEORY IN RISK ANALYSIS

Risk analysis has been found useful in focusing managerial attention to critical areas of construction projects. Established methods for risk analysis include expectation variance criterion, probability analysis, Monte Carlo simulation and beta analysis. All the afore-stated methods provide useful insights in instances where the probability density function can be precisely defined. However, situations exist in the

construction industry where such assumptions are inadequate or inappropriate for describing events. In construction risk analysis, fuzziness is considered an attribute inherent in the mechanics of uncertain events (Ayyub 2001). It is envisaged that recognition of fuzziness could enhance structural explicitness in construction risk models (Ross 2004). In canvassing for explicitness in the depiction of uncertain events, Hirota (1980) constructs the graphs of membership functions representing the probabilistic and fuzzy sets over a possible range of values. As seen in *Figure 1*, below, Hirota (1980) posits that if the membership function of a fuzzy set is well-defined, it can clearly represent the mechanics of uncertain situations. Hirota (1980) also concludes that probabilistic sets only consider the “rough tendency” of mean values and variances which are derivatives of its n^{th} moment analysis leading to the lack of clear distinction seen in the profile graph of the probabilistic sets in *Figure 2*.

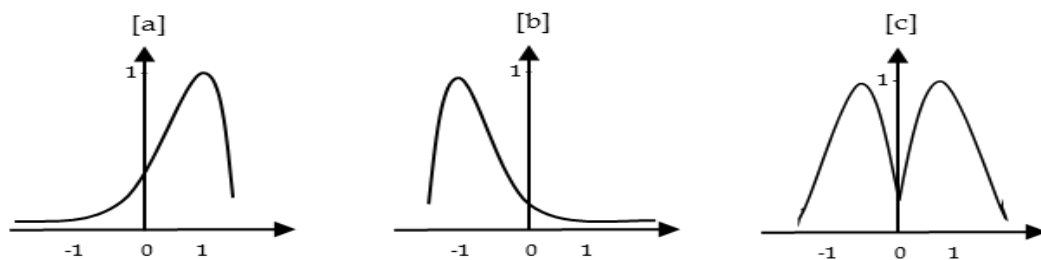


Figure 1: Fuzzy sets [a] numbers nearly early equal one [b] numbers nearly equal minus one [c] the union of [a] and [b] above. (Source: Hirota 1980)

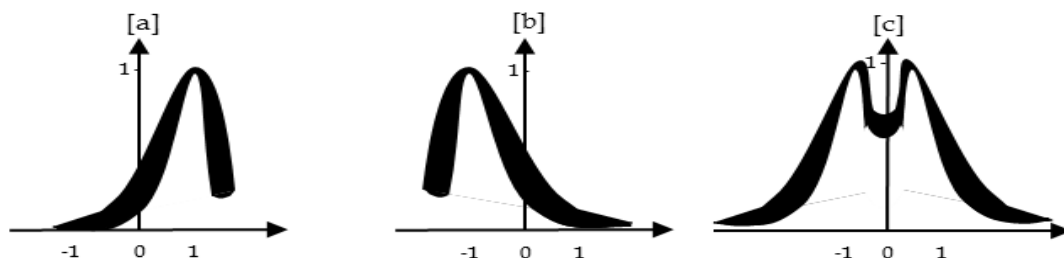


Figure 2: Probabilistic sets [a] numbers nearly early equal one [b] numbers nearly equal minus one [c] the union of [a] and [b] above. (Source: Hirota 1980)

In many of such situation where there is a lack of clear distinction in construction events, fuzzy set analysis has been found useful in modelling the inherent risks and uncertainties. An illustrative review of the application of fuzzy set theory across certain areas of the construction project lifecycle is outlined below.

Investment appraisal

Investment appraisal is considered the most important stage of a project (Shaopei 1998). At the investment appraisal stage of a construction project, fuzzy set theory has been used in event forecasting, cost estimation and overall project evaluation. One crucial challenge in the investment appraisals of construction projects is the virtual indeterminacy in the directions and movements of construction markets. In applying fuzzy set theory, single figures-of-merit say for NPV, IRR or any other numerical profitability indicator could be fuzzified through any of the appropriate value assignment methods. For example, Byrne (1997) applied fuzzy set theory to real estate investment situations by assuming a $\pm 10\%$ cost contingency to crisp values of discounted cash flow figures. Other portfolio studies have utilized subjective values obtained as linguistic variables to represent the lack of distinction in the estimates of uncertain events. It was hinted by Dikmen *et al.* (2007) that fuzzy set theory has been increasingly significant in investment appraisal situations. However, Rebiasz (2007)

has raised concerns on the economy of using fuzzy set theory in providing a rich picture in the early stages of a construction project. It was considered that the sparse information available at this stage of a construction project might not be well suited for enhancing robust analysis in evaluating the fuzziness in risk and uncertain events.

Network Scheduling

The critical path method (CPM) and the program evaluation and review technique (PERT) are two common techniques for scheduling networks in construction projects. The assessment of criticality of a project network provides information for project control. In classical set computations, a project network of zero float is judged critical (Long & Ohsato 2008). While fuzzy set theory acknowledges this proposition, fuzzy set theory adds that every project path has a degree of criticality (called critical-potential) which can be defined as the possibility degree of zero range of uncertainty (Fetz *et al.* 2005). In classical risk analysis, the critical path of a project network can be identified and adjusted by backward and forward recursion of activity times (Fetz *et al.* 2005). One approach to the fuzzification of crisp activity duration values is the corresponding alteration of forward and backward passes based on the modification in fuzzy arithmetic. A distinct feature in fuzzy arithmetic is that fuzzy subtraction is not the inverse of fuzzy addition (Lorterpong & Moselhi 1996). Chen and Huang (2007) further discovered that criticality of a path rises as the fuzzy float time decreases. Shih-pen (2007) developed a method to estimate the relative degree of criticality of activities on a project network. This calculus provides a comparative assessment in the criticality of project paths on a scheduled network. Extensive work has been done in analysing risks in scheduled networks using fuzzy set theory. However, the extent to which fuzziness is conserved is still an arbitrary procedure.

Project monitoring and Control

Monitoring and control are dynamic initiatives to guide the execution of construction projects. Monitoring and control might involve processes that range from changing overall method of construction, adjustment of internal structure, alteration of temporal and causal dependencies of activities as well as resource modification. In construction projects, monitoring and control is often carried out in order to achieve cost-time optimization and quality assurance. Whilst quality assurance is usually a more subjective and routine exercise, cost-time optimization is a crucial risk analysis function that generally involves a broad range of initiatives. In classical risk analysis, cost-optimization is usually achieved by time-cost trade off, resource allocation and resource levelling. The principles of fuzzy set theory can also be useful in providing instructive guidance for project monitoring and control. In respect of network scheduling, the strategic initiative explicitly steered by fuzzy set theory is that activity durations with lower degree of criticality should be minimized before activities with higher degree of criticality (Chen and Huang 2007). The caveat expressed in situations where equal degree of criticality occurs, is that resource constraints should dictate the optimization initiative. Long and Ohsato (2008) specify that fuzzy set theory could assist dynamic control of projects by enhancing proactive and periodical updates as the project progress. In a general sense, this suggests that adopting an “as soon as possible” implementation approach can help in curtailing construction project risks.

RESEARCH METHOD

This research constitutes part of a larger study that aims to investigate the specific insights that fuzzy set theory could bring into the construction risk analysis discipline.

The participants were chosen through purposive sampling due to the difficulty in finding construction professionals familiar with the principles of fuzzy set theory. In total, 77 copies of the questions for the structured interview were sent through electronic mail to the selected respondents. An undisclosed number was also circulated to members of the Royal Institution of Chartered Surveyors (RICS), through the public liaison representatives of the RICS Office, Edinburgh, United Kingdom. Consequent upon the low interest in participating in the interview, construction professionals with relevant publications on fuzzy set theory were contacted to shed light on their experience with fuzzy set theory. The participants from the industry were contacted through informal links to construction establishments. All the respondents that agreed to participate in the study were duly interviewed.

Structured interviews were eventually conducted with six construction professionals in England and Scotland. Four of the participants were university academics in construction management and the other two were an off-shore structural engineer and a construction site manager. All of the university academics that participated in the study were Heads of school of built environment or civil engineering in universities across the United Kingdom. Two of the participants had over 25 years' experience in the construction industry, another two had over 15 years' experience and the last two had just over 5 years' experience in the construction industry. The interview questions were sent through e-mail and the resulting conversations were recorded and transcribed.

The outline design set out to elicit the background information of respondents and the organization in regards to their experience in the construction industry, title and the organization's main construction activity. Other sections were aimed at aligning the literature review with current industry opinion and to investigate the informed views of individual construction practitioners.

The findings obtained from the structured interviews were subjected to intensive analysis through influence diagramming and concept filtering. Some empirical hypotheses were tested based on the numerical facts obtained from the investigation.

PRELIMINARY FINDINGS AND DISCUSSIONS

The preliminary findings obtained from the investigation suggest the appropriateness of fuzzy set theory as a valid mathematical modelling framework for analysing construction risks. Many of the interviewees suggested that fuzzy set theory should be applied to construction risks whose structure are well-defined and whose nature are considerably complex and can be quantified within a range of options. It was however instructive that fuzzy set theory will not function best as a stand-alone mathematical framework. Its utility and practicality is enhanced by combining the logic of fuzzy set theory with insights drawn from pre-existing mathematical formulations such as Probability Theory, Analytic Hierarchy Process, Latin Hyper Cube Sampling and Monte-Carlo Simulation. It is contingent that this practical dependence is indicative of its volatility and vulnerability as a conceptual approach within the risk analysis function. All of the interviewees confirmed that incorporating "matter of degree" into decision analysis is a primary essence of fuzzy set theory in construction risk analysis. This study discovered that fuzzy set theory facilitates an enhancement in reasoning for making rational decisions in an environment of uncertainty, incomplete information, equivocation and conflicting information. This study equally confirms that fuzzy set theory have much less axiomatic limitations than probability theory.

The study has also revealed that fuzzy set theory has far-reaching applications in construction risk analysis. Many of the interviewees however noted that the use of a sophisticated framework like fuzzy set theory for all risk analysis problems might suggest a lack of consideration for the economic aspects of construction projects. For instance, Rebiasz (2007) illustrated the fuzzy set theory evaluation of risks in the production capacity of a pre-existing business and found that the information concerning probable values of NPV is more useful than information obtained through fuzzy set theory. Although, the fuzzy set computations were more time-consuming and rigorous in comparison to those from probabilistic techniques (Rebiasz 2007), it proved to be of lesser use. Some simpler and easier mathematical models might achieve same results in certain instances. For example, in projects which are highly repetitive and small in scope such as household renovations, roadwork maintenance and minor dam rehabilitations, fuzzy set theory appears to be a superfluous model when considering the economics of scale, time and resources. Dubois *et al.* (2004) had earlier stated that the transformation of probability distribution to fuzzy set distribution leads to loss of information which invariably increases the co-efficient of uncertainty in certain situations. It is deducible that discretion should be exercised by the construction risk analyst as to situations where fuzzy set theory conveys great utility rather than a blind-adherence to its use.

In respect of the construction project lifecycle, many of the interviewees recognized that fuzzy set theory generally has lesser relevance in the earlier and much later stages of a typical but complex construction project. In the earlier stages of the project, dearth of information results in the absence of a robust structural framework for risk analysis. Equally, in the much later stages of a typical construction project (say project termination stage); risk generally has a declining profile (Ayyub 2003). Therefore, the utility of the risk analysis function is basically diminished. Many of the interviewees hinted that fuzzy set theory generally has great utility in the detailed design and project execution phase. In activity duration modelling, the fuzzy set theory representation of “as soon as possible” to execute activities in a construction project planning is considered to be a better framework for achieving a more effectively and efficiently managed project. The study discovered that the “Last Planner System” of project planning described by Ballard (2000) finds appropriate mathematical expression in the conceptual logic of fuzzy set theory.

The discussions stemming from the investigation have suggested that risk analysis as a discipline is progressing on a distinctive continuum. It was discovered that as the structure of the problem becomes better understood, considerable insights can be drawn from fuzzy set theory. The study equally finds that the explicit inclusion of dynamism and continuity in various facets of the construction lifecycle conveys a more realistic conception of the character of risks and uncertainties. Contextually, the uniqueness of construction products already restricts the transferability of knowledge in projects. Inclusion of greater and misdirected subjectivity in construction risk analysis holds potential of increasing the uncertainties faced by construction professionals. These higher potential of uncertainties invariably create a vulnerable and volatile situation in construction risk analysis. Consequently, it was suggested that this vulnerability and volatility of fuzzy set theory discourages construction industries and organizations from embracing the principles and applications of fuzzy set theory.

Finally the research exercise indicated the usefulness of fuzzy set theory but also confirms the cynicism towards its use. All of the interviewees confirmed a low level of awareness of the principles of fuzzy set theory among the professionals of the

construction sector. Although all of the respondents that participated in the interview were leading experts, three of them were initially reluctant to participate in the interview. Also, some responses reflected an educated guess rather than an intense familiarity with the subject matter. It was suggested by some of the interviewees that more attention needs to be given to enhancing the overall analytical capabilities of future construction professionals.

CONCLUSIONS AND FURTHER WORK

The study confirmed that fuzzy set theory represents a valid mathematical modelling technique for analysing risk in construction projects. Fuzzy set theory is applicable in many aspects of the construction project where risk attributes can be quantified within a range of options and whose nature is considerably complex. Analysing construction risks with fuzzy sets could enhance reasoning about the project delivery sequence. This implies that time-targets are better met; construction costs will be more accurate; safety and environmental sustainability requirements will be better achieved. This will hopefully lead to a situation in which the overall construction product becomes more innovative and resourceful leading to the creation of a competitive edge. In decision-making, it was found that fuzzy set theory could promote explicitness in many decision analysis functions. However, the actual decision-support function is still primarily dependent on the strategy of the respective organization.

In construction risk analysis, fuzzy set theory could be limited by the operational scope of the project. It was also discovered that fuzzy set theory as a stand-alone mathematical tool might not be sufficient to complete the risk analysis process. This was due to the vulnerability, volatility and computational intensiveness of fuzzy set theory. These limitations have potentials of discouraging construction practitioners from embracing fuzzy set theory as a more widely- acclaimed risk analysis tool. It was also found that some areas of fuzzy set theory still require further research and investigation. Specifically, these areas include the geometry of membership functions, defuzzification of fuzzy variables and informed guidance on the suitability of fuzzy sets for various categories of uncertain events in construction projects. Equally, the appropriateness of fuzzy sets needs to be examined in concert with the current human resource base. In considering broad-based issues of construction risk where multiple organizations are often responsible for project delivery, subjectivity often creates a larger scope of consideration in the decision-making paradigm.

The subsequent stages of this study will involve the utilisation of fuzzy set theory for risk analysis in an actual construction project. Equally, further research is planned to increase the number of participants from the industry in order to capture a more representative data set. This exercise will hopefully bridge the gap between fuzzy set theory and the construction industry's practice of risk analysis. It is also hoped that further research will attempt to establish best practice for fuzzy set theory in construction risk analysis. Following this will be a triangulation of the data obtained from both research methodologies.

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