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Fiber Motion in Dusty Fluid Turbulent Flow with Two-point Correlation

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Abstract

The equation of fiber motion in dusty fluid turbulent flow has been derived in terms of correlation tensors of second order. In presence of dust particles, mathematical modeling of fiber suspensions in the turbulent flow is discussed including the correlation between the pressure fluctuations and velocity fluctuations at two points of the flow field, where the correlation tensors are the functions of space coordinates, distance between two points and the time.

Keywords: Fiber motion; Dusty fluid; Turbulent flow; Correlation.

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1. Introduction

The behavior of dust particles in a turbulent fluid depends on the concentration of the particles and on the size of the particles with respect to the scale of turbulent fluid. A fiber suspension in a turbulent flow affect the transport, rheology and light scattering properties of suspensions that are of great interest in many areas of science and industry. At great concentration there is an interaction between the particles through collisions and through the effects on the flow of the fluid in the neighborhood of the particles. Hinze [1] obtained an expression for correlation between pressure fluctuations and velocity fluctuations in turbulent motion. Saffman [2] observed the effect of dust particles of an incompressible flow and derived an equation that described the motion of a fluid containing small dust particles. Anderson [3] discussed on some observation of fiber suspensions in turbulent motion. Batchelor [4] obtained the equations of motion of fiber suspensions in the flow. Agermann and Kohler [5] studied the rotational and translational dispersion of fibers in turbulent flow by assuming the dimension of fibers to be less than that of smallest eddies in the flow. Kishore and Sinha [6] derived an analytical expression for the rate of change of vorticity covariance in dusty fluid turbulent flow. Olson and Zhu Li [7] derived the new equation of turbulent fiber suspensions and its solution and application to the pipe flow. In

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view of all these works the main aim of the present study is to derive an equation for fiber motion in dusty fluid turbulent flow with the aid of pressure-velocity correlation.

2. Mathematical Model of the Problem

Let us assume that the fluid is incompressible. The equation of motion and continuity for fiber suspensions in turbulent flow of viscous incompressible fluid are [7]:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \upsilon \frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{\mu_f}{\rho}\frac{\partial}{\partial x_j}\left[a_{ijlm}\varepsilon_{lm} - \frac{1}{3}\left(I_{ij}a_{lm}\right)\varepsilon_{lm}\right] \tag{1}$$

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{2}$$

In the presence of dust particles the equations of motion are given by [6]

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \upsilon \frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{KN}{\rho}\left(v_i - u_i\right) + \frac{\mu_f}{\rho}\frac{\partial}{\partial x_j}\left[a_{ijlm}\varepsilon_{lm} - \frac{1}{3}\left(I_{ij}a_{lm}\right)\varepsilon_{lm}\right] \tag{3}$$

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{4}$$

$$\frac{\partial v_i}{\partial t} + v_j \frac{\partial u_i}{\partial x_j} = -\frac{K}{m_s}\left(u_i - v_i\right) \tag{5}$$

where $u_i(x,t)$ is the fluid velocity components; $v_i(x,t)$, the solid particles (dust) velocity components; $p(x,t)$, the unknown pressure field; $m_s = \frac{4}{3}\pi R_s^3 \rho_s$, the mass of a single spherical dust particles of radius R_s; $\upsilon =$constant is the molecular kinematic viscosity; $K = 6\pi R_s \rho v$, the Stoke's drag formula; *N*, the number density of dust particles; $\frac{KN}{\rho} = f$, has dimension of frequency; μ_f, the apparent viscosity of fiber suspensions; *ρ*, the density of the fluid particle; $\varepsilon_{lm} = \frac{1}{2}\left(\frac{\partial u_l}{\partial x_m} + \frac{\partial u_m}{\partial x_l}\right)$ is the tensor of strain rate; I_{ij}, the turbulent intensity of suspensions; a_{lm} and a_{ijlm} are the second and fourth-orientation tensors of the fiber.

We assume that the mean velocity $\overline{U_i}$ is constant throughout the region considered and independent of time and we put

$$\left(U_i = \overline{U}_i + u_i\right)_A,$$

$$\left(U_j = \overline{U}_j + u_j\right)_B.$$

The value of each term can be obtained by taking the equations of motion for u_j at the point *B* and for u_i at the point *A*.

The equation of motion for u_i at the point A is obtained from Eq. (3),

$$\frac{\partial u_i}{\partial t}+\left(\overline{U}_k+u_k\right)\frac{\partial u_i}{\partial x_k}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\upsilon\frac{\partial^2 u_i}{\partial x_k\partial x_k}+f\left(v_i-u_i\right)+\frac{\mu_f}{\rho}\frac{\partial}{\partial x_k}\left[a_{iklm}\varepsilon_{lm}-\frac{1}{3}\left(I_{ik}a_{lm}\right)\varepsilon_{lm}\right] \quad (6)$$

For an incompressible fluid $\left(u_i\frac{\partial u_k}{\partial x_k}\right)_A=0$ so that Eq. (6) can be written as

$$\frac{\partial}{\partial t}\left(u_i\right)_A+\left[\overline{U}_k+\left(u_k\right)_A\right]\left(\frac{\partial}{\partial x_k}\right)_A\left(u_i\right)_A+\left(u_i\frac{\partial u_k}{\partial x_k}\right)_A=-\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A+\upsilon\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_A\left(u_i\right)_A$$
$$+f\left(v_i-u_i\right)_A+\frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_A\left[a_{iklm}\varepsilon_{lm}-\frac{1}{3}\left(I_{ik}a_{lm}\right)\varepsilon_{lm}\right]_A \quad (7)$$

Multiplying Eq. (7) by $\left(u_j\right)_B$ we obtain

$$\left(u_j\right)_B\frac{\partial}{\partial t}\left(u_i\right)_A+\left[\overline{U}_k+\left(u_k\right)_A\right]\left(\frac{\partial}{\partial x_k}\right)_A\left(u_i\right)_A\left(u_j\right)_B+\left(u_i\right)_A\left(\frac{\partial}{\partial x_k}\right)_A\left(u_k\right)_A\left(u_j\right)_B$$
$$=-\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A\left(u_j\right)_B+\upsilon\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_A\left(u_i\right)_A\left(u_j\right)_B+f\left(v_i-u_i\right)_A\left(u_j\right)_B$$
$$+\frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_A\left[a_{iklm}\varepsilon_{lm}-\frac{1}{3}\left(I_{ik}a_{lm}\right)\varepsilon_{lm}\right]_A\left(u_j\right)_B \quad (8)$$

where $\left(u_j\right)_B$ can be treated as a constant in a differential process at the point *A*.

Similarly, the equation of motion for u_j at the point *B* is obtained as

$$\frac{\partial u_j}{\partial t}+\left(\overline{U}_k+u_k\right)\frac{\partial u_j}{\partial x_k}=-\frac{1}{\rho}\frac{\partial p}{\partial x_j}+\upsilon\frac{\partial^2 u_j}{\partial x_k\partial x_k}+f\left(v_j-u_j\right)+\frac{\mu_f}{\rho}\frac{\partial}{\partial x_k}\left[a_{jklm}\varepsilon_{lm}-\frac{1}{3}\left(I_{jk}a_{lm}\right)\varepsilon_{lm}\right]$$

Since, for an incompressible fluid $\left(u_j\frac{\partial u_k}{\partial x_k}\right)_B=0$ then the above equation can be written as

$$\frac{\partial}{\partial t}\left(u_j\right)_B+\left[\overline{U}_k+\left(u_k\right)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B\left(u_j\right)_B+\left(u_j\frac{\partial u_k}{\partial x_k}\right)_B=-\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B+\upsilon\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_B\left(u_j\right)_B$$
$$+f\left(v_j-u_j\right)_B+\frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_B\left[a_{jklm}\varepsilon_{lm}-\frac{1}{3}\left(I_{jk}a_{lm}\right)\varepsilon_{lm}\right]_B \quad (9)$$

Multiplying Eq. (9) by $(u_i)_A$, we get

$$(u_i)_A \frac{\partial}{\partial t}(u_j)_B + \left[\overline{U}_k + (u_k)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B (u_j)_B (u_i)_A + (u_j)_B \left(\frac{\partial}{\partial x_k}\right)_B (u_k)_B (u_i)_A = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B (u_i)_A$$

$$+\upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B (u_j)_B (u_i)_A + f(v_j - u_j)_B (u_i)_A + \frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_B \left[a_{jklm}\varepsilon_{lm} - \frac{1}{3}(I_{jk} a_{lm})\varepsilon_{lm}\right]_B (u_i)_A \tag{10}$$

where $(u_i)_A$ can be treated as a constant in a differential process at the point *B*.

Addition of Eqs. (8) and (9) gives the result

$$\frac{\partial}{\partial t}(u_i)_A (u_j)_B + \left[\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A (u_k)_A (u_j)_B + \left(\frac{\partial}{\partial x_k}\right)_B (u_i)_A (u_k)_B (u_j)_B\right] + \overline{U}_k\left[\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A (u_j)_B\right]$$

$$+\overline{U}_k\left[\left(\frac{\partial}{\partial x_k}\right)_B (u_i)_A (u_j)_B\right] = -\frac{1}{\rho}\left[\left(\frac{\partial}{\partial x_i}\right)_A p_A (u_j)_B + \left(\frac{\partial}{\partial x_j}\right)_B p_B (u_i)_A\right]$$

$$+ f\left[(v_i - u_i)_A (u_j)_B + (v_j - u_j)_B (u_i)_A\right] + \upsilon\left[\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A + \left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B\right](u_i)_A (u_j)_B$$

$$+\frac{\mu_f}{\rho}\left[\left(\frac{\partial}{\partial x_k}\right)_A \left(a_{iklm}\varepsilon_{lm} - \frac{1}{3} I_{ik} a_{lm}\varepsilon_{lm}\right)_A (u_j)_B + \left(\frac{\partial}{\partial x_k}\right)_B \left(a_{jklm}\varepsilon_{lm} - \frac{1}{3} I_{jk} a_{lm}\varepsilon_{lm}\right)_B (u_i)_A\right] \tag{11}$$

To find the relation of turbulent fiber motions in presence of dust particles at the point *B* to those at point A, it will give no difference if we take one point as the origin of *A* or *B* of the coordinate system. Let us consider the point *A* as the origin and can write

$$\zeta_k = (x_k)_B - (x_k)_A$$

Then we obtain,

$$\left(\frac{\partial}{\partial x_k}\right)_A = -\frac{\partial}{\partial \zeta_k}, \qquad \left(\frac{\partial}{\partial x_k}\right)_B = \frac{\partial}{\partial \zeta_k}, \text{ and } \left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A = \left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B = \frac{\partial^2}{\partial \zeta_k \partial \zeta_k}.$$

Using the above relations and taking ensemble average on both sides, Eq. (11) becomes

$$\frac{\partial}{\partial t}\overline{(u_i)_A (u_j)_B} - \frac{\partial}{\partial \zeta_k}\overline{(u_i)_A (u_k)_A (u_j)_B} + \frac{\partial}{\partial \zeta_k}\overline{(u_i)_A (u_k)_B (u_j)_B} = -\frac{1}{\rho}\left[-\frac{\partial}{\partial \zeta_i}\overline{p_A (u_j)_B} + \frac{\partial}{\partial \zeta_j}\overline{p_B (u_i)_A}\right]$$

$$-\frac{\mu_f}{\rho}\frac{\partial}{\partial \zeta_k}\left[\overline{(a_{iklm}\varepsilon_{lm})_A (u_j)_B} - \frac{1}{3}\overline{(I_{ik} a_{lm}\varepsilon_{lm})_A (u_j)_B} - \overline{(a_{jklm}\varepsilon_{lm})_B (u_i)_A} + \frac{1}{3}\overline{(I_{jk} a_{lm}\varepsilon_{lm})_B (u_i)_A}\right]$$

$$+2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}\overline{(u_i)_A(u_j)_B}+f\left[\overline{(v_i)_A(u_j)}_B-2\overline{(u_i)_A(u_j)}_B+\overline{(u_i)_A(v_j)}_B\right] \tag{12}$$

Eq. (12) represents the mean motion equation of fiber suspensions in turbulent flow in presence of dust particles and the pressure-velocity correlation.

It is noted that the coefficient of $\overline{U}_k$ has been vanished. Eq. (12) describes the motions of fiber suspensions in turbulent flow in presence of dust particles, where the motions with respect to a coordinate system moving with the mean velocity $\overline{U}_k$. Eq. (12) contains the double velocity correlation $\overline{(u_i)_A(u_j)}_B$, double velocity correlation between dust particles and the fluid such as $\overline{(v_i)_A(u_j)}_B$, double correlations such as $\overline{p_A(u_j)}_B$, triple correlations such as $\overline{(u_i)_A(u_k)_A(u_j)}_B$ where all the terms apart from one another. The correlations $\overline{p_A(u_j)}_B$ and $\overline{p_B(u_i)}_A$ form the tensors of the first order, because pressure is a scalar quantity and the triple correlations $\overline{(u_i)_A(u_k)_A(u_j)}_B$ and $\overline{(u_i)_A(u_k)_B(u_j)}_B$ form the tensors of third order.

We designate the first order correlations by $(k_{p,j})_{A,B}$, second order correlations by $(Q_{i,j})_{A,B}$ and third order correlations by $(s_{ik,j})_{A,B}$.

Therefore, we set $(k_{i,p})_{A,B}=\overline{(u_i)_A p_B}$, $(k_{p,j})_{A,B}=\overline{p_A(u_j)}_B$,

$(s_{ik,j})_{A,B}=\overline{(u_i)_A(u_k)_A(u_j)}_B$, $(s_{i,kj})_{A,B}=\overline{(u_i)_A(u_k)_B(u_j)}_B$, $(Q_{i,j})_{A,B}=\overline{(u_i)_A(u_j)}_B$,

$(F_{i,j})_{A,B}=\overline{(v_i)_A(u_j)}_B$, and $(G_{i,j})_{A,B}=\overline{(u_i)_A(v_j)}_B$.

where the index *p* indicates the pressure and is not a dummy index like *i* or *j* so that the summation convention does not apply to *p*.

Also the term $\overline{(a_{jklm}\varepsilon_{lm})_B(u_i)}_A$ and $\overline{(I_{jk}a_{lm}\varepsilon_{lm})_B(u_i)}_A$ form the correlations of third order, we designate these by $D_{i,jk}$ and $H_{i,jk}$, respectively.

Thus we set, $(D_{i,jk})_{A,B}=\overline{(u_i)_A(a_{jklm}\varepsilon_{lm})}_B$, $(D_{ik,j})_{A,B}=\overline{(a_{iklm}\varepsilon_{lm})_A(u_j)}_B$,

$(H_{i,jk})_{A,B}=\overline{(u_i)_A(I_{jk}a_{lm}\varepsilon_{lm})}_B$, $(H_{ik,j})_{A,B}=\overline{(I_{ik}a_{lm}\varepsilon_{lm})_A(u_j)}_B$.

If we use the above relations of first, second and third order correlations in Eq. (12) then we obtain

$$\frac{\partial}{\partial t}Q_{i,j}-\frac{\partial}{\partial\zeta_k}S_{ik,j}+\frac{\partial}{\partial\zeta_k}S_{i,kj}=-\frac{1}{\rho}\left(-\frac{\partial}{\partial\zeta_i}K_{p,j}+\frac{\partial}{\partial\zeta_j}K_{i,p}\right)+2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}Q_{i,j}$$
$$+f\left(F_{i,j}-2Q_{i,j}+G_{i,j}\right)+\frac{\mu_f}{\rho}\left[-\frac{\partial}{\partial\zeta_k}\left(D_{ik,j}-\frac{1}{3}H_{ik,j}\right)+\frac{\partial}{\partial\zeta_k}\left(D_{i,jk}-\frac{1}{3}H_{i,jk}\right)\right]$$

or, $$\frac{\partial}{\partial t}Q_{i,j} - \frac{\partial}{\partial \zeta_k}S_{ik,j} + \frac{\partial}{\partial \zeta_k}S_{i,kj} = -\frac{1}{\rho}\left(-\frac{\partial}{\partial \zeta_i}K_{p,j} + \frac{\partial}{\partial \zeta_j}K_{i,p}\right) + 2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j}$$
$$+ f\left(F_{i,j} - 2Q_{i,j} + G_{i,j}\right) + \frac{\mu_f}{\rho}\frac{\partial}{\partial \zeta_k}\left[\left(D_{i,jk} - D_{ik,j}\right) + \frac{1}{3}\left(H_{ik,j} - H_{i,jk}\right)\right] \quad (13)$$

where all the correlations refer to the two points *A* and *B*.

Now for an isotropic turbulence of an incompressible flow, the double pressure-velocity correlations are zero, that is

$$\left(k_{p,j}\right)_{A,B} = 0, \left(k_{i,p}\right)_{A,B} = 0$$

In an isotropic turbulence it follows from the condition of invariance under reflection with respect to point *A*,

$$\overline{(u_i)_A (u_k)_B (u_j)_B} = -\overline{(u_k)_A (u_j)_A (u_i)_B}$$

or, $\left(s_{i,kj}\right)_{A,B} = -\left(s_{kj,i}\right)_{A,B}$,

and hence $\left(D_{i,jk}\right)_{A,B} = -\left(D_{jk,i}\right)_{A,B}, \left(H_{i,jk}\right)_{A,B} = -\left(H_{jk,i}\right)_{A,B}$.

Thus Eq. (13) can be written as

$$\frac{\partial}{\partial t}Q_{i,j} - \frac{\partial}{\partial \zeta_k}\left(S_{ik,j} + S_{kj,i}\right) = 2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j} + f\left(F_{i,j} - 2Q_{i,j} + G_{i,j}\right)$$
$$+ \frac{\mu_f}{\rho}\left[-\frac{\partial}{\partial \zeta_k}\left(D_{jk,i} + D_{ik,j}\right) + \frac{1}{3}\frac{\partial}{\partial \zeta_k}\left(H_{ik,j} + H_{jk,i}\right)\right] \quad (14)$$

The term $\frac{\partial}{\partial \zeta_k}\left(S_{ik,j} + S_{kj,i}\right)$, $\frac{\partial}{\partial \zeta_k}\left(D_{jk,i} + D_{ik,j}\right)$ and $\frac{\partial}{\partial \zeta_k}\left(H_{ik,j} + H_{jk,i}\right)$ form the tensors of second order, we designate these by $S_{i,j}, D_{i,j}$ and $H_{i,j}$ respectively, that is

$$S_{i,j} = \frac{\partial}{\partial \zeta_k}\left(S_{ik,j} + S_{kj,i}\right), D_{i,j} = \frac{\partial}{\partial \zeta_k}\left(D_{jk,i} + D_{ik,j}\right)$$

And $$H_{i,j} = \frac{\partial}{\partial \zeta_k}\left(H_{ik,j} + H_{jk,i}\right).$$

Therefore Eq. (14) gives the result

$$\frac{\partial}{\partial t}Q_{i,j} - S_{i,j} = 2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j} + f\left(F_{i,j} - 2Q_{i,j} + G_{i,j}\right) - \frac{\mu_f}{\rho}\left(D_{i,j} - \frac{1}{3}H_{i,j}\right) \quad (15)$$

This is the equation of motion of fiber suspensions in dusty fluid turbulent flow in terms of correlation tensors of second order.

In the absence of dust particles, $f = 0$, then Eq. (15) reduces to

$$\frac{\partial}{\partial t}Q_{i,j} - S_{i,j} = 2\upsilon \frac{\partial^2}{\partial\zeta_k\partial\zeta_k}Q_{i,j} - \frac{\mu_f}{\rho}\left(D_{i,j} - \frac{1}{3}H_{i,j}\right) \tag{16}$$

Eq. (16) describes the turbulent motion of fiber suspensions in terms of the correlation tensors of second order.

If there are no effects of fiber suspension in the flow field then $\mu_f = 0$ so that Eq. (16) takes the form

$$\frac{\partial}{\partial t}Q_{i,j} - S_{i,j} = 2\upsilon \frac{\partial^2}{\partial\zeta_k\partial\zeta_k}Q_{i,j} \tag{17}$$

Eq. (17) represents the turbulent motion in terms of correlation tensors of second order which is the same as obtained in Hinze [1].

3. Discussion and Conclusion

The equation of motion of fiber suspensions in dusty fluid turbulent flow has been derived by averaging procedure, which includes the effect of dust particles and the correlations between the pressure fluctuations and velocity fluctuations at two points of the flow field. Fiber suspensions in a turbulent fluid undergo mean motion due to the mean fluid velocity and random motion due to the fluctuating component of fluid velocity. The resulting Eq. (15) represents the equation of turbulent motion in terms of correlation tensors of second order in presence of dust particles. In this equation, all the terms $Q_{i,j}, S_{i,j}, F_{i,j}, G_{i,j}, D_{i,j}, H_{i,j}$ are the second order correlation tensors where, $Q_{i,j}$ and $S_{i,j}$ represents the velocity correlations at the two points A and B of the flow field, $F_{i,j}$ and $G_{i,j}$ represents velocity correlations between the fluid velocity components and solid particles (dust) velocity components at the two points whereas $D_{i,j}$ and $H_{i,j}$ represent velocity correlations of suspending fluid. But in absence of fiber suspensions in the fluid and without any effect of dust particles to the fluid velocity, the resulting Eq. (15) reduces to the Eq. (17) which represents the turbulent motion.

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