TALKING ACROSS CULTURES: AN INTERNATIONAL STUDY OF YOUNG CHILDREN'S MATHEMATICAL EXPLANATIONS

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The research that is described in this paper is part of a small-scale collaborative study, Talking Across Cultures , investigating children's mathematical explanations during whole-class discussion in three mathematics lessons in each of Australia, Hungary and Japan, during children's first year at school. This paper outlines the methodology of the project and describes examples of successful strategies for encouraging and supporting children's mathematical discussion used by a teacher in each of the three countries. Commonalties and differences between these strategies and barriers to their use elsewhere are discussed.

BACKGROUND

> Teaching is a cultural activity. Because cultural activities vary little within society, they are often transparent and unnoticed. ... Comparative research is a powerful way to unveil unnoticed but ubiquitous practices. (Stigler, Galimore & Hiebert, 2000, p. 87)

Cross-cultural comparative research provides a powerful means of achieving better understanding of one's own practice and looking for ways of extending its boundaries. Clarke (2002) describes the purpose of studying international classroom practices as not merely to mimic them, but rather to support reflection on our own practice. Thus it is important to distinguish between those classroom practices that are specifically cultural, those that are based on deliberate pedagogical decisions, and those that are the unintended consequence of other actions and decisions.

The research reported here forms part of a small-scale collaborative study, Talking Across Cultures, that is investigating children's mathematical explanations during the whole-class discussion phase of three mathematics lessons in each of Australia, Hungary and Japan, during children's first year at school. As well as investigating the types of explanations given by these children, the study is also trying to identify strategies their teachers use to support high quality mathematical explanations.

This study was prompted not only by larger-scale studies, such as the TIMSS Video study (see, for example, Hollingsworth, Lokan, & McCrae, 2003), but also previous research by the Australian authors of this paper, where they found widespread agreement with the notion of mathematics classrooms being places "where students construct powerful mathematical ideas through ... participating in whole-class dialogue and argumentation — together with a realisation that current Australian primary classroom practice falls far short of this goal" (Groves, Doig & Splitter, 2000).

While Australia has performed well overall in recent international studies of performance in mathematics, the Australian video-data from the TIMSS Video Study (Hollingsworth, Lokan, & McCrae, 2003) showed that about three-quarters of problems set for solution by students were deemed as being low in procedural complexity, and repetitious. This was in marked contrast to the problems set for students in high achieving countries, where Japanese teachers, for example, used only 17% of this type of problem. Hollingsworth and her colleagues suggested that there is a "syndrome of shallow teaching" in Australia (p. xxi), the implication being one of shallow learning, typified by a lack of complex, higher-order thinking.

Moreover, while there has been considerable recent emphasis on the improvement of classroom environments in Australian schools, there has been a lot less emphasis on approaches to teaching and learning that stress higher-order thinking and overall intellectual demand and expectations (Luke et al, 2003). Classrooms in Australia, Hungary and Japan were chosen as the sites for this study because, while all three countries perform well on international studies of performance in mathematics, Hungary and Japan perform particularly well in areas that require higher-order thinking — for example, problem solving.

According to Stigler et al (1999), one of the problems in effecting change in teaching practice is that "teachers lack a set of shared referents for the words they use to describe classroom [practice]" (p. 5). They suggest that the development of a video library of examples of quality practice, linked with definitions and indicators of quality, would have immediate practical value for teachers.

Our project has collected and analysed, data from lessons in the classrooms of three "expert" teachers in each of Australia, Japan and Hungary. Data collection included video recording of lessons, and audio recording of interviews with each teacher. Teacher interviews were audio-taped and transcribed. Data analysis was carried out both individually by the researchers and during face-to-face meetings, including a week-long meeting for all researchers in Australia.

In this paper we focus on one lesson from each of the countries and attempt to illustrate some of the strategies used by each of the three teachers to support children's development of mathematically sophisticated explanations, and use this as a basis for reflecting on some aspects of Australian classroom practice.

THE JAPANESE LESSON

The lesson discussed here was part of a sequence of lessons on subtraction, taking place about eight months after the beginning of the children's first year at school.

The lesson began with the teacher reminding children of the previous day's lesson on 13 – 9, then spending about five minutes handing back workbooks while reading aloud the comments children had written at the end of that lesson. Children were told that in this lesson they would use their prior knowledge to find 14 – 8. They could use magnetic tiles if they wished and any strategy they chose.

After about ten minutes, the teacher identified four different solution strategies used by the children. The teacher then asked all of the children to identify the strategies they had used and place their own magnetic name cards on the blackboard under that strategy. He drew a circle on the board and told those children who could not match their solution with any of the four strategies to place their names in the circle. These children were told that, if they later decide that their strategy belonged to one of the four on the board, they could move their name cards, or perhaps they had come up with a new idea. After name cards were placed on the board, the teacher asked children to explain their strategies.

Approximately six minutes were spent discussing each of the strategies. At the end of this time, only two children still had name cards in the circle. As time was running out, the teacher said they would talk about those children's solutions next time, but in the meantime asked all children to look at the four strategies and comment. Most of the comments were recorded on the board and labelled with the children's names, with about a dozen names being on the board by the end of the lesson. The teacher ended the lesson by summarising what they had done. He then asked the children to spend a few minutes reflecting on the lesson in their workbooks.

We identified a number of strategies to support children's mathematical explanations in this lesson.

Interweaving the concrete with the abstract. A major challenge for teachers is to connect conventional mathematical activity with students' real experiences. A significant feature of this lesson was the way in which the teacher combined children's verbal explanations of their solution strategies with his and the children's manipulation of magnetic tiles and written and symbolic representations of children's actions. So, for example, when a girl effectively stated that if $b + c = a$ then $a - b = c$, the teacher asked her whether she could connect this to the solution using the tiles. Similarly, solution strategies devised by the children were specifically named for later use — e.g. the "Subtraction, addition" strategy.

Public and permanent recording of explanations. In this lesson, the teacher and children developed a shared vocabulary for the various solution strategies. The teacher made sure that explanations were recorded in such a way that they could be retrieved when needed, so that these became the protocols for supporting students' induction into the mathematical discourse relating to solutions strategies for simple subtractions. The blackboard provided a public and semi-permanent way of recording the written and symbolic representations of children's solutions as well as the results of children's actions. At the end of the lesson, the teacher specifically asked the students to look at the four strategies, which were clearly organised on the blackboard, and comment on them. Children used their workbooks to record their working and reflections on each lesson, together with the date. Thus they were able to refer to previous work with ease — for example, one child recalled that four weeks earlier they had found that 8 + 6 = 14, and so 14 – 8 = 6.

Giving children ownership of ideas. While there was no imperative to include all children in discussions, by the end of the lesson about a third of the children had their name cards attached to strategies or comments on the board. Moreover, the teacher had flagged that in the next lesson they would explore one child's remark that if 14 – 8 = 6, then we must also have 14 – 6 = 8. Children's strategies were publicly acknowledged and referred to across lessons.

Promoting high level written explanations. According to Fujii (2004), Professor Takashi Nakamura (University of Yamanashi) classifies children's written comments about lessons into four levels: affective (related to enjoyment); descriptive ("this is what I did"); comparative (comparing their own solutions with those of other children, perhaps friends); and those that involve generalisation or specialisation beyond the classroom context. According to Fujii, the level at which students respond to requests to reflect on lessons depends on the exact form of the teacher's question. If the teacher asks "What did you learn" the answer will be cognitive. On the other hand, if the teacher asks "How did you feel about this lesson" the answer, obviously will be affective. This teacher regards written explanations as significant and uses a special Japanese word that combines comment and impression to ask students to reflect on the lesson.

THE AUSTRALIAN LESSON

This lesson took place around the middle of the children's first year at school. The major focus of the lesson was the Fireman's ladder problem, in which children were told that "a fireman was standing on the middle rung of a ladder. He goes up three more rungs to get to the top. How many rungs altogether on the ladder?" They could then replace three rungs with five or any other number.

The lesson began with children sitting in a circle on the floor, taking turns to throw a die and double the number thrown. After about ten minutes, the teacher began to introduce the Fireman's ladder problem. During this introduction, she spent about ten minutes eliciting from children what they understood by "the middle". Children then worked by themselves or together in small groups for about 20 minutes to solve the problem and its extensions. The lesson ended with a discussion, again of about 20 minutes, where children explained their solutions and engaged in considerable debate about different answers obtained.

Even in the preliminary part of the lesson, it was clear that the teacher has both an agenda regarding the way in which she expects the children to work mathematically and a range of explicit or implicit strategies that she uses to support children's mathematical explanations.

For example in the early part of the lesson that focussed on doubling, her comments and questions included the following:

> T: Talking about doubling. I was thinking that if someone doesn't remember how to double, and we were just rolling one dice and let's say we rolled it on six. How would you tell them to work out the double if they didn't remember it straight away? ...
>
> How did you do it? What did you do in your head? ... Who can explain what she did? ...
>
> What is doubling? What do we do when we are doubling? ...

If you had 100 and you had to double it, or you had 50, what's the **idea**? What do you have to do? ...

It doesn't matter whether you know the answer or not. What do you have to do? ...

Don't tell him the answer. Tell him how to do it. ...

[B1 is doubling 4]

T (to B2): Why did you tell him to count another four?

B2: Because ... doubling has to be the same number. Otherwise it isn't doubling.

Later, finishing explaining to the children what needs to be done for the *Frieman's ladder* problem, she says:

T: Here's what you can do. To work that out you can use materials in our room. You can use things we usually use. You can use paper to draw it. You can use ... sticks to make the ladder .. or anything else in the room you think would be good to make a ladder with ....

At the end though, we ae going to work out how you did it ... we're going to talk about how you did it.

Some of the the key features of this lesson and strategies used by this teacher to support young children's mathematical explanations are the following.

Focussing on the conceptual. Two long segments of the lesson focussed on the meaning of doubling and the meaning of "the middle". Even when children were able to find the double of a number or the middle person in a line, the teacher persisted with asking questions like "What **is** doubling?" and "How can we tell it is the middle?" The focus was explicitly on the concepts involved and not the calculations or the answers – for example, "It doesn't matter whether you know the answer or not. What do you have to do?"

Making thinking public. This teacher frequently asks young children questions like "What did you do in your head?" She also has many strategies for making sure that children listen to and engage with other children's thinking and explanations.

Having and flagging high expectations. The effect of the teacher making comments such as "at the end though, we are going to work out how you did it ... we're going to talk about how you did it" is two-fold. Firstly, it focuses children on their thinking and how they will explain it before they start working on the problem. Secondly, it flags the expectation that all children will have been able to work out the solution and that they will be able to explain their thinking. .

THE HUNGARIAN LESSON

This lesson took place during the first weeks of the school year, so children were still learning about what it meant to be at school. At this stage of the year, the teacher was trying to establish both the social and the socio-mathematical norms (see, for example, Yackel & Cobb, 1996) associated with achieving quality dialogue. A brief excerpt from the lesson follows.

T: You will see a picture above and a picture below and I ask you to look at it very carefully. Where do you see differences between the two pictures? Look at it very carefully.

Show by holding up your fingers the number of differences you found. [There are a total of seven differences.]

I can see four, two, we will see all. It's not easy to count all the differences, I know. Show me Iván, yes? We have different opinions. We will discuss it soon. We will wait for Máté. Six? Seven? We will see. Máté? Three? Four? OK. Let's see. Dávid?

Dávid: I have found two.

T: Tell us just one so the others can have a turn too.

Dávid: There is no car on the other paper.

T: I will circle it here. Rebecca?

Rebecca: There is a person here, but no person there.

... [Finally the children found all the differences.]

T: Which picture could have been drawn first? The lower one or the other one? Tomo?

Tomo: The lower one.

T: Why?

Tomo: ...

T: Who can help Tomo? Máté, what do you think?

Máté: On one of the pictures it is evening and on the other it is not. On one picture there are people in the lake and on the other there aren't.

T: Well I like to swim in the evenings too, so it could have been .... What else changed on the picture? We didn't speak about this. What happened to the car?

Tomo: Father came home by car. Father and mother came home ....

In this classroom there was free discussion, mainly between the teacher and the children. The teacher was trying to create a working climate in the classroom. Free discussion was important for her. The activity she organised was flexible enough to be able to involve the children and encourage them to speak in the classroom, even during their first days at school.

Free discussion is an important element of the didactical contract (Brousseau, 1997) in this classroom, as shown by this excerpt from the section of the interview with the teacher where she was asked what she saw as the purpose of whole-cass discussion in her lessons.

I: Several times we could hear discussion in the classroom. Did it have any purpose? What, if any, was the reason for this discussion?

T: Do you mean the mean the mathematical content?

I: Yes. Mathematical content and pedagogical content too.

T: We respect each other's opinion. That is the most important part. This is the rule: we have to listen to each other. We can judge their opinion. It was also nice to see that someone made a mistake and realised very fast that he was making a mistake. We had a chance to correct each other. And it's not just me, but the children. So we listen to each other. We can talk to each other, discuss with each other. But this time [so early in the year] it was just me respecting the children's opinions.

CONCLUSIONS

It is important to distinguish between those classroom practices that are specifically cultural, those that are based on deliberate pedagogical decisions, and those that are the unintended consequence of other actions and decisions.

It is relatively easy to see similarities and differences between the three lessons and find reasons for these that relate to the school and community norms that operate in the different countries. For example, in terms of public and permanent recording of explanations and the use of the blackboard, Australian teachers are severely hampered by the fact that many do not even have a blackboard to use. This is an unexpected consequence of attempts to make teaching more child-centred. Another example of the way in which cultural differences shape what happens in the classroom can be seen when early in the Hungarian lesson the teacher says "let's see who can be the most clever today" — something that would be most unlikely to happen in either an Australian or Japanese classroom for quite different reasons. In Japan, the focus is on perseverance, while in Australia talking about "being clever" would be seen by many teachers as elitist.

Instead of looking at these differences in the first instance, the research team identified common features of lessons across the three countries that appeared to support high quality mathematical explanations. These were then used to create a framework for describing characteristics of mathematically progressive discussion. This framework, which shares the main features of

Communities of Inquiry (see, for example, Groves, Doig & Splitter, 2000; Groves & Doig, 2004), has three major aspects which were evident to differing extents in the three lessons above, namely:

- community (dealing with aspects such as a common purpose, children's ideas being valued as a source of knowledge, seeing ones-self as a mathematical thinker);
- mathematics (e.g. problematising the curriculum, conceptually complex but accessible mathematics, building connections); and
- inquiry (e.g. facilitating progressive dialogue, inviting alternative explanations, identifying assumptions, giving explanations and providing evidence).

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