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Fiber Suspensions in Turbulent flow with Two-Point Correlation

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Abstract

The equation of motion for turbulent flow of fiber suspensions has been derived in terms of correlation tensors of second order. Mathematical modeling of fiber suspensions in the turbulent flow is discussed including the correlation between the pressure fluctuations and velocity fluctuations at two points of the flow field, where the correlation tensors are the functions of space coordinates, distance between two points and the time.

Keywords: Fiber suspension, Turbulent flow, Correlation.

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Introduction

The turbulent flow of fiber suspensions can be found in many areas of industry, such as the production of the composite materials, environmental engineering, chemical engineering, textile industry, paper making and so on. Fiber suspensions property has a significant effect on the quality of the products. So the fiber suspensions in a turbulent flow would be a better discussion. The interaction between the fluid and the fiber in a flow is complicated and it is more complicated if the flow is turbulent. The motion between a fluid particle and suspended fibers in order to behavior of turbulence with the correlations between pressure fluctuations and velocity fluctuations based on the basic fluid dynamics. The fiber orientation is an important physical quantity and do not only refer to rheology of fiber suspensions. Hinze (1959) derived an expression for turbulent motion with the correlation between pressure fluctuations and velocity fluctuations at two points of the flow field. Anderson (1966) discussed on some observation of fiber suspensions in turbulent motion. Batchelor (1971) obtained the equations of motion of fiber suspensions in the flow. Zhang and Lin (2004) studied on the motion of particles in the turbulent pipe flow of fiber suspensions. Lin *et al* (2005) derived the new equation of turbulent fiber suspensions and its solution. They also verified the equations and their solutions by applying to a turbulent pipe flow of fiber suspensions. However, there are few studies relevant to the turbulent fiber suspension although it is prevalent in the industry. The main aim of this study is to derive an equation of motion for turbulent flow of fiber suspensions with two-point correlation between pressure fluctuations and velocity fluctuations.

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Mathematical model of the problem

The equations of motion and continuity for turbulent flow of a viscous incompressible fluid are

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\upsilon\frac{\partial^2 u_i}{\partial x_j\partial x_j} \quad (1)$$

$$\frac{\partial u_i}{\partial x_i}=0 \quad (2)$$

For fiber suspensions into the flow, the equation of motion is given by

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\upsilon\frac{\partial^2 u_i}{\partial x_j\partial x_j}+\frac{\mu_f}{\rho}\frac{\partial}{\partial x_j}\left[a_{ijlm}\varepsilon_{lm}-\frac{1}{3}\left(I_{ij}a_{lm}\right)\varepsilon_{lm}\right] \quad (3)$$

where, u_i are the fluid velocity components, p is the unknown pressure field, υ is the kinematical viscosity of the suspending fluid, μ_f is the apparent viscosity of fiber suspensions, ρ is the density of the fluid particle, $\varepsilon_{lm}=\frac{1}{2}\left(\frac{\partial u_l}{\partial x_m}+\frac{\partial u_m}{\partial x_l}\right)$ is the tensor of strain rate, I_{ij} is the turbulent intensity of suspensions, a_{lm} and a_{ijlm} are the second and fourth-orientation tensors of the fiber respectively and t is the time.

We assume that the mean velocity $\overline{U}_i$ is constant throughout the region considered and independent of time and we put

$$\left(U_i = \overline{U}_i + u_i\right)_A, \quad \left(U_j = \overline{U}_j + u_j\right)_B$$

The value of each term can be obtained by using the equations of motion for u_j at the point B and for u_i at the point A. The equation of motion for u_i at the point A, obtain from equation (3) takes the following form

$$\frac{\partial u_i}{\partial t} + \left(\overline{U}_k + u_k\right)\frac{\partial u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \upsilon\frac{\partial^2 u_i}{\partial x_k \partial x_k} + \frac{\mu_f}{\rho}\frac{\partial}{\partial x_k}\left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}\left(I_{ik}a_{lm}\right)\varepsilon_{lm}\right] \tag{4}$$

Since, for an incompressible fluid , $\left(u_i \frac{\partial u_k}{\partial x_k}\right)_A = 0$, then the equation (4) can be written as

$$\frac{\partial}{\partial t}\left(u_i\right)_A + \left[\overline{U}_k + \left(u_k\right)_A\right]\left(\frac{\partial}{\partial x_k}\right)_A\left(u_i\right)_A + \left(u_i \frac{\partial u_k}{\partial x_k}\right)_A = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A\left(u_i\right)_A + \frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_A\left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}\left(I_{ik}a_{lm}\right)\varepsilon_{lm}\right]_A \tag{5}$$

Multiplying equation (5) by $(u_j)_B$, we obtain

$$\left(u_i\right)_A\frac{\partial}{\partial t}\left(u_j\right)_B + \left[\overline{U}_k + \left(u_k\right)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B\left(u_j\right)_B\left(u_i\right)_A + \left(u_j\right)_B\left(\frac{\partial}{\partial x_k}\right)_B\left(u_k\right)_B\left(u_i\right)_A = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A\left(u_j\right)_B + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A\left(u_i\right)_A\left(u_j\right)_B + \frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_A\left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}\left(I_{ik}a_{lm}\right)\varepsilon_{lm}\right]_A\left(u_j\right)_B \tag{6}$$

Where, $(u_j)_B$ can be treated as a constant in a differential process at the point A .

Similarly, the equation of motion for u_j at the point B is given by

$$\frac{\partial u_j}{\partial t} + \left(\overline{U}_k + u_k\right)\frac{\partial u_j}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_j} + \upsilon\frac{\partial^2 u_j}{\partial x_k \partial x_k} + \frac{\mu_f}{\rho}\frac{\partial}{\partial x_k}\left[a_{jklm}\varepsilon_{lm} - \frac{1}{3}\left(I_{jk}a_{lm}\right)\varepsilon_{lm}\right] \tag{7}$$

Since, for an incompressible fluid $\left(u_j \frac{\partial u_k}{\partial x_k}\right)_B = 0$ then the equation (7) can be written as

$$\frac{\partial}{\partial t}\left(u_j\right)_B + \left[\overline{U}_k + \left(u_k\right)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B\left(u_j\right)_B + \left(u_j \frac{\partial u_k}{\partial x_k}\right)_B = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B\left(u_j\right)_B + \frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_B\left[a_{jklm}\varepsilon_{lm} - \frac{1}{3}\left(I_{jk}a_{lm}\right)\varepsilon_{lm}\right]_B \tag{8}$$

Multiplying equation (8) by (u_i) A, we get

$$\left(u_i\right)_A\frac{\partial}{\partial t}\left(u_j\right)_B + \left[\overline{U}_k + \left(u_k\right)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B\left(u_j\right)_B\left(u_i\right)_A + \left(u_j\right)_B\left(\frac{\partial}{\partial x_k}\right)_B\left(u_k\right)_B\left(u_i\right)_A = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B\left(u_i\right)_A + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B\left(u_j\right)_B\left(u_i\right)_A + \frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_B\left[a_{jklm}\varepsilon_{lm} - \frac{1}{3}\left(I_{jk}a_{lm}\right)\varepsilon_{lm}\right]_B\left(u_i\right)_A \tag{9}$$

where $(u_i)A$ can be treated as a constant in a differential process at the point B .

Addition of the equations (6) and (9) gives the result

$$\frac{\partial}{\partial t}\left(u_i\right)_A\left(u_j\right)_B + \left[\left(\frac{\partial}{\partial x_k}\right)_A\left(u_i\right)_A\left(u_k\right)_A\left(u_j\right)_B + \left(\frac{\partial}{\partial x_k}\right)_B\left(u_i\right)_A\left(u_k\right)_B\left(u_j\right)_B\right] + \overline{U}_k\left[\left(\frac{\partial}{\partial x_k}\right)_A\left(u_i\right)_A\left(u_j\right)_B + \left(\frac{\partial}{\partial x_k}\right)_B\left(u_i\right)_A\left(u_j\right)_B\right]$$

$$= -\frac{1}{\rho}\left[\left(\frac{\partial}{\partial x_i}\right)_A p_A (u_j)_B + \left(\frac{\partial}{\partial x_j}\right)_B p_B (u_i)_A\right]$$

$$+ \upsilon\left[\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A + \left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B\right](u_i)_A (u_j)_B$$

$$+ \frac{\mu_f}{\rho}\left[\left(\frac{\partial}{\partial x_k}\right)_A \left(a_{iklm}\varepsilon_{lm} - \frac{1}{3} I_{ik} a_{lm}\varepsilon_{lm}\right)_A (u_j)_B + \left(\frac{\partial}{\partial x_k}\right)_B \right.$$

$$\left. \left(a_{jklm}\varepsilon_{lm} - \frac{1}{3} I_{jk} a_{lm}\varepsilon_{lm}\right)_B (u_i)_A\right] \quad (10)$$

To find the relation of turbulent fiber motions at the point B to those at point A , it will give no difference if we take one point as the origin of A or B of the coordinate system. Let us consider the point A as the origin. In order to differentiate between the effects of distance and location, we introduce as new independent variables

$$\zeta_k = (x_k)_B - (x_k)_A$$

Then we obtain, $$\left(\frac{\partial}{\partial x_k}\right)_A = -\frac{\partial}{\partial \zeta_k},$$

$$\left(\frac{\partial}{\partial x_k}\right)_B = \frac{\partial}{\partial \zeta_k}$$

$$\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A = \left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B = \frac{\partial^2}{\partial \zeta_k \partial \zeta_k}$$

Using the above relations in equation (10) and taking ensemble average on both sides, we obtain

$$\frac{\partial}{\partial t}\overline{(u_i)_A (u_j)_B} - \frac{\partial}{\partial \zeta_k}\overline{(u_i)_A (u_k)_A (u_j)_B} + \frac{\partial}{\partial \zeta_k}$$

$$\overline{(u_i)_A (u_k)_B (u_j)_B} = -\frac{1}{\rho}\left[-\frac{\partial}{\partial \zeta_i}\overline{p_A (u_j)_B} + \frac{\partial}{\partial \zeta_j}\overline{p_B (u_i)_A}\right]$$

$$+ 2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}\overline{(u_i)_A (u_j)_B} - \frac{\mu_f}{\rho}\frac{\partial}{\partial \zeta_k}\left[\overline{(a_{iklm}\varepsilon_{lm})_A (u_j)_B}\right.$$

$$\left. -\frac{1}{3}\overline{(I_{ik} a_{lm}\varepsilon_{lm})_A (u_j)_B}\right]$$

$$+ \frac{\mu_f}{\rho}\frac{\partial}{\partial \zeta_k}\left[\overline{(a_{jklm}\varepsilon_{lm})_B (u_i)_A} - \frac{1}{3}\overline{(I_{jk} a_{lm}\varepsilon_{lm})_B (u_i)_A}\right] \quad (11)$$

This equation represents the equation of mean motion of fiber suspensions in turbulent flow with pressure-velocity correlation.

It is clear that the coefficient of $\overline{U}_k$ has been vanished. The equation (11) describes the turbulent motion of fiber suspensions, where the motions with respect to a coordinate system moving with the mean velocity . $\overline{U}_k$

Equation contains the double velocity correlation $\overline{(u_i)_A (u_j)_B}$,

double correlations such as $\overline{p_A (u_j)_B}$, triple correlations such as $\overline{(u_i)_A (u_k)_A (u_j)_B}$ where all the terms apart from one another. The correlations $\overline{p_A (u_j)_B}$ and $\overline{p_B (u_i)_A}$ form the tensors of first order, because pressure a scalar quantity and the triple correlations $\overline{(u_i)_A (u_k)_A (u_j)_B}$ and

$\overline{(u_i)_A (u_k)_A (u_j)_B}$ form the tensors of third order.

We designate the first order correlations by $(K_{p,j})_{A,B}$, second order correlations by $(Q_{i,j})_{A,B}$, and third order correlations by $(S_{i,j})_{A,B}$,

Therefore, we set

$$(k_{i,p})_{A,B} = \overline{(u_i)_A p_B},\ (k_{p,j})_{A,B} = \overline{p_A (u_j)_B}$$

$$(Q_{i,j})_{A,B} = \overline{(u_i)_A (u_j)_B}$$

$$(s_{ik,j})_{A,B} = \overline{(u_i)_A (u_k)_A (u_j)_B},\ (D_{ik,j})_{A,B} = \overline{(a_{iklm}\varepsilon_{lm})_A (u_j)_B}$$

Where, the index indicates the pressure and is not a dummy index like i or j so that the summation convention does not apply to p.

Also the term $\overline{(a_{jklm}\varepsilon_{lm})_B (u_i)_A}$ and $\overline{(I_{jk} a_{lm}\varepsilon_{lm})_B (u_i)_A}$ form the correlation tensors of third order, we designate these by $D_{i,jk}$ and $H_{i,jk}$ respectively.

Thus we set

$$(D_{i,jk})_{A,B} = \overline{(u_i)_A (a_{jklm}\varepsilon_{lm})_B},$$

$$(D_{ik,j})_{A,B} = \overline{(a_{iklm}\varepsilon_{lm})_A (u_j)_B}$$

$$\left(H_{i,jk}\right)_{A,B} = \overline{\left(u_i\right)_A\left(I_{jk}a_{lm}\varepsilon_{lm}\right)_B},$$
$$\left(H_{ik,j}\right)_{A,B} = \overline{\left(I_{ik}a_{lm}\varepsilon_{lm}\right)_A\left(u_j\right)_B}.$$

If we use the above relations of first, second and third order correlations in equation (11), then we obtain

$$\frac{\partial}{\partial t}Q_{i,j} - \frac{\partial}{\partial \zeta_k}S_{ik,j} + \frac{\partial}{\partial \zeta_k}S_{i,kj} = -\frac{1}{\rho}\left(-\frac{\partial}{\partial \zeta_i}K_{p,j} + \frac{\partial}{\partial \zeta_j}K_{i,p}\right) + 2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j}$$
$$+\frac{\mu_f}{\rho}\frac{\partial}{\partial \zeta_k}\left[\left(D_{i,jk} - D_{ik,j}\right) + \frac{1}{3}\left(H_{ik,j} - H_{i,jk}\right)\right] \tag{12}$$

where all the correlations refer to the two points A and B .

Now for an isotropic turbulence of an incompressible flow, the double pressure-velocity correlations are zero, that is,

$$\left(k_{p,j}\right)_{A,B} = 0,\ \left(k_{i,p}\right)_{A,B} = 0$$

In an isotropic turbulence it follows from the condition of invariance under reflection with respect to point ,

$$\overline{\left(u_i\right)_A\left(u_k\right)_B\left(u_j\right)_B} = -\overline{\left(u_k\right)_A\left(u_j\right)_A\left(u_i\right)_B}$$

or $\left(s_{i,kj}\right)_{A,B} = -\left(s_{kj,i}\right)_{A,B}$

and hence $\left(D_{i,jk}\right)_{A,B} = -\left(D_{jk,i}\right)_{A,B}$ $\left(H_{i,jk}\right)_{A,B} = -\left(H_{jk,i}\right)_{A,B}$

After substitution these above relations, equation(12) becomes

$$\frac{\partial}{\partial t}Q_{i,j} - \frac{\partial}{\partial \zeta_k}\left(S_{ik,j} + S_{kj,i}\right) = 2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j}$$
$$+\frac{\mu_f}{\rho}\left[-\frac{\partial}{\partial \zeta_k}\left(D_{jk,i} + D_{ik,j}\right) + \frac{1}{3}\frac{\partial}{\partial \zeta_k}\left(H_{ik,j} + H_{jk,i}\right)\right] \tag{13}$$

The term $\frac{\partial}{\partial \zeta_k}\left(S_{ik,j} + S_{kj,i}\right)$, $\frac{\partial}{\partial \zeta_k}\left(D_{jk,i} + D_{ik,j}\right)$ and $\frac{\partial}{\partial \zeta_k}\left(H_{ik,j} + H_{jk,i}\right)$ form the tensors of second order, we designate these by $S_{i,j}$, $D_{i,j}$ and $H_{i,j}$ respectively, that is

$$S_{i,j} = \frac{\partial}{\partial \zeta_k}\left(S_{ik,j} + S_{kj,i}\right)\ ,\quad D_{i,j} = \frac{\partial}{\partial \zeta_k}\left(D_{jk,i} + D_{ik,j}\right)$$

and $H_{i,j} = \frac{\partial}{\partial \zeta_k}\left(H_{ik,j} + H_{jk,i}\right)$.

Therefore equation (13) gives the result

$$\frac{\partial}{\partial t}Q_{i,j} - S_{i,j} = 2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j} - \frac{\mu_f}{\rho}\left(D_{i,j} - \frac{1}{3}H_{i,j}\right) \tag{14}$$

Equation (14) is the equation of motion for turbulent flow of fiber suspensions in terms of correlation tensors of second order.

If there are no effects of fiber suspension in the flow then the apparent viscosity of the fluid vanishes, that is, μ_f so that the equation (14) takes the form

$$\frac{\partial}{\partial t}Q_{i,j} - S_{i,j} = 2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j} \tag{15}$$

This equation represents the turbulent motion interms of correlation tensors of second order, which is the same as obtained by J. O. Hinze.

Discussion and Conclusion

The equation of motion for turbulent flow of fiber suspensions has been derived by averaging procedure, which includes the effect of fiber suspensions and the correlation between the pressure fluctuations and velocity fluctuations at two points of the fluid flow. The discussion provides the equation of fiber mean motion, as well as for the resulting turbulent fiber motion. The interaction between the turbulent fluid and the fiber based on the Reynolds number. The occurrence of the turbulent flow will depend on the values of the non-dimensional number known as critical Reynolds number, which varies from 2000 to 2300. The flow will be turbulent if the Reynolds number (Re) is greater than the critical Reynolds number (R_{cr}), so that the turbulent flow occurs at high Reynolds number. If the Reynolds number increases from 1600 to 2500 then the flow converts to turbulent flow from laminar flow, the orientation distribution of fiber changes in a range. It is clear that turbulence has effect on the orientation distribution of fiber.

Fiber suspensions in a turbulent fluid undergo mean motion due to the mean fluid velocity and random motion due to the fluctuating component of fluid velocity. The velocity of fiber fluctuates around the mean velocity of flow. Fluctuation velocity of turbulence at the two points A and B of the flow field leads to a weakening of the concentration of the fiber orientation distribution on small angle. This concentration leads to be weaker and orientation distribution of fiber becomes more uniform as Reynolds numbers increases and flow fluctuation velocity strengths. The velocity of fiber has the same fluctuation property as fluid velocity due to the strong following ability of fiber. The fluctuation velocity of fiber on flow direction is more energetic than that on lateral direction. As Reynolds number increases, the intensity of fluctuation velocity enhances, flow velocity gradient becomes more irregular and orientation distribution of fiber becomes wider. Thus the resulting equation demonstrates that as Reynolds number increases, the fluctuation velocity of turbulence at two points in the flow field becomes to be weaker, fiber orientation distribution tends to be more uniform and fluctuation velocity of fluid flow strengthens.

Nomenclature

u_i Fluid velocity components

p Unknown pressure field

υ Kinematical viscosity of the suspending fluid

μ_j Apparent viscosity of fiber suspensions

ρ Density of the fluid particle

ε_{lm} Tensor of strain rate

I_{ij} Turbulent intensity of suspensions

$\alpha_{lm}, \alpha_{ijlm}$, Second and Fourth-orientation tensors of the fiber respectively

t Time

U_i Mean velocity

ζ_k Independent variables

A,B Points where the correlation calculated

$\overline{(u_i)_A (u_j)_B}$ Double velocity correlations form the tensors of second order at the points A and B

$\overline{p_A (u_j)_B}$ Double pressure-velocity correlations form the tensors of first order at the points A and B

$\overline{(u_i)_A (u_k)_A (u_j)_B}$ Triple correlations form the tensors of third order at the points A and B

$(s_{ik,j})_{A,B}$ $(Q_{i,j})_{A,B}$, $(s_{ik,j})_{A,B}$ First, second and third order correlations respectively at the points A and B

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