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Towards mathematical literacy in the 21st century: Perspectives from Indonesia

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Abstract

The notion of mathematical literacy advocated by PISA (OECD, 2006) offers a broader conception for assessing mathematical competences and processes with the main focus on the relevant use of mathematics in life. This notion of mathematical literacy is closely connected to the notion of mathematical modelling whereby mathematics is put to solving real world problems. Indonesia has participated as a partner country in PISA since 2000. The PISA trends in mathematics from 2003 to 2009 revealed unsatisfactory mathematical literacy among 15-year-old students from Indonesia who lagged behind the average of OECD countries. In this paper, exemplary cases will be discussed to examine and to promote mathematical literacy at teacher education level. Lesson ideas and instruments were adapted from PISA released items 2006. The potential of such tasks will be discussed based on case studies of implementing these instruments with samples of pre-service teachers in Yogyakarta.

Introduction

The notion of mathematical literacy advocated by the Programme for International Student Assessment (PISA) has gained wide acceptance globally. Mathematical literacy goes beyond curricular mathematics and covers a broader conception of what constitutes mathematics. The main focus of PISA assessment is on measuring the potential of 15-year-old students in activating their mathematical knowledge and competencies to solve problems set in real-world situations. PISA's (OECD, 2006) definition of mathematical literacy captures this:

Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements, and to engage in mathematics in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen. (OECD, 2006, p. 72).

Indonesia has participated as a partner country in PISA since the start in 2000. The trend from PISA results in mathematics from 2000 to 2009 consistently revealed poor performance. Indonesia is ranked among the lowest performing countries that performed below the OECD average (Table 1). Figure 1 presents changes in some of countries performance from PISA 2003 to 2009. The comparison between 2003 and 2009 results showed that Indonesian 15-year-olds improved their performance by 11 score points. However, a worrying note from the 2009 PISA results was that almost 80% of the Indonesian sample performed below the baseline of level 2 of mathematical literacy. At Level 2, students are expected to show ability to use basic algorithms, formulas, procedures or conventions and use direct reasoning and interpretations of the results (Table 2). The fact that majority of

Indonesian students performed below the baseline shows a serious problem with maintaining basic skills in mathematics. Clearly there is a strong impetus to address this problem by improving the quality of mathematics teaching and learning so that more students are mathematically literate.

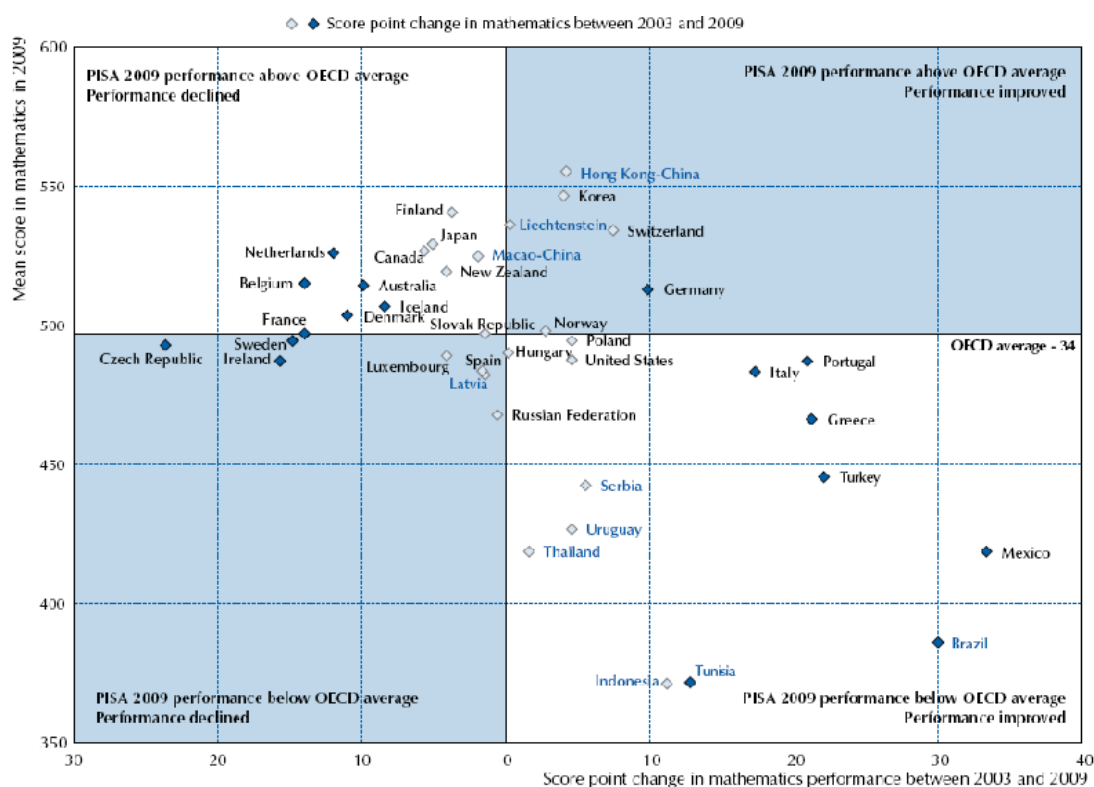


Figure 1. Change in mathematics performance from 2003 to 2009 (OECD, 2010, vol. 5, p. 61)

Table 1

Mean scores of mathematical literacy of Indonesian samples in comparison to OECD average [From OECD 2000, Table 3.3., p. 287; OECD 2003, Table 2.5c, p. 356 ; OECD, 2010, Table V.3.1., p. 156]

Country	Mathematics literacy mean scores (S.E.)			
	2000	2003	2006	2009
Indonesia	367 (4.5)	360 (3.9)	391 (5.6)	371 (3.7)
OECD average	500 (0.6)	500 (0.6)	497 (0.6)	499 (0.6)

Concern over a lack of mathematical literacy among Indonesian students has prompted initiatives to raise more awareness on mathematical literacy. In primary school levels, a reform movement was aimed at placing more emphasis on teaching mathematics connected to real life with PMRI (Sembiring, Hoogland, Dolk, 2010). Professional development sessions for secondary school teachers have started a few years ago to carry forward the realistic approach of teaching mathematics. Last year, the national council of

research and development in Jakarta, 'Balitbang', offered a grant to construct test items for secondary schools similar to PISA items using Indonesian contexts. Some of the items are available from <http://pisaindonesia.wordpress.com/aktivitas-pisa-indonesia>.

Presently, mathematical literacy contests for secondary school students are being held concurrently in 7 cities in Indonesia to improve mathematical literacy of secondary school students. In addition to such events, a sustainable program at teacher education level is needed to build capacity of future teachers in planning and carrying out lessons that support the development of mathematical literacy.

Mathematical Literacy and Mathematical Modelling

The notion of mathematical literacy is closely connected to the notion of mathematical modelling (Kaiser & Willander, 2005; de Lange, 2006; Stacey, 2009). Mathematical modelling involves cyclical processes which start with a problem situated in a 'real-world' context which is translated and formulated as a mathematical problem. The process of formulating mathematical problems from real-world problems involves simplifying the real-world situations by making assumptions in order to derive mathematical solutions. This process is often referred to as '*mathematisation*' process (de Lange, 2006). The cycle of mathematical modelling ends with interpretations of mathematical solutions in reference to the real-world situations. Evidently, both mathematical modelling and mathematical literacy place the functionality of mathematics in solving real-life situations at the centre of mathematical learning. The descriptors for the top levels (i.e., level 6) of proficiency in mathematics explicitly pinpoint ability to work with models for complex situations and to generalize and utilize information based on the models (Table 2).

Table 2

Descriptions of mathematical literacy of students at level 2, and 6 [From OECD 2010, p. 130]

Level	Lower score limit	What students can typically do
6	669	At Level 6 students can conceptualise, generalise, and utilise information based on their investigations and modelling of complex problem situations. They can link different information sources and representations and flexibly translate between them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply this insight and understandings along with a mastery of symbolic and formal mathematical operations and relationships to develop new approaches and strategies for attacking novel situations. Students at this level can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments, and the appropriateness of these to the original situations.
2	420	At Level 2 students can interpret and recognize situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures, or conventions. They are capable of direct reasoning and literal interpretations of the results.

Real-world contexts and situations are integral elements of mathematical literacy. However, there is no conclusive voice as to whether real-world contexts in mathematical problem afford or inhibit students' mathematical learning (Rittle-Johnson & Koedinger, 2005; Feijs & de Lange, 2004; Sembiring, Hadi & Dolk, 2008, Widjaja, Dolk, & Fauzan, 2010). Contexts might present a barrier in solving mathematical problems. Movshovitz-Hadar, Zaslavsky, and Inbar (1987) noted that contextual mathematical problems demand linguistic skills which present a barrier on mathematical performance. Prior studies revealed that contexts might not be activated by students due to a tendency for direct translation from a problem into mathematical formulas (Busse, 2005; van den Heuvel-Panhuizen, 1999). However, real-world contexts carry a lot of potential for learning, and allow for multiple pathways to derive at mathematical solutions. Hence the use of real-world contexts cultivates flexible thinking (English, 2010; Lave & Wenger, 1994). Similarly, Widjaja, Dolk and Fauzan (2010) found that meaningful contexts allowed students to relate with their personal experiences which afforded them to solve problems at different levels of mathematical sophistications.


Mathematical Tasks

Two contextualized tasks adapted from the PISA 2006 released items will be discussed (Appendix A, Appendix B). These tasks were assigned to cohorts of Indonesian pre-service teachers as part of their module on '*Teaching strategy of mathematics in secondary school*'. The goal is to expose pre-service teachers to contextualized mathematical tasks and mathematical modelling processes. This exposure is important as future reference

in fostering their students' mathematical literacy. The goal of task one given the condition to choose a maximum of 2 toppings was to find combinations of pizzas. Task one was given as a written quiz to be solved in 20 minutes for a class of Indonesian pre-service teachers (Figure 2).

Task two was adapted as a modelling task to investigate a relationship between a person's leg length and his or her pace length. Task two can be considered an extension of the original item whereby a relationship between pace length and the number of steps per minute was given as a mathematical formula. Task two was assigned as group work (four pre-service teachers) to be completed in two weeks. The choice of method of investigation and location for data collection are left open for groups to decide.

Masalah Pizza



[Picture taken from: <http://en.wikipedia.org/wiki/Pizza>]

Yummy Pizza Shop ingin mengembangkan usahanya. Toko pizza ini menawarkan tiga ukuran pizza: kecil, sedang dan besar.

Pelanggan dapat memilih **8 topping berbeda**: keju, jamur, nanas, sosis, seafood, ayam, sapi dan sayuran.

Setiap pizza disajikan dengan saus tomat dan taburan keju.

Harga pizza ditentukan oleh ukuran dan banyaknya topping yang dipesan.

Ari ingin memesan pizza ukuran sedang untuk makan malamnya. Karena keterbatasan anggaran, dia hanya dapat memilih **maksimum 2 topping berbeda**. Berapa banyak variasi pizza yang dapat Ari pilih? Jelaskan jawabanmu selengkap mungkin.

Tuliskan dalam lembar terpisah yang disediakan.

[Problem adapted from: Sáenz, C. (2009). The role of contextual, conceptual and procedural knowledge in activating mathematical competencies (PISA). *Educational Studies in Mathematics*, 71(2), 123–143.]

Figure 2. Task 1 is adapted from PISA 2006 released item and Sáenz (2009) as a written quiz item

Hubungan antara panjang kaki dan panjang langkah

Apakah ada hubungan antara panjang kaki dan panjang langkah seseorang?

1. Tentukan faktor dan variabel dalam masalah ini
2. Kumpulkan data untuk membantu kalian menemukan model matematika. Catat data yang dikumpulkan
3. Apa model yang dapat menjelaskan hubungan antara panjang kaki dan panjang langkah? (Coba cari apakah ada model yang sudah baku).
4. Jelaskan metode kalian dalam menemukan hubungan ini.
5. Setelah menemukan penyelesaian, interpretasikan model yang kalian temukan.
6. Selidiki kembali asumsi yang kalian buat di awal dan berikan masukan untuk perbaikan model kalian.

Figure 3. Task 2: A modelling task to investigate the relationships between pace and leg length

Findings

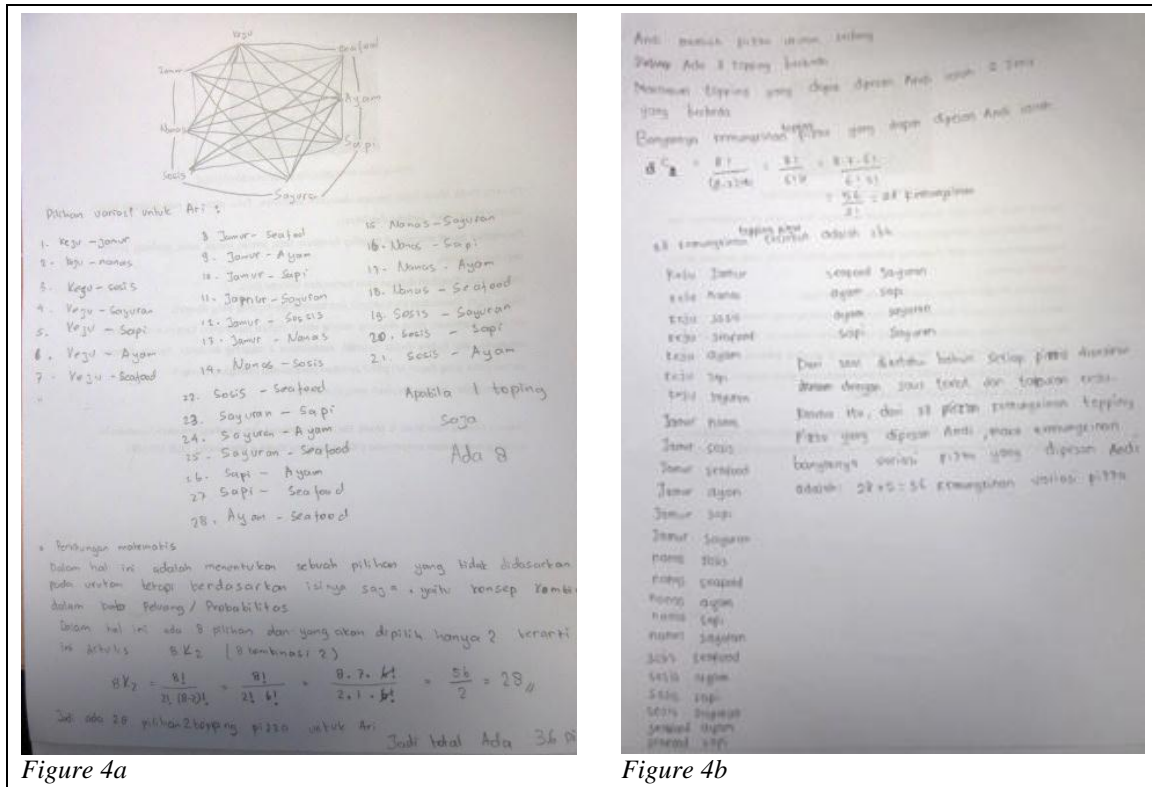


Figure 4. Samples of responses to Task 1

Pre-service teachers' responses to task one showed that pre-service teachers came up with different interpretations for the condition of "maximum of two different toppings" as illustrated in a few samples given in Figure 4. A variety of strategies were displayed, e.g., make a list, create a diagram, and use a formula. Both solutions displayed knowledge of relevant formula to solve the problem as well as correct listing of combinations of pizza with two toppings. However, the solution in Figure 4b might suggest that this pre-service teacher could not interpret her solution to the real-world context. The most common incorrect interpretation was disregarding the possibility of having only 1 topping. In this case, only 28 combinations were identified. Some pre-service teachers applied the formulae for finding combinations without reference to the given contexts. For instance, one pre-service teacher found 30 combinations by adding $C_2^8 = \frac{8!}{2!(8-2)!} = \frac{8!}{2!(6)!} = \frac{8 \times 7}{2} = 28$ and $C_1^2 = \frac{2!}{1!(2-1)!} = \frac{2!}{1!(1)!} = 2$. This is an example of what Busse (2005) labelled as 'mathematically bound'.



Figure 5. Investigations of relationships between pace and leg length

Implementations of task two with groups of pre-service teachers revealed the potential and challenges faced by pre-service teachers. As illustrated in Figure 5, different ways of investigating the relationships between leg length and pace length were observed. One group decided to collect data by measuring the footsteps of people who walked on the beach. Another group chose to collect data on campus but required volunteers (classmates) to step their feet on paints and walk along the white cloth to obtain a more accurate measurements of pace lengths. The pace lengths were calculated after taking average of four footsteps. Both groups noticed that there were variations among data and it was not a straightforward linear relation based on plotting of the points. A person's mood when walking (e.g., no hurry or in hurry), and locations (e.g., beach or campus) were offered as reasons for non-uniform pace lengths. An assumption such as constant pace length was not made but an average of pace lengths was taken instead. Using the line of best fit, a linear model to explain the relationship

between leg length and pace length was derived. Different linear models were offered, $y = 0.526x + 12.86$ by the group who collected data on the beach and $y = 0.641x - 7.138$, by the group who collected data on campus, with x represents leg length and y represents pace length.

Conclusions

Two tasks from PISA 2006 items were adapted to be use with Indonesian pre-service teachers. The initial finding suggested that contextualized tasks provide opportunities for various strategies. Such tasks allow pre-service teachers to experience the potentials power of mathematics in real-world contexts. Introducing pre-service teachers with contextual tasks and mathematical modelling as part of their training is expected to build capacity of the future teachers to in planning and carrying out lessons that support the development of mathematical literacy. Commitment to place more emphasis on learning processes which present mathematical problems in real-world settings as part of teacher training program is strongly needed. Providing pre-service teachers with such learning experience during their training will better equipped them to make use of their mathematical knowledge and skills in their life.

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Appendix A: PISA 2006 released item (OECD, 2006, p. 71)

M510: Choices

Question 1: CHOICES

M510Q01

In a pizza restaurant, you can get a basic pizza with two toppings: cheese and tomato. You can also make up your own pizza with **extra** toppings. You can choose from four different extra toppings: olives, ham, mushrooms and salami.

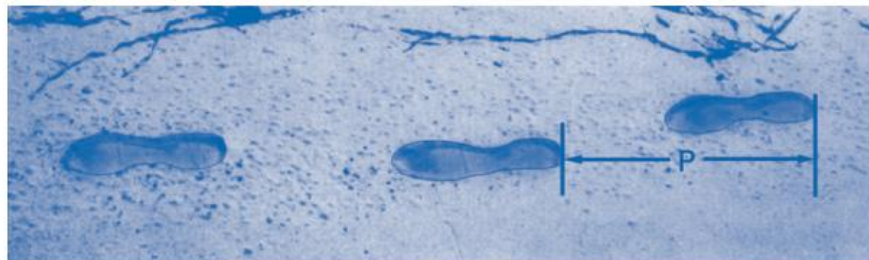
Ross wants to order a pizza with two different **extra** toppings.

How many different combinations can Ross choose from?

Answer:combinations.

Appendix B: PISA 2006 released item (OECD, 2006, p. 8)

MATHEMATICS EXAMPLE 26: WALKING



The picture shows the footprints of a man walking. The pacer length P is the distance between the rear of two consecutive footprints.

For men, the formula, $\frac{n}{P} = 140$, gives an approximate relationship between n and P where,

n = number of steps per minute, and

P = pace length in metres.

Question 1: WALKING

Bernard knows his pacer length is 0.80 metres. The formula applies to Bernard's walking.

Calculate Bernard's walking speed in metres per minute and in kilometres per hour. Show your working out.